Some thoughts on entropy bounds, swampland and inflation

Shinji Mukohyama
(YITP, Kyoto U)

ref.) arXiv:1905.10950 w/ Shuntaro Mizuno, Shi Pi and Yun-Long Zhang

BH entropy

$$S_{BH} = \frac{k_B c^3}{4\hbar G_N} A_H$$

- Gravity (G_N) & quantum mechanics ($\hbar$) & statistical mechanics (k_B) are involved!
- **Thermodynamic entropy: S = ln(# of states).** Can be understood microscopically.
- **BH entropy: S = ln(# of states)?** Can we understood it microscopically?
- We might be able to learn something about **quantum gravity** from BH entropy.
- BH entropy is also expected to be a key to understand **information loss problem**.

BH entropy

$(c = \hbar = G_N = k_B = 1)$

- Schwarzschild BH
  energy $E_{BH} = M_{BH}$
  temperature $T_{BH} = T_{Hawking}$

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{r_H}{r} \qquad r_H = 2M_{BH}$$

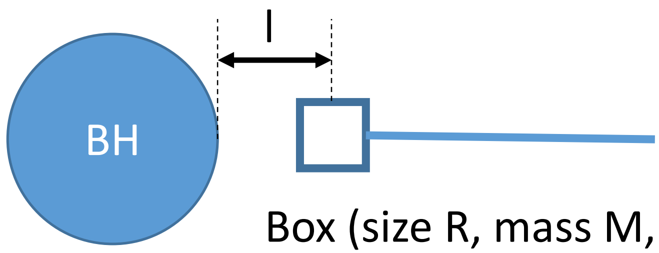
$$T_{Hawking} = \frac{\kappa}{2\pi} = \frac{f'(r_H)}{4\pi} = \frac{1}{8\pi M_{BH}}$$

- 1st law (Bardeen-Carter-Hawking 1973)
  $T_{BH}\, dS_{BH} = dE_{BH}$
  $dS_{BH} = dE_{BH} / T_{BH} = 8\pi M_{BH} dM_{BH} = d(4\pi M_{BH}{}^2)$
  $S_{BH} = 4\pi M_{BH}{}^2 = A_H/4$

$$S_{BH} = \frac{k_B c^3}{4\hbar G_N} A_H$$

- (classical) 2nd law
  $\Delta S_{BH} \geqq 0$
- (semi-classical) generalized 2nd law (GSL)
  $\Delta S_{tot} \geqq 0$, where $S_{tot} = S_{BH} + S_{matter}$
- Quantum gravity probably breaks GSL @ Page time

Bekenstein bound (1981)



$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$T_{BH} = \frac{\kappa}{2\pi} = \frac{f'(r_H)}{4\pi}$$

- Near horizon behavior (r : box's position)

$$f(r) \approx f'(r_H)(r - r_H) = 4\pi T_{BH}(r - r_H)$$
$$\approx (2\pi T_{BH} l)^2$$

$$\left(l = \int_{r_H}^{r} \frac{dr'}{\sqrt{f(r')}} \simeq \frac{1}{\sqrt{4\pi T_{BH}}} \int_{r_H}^{r} \frac{dr'}{\sqrt{r' - r_H}} = \sqrt{\frac{r - r_H}{\pi T_{BH}}} \right)$$

- Box's energy measured @ infinity

$$E = M\sqrt{f(r)} \simeq 2\pi M T_{BH} l$$

- 1st law with $\Delta M_{BH} = E$

$$\Delta S_{BH} = \frac{\Delta M_{BH}}{T_{BH}} = \frac{E}{T_{BH}} \approx 2\pi M_{BH} l$$

- Total entropy

$$\Delta S_{tot} = \Delta S_{BH} - S \approx 2\pi M l - S$$

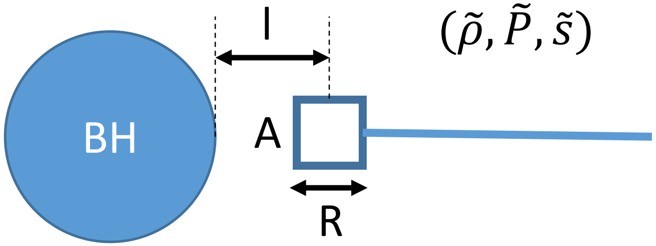
- **GSL ($\Delta S_{tot} \geqq 0$) for $^\forall l \geqq R$ requires**

$$\boxed{S \leq 2\pi M R}$$

Unruh-Wald argument (1982)

Thermal atmosphere around BH causes a buoyancy force

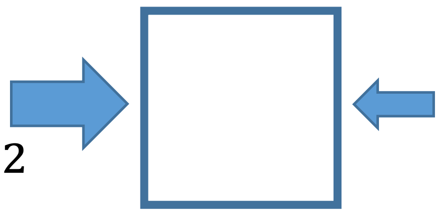
$(\tilde{\rho}, \tilde{P}, \tilde{s})$



Box filled with a gas (ρ, P, s)

- Buoyancy force

$\left(A\tilde{P}\sqrt{f}\right)_{l-R/2}$



$\left(A\tilde{P}\sqrt{f}\right)_{l+R/2}$

$$f_b(l) = \left(A\tilde{P}\sqrt{f}\right)_{l-R/2} - \left(A\tilde{P}\sqrt{f}\right)_{l+R/2}$$

- Work done against the buoyancy force

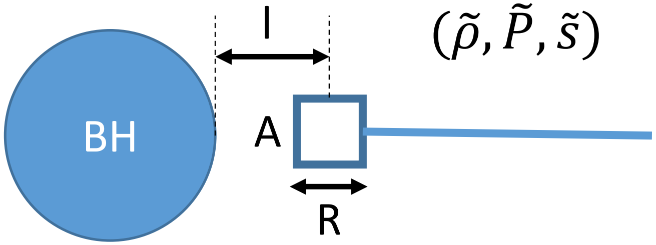
$$W_b(l) = -\int_{\infty}^{l} f_b(l')dl' = \int_{box} \tilde{P}\sqrt{f}\,dV$$

- Box's energy measured @ infinity

$$E_{box} = \int_{box} \rho\sqrt{f}\,dV$$

Unruh-Wald argument (1982)

Thermal atmosphere around BH causes a buoyancy force



Box filled with a gas (ρ, P, s)

- 1st law with $\Delta M_{BH} = E_{box} + W_b$

$$\Delta S_{BH} = \frac{\Delta M_{BH}}{T_{BH}} = \frac{1}{T_{BH}} \int_{box} (\rho + \tilde{P}) \sqrt{f} dV$$

- Total entropy

$$\Delta S_{tot} = \Delta S_{BH} - S = \int_{box} \left[\frac{1}{\tilde{T}} (\rho + \tilde{P}) - s \right] dV$$

$$= \int_{box} \frac{1}{\tilde{T}} [(\rho - \tilde{T} s) - (\tilde{\rho} - \tilde{T} \tilde{s})] dV \geq 0$$

$\tilde{T} \equiv \frac{T_{BH}}{\sqrt{f}}$: Tolman temperature

s : entropy density of gas

Gibbs-Duhem relation $\tilde{\rho} = \tilde{T} \tilde{s} - \tilde{P}$

The thermal state minimizes $\rho - \tilde{T} s$

➡ **Bekenstein bound is NOT needed for the validity of GSL!**

This argument can be extended to a charged BH (Shimomura-Mukohyama 2000) & a rotating BH (Gao-Wald 2001).

Casini's "proof" of Bekenstein bound (2008)

- Relative entropy

$$S(\rho_1|\rho_2) \equiv Tr(\rho_1 \ln\rho_1) - Tr(\rho_1 \ln\rho_2)$$

non-negativity of relative entropy

$S(\rho_1|\rho_2) \geqq 0$, where equality holds iff $\rho_1 = \rho_2$

(proof)

$\{|a_i\rangle\}\&\{|b_i\rangle\}$: complete orthonormal sets of eigenvectors of ρ_1 & ρ_2

$$\rho_1 = \sum_i |a_i\rangle a_i \langle a_i| \qquad \rho_2 = \sum_i |b_i\rangle b_i \langle b_i|$$

$$S(\rho_1|\rho_2) = Tr(\rho_1 \ln\rho_1) - Tr(\rho_1 \ln\rho_2) + Tr\rho_2 - Tr\rho_1 = \sum_{i,j} \left|\langle a_i|b_j\rangle\right|^2 (a_i \ln a_i - a_i \ln b_j + b_j - a_i) \geq 0$$

Q.E.D.

- Setup

V : a spatial region on a Cauchy surface
-V : complementary set of V
ρ : a quantum state
ρ^0 : vacuum

$$\rho_V \equiv Tr_{-V}\rho$$

$$\rho_V^0 \equiv Tr_{-V}\rho^0$$

- **Local Hamiltonian** K (**modular Hamiltonian** in continuum theory)

$$\rho_V^0 = \frac{e^{-K}}{Tre^{-K}}$$

e.g.) $K = 2\pi \int dxdy \int_0^\infty dz\, zH(x,y,z) = \int d^3x \frac{H(x,y,z)}{T_{Rindler}(z)}$ for V = half space

Casini's "proof" of Bekenstein bound (2008)

- "Proof"

$$0 \le S(\rho_V|\rho_V^0) \equiv Tr(\rho_V \ln\rho_V) - Tr(\rho_V \ln\rho_V^0)$$

$\ln\rho_V^0 = -K - \ln(Tre^{-K})$

$$= Tr(\rho_V \ln\rho_V) + Tr(K\rho_V) + \ln(Tre^{-K})\, Tr\rho_V$$

$Tr\rho_V = 1 = Tr\rho_V^0$

$\ln(Tre^{-K})\, Tr\rho_V = Tr[\rho_V^0 \ln(Tre^{-K})]$

$\ln(Tre^{-K}) = -K - \ln\rho_V^0$

$$= Tr(\rho_V \ln\rho_V) - Tr(\rho_V^0 \ln\rho_V^0) + Tr(K\rho_V) - Tr(K\rho_V^0)$$

$Tr(\rho_V \ln\rho_V) = -S(\rho_V)$, $Tr(\rho_V^0 \ln\rho_V^0) = -S(\rho_V^0)$

$$\Rightarrow \quad S(\rho_V) - S(\rho_V^0) \le Tr(K\rho_V) - Tr(K\rho_V^0)$$

$S(\rho_V) - S(\rho_V^0) = S$, $Tr(K\rho_V) - Tr(K\rho_V^0) = O(1) \times MR$

"Q.E.D."

- **This is basically Bekenstein bound** $\boxed{S \le 2\pi MR}$

- **Therefore, despite the doubt on its derivation/motivation, the bound itself seems correct if interpreted properly!**
- Perhaps we should be cautious but, at the same time, open-minded to new ideas and conjectures!

Swampland conjectures (Ooguri-Vafa 2007, + α 2018)

- **Distance conjecture**

$$L_{\rm kin} = -\frac{1}{2}\gamma_{ab}(\phi^c) g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b \qquad V(\phi^c) = 0$$

$\Delta\phi$: geodesic distance in the moduli space
→ towers of light states with mass

$$m \sim e^{-a\Delta\phi} \qquad a\ (>0) = O(1)$$

- **Assumption I :** The distance conjecture holds not only in the moduli space with $V(\phi^c) = 0$ but also in the field space with $V(\phi^c) \neq 0$.
[This is in conflict with e.g. monodromy inflation.]

- # of particle species below the cutoff of an EFT

$$N \sim n(\phi) e^{b\phi}, \qquad \frac{dn}{d\phi} > 0$$

$n(\phi)$: effective # of towers

- **Ansatz** : entropy of the towers of particles in accelerating universe

$$S_{\rm tower}(N,R) \sim N^{\delta_1} R^{\delta_2}$$

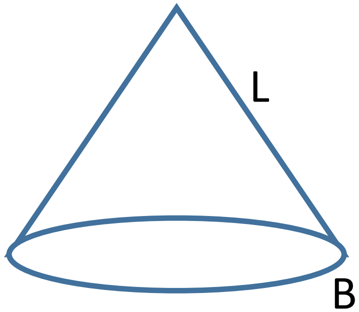
$\delta_{1,2}$ (>0) = O(1)

N: # of particle species
R = 1/H : AH radius

Covariant entropy bound (Bousso 1999)

$$S \le \frac{A}{4}$$

S : entropy on L
A : area of B



L (light-sheet) : a hypersuraface generated by null geodesics that are orthogonal to B and that have non-positive expansion

B : a spacelike 2-surface

- Bekenstein bound is not covariant and it assumes constant and finite size, negligible gravity, and no negative energy.
- Bousso bound is covariant and can be applied to gravitational collapse and FLRW universes.

Swampland conjectures (Ooguri-Vafa 2007, + α 2018)

- Covariant entropy bound, conservatively applied to quasi de Sitter

If $$\left|\frac{\dot{H}}{H^2}\right| \lesssim c_1\,, \quad \text{(1)} \qquad \frac{\min m^2_{\text{scalar}}}{H^2} \gtrsim -c_2 \quad \text{(2)}$$ then $$S \leq \pi/H^2$$

$c_{1,2}$ (>0) = O(1)

+ the entropy ansatz with R = 1/H → $$N \lesssim H^{-(2-\delta_2)/\delta_1}$$

- **Assumption II :** The upper bound on N is an increasing function of the horizon radius and is saturated for large N.

$$\Rightarrow \quad N \sim \left(\frac{1}{H}\right)^{\frac{2-\delta_2}{\delta_1}}\,, \quad \delta_1 > 0\,, \quad 0 < \delta_2 < 2$$

- Equate the two expressions for N, considering ϕ as a time variable

$$\Rightarrow \quad \ln n(\phi) \sim -b\phi - \frac{2-\delta_2}{2\delta_1}\ln H^2$$

$$\frac{dn}{d\phi} > 0 \quad \Rightarrow \quad \left|\frac{1}{H^2}\frac{d(H^2)}{d\phi}\right| \gtrsim c_0\,, \quad \text{(3)} \qquad c_0 \equiv \frac{2b\delta_1}{2-\delta_2}$$

if (1)&(2) hold

- **If (3) does not hold then either (1) or (2) should be violated**

$$\left|\frac{1}{H^2}\frac{d(H^2)}{d\phi}\right| \gtrsim c_0\,, \quad \text{or} \quad \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \quad \text{or} \quad \frac{\min m^2_{\text{scalar}}}{H^2} \lesssim -c_2$$

Swampland conjectures (Ooguri-Vafa 2007, + α 2018)

- This is **the (refined) de Sitter swampland conjecture** rewritten in a way that is useful for extensions

$$\left|\frac{1}{H^2}\frac{d(H^2)}{d\phi}\right| \gtrsim c_0\,, \quad \text{or} \quad \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \quad \text{or} \quad \frac{\min m^2_{\text{scalar}}}{H^2} \lesssim -c_2$$

- For a single-field slow-roll inflation with a canonical kinetic term,

$$\left|\frac{V'}{V}\right| > c\,, \quad \text{or} \quad \frac{V''}{V} < -c' \qquad c \equiv \min(c_0, \sqrt{2c_1}) \qquad c' \equiv c_2/3$$

  this is what is usually known as the (refined) de Sitter conjecture.
- The de Sitter conjecture would be a serious challenge to the standard single-field slow-roll inflation (or to string theory).
- **On the other hand, our universe may be fine-tuned. An "O(1)" number may be as small as 10^{-120} in our universe (the c.c. problem).**
- Anyway, I think it is important/interesting to push forward the idea as far as we can go.

Extension to DBI scalar (arXiv:1905.10950 w/ Shuntaro Mizuno, Shi Pi and Yun-Long Zhang)

- String theory allows for not only canonical scalar but also DBI scalar (representing the position of a D-brane in extra-dimensions)

$$I_{\rm DBI} = \int d^4x\sqrt{-g}\left\{T(\varphi)\left[-\sqrt{1-\frac{2X}{T(\varphi)}}+1\right]-U(\varphi)\right\} \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

- **Can we extend the swampland conjectures to a DBI scalar and, more generally, to a k-essence type scalar with Lagrangian P(X , φ) ?**
- There seems at least three options:

A) Expand the action w.r.t. X as $P(X,\varphi) = P_0(\varphi) + P_1(\varphi)X + \mathcal{O}(X^2)$
and then make the following identification

$$V(\phi) \;\Leftrightarrow\; -P_0(\varphi)\,, \quad d\phi \;\Leftrightarrow\; \sqrt{P_1(\varphi)}d\varphi$$

B) Introduce perturbation as
calculate the quadratic action as
and then make the identification

$$\varphi = \varphi^{(0)}(t) + \pi(t,\vec{x})$$

$$P(X,\varphi) \ni \frac{1}{2}\mathcal{K}_{\parallel}\dot{\pi}^2 - \frac{1}{2a^2}\mathcal{K}_{\perp}\delta^{ij}\partial_i\pi\partial_j\pi$$

$$d\phi \;\Leftrightarrow\; \sqrt{\mathcal{K}_{\parallel}}d\varphi \qquad \mathcal{K}_{\parallel} = (2P_{,XX}X + P_{,X})^{(0)}$$

C) Make the identification $d\phi \;\Leftrightarrow\; \sqrt{\mathcal{K}_{\perp}}d\varphi$ $\qquad \mathcal{K}_{\perp} = P_{,X}^{(0)}$

- **None of the three options is convincing…**

2-field model with hyperbolic field space

- Distance conjecture → negatively curved moduli/field space
  simplest : 2d hyperbolic field space

$$\gamma_{ab}(\phi^c)d\phi^a d\phi^b = d\chi^2 + e^{2\beta\chi}d\varphi^2$$

- **Simple 2-field model**

$$I = \int d^4x\sqrt{-g}\left\{-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}e^{2\beta\chi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - T(\varphi)\left[\cosh(2\beta\chi)-1\right] - U(\varphi)\right\}$$

- χ-eom **for large β^2**

$$\cancel{\Box\chi} + 2\beta e^{2\beta\chi}X - 2\beta T(\varphi)\sinh(2\beta\chi) = 0 \quad \Rightarrow \quad \chi \simeq \frac{1}{2\beta}\ln\gamma$$

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{2X}{T(\varphi)}}} \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

  χ has a large mass $\partial_\chi^2 V|_{2\beta\chi=\ln\gamma} = \frac{4T}{\gamma}\beta^2$ ⇒ χ can be integrated out

- Effective single-field action

$$I_{\text{eff}} = \int d^4x\sqrt{-g}\left\{T(\varphi)\left[-\sqrt{1-\frac{2X}{T(\varphi)}}+1\right] - U(\varphi)\right\}$$

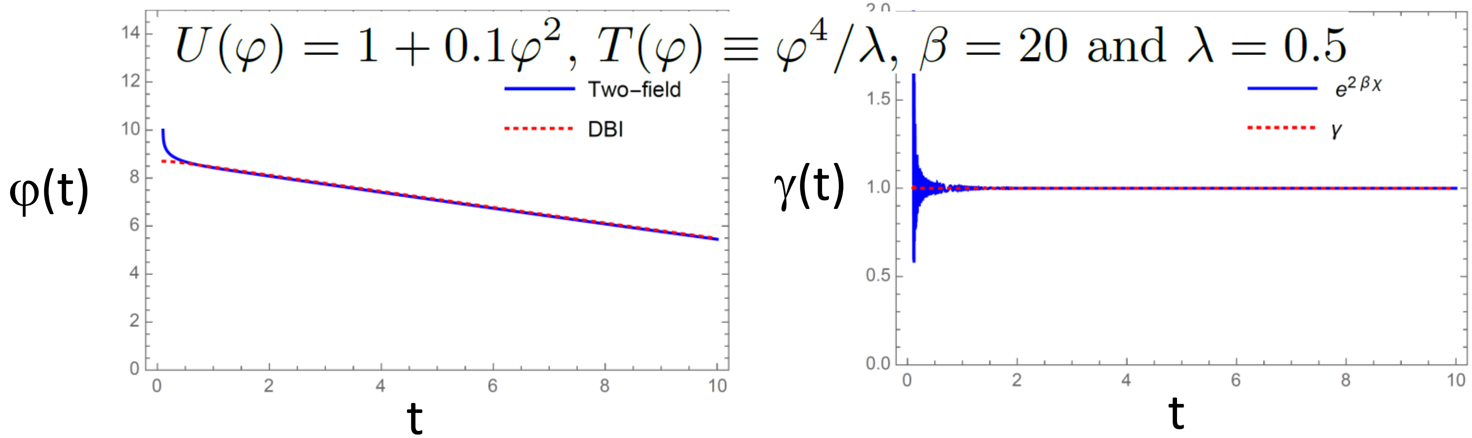
This is a DBI action!

c.f. This is a special case of the gelaton (Tolley & Wyman 2010; Edler & Joyce & Khoury & Tolley 2015).

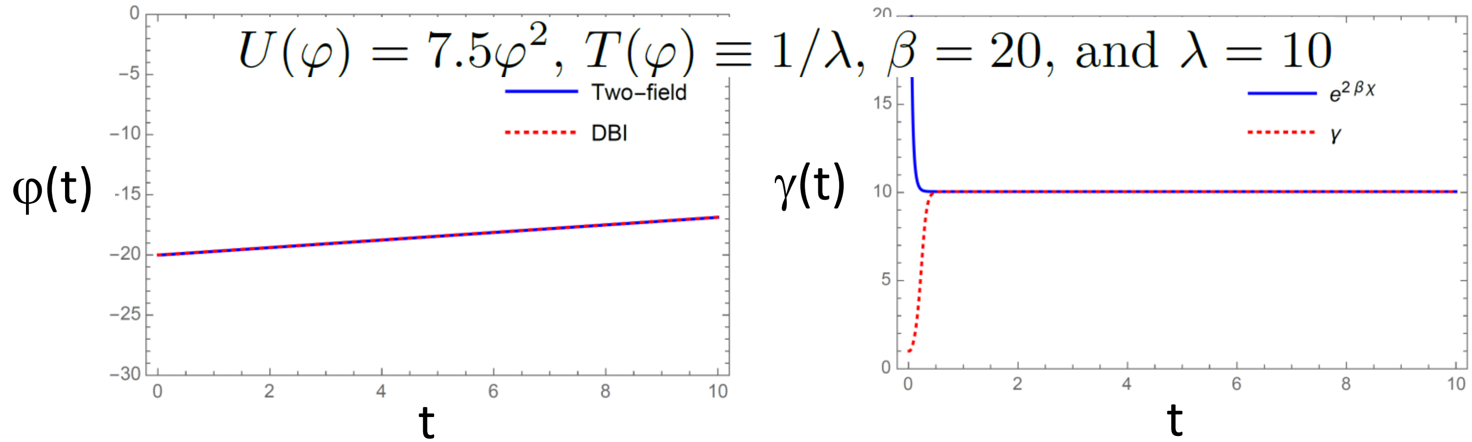
2-field model with hyperbolic field space

- The 2-field model and the single-field DBI model agree very well!

$$U(\varphi) = 1 + 0.1\varphi^2,\ T(\varphi) \equiv \varphi^4/\lambda,\ \beta = 20 \text{ and } \lambda = 0.5$$



$$U(\varphi) = 7.5\varphi^2,\ T(\varphi) \equiv 1/\lambda,\ \beta = 20, \text{ and } \lambda = 10$$



- For the 2-field model we know how to use the swampland conjecture.
- **Perhaps we can obtain the swampland conjecture for the single-field DBI model by taking $\beta^2 \rightarrow \infty$ limit**

2-field model with hyperbolic field space

- Geodesic distance in the field space **for large β²**

$$d\phi = \sqrt{\gamma_{ab}(\phi^c)d\phi^a d\phi^b} = \sqrt{d\chi^2 + e^{2\beta\chi}d\varphi^2} \simeq \left[\frac{\dot{\gamma}^2}{4\beta^2\gamma^2\dot{\varphi}^2} + \gamma\right]^{\frac{1}{2}} d\varphi \simeq \sqrt{\gamma}d\varphi$$

Thus the fist condition in the dS conjecture is

$$\left|\frac{1}{H^2}\frac{d(H^2)}{d\phi}\right| \gtrsim c_0 \quad \Longleftrightarrow \quad \boxed{\frac{1}{\sqrt{\gamma}}\left|\frac{1}{H^2}\frac{d(H^2)}{d\varphi}\right| \gtrsim c_0}$$

- Squared masses of scalar perturbation modes **for large β²**

$$I^{(2)} = \frac{1}{2}\int dta^3 \left[\dot{Y}^{\mathrm{T}}\mathcal{K}\dot{Y} + \dot{Y}^{\mathrm{T}}\mathcal{M}Y + Y^{\mathrm{T}}\mathcal{M}^{\mathrm{T}}\dot{Y} - Y^{\mathrm{T}}\left(-\mathcal{K}\frac{\vec{\nabla}^2}{a^2} + \mathcal{V}\right)Y\right]$$

$$\varphi = \varphi^{(0)}(t) + \delta\varphi(t,\vec{x})$$
$$\chi = \chi^{(0)}(t) + \delta\chi(t,\vec{x})$$

dynamical

$$\chi^{(0)} \simeq \frac{1}{2\beta}\ln\gamma^{(0)} \qquad \gamma^{(0)} \equiv \frac{1}{\sqrt{1 - \frac{(\dot{\varphi}^{(0)})^2}{T(\varphi^{(0)})}}} \qquad Y = \begin{pmatrix}\delta\varphi \\ \delta\chi\end{pmatrix}$$

non-dynamical

$$g_{\mu\nu}dx^\mu dx^\nu = -[1 + 2\Phi(t,\vec{x})]dt^2 + 2N(t)a(t)\partial_i B(t,\vec{x})dtdx^i + a(t)^2 dx^i dx^j$$

$$\det\left[m^2\mathcal{K} - 2im\mathcal{M} - \mathcal{V}\right] = 0$$

$$m_+^2 = 4T(\varphi)\gamma\beta^2 + \mathcal{O}(\beta^0)$$
$$m_-^2 = \Omega + \mathcal{O}(\beta^{-2}) + \mathcal{O}(M_{\mathrm{Pl}}^{-2})$$

$$\Omega = \frac{1}{\gamma^3}U'' + \frac{(\gamma-1)^2}{2\gamma^4}T'' - \frac{1}{16\gamma^4 T}\left[\gamma^2 T' + 2\gamma(T' - U') - 3T'\right]^2$$

Thus the last condition in the dS conjecture is

$$\frac{\min m^2_{\mathrm{scalar}}}{H^2} \lesssim -c_2 \quad \Longleftrightarrow \quad \boxed{\frac{\Omega}{H^2} \lesssim -c_2}$$

De Sitter swampland conjecture for a DBI scalar

(arXiv:1905.10950 w/ Shuntaro Mizuno, Shi Pi and Yun-Long Zhang)

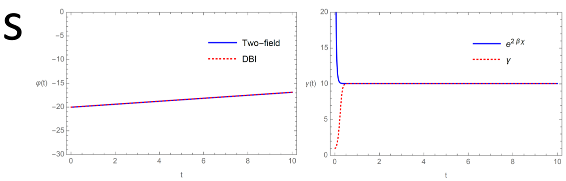
- For the 2-field system (φ , χ) in $\beta^2 \rightarrow \infty$ limit

$$\left|\frac{1}{H^2}\frac{d(H^2)}{d\phi}\right| \gtrsim c_0\,, \quad \text{or} \quad \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \quad \text{or} \quad \frac{\min m^2_{\rm scalar}}{H^2} \lesssim -c_2$$

$$\Leftrightarrow \quad \boxed{\frac{1}{\sqrt{\gamma}}\left|\frac{1}{H^2}\frac{d(H^2)}{d\varphi}\right| \gtrsim c_0\,, \quad \text{or} \quad \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \quad \text{or} \quad \frac{\Omega}{H^2} \lesssim -c_2}$$

$$\Omega = \frac{1}{\gamma^3}U'' + \frac{(\gamma-1)^2}{2\gamma^4}T'' - \frac{1}{16\gamma^4 T}\left[\gamma^2 T' + 2\gamma(T'-U') - 3T'\right]^2$$

- In $\beta^2 \rightarrow \infty$ limit, the 2-field model is equivalent to the single-field DBI and thus the above condition may be considered as de Sitter swampland conjecture for a DBI scalar



$$I_{\rm DBI} = \int d^4x\sqrt{-g}\left\{T(\varphi)\left[-\sqrt{1-\frac{2X}{T(\varphi)}}+1\right] - U(\varphi)\right\}$$

- This would ensure the equivalence between the de Sitter swampland conjectures in the 2-field model and the single-field DBI model

- The limit $\gamma \rightarrow 1$ with φ & X and (ln T)' & (ln T)'' kept finite recovers canonical one

$$\left|\frac{1}{H^2}\frac{d(H^2)}{d\varphi}\right| \gtrsim c_0\,, \quad \text{or} \quad \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \quad \text{or} \quad \frac{U''}{H^2} \lesssim -c_2$$

Extension to general P(X, φ)

- Equivalent Lagrangian

$$L = P(\chi,\varphi) + \lambda(\chi - X) = P(\chi,\varphi) + P_{,\chi}(\chi,\varphi)(X-\chi)$$

- Adding a small kinetic term of χ

$$\tilde{L} = L + Z^2 g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi/2$$

- Geodesic distance in the field space

$$d\phi = \sqrt{P_{,\chi}(\chi,\varphi) + Z^2(d\chi/d\varphi)^2}\,d\varphi \quad \xrightarrow{Z\to 0} \quad d\phi = \sqrt{P_{,X}(X,\varphi)}\,d\varphi$$

- Scalar perturbations in the k=0 sector contain
  two fast modes $\sim e^{\pm m_+ t}$ with $m_+^2 = \mathcal{O}(Z^{-2}) > 0$
  two slow modes $\sim e^{\pm m_- t}$ with $m_-^2 = \mathcal{O}(Z^0)$

- De Sitter swampland conjecture for P(X , φ)

$$\frac{1}{\sqrt{P_{,X}(X,\varphi)}}\left|\frac{1}{H^2}\frac{d(H^2)}{d\varphi}\right| \gtrsim c_0\,, \quad \text{or} \quad \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \quad \text{or} \quad \frac{m_-^2}{H^2} \lesssim -c_2$$

Summary

- Analogy between thermodynamics & properties of BH
  → BH entropy $S_{BH} = A_H/4$ (A_H: horizon area)
- Bekenstein bound was "derived" by a gedanken experiment
  $$S \leq 2\pi MR$$
- **Bekenstein's "derivation" was refuted** by Unruh & Wald. Nonetheless **the bound seems correct (if interpreted properly)** and was "proven" by Casini.
- **The distance conjecture + the covariant entropy bound** motivate **the de Sitter swampland conjecture** under a number of speculations. Some of the speculations may be doubtful but the conjecture itself may be correct (as in the case of Bekenstein bound).
- Note that **in our universe "O(1)" numbers may be small (could be as small as 10^{-120} as in the case of c.c.).**
- The conjecture was formulated for scalars with linear kinetic terms but string theory allows for DBI scalars with nonlinear kinetic terms.
- We therefore **extended the de Sitter conjecture to a DBI scalar** by considering a model of two scalars with a hyperbolic field space that reduces to a single-field DBI and applying the conjecture to the 2-field model.
- We also considered **extension to a general $P(X, \varphi)$**.