Hybrid Higgs Inflation
—Observational Constraints and Reheating—

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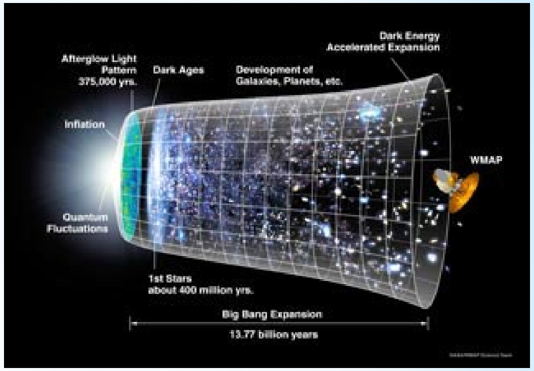
Big mystery in cosmology

Acceleration of cosmic expansion

- Inflation: early stage of the Universe

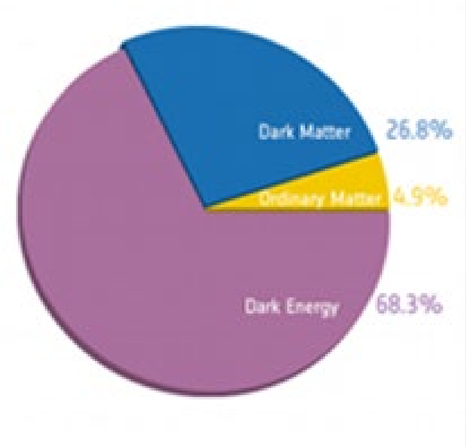


What is an inflaton ?

- Present Acceleration

cosmological constant

$$\Lambda \sim 10^{-120} m_{PL}^2$$



⇒
- Dark Energy
- Modified gravity

Inflation

So many models

The origin ?

Standard Model

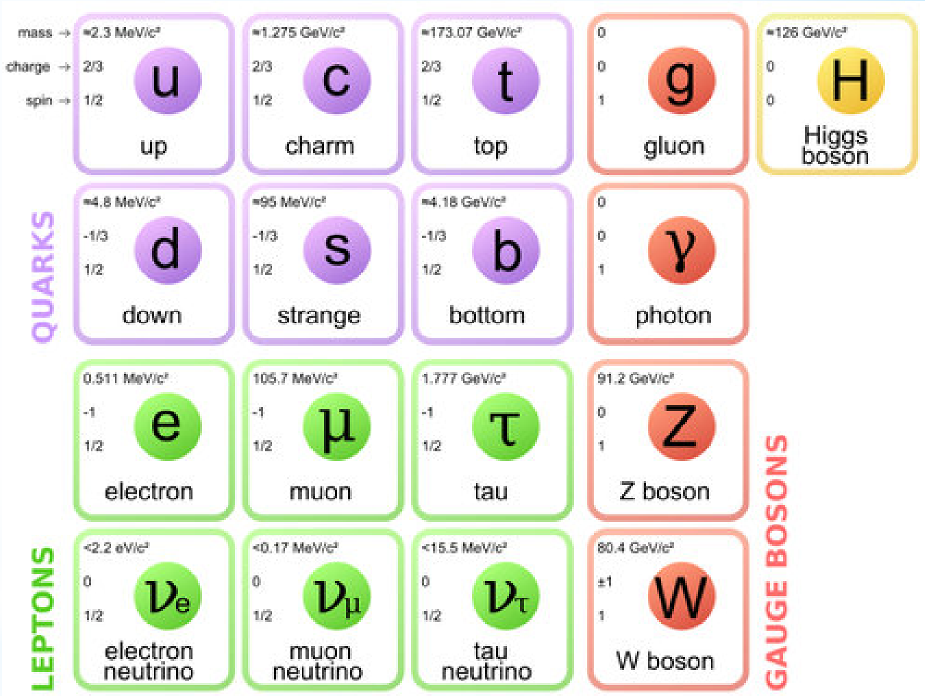
Scalar field＝Higgs

Gravity is modified

⇩

Higgs inflation

original (conventional)
new
hybrid
generalized



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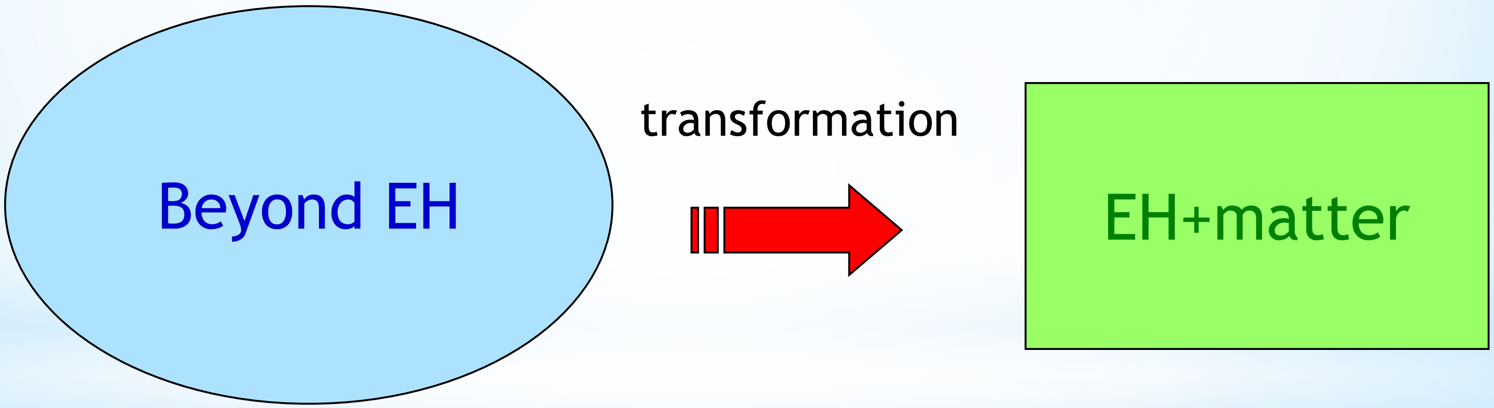
Toward the EH action :

If we can find an equivalent gravitational theory only with the EH action by some transformation, it makes our discussion simpler.



Basic equations are very complicated

well known

1. A scalar-tensor type theory

KM (1989)

$$S = \int d^D x \sqrt{-g} \left[f(\phi) R - \frac{\epsilon_\phi}{2} (\nabla \phi)^2 - V(\phi) \right]$$

$$\hat{g}_{\mu\nu} = e^{2\omega} g_{\mu\nu}$$

a conformal transformation

$$\omega = \frac{1}{D-2} \ln(2\kappa^2 |f(\phi)|)$$

$$S = \int d^D x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} R(\hat{g}) - \frac{1}{2} (\nabla \sigma)^2 - U(\sigma) \right]$$

$$\kappa\sigma = \int d\phi \left[\frac{\epsilon_\phi (D-2) f(\phi) + 2(D-1)(f'(\phi))^2}{2(D-2) f^2(\phi)} \right]^{1/2}$$

$$U(\sigma) = \epsilon_f [2\kappa^2 |f(\phi)|]^{-D/(D-2)} V(\phi)$$

(original) Higgs inflation

Bezrukov, Shaposhnikov (2008)

Spokoiny (1984); Salopek, Bond, Bardeen (1989);
Futamase, KM (1989); Fakir, Unruh (1990)

Higgs field: +non-minimal coupling $(\xi < 0)$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\xi\phi^2 R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

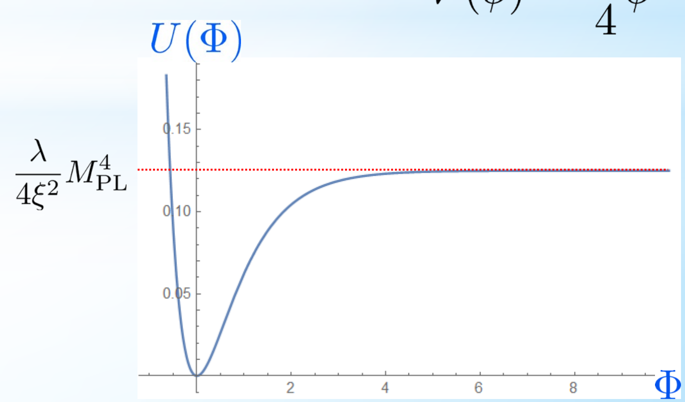
conformal transformation $\tilde{g}_{\mu\nu} = (1 - \xi\kappa^2\phi^2)g_{\mu\nu}$

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}(\tilde{\nabla}\Phi)^2 - U(\Phi)\right]$$

$$V(\phi) = \frac{\lambda}{4}\phi^4$$

$$\frac{d\Phi}{d\phi} = \frac{1}{\sqrt{(1 - \xi\kappa^2\phi^2)}}$$

$$U(\Phi) = \frac{1}{(1 - \xi\kappa^2\phi^2)^2}V(\phi)$$



2. F(R , ϕ) theory

$$S=\int d^Dx\sqrt{-g}\left[F(R,\phi)-\frac{\epsilon_\phi}{2}(\nabla\phi)^2\right]$$

KM (1989)

higher derivarives

Jakubiec, Kijowski (1987);
Magnano, Ferraris, Francaviglia, (1987);
Ferraris, Francaviglia, Magnano, (1988)

$$\widehat{g}_{\mu\nu}=e^{2\omega}g_{\mu\nu}$$ a conformal transformation

$$\omega=\frac{1}{D-2}\ln\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right] \qquad \kappa\sigma=\sqrt{\frac{D-1}{D-2}}\ln\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]$$

"new degree of freedom"

$$S=\int d^Dx\sqrt{-\widehat{g}}\left[\frac{1}{2\kappa^2}R(\widehat{g})-\frac{1}{2}(\widehat{\nabla}\sigma)^2 -\frac{\epsilon_\phi\epsilon_F}{2}e^{-\sqrt{\frac{D-1}{D-2}}\kappa\sigma}(\widehat{\nabla}\phi)^2-U(\phi,\sigma)\right]$$

$$U(\phi,\sigma)=\epsilon_F\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]^{-D/(D-2)}\left(R\frac{\partial F}{\partial R}-F(R)\right)$$

A simple example

KM (1988)

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R + \alpha R^2\right]$$

: Starobinski inflation

It contains higher derivatives

conformal transformation $\tilde{g}_{\mu\nu} = (1 + 2\alpha R) g_{\mu\nu}$

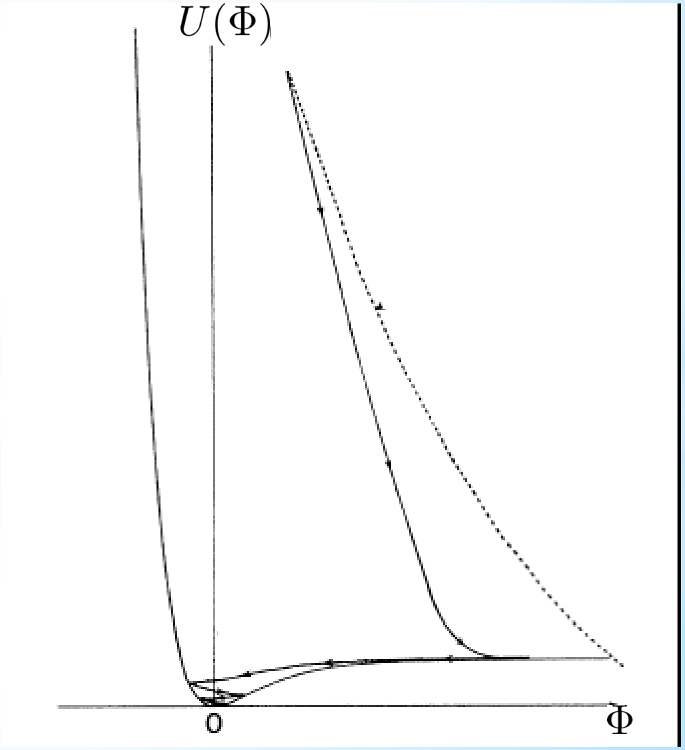
$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}(\tilde{\nabla}\Phi)^2 - U(\Phi)\right]$$

GR + a scalar field with a potential U(Φ)

$$\kappa\Phi = \sqrt{\frac{3}{2}}\ln\left[1 + 2\alpha R\right]$$

$$U(\Phi) = \frac{1}{8\alpha}\left(1 - e^{-\sqrt{\frac{3}{2}}\kappa\Phi}\right)^2$$

It is easy to judge
whether inflation occurs or not



3. F($R_{\mu\nu}$) theory

Jakubiec, Kijowski , GRG 19 (1987) 719 ;
Magnano, Ferraris, Francaviglia, GRG 19 (1987) 465 ;
Ferraris, Francaviglia, Magnano, CQG. 5 (1988) L95

$$S = \int d^4x \sqrt{-g} F(g^{\mu\nu}, R_{\mu\nu})$$

$$\sqrt{-q} q^{\mu\nu} = 2\kappa^2 \sqrt{-g} \frac{\partial F}{\partial R_{\mu\nu}}$$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-q} \Big[R(q, \partial q, \partial q^2) + q^{\mu\nu} (C^{\rho}{}_{\rho\sigma} C^{\sigma}{}_{\mu\nu} - C^{\rho}{}_{\sigma\mu} C^{\sigma}{}_{\rho\nu}) - q^{\mu\nu} \mathcal{R}_{\mu\nu} + \frac{\sqrt{-g}}{\sqrt{-q}} F(\mathcal{R}_{\mu\nu}(g, q), g^{\alpha\beta}) \Big] + S_{\rm matter}(g^{\alpha\beta}, \psi)$$

$$C^{\rho}{}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\nabla^{(q)}_{\mu} g_{\nu\sigma} + \nabla^{(q)}_{\nu} g_{\mu\sigma} - \nabla^{(q)}_{\sigma} g_{\mu\nu} \right)$$

$$R_{\mu\nu} = \mathcal{R}_{\mu\nu}(g^{\alpha\beta}, q^{\gamma\delta})$$

The EH gravitational action + spin 2 field ($g^{\mu\nu}$) + other matter fields

new Higgs inflation Germani, Kehagias (2010)

Higgs field: + derivative coupling

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \alpha G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right) - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right]$$

The EH gravitational action ($q^{\mu\nu}$) + spin 2 field ($g^{\mu\nu}$) + other matter fields

Behavior ?

The previous method may not work

Instead, we may use a disformal transformation

disformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 \left(g_{\mu\nu} + \beta^2 \nabla_\mu\phi\nabla_\nu\phi\right)$$

$$\Omega^2 = \frac{(2-\lambda^2)}{2(1-\lambda^2)^{\frac{1}{2}}}$$

$$\lambda^4(1-\lambda^2) = 4\alpha^2 X^2 \qquad X = -\frac{1}{2}(\nabla\phi)^2$$

$$\beta^2 = \alpha(1-\lambda^2)^{-\frac{1}{2}}$$

The EH gravitational action

+ Higgs field ϕ with higher-derivatives

The higher-derivative terms are too complicated

It may be better to analyze it in the original frame

However, if we can **ignore the higher-derivative terms**,
The analysis in the disformal frame becomes easy

Germani, Martucci, Moyassari (2012)

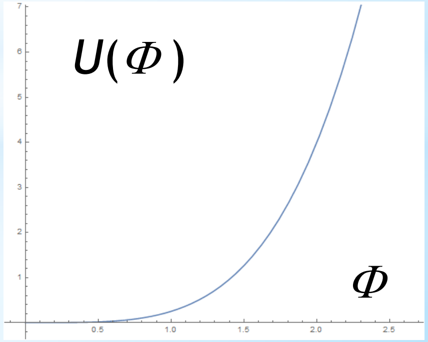
Slow-rolling inflationary phase

$$\mathcal{L}_{\rm Higgs} = -\sqrt{-\tilde{g}}\left[\left(\frac{1+\alpha V(\phi)}{2}\right)(\tilde{\nabla}\phi)^2 + V(\phi)\right] + \cdots \qquad \alpha = \frac{1}{M^2 M_{\rm PL}^2}$$

$$= -\sqrt{-\tilde{g}}\left[\frac{1}{2}(\tilde{\nabla}\Phi)^2 + U(\Phi)\right] + \cdots$$

higher-derivative terms

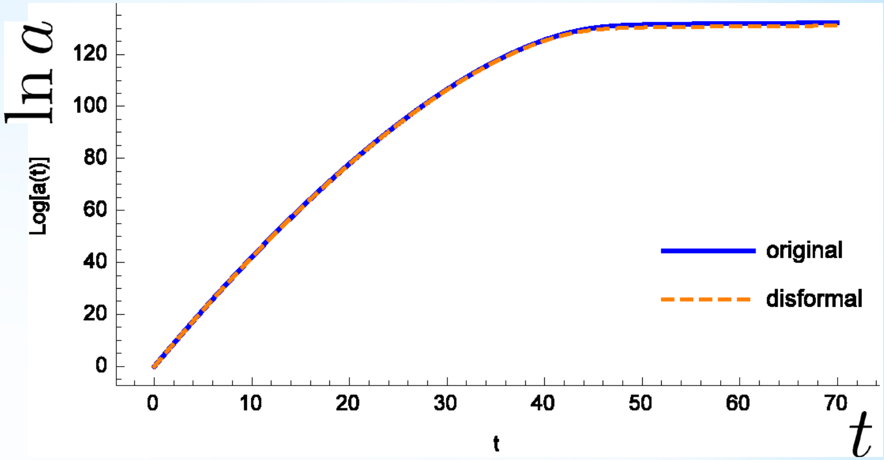
$$U(\Phi) = \begin{cases} \frac{\lambda}{4}\Phi^4 & \Phi \ll \Phi_{cr} \\ 3\sqrt[3]{\frac{3\lambda}{4}} M_{\rm PL}^4 \left(\frac{M}{M_{\rm PL}}\right)^{4/3}\left(\frac{\Phi}{M_{\rm PL}}\right)^{4/3} & \Phi \gg \Phi_{cr} \end{cases}$$



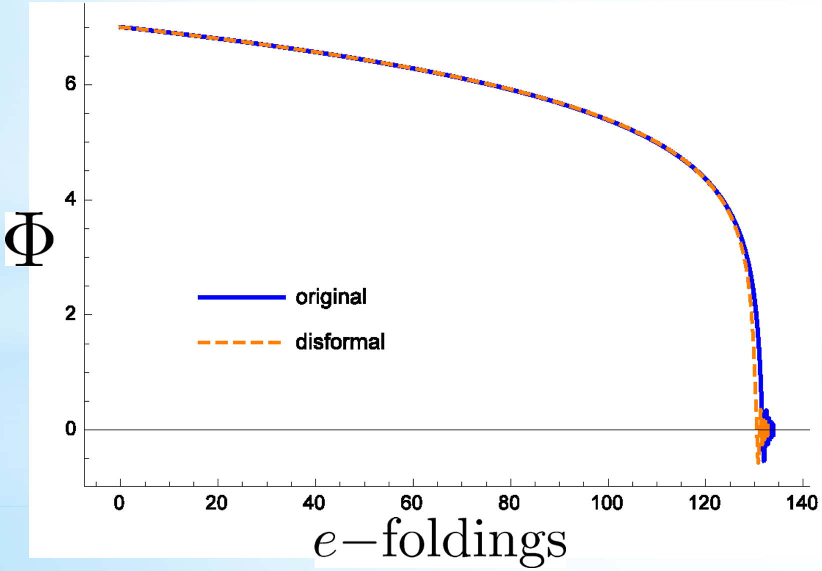
How well this truncation describes the original model ?

The original model
vs
the truncated one
after disformal transformation

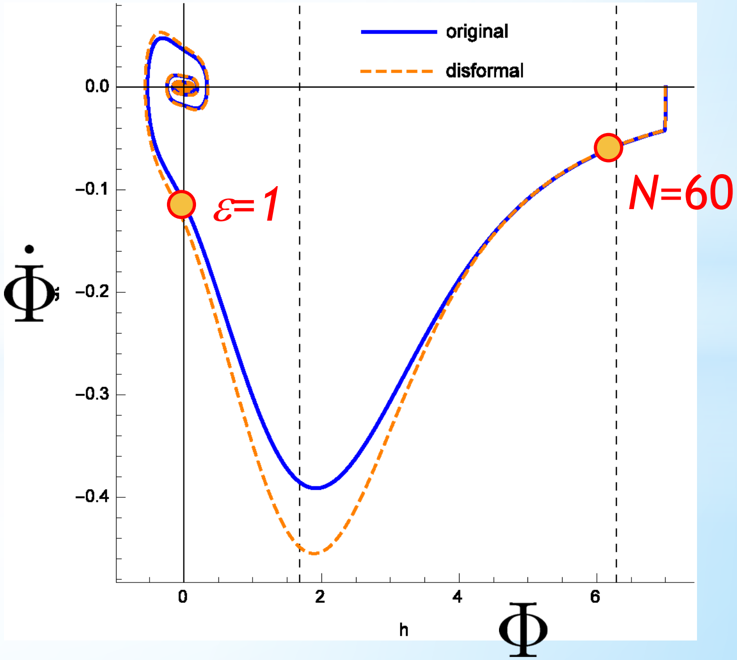
scale factor



Higgs field

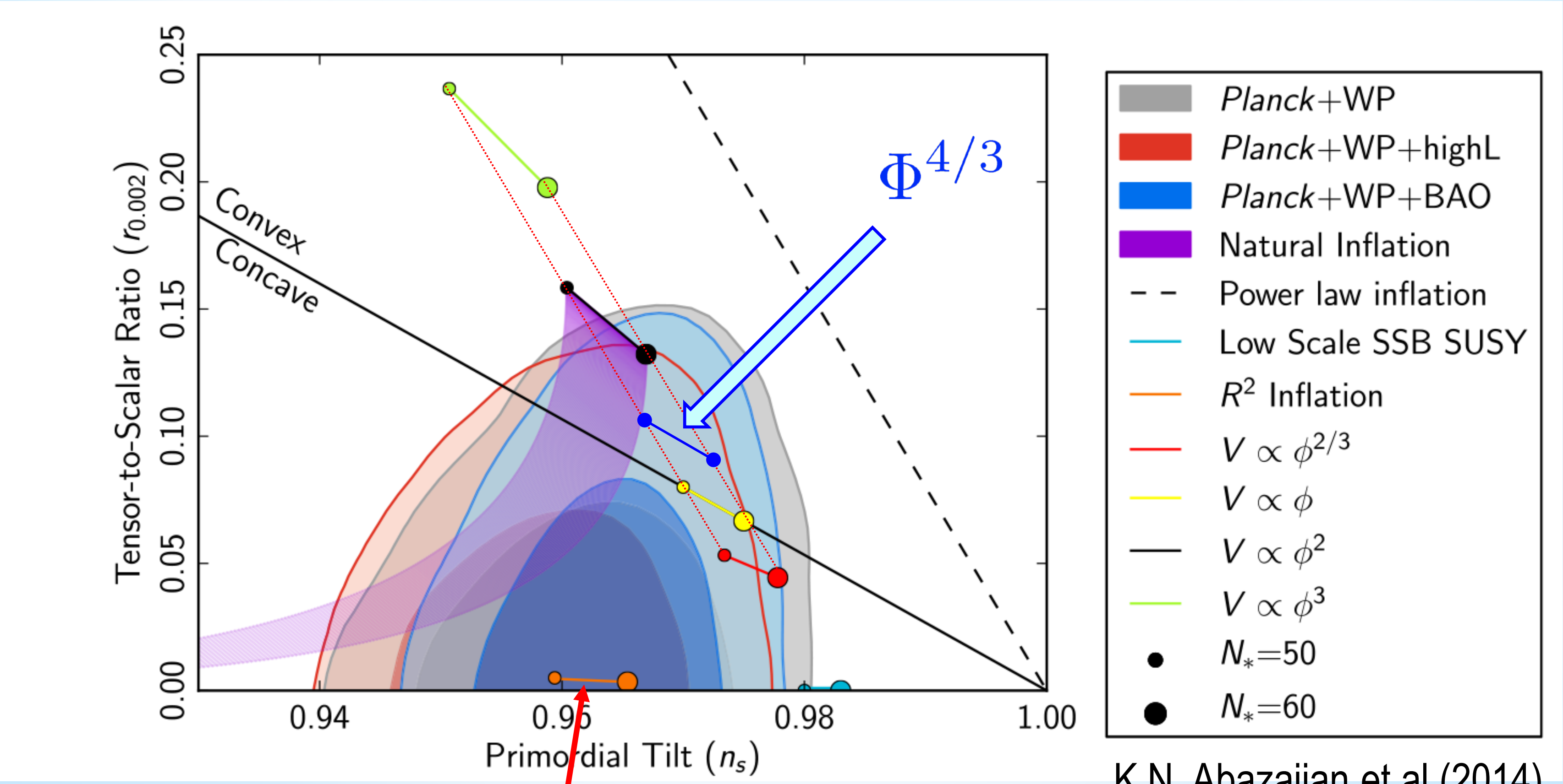


phase space of Higgs field



Observational constraint

perturbations



K.N. Abazajian et al (2014)

R^2(or original Higgs)

“original” Higgs Inflation

defect

$|\xi|$ is too large ($\xi \cong -10^5$)

r might be too small

new Higgs Inflation

defect

Observationally marginal

Hybrid ?

Hybrid Higgs Inflation (conventional+new)

Sato , KM (2018)

Kamada et al (2012): generalized Higgs inflation

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \alpha G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right) - \frac{1}{2}\xi\phi^2 R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

$$\alpha = \frac{1}{M^2 M_{\rm PL}^2}$$

disformal transformation

EH action +

$$\mathcal{L}_{\rm Higgs} = -\sqrt{-g}\left[\left(\frac{(1-\xi(1-6\xi)\kappa^2\phi^2) + \alpha V(\phi)}{2(1-\xi\kappa^2\phi^2)^2}\right)(\nabla\phi)^2 + \frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}\right] + \cdots$$

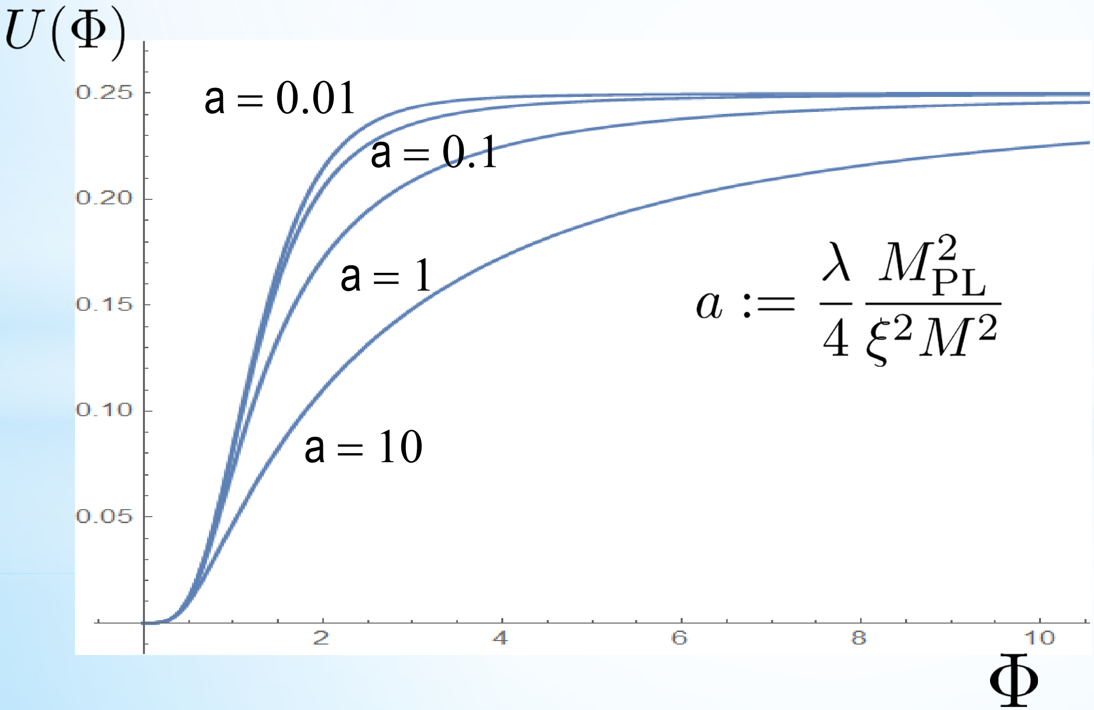
$$= -\sqrt{-g}\left[\frac{1}{2}(\nabla\Phi)^2 + U(\Phi)\right] + \cdots$$

higher-derivative terms

$$\Phi = \int \frac{\sqrt{(1-\xi(1-6\xi)\kappa^2\phi^2) + \alpha V(\phi)}}{(1-\xi\kappa^2\phi^2)}d\phi \qquad U(\Phi) = \frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}$$

$$V = \frac{\lambda}{4}\phi^4 \qquad \alpha = \frac{1}{M^2 M_{\rm PL}^2}$$

$$U(\Phi) = \frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}$$
$$= \frac{\lambda}{4\xi^2}M_{\rm PL}^4\left[1 - \frac{\lambda M_{\rm PL}^4}{2|\xi|^3 M^2}\frac{1}{\Phi^2} + \cdots\right]$$



$a \ll 1$

the original Higgs inflation

$a \sim O(1)$

$U \propto 1 - c_0\Phi^{-2}$

➢ cosmological perturbations in the disformal frame

Scalar perturbation

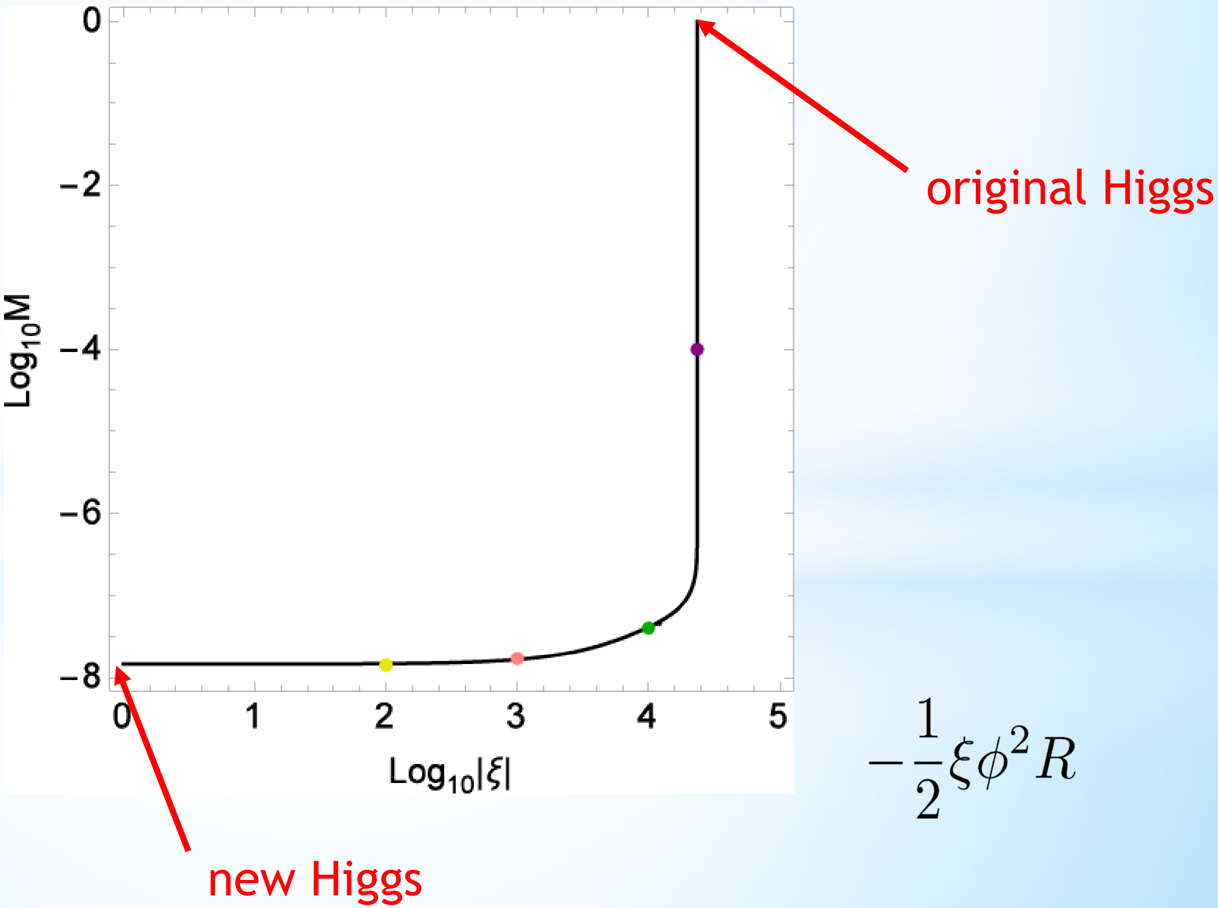
N. Makino, M. Sasaki(1991)
J.-O. Gong et al (2011)
Y. Watanabe et al(2015)
H. Motohashi, J. White (2016)

$$P_\zeta \approx 10^{-9}$$ at N=60

⇩

constrain the relation between M and ξ

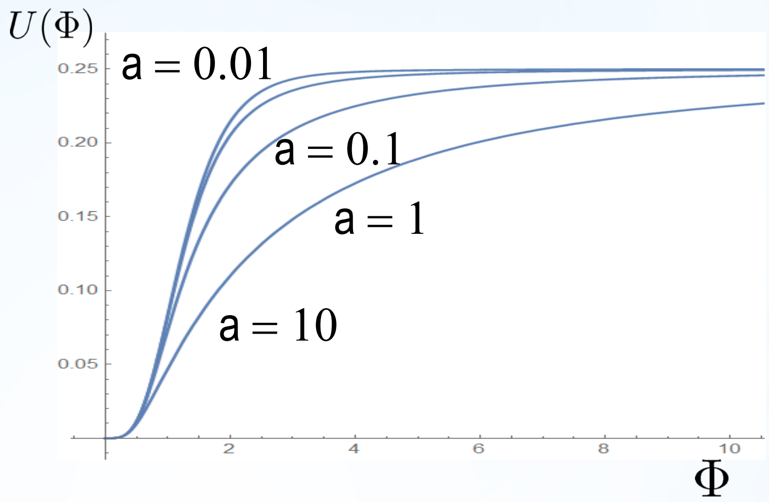
$$\frac{1}{M^2 M_{\rm PL}^2} G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$



$$-\frac{1}{2}\xi\phi^2 R$$

Truncated model

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\Phi)^2 - U(\Phi) + \cdots\right]$$



$$a := \frac{\lambda}{4}\frac{M_{\rm PL}^2}{\xi^2 M^2}$$

$$\epsilon = \frac{1}{2}\left(\frac{1}{U}\frac{dU}{d\phi}\right)^2$$

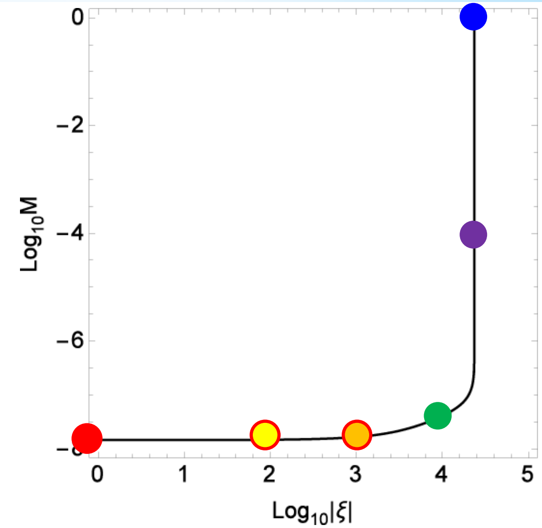
$$\eta = \frac{1}{U}\frac{d^2U}{d\phi^2}$$

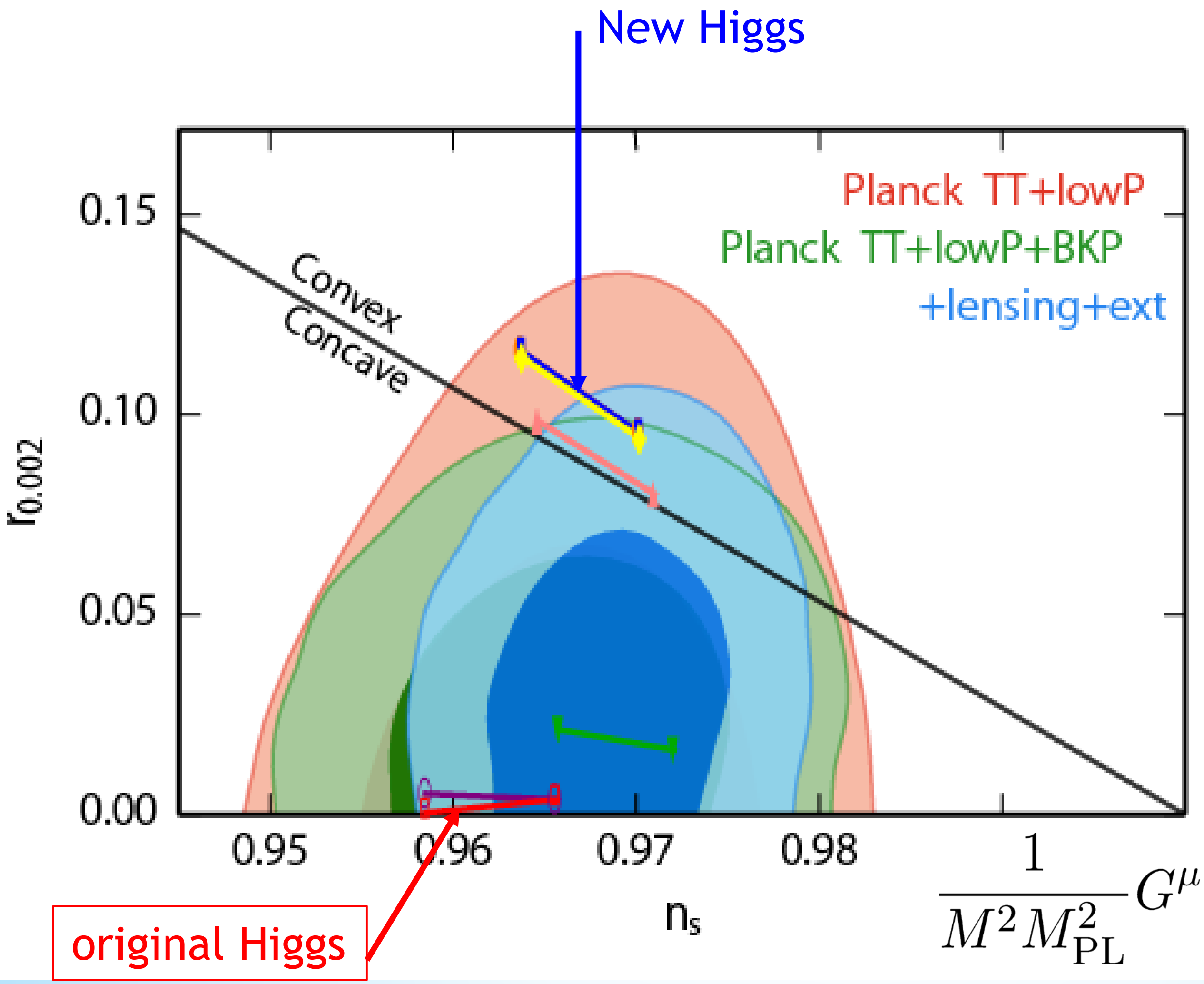
⇨

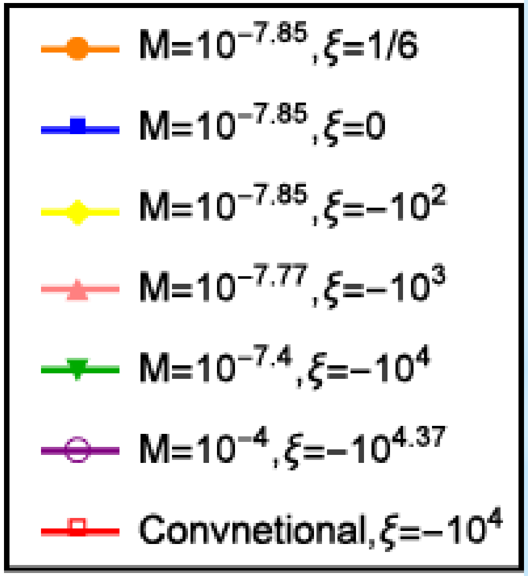
$$n_s \simeq 1 - 6\epsilon + 2\eta$$

$$r \equiv \frac{P_T}{P_\zeta} \simeq 16\epsilon$$

➢ Hybrid Higgs Inflation







$$\frac{1}{M^2 M_{\rm PL}^2} G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \quad -\frac{1}{2}\xi\phi^2 R$$

The truncation is valid ?

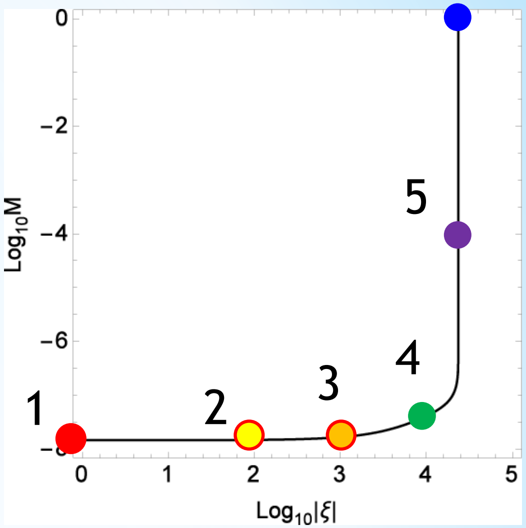
accuracy

$$\Delta n_s \equiv \frac{|n_s - n_{s\,ADM}|}{n_{s\,ADM}} \times 100$$

$$\Delta r \equiv \frac{|r - r_{ADM}|}{r_{ADM}} \times 100$$

$n_{s\mathrm{ADM}}$
r_{ADM}

Perturbations in original frame based on ADM decomposition



model	$\Delta n_s[\%]$		$\Delta r[\%]$	
	N=60	N=50	N=60	N=50
①	5.8×10^{-2}	9.3×10^{-2}	3.4	4.2
②	7.5×10^{-2}	1.1×10^{-1}	3.6	4.5
③	7.6×10^{-2}	1.1×10^{-1}	3.6	4.6
④	1.2×10^{-1}	1.8×10^{-1}	5.1	6.5
⑤	3.6×10^{-1}	9.3×10^{-2}	3.5	4.4

Other hybrid type inflation

- Non-minimal coupling (Higgs inflation) + R^2 (Starobinsky inflation)

  KM, J.A. Stein-Schabes, T. Futamase (1989)

  M. He, A.A. Starobinsky, J. Yokoyama (2018)
  M. He, R.Jinno,K. Kamada, S.C. Park, A.A. Starobinsky, J. Yokoyama (2019)

  similar to Higgs inflation

  equivalent to two field inflation (entropy perturbations)

- Hybrid Higgs inflation with R^2 term

  R. Diedrichs, S. Sato, KM (in progress)

  similar to hybrid Higgs inflation

  equivalent to two field inflation (entropy perturbations)

Reheating (in progress)

In oscillation phase after inflation,
we cannot ignore higher-derivative terms

⇨ **Analysis in the original Jordan frame**

1. stability

2. particle production

FLRW background

$$H^2 = \frac{1}{3(M_P^2 - \xi h^2)} \left[\left(1 + \frac{9H^2}{M^2} \right) \frac{\dot{h}^2}{2} + 6\xi h \dot{h} H + V(h) \right],$$

$$\mathcal{M} \begin{pmatrix} \ddot{h} \\ \dot{H} \end{pmatrix} = - \begin{pmatrix} 3H\dot{h} \left(1 + 3\frac{H^2}{M^2}\right) + 12\xi h H^2 + \frac{dV}{dh} \\ \left(1 + 3\frac{H^2}{M^2} - 2\xi\right) \dot{h}^2 + 2\xi H h \dot{h} \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} 1 + 3\frac{H^2}{M^2} & 6\left(\frac{H\dot{h}}{M^2} + \xi h\right) \\ -2\left(\frac{H\dot{h}}{M^2} + \xi h\right) & 2(M_P^2 - \xi h^2) - \frac{\dot{h}^2}{M^2} \end{pmatrix}$$

Perturbations

$$ds^2 = -N^2 dt^2 + \gamma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right),$$

$$N = 1 + \alpha, \qquad N_i = \partial_i \beta \qquad \gamma_{ij} = a^2(t) e^{2\zeta} \left(\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj} \right)$$

$$h_{ii} = h_{ij,j} = 0$$

Scalar perturbations:

$$f(h) = 1 - \xi \frac{h^2}{M_P^2}$$

$$S_S^{(2)} = \frac{M_P^3}{2} \int dt d^3x a^3 \left[G_S \dot{\zeta}^2 - \frac{F_S}{a^2} (\zeta_{,i})^2 \right] \qquad \varpi_M^2 = \frac{\dot{h}^2}{M^2 M_P^2}$$

$$\begin{aligned} G_S =& 3\left(f(h) - \frac{\varpi_M^2}{2} \right) \\ &+ \frac{\left(f(h) - \frac{\varpi_M^2}{2} \right)^2 \times \left(-3H^2 f(h) + (9 + \frac{M^2}{2})\varpi_M^2 + \frac{6H\xi h\dot{h}}{M_P^2} \right)}{\left[H(f(h) - \frac{3\varpi_M^2}{2}) - \xi \frac{h\dot{h}}{M_P^2} \right]^2} \end{aligned}$$

$$F_S = \frac{1}{a} \frac{d}{dt} \left(\frac{a \left(f(h) - \frac{\varpi_M^2}{2} \right)^2}{H(f(h) - \frac{3\varpi_M^2}{3}) - \xi \frac{h\dot{h}}{M_P^2}} \right) - \left(f(h) + \frac{\varpi_M^2}{2} \right)$$

The squared sound speed $\qquad c_S^2 = \frac{F_S}{G_S}.$

Tensor perturbations:

$$S_T^{(2)} = \frac{M_P^3}{2} \int dt d^3 x a^3 \left[G_T \dot{h}_{ij}^2 - \frac{F_T}{a^2} \left(\nabla h_{ij}\right)^2 \right]$$

$$G_T = f(h) - \frac{\varpi_M^2}{2} \qquad f(h) = 1 - \xi \frac{h^2}{M_P^2} \qquad \varpi_M^2 = \frac{\dot{h}^2}{M^2 M_P^2}$$

$$F_T = f(h) + \frac{\varpi_M^2}{2}$$

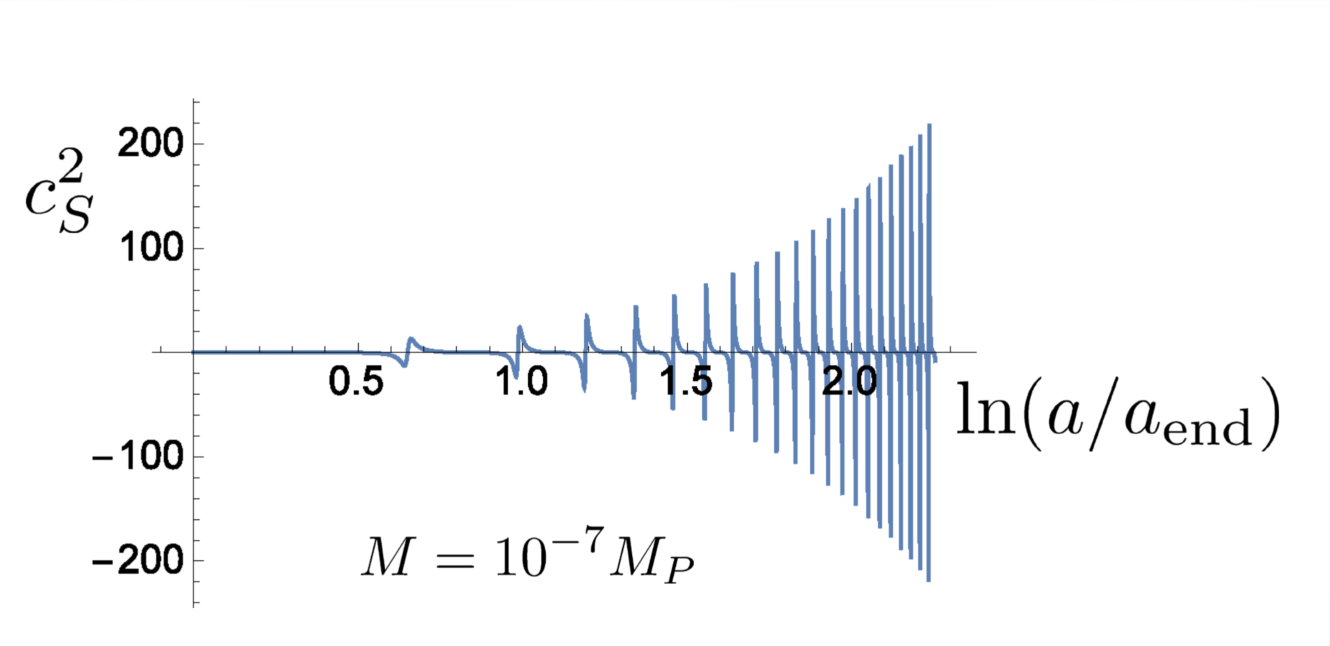
The squared sound speed $c_T^2 = \frac{F_T}{G_T}.$

stability

$F_S \geq 0$ $\quad c_S^2 \geq 0$ ⇨ ?

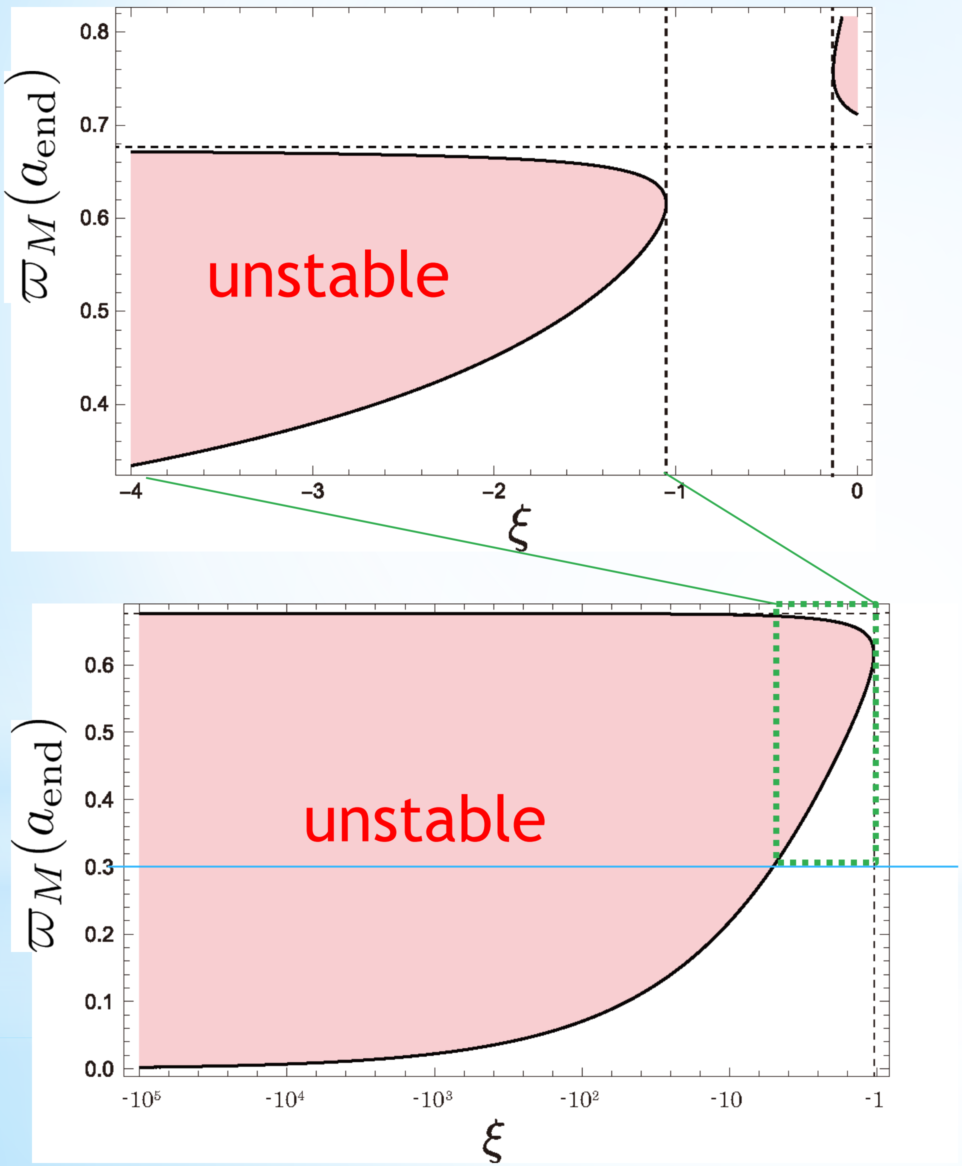
$F_T \geq 0$ $\quad c_T^2 \geq 0$ ⇨ tensor perturbations : stable

In new Higgs inflation model ($\xi = 0$),
gradient instability appears in oscillating phase



But, Higgs inflation model is stable

Non-minimal coupling may stabilize the perturbations



Stability depends on

ξ and $\varpi_M(a_{\rm end})$

$$\varpi_M^2 = \frac{\dot{h}^2}{M^2 M_P^2}$$

Oscillation of Higgs field

$$h \approx h_e \left(\frac{a_e}{a}\right) \mathrm{cn}(\tilde{t}, k_e)$$

Jacobi elliptic function

$$\frac{d\tilde{t}}{dt} \equiv \sqrt{\frac{\left(6\xi - 1 + \frac{3H^2}{M^2}\right)\left(\frac{a}{a_e}\right)^2 \left(2H^2 + \dot{H}\right) - \frac{12H^2}{M^2} H^2 + \lambda h_e^2}{1 + \frac{3H^2}{M^2}}}$$

$$k_e^2 \equiv \frac{\lambda h_e^2}{2} \left[\left(6\xi - 1 + \frac{3H^2}{M^2}\right)\left(\frac{a}{a_e}\right)^2 \left(2H^2 + \dot{H}\right) - \frac{12H^2}{M^2} H^2 + \lambda h_e^2\right]^{-1}$$

⇨ Particle production via interaction between Higgs and other particles

Particle decay

Toy model

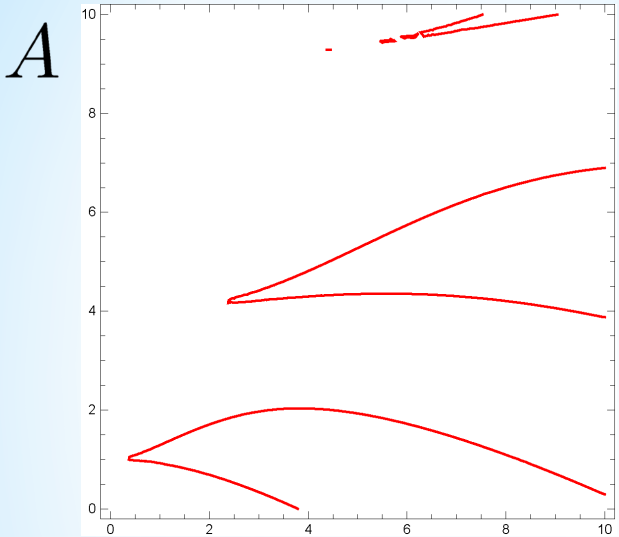
Decay of Higgs field into two massless scalar fields $\mathcal{L}_{\rm int} = \frac{1}{2} g^2 h \phi^2$

$$\frac{d^2\tilde{\phi}_k}{d\eta^2} + \left[A + m_\phi^2\right]\tilde{\phi}_k = 0$$

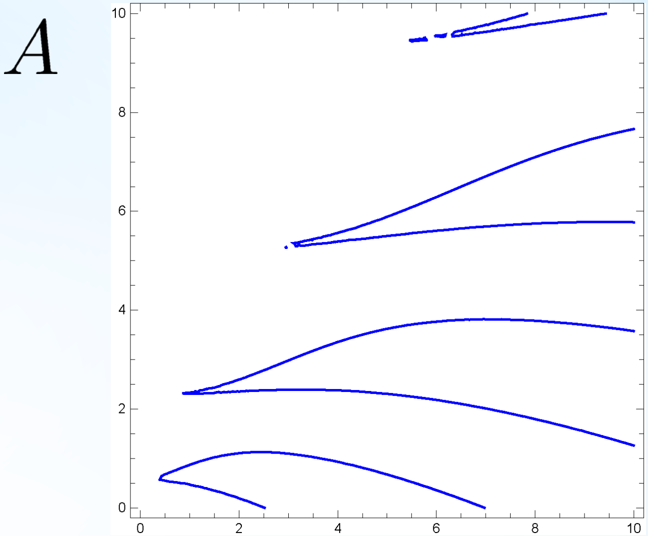
$$A = \frac{a_e^2}{a^2}k^2 - 2\frac{a^2}{a_e^2}H^2 + \frac{a^2}{a_e^2}\dot{H} \approx \frac{a_e^2}{a^2}k^2 \qquad \tilde{\phi} = \left(\frac{a}{a_e}\right)\phi$$

$$m_\phi^2 = \frac{a}{a_e} g^2 \mathrm{cn}\left(m\eta, k_e\right)$$

Instability chart



$$k_e = 0 \qquad q = \frac{a}{a_e} g^2$$



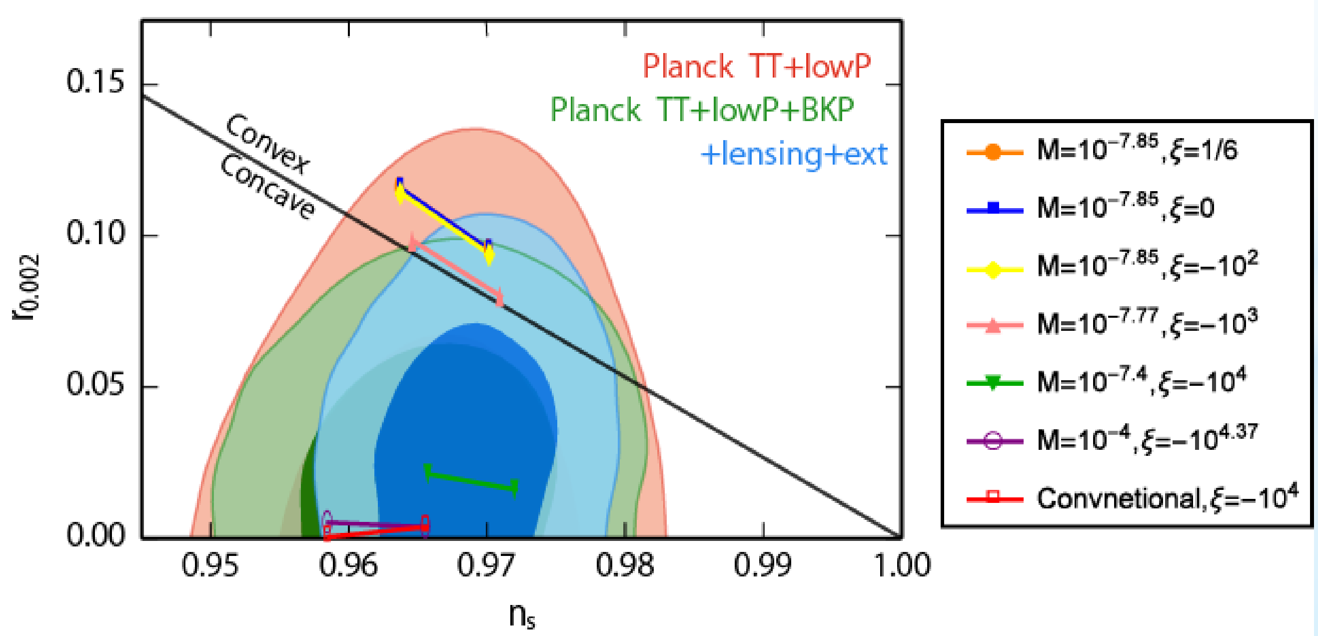
$$k_e = \frac{1}{\sqrt{2}} \qquad q = \frac{a}{a_e} g^2$$

exponential growth in unstable region

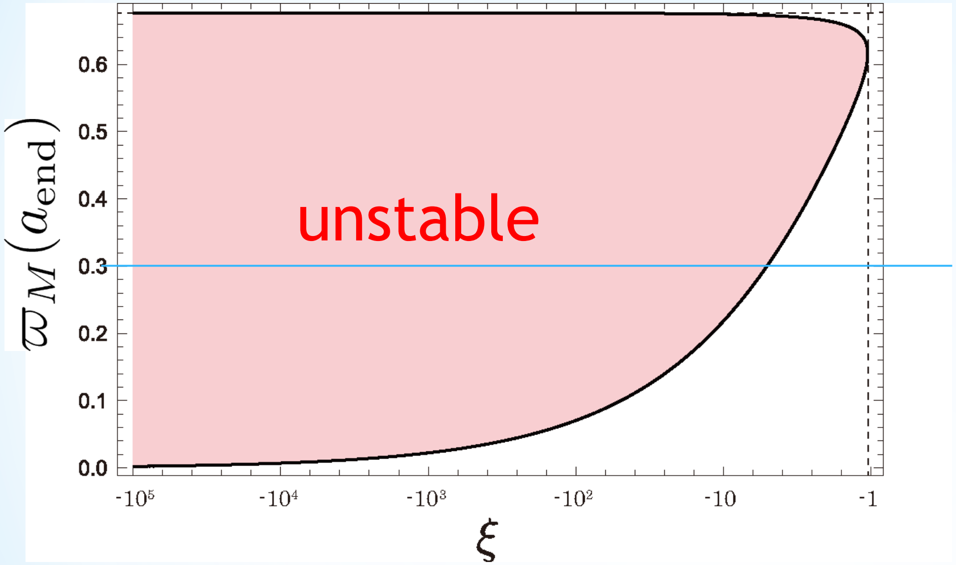
⇨ More realistic interactions based on the standard model (in progress)

Summary

- We propose a hybrid type of the conventional Higgs inflation and new Higgs inflation models.
- Although the primordial tilt n_s in the hybrid model barely changes, the tensor-to-scalar ratio r moves from the value in new Higgs inflationary model to that in the conventional Higgs inflationary model as $|\xi|$ increases.



- For some parameter region (ξ, M), the perturbations are stable



- The analysis of particle production for reheating is possible (in progress)

Thank you for your attention