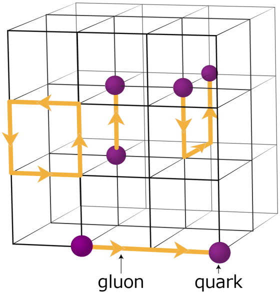
Quy Nhon, August 2018

Windows on the Universe

Recent developments in Lattice QCD

Agostino Patella
Humboldt University, Berlin

Lattice QCD



- A regularization of QCD (it is QCD, not a model of QCD). The lattice spacing a is the UV cutoff.
- The only known consistent way to define QCD at all energy scales.
- When restricted to a finite box, suitable for numerical calculation of the path integral
- Limits to be taken in numerical calculations

$$a \to 0\ , \qquad L \to \infty$$

Main challenges:

- Precision!
  Dam Thanh Son: *HEP: calculations involving strong interactions are difficult, limited precision*
- Reduce statistical error (larger computers, smarter algorithms).
- Have better control of systematic errors (limits).
- Enlarge the set of observables that can be calculated.

Part I – What can we calculate on the Lattice?

A selection of observables (with an eye to the future)

Disclaimer: this list is by no means complete!

Bread and butter observables

Masses of strongly-stable hadrons

- Examples: π, K, p/n, Λ, Ξ, Ω, exotic states.
- In principle: deuterium, and nucleon binding energy.
- Challenges: signal-to-noise-ratio problem, excited state contamination

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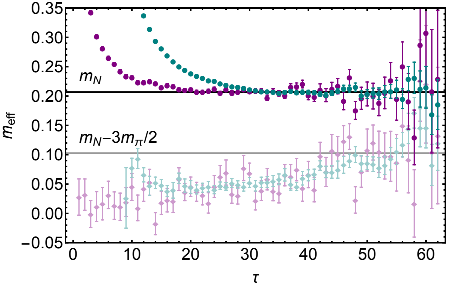
Masses are extracted from two-point functions, e.g.

$$\langle \phi(t)\phi(0)\rangle \overset{t\to\infty}{\simeq} Ae^{-Mt} + O(e^{-\Delta Mt})$$

$$-\frac{d}{dt}\ln\langle \phi(t)\phi(0)\rangle \overset{t\to\infty}{\simeq} M + O(e^{-\Delta Mt})$$



($M_\pi \simeq 350$ MeV) Detmold, Endres, *Signal/noise optimization strategies for stochastically estimated correlation functions*, PoS LATTICE **2014** (2015) 170, arXiv:1409.5667.

Bread and butter observables

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Physics-based algorithmic improvements (just waiting for larger computers would not be enough)

Cè, Giusti, Schaefer, *Domain decomposition, multi-level integration and exponential noise reduction in lattice QCD*, Phys. Rev. D93 (2016) no.9, 094507, arXiv:1601.04587

Cè, Giusti, Schaefer, *A local factorization of the fermion determinant in lattice QCD*, Phys. Rev. D95 (2017) no.3, 034503 arXiv:1609.02419.

Bread and butter observables

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Matrix elements of single-hadron states

- LO EW leptonic decay rates: e.g. F_π, F_K $\quad F_\pi \propto \langle 0|J_0^L|\pi\rangle$
- LO EW semi-leptonic decay form factors: e.g. $\langle K(\mathbf{0})|J_0^L|\pi(\mathbf{p})\rangle$
- Nucleon axial charge $\quad g_A \propto \langle N|J_0^A|N\rangle$
- Strange quark content of the nucleon $\quad \langle N|\bar{s}s|N\rangle$
- Challenges: signal-to-noise-ratio problem, excited state contamination, renormalization of operators.
- Challenges for some of these observables: inclusion of isospin-breaking corrections.

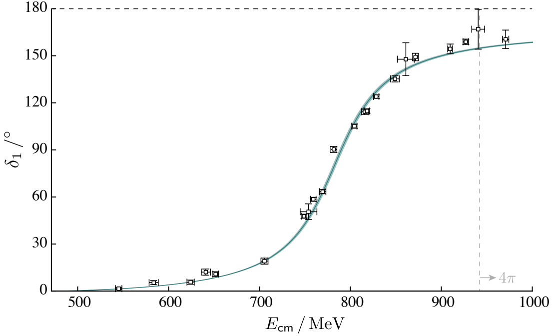
Scattering amplitudes and resonances

- These are peaks in differential cross sections, e.g.

$$\pi + \pi \to \rho \to \pi + \pi$$

- Challenges: extract real-time dynamics from Euclidean QFT.
- Only in simple cases... but theory has developed a lot recenty
Lüscher, Rummukainen, Gottlieb, Kim, Sachrajda, Sharpe, Yamazaki, Bernard, Lage, Meissner, Rusetsky, Hansen, Briceño, Davoudi, Li, Liu

$M_\pi = 236$ MeV [Wilson *et al.*, 2015, arXiv:1507.02599]



Strong coupling

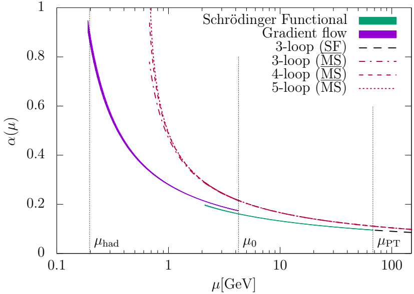
- Determination by ALPHA collaboration Bruno *et al.* (ALPHA coll.), *QCD Coupling from a Nonperturbative Determination of the Three-Flavor Λ Parameter*, Phys. Rev. Lett. 119 (2017) no.10, 102001, arXiv:1706.03821

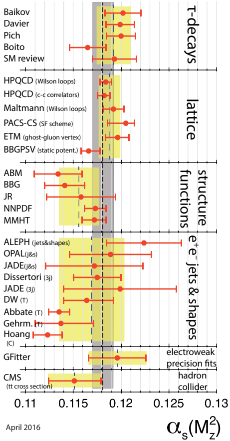
$$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.11852(84)$$

- ... to be compared with 2016 PDG average

$$\alpha_{\overline{\rm MS}}(M_Z) = 0.1181(11)$$

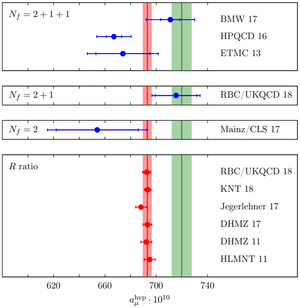
- Challenges: cover a few orders of magnitude in energy scales, cross the charm threshold non-perturbatively





Observables I am not going to talk about

Meyer, Wittig, arXiv:1807.09370



HVP contribution to muon anomalous magnetic moment

- Precision of current determinations $\simeq$ 2%, goal precision $\simeq$ 0.5–1%
- Challenges: reaching energy scales of order of (a fraction of) m_μ, isospin breaking corrections.

Zero-density finite-temperature properies

- Thermodynamics properties: average energy and enthropy.
- Hydrodynamic properties, i.e. correlation functions of the energy-momentum tensor.
- Topological susceptibility (axion physics).

Heavy quark physics, and much more...

Part II – Isospin-breaking corrections

Isospin-breaking effects

- Most lattice QCD simulations are performed in the isosymmetric limit. However in the real world up and down quark have different masses and charges.
- Isospin-breaking effects are typically a few percent effects:

$$\frac{m_u - m_d}{M_p} \simeq 0.3\% \qquad \alpha_{\mathrm{EM}} = 0.7\% \qquad \frac{M_n - M_p}{M_n} \simeq 0.1\%$$

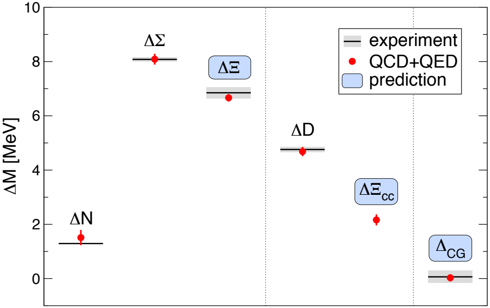
- From FLAG16 [Aoki *et al.*, arXiv:1607.00299] and [PDG review, Rosner *et al.*, 2016], [Cirigliano *et al.*, Rev. Mod. Phys. **84**, 399 (2012)]

$$\begin{array}{lll} f_{\pi^\pm} = 130.2(1.4)\ \mathrm{MeV} & \mathrm{err} = 1\% & \delta^{\chi\mathrm{PT}}_{\mathrm{QED}}(\pi^- \to \ell^- \bar{\nu}) = 1.8\% \\ f_{K^\pm} = 155.6(0.4)\ \mathrm{MeV} & \mathrm{err} = 0.3\% & \delta^{\chi\mathrm{PT}}_{\mathrm{QED}}(K^- \to \ell^- \bar{\nu}) = 1.1\% \\ f_+(0) = 0.9704(24)(22) & \mathrm{err} = 0.5\% & \delta^{\chi\mathrm{PT}}_{\mathrm{QED}}(K \to \pi \ell \bar{\nu}) = [0.5, 3]\% \end{array}$$

- Lattice QCD+QED provides a way to calculate isospin breaking effects from first principles. This is pioneering work, which will become more and more relevant in the coming years.

Isospin-breaking splitting of baryonic spectrum

Borsanyi *et al.* (BMW coll.), *Ab initio calculation of the neutron-proton mass difference*, Science 347 (2015) 1452-1455, arXiv:1406.4088



Very important proof of concept: it is possible to resolve isospin-breaking corrections.

Radiative corrections to decay rates

Giusti *et al.* (RM+SOTON coll.), *First lattice calculation of the QED corrections to leptonic decay rates*, Phys. Rev. Lett. **120** (2018) no.7, 072001, arXiv:1711.06537

- Decay rate of hadron h with emission of real soft photons with energy less than ΔE_γ

$$\Gamma(P^\pm \to \mu^\pm \nu_\mu[\gamma]) = \Gamma_P^0\,[1 + \delta R_P]$$

- Ratio of inclusive decay rates with $\Delta E_\gamma^K \simeq 230$ MeV and $\Delta E_\gamma^\pi \simeq 30$ MeV

$$\frac{\Gamma_K}{\Gamma_\pi} = \left|\frac{V_{us}}{V_{ud}}\,\frac{f_k^{(0)}}{f_\pi^{(0)}}\right| \frac{M_\pi^3}{M_K^3} \left(\frac{M_K^2 - m_\pi^2}{M_\pi^2 - m_\mu^2}\right)^2 (1 + \delta R_{K\pi})$$

- Determination of RM+SOTON collaboration

$$\delta R_{K\pi} = \delta R_K - \delta R_\pi = -0.0122(10)_{stat}(2)_{input}(8)_{chir}(5)_{FVE}(4)_{disc}(6)_{qQED} = -0.0122(16)$$

- ... to be compared with PDG estimate

$$\delta R_{K\pi} = 0.0112(21)$$

Theoretical aspects and perspectives

- Including QED effects in lattice simulations is particularly complicated.
- If we want to measure the mass of the proton on the lattice, we need to be able to put a nonzero charge in a finite box.
- On a torus with periodic boundary conditions, Gauss law forbids a nonzero charge.

$$\partial_k E_k(x) = \rho(x) \quad \Rightarrow \quad Q = \int d^3x \; \rho(t, \mathbf{x}) = \int d^3x \; \partial_k E_k(t, \mathbf{x}) = 0$$

- Various recipes have been used to work around this issue, e.g. imposing constraints on the photon field

$$\int d^3x \; A_\mu(x_0, \mathbf{x}) = 0$$

- A formulation that respects fundamental axioms of QFT (in particular locality) is desirable.

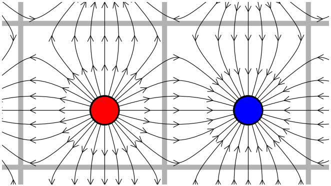
Theoretical aspects and perspectives

C-parity boundary conditions on the torus

Wiese, *C periodic and G periodic QCD at finite temperature*, Nucl. Phys. B375, 45 (1992)

Lucini, AP, Ramos, Tantalo, *Charged hadrons in local finite-volume QED+QCD with $C^\star$ boundary conditions*, JHEP **1602** (2016) 076, arXiv:1509.01636

$$A_\mu(x+L\mathbf{k}) = -A_\mu(x) \qquad \psi(x+L\mathbf{k}) = C^{-1}\bar{\psi}^T(x) \qquad \bar{\psi}(x+L\mathbf{k}) = -\psi^T(x)C$$



Electric flux can escape the torus and flow into the image charge

$$Q(t) = \int d^3x\ \rho(t,\mathbf{x}) = \int d^3x\ \partial_k E_k(t,\mathbf{x}) \neq 0$$

Challenge for the future: incorporate a theoretically-solid treatment of QED corrections into numerical simulations.

Conclusions

- These are exciting times for Lattice QCD!
- A way to reduce the signal-to-noise ratio problem has been proposed, which opens the way to more-reliable determinations of neuclon properties.
- A lot of progress has been made towards the calculation of scattering amplitudes.
- A new, very robust, determination of the strong coupling is available. Its precision is better than the PDG average.
- Some observables have reached the percent precision. At this level pf precision, isospin-breaking corrections must be included.

Backup slides

Bloch-Nordsieck prescription

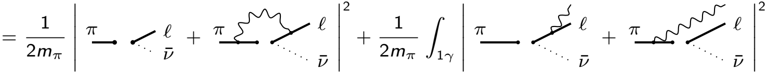
Physics interpretation: from the experimental point of view it is impossible to differentiate between

$$h \to \ell + \bar{\nu} \ ,$$
$$h \to \ell + \bar{\nu} + N\gamma \ ,$$

- if each photon is emitted with a lower energy than the detector resolution ΔE;
- and the total energy carryed away by the undetected photons is (roughly) less than the resolution ΔE with which we can reconstruct the lepton energy.

The physical quantity is the decay rate integrated over soft photons, which is finite.[1]

$$\Gamma(\Delta E) = \lim_{m_\gamma \to 0} \frac{1}{2m_\pi} \sum_{N=0}^{\infty} \frac{1}{N!} \int_{\substack{k_\alpha < \Delta E \\ \sum_\alpha k_\alpha < \Delta E}} \mathrm{d}\Phi_{N\gamma} \ \left| \langle \pi | \mathcal{H}_{\mathrm{W}} | \ell, \bar{\nu}, N\gamma \rangle \right|^2 =$$



The logarithm in the photon mass is traded for a logarithm in the energy resolution:

$$\langle \pi | \mathcal{H}_{\mathrm{W}} | \ell, \bar{\nu} \rangle = \left[1 + \frac{\alpha_{\mathrm{EM}}}{2} R \ln \frac{m_\gamma}{\Lambda} \right] A_{\mathrm{LO}} + A_{\mathrm{NLO}, k^2 > \Lambda^2} + O(\alpha_{\mathrm{EM}}^2)$$

$$\Rightarrow \qquad \Gamma(\Delta E) = \left[1 + \alpha_{\mathrm{EM}} \, \mathrm{Re} \, R \ln \frac{\Delta E}{\Lambda} \right] \Gamma_{\mathrm{LO}} + \Gamma_{\mathrm{NLO}, k^2 > \Lambda^2} + O(\alpha_{\mathrm{EM}}^2)$$

[1] The diagrammatic expansion is wrong. I am deliberately neglecting the wave-function renormalization for sake of presentation.

Large collinear logarithms

We consider the (phenomenologically irrelevant) decay process

$$B^- \to e^- + \bar{\nu}_e$$

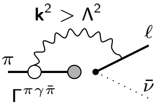
$$\Gamma(\Delta E) = \left[1 + \alpha_{\mathrm{EM}} \mathrm{Re}\, R \ln \frac{\Delta E}{\Lambda}\right] \times \Gamma_{\mathrm{LO}} + \Gamma_{\mathrm{NLO},\, k^2>\Lambda^2} + O(\alpha_{\mathrm{EM}})$$

Γ_{LO}: tree-level decay rate with $m_\gamma = 0$

Back of the envelope calculation
$\mathrm{Re}\, R \simeq -2 + \ln(m_B^2/m_e^2) \simeq 16.5$
$\Lambda \simeq m_B, \;\; \Delta E/m_B \simeq 10\%$

- $\alpha_{\mathrm{EM}}\, \mathrm{Re}\, R \ln \Delta E/m_B = 28\%$
- $\frac{1}{2}\left(\alpha_{\mathrm{EM}}\, \mathrm{Re}\, R \ln \Delta E/m_B\right)^2 = 4\%$

$\Gamma_{\mathrm{NLO},\, k^2>\Lambda^2}$: 1-loop decay rate with $k^2 > \Lambda^2$ restriction on photon loop momenta



$$\subset \int_{\mathbf{k}^2>\Lambda^2} \frac{\mathrm{d}^3 k}{(2\pi)^3 2k} \left[\frac{\gamma_\mu \Gamma_\mu^{B\gamma\bar{B}}(\bar{p}_B, k)}{(\bar{p}_B k)\left\{|\mathbf{k}|\sqrt{m_e^2 + \mathbf{p}_e^2} - \mathbf{k}\mathbf{p}_e\right\}} \right]_{k_0 = i|\mathbf{k}|} \propto \ln \frac{m_B}{m_e}, \;\; \ln \frac{\Lambda_{\mathrm{QCD}}}{m_e}$$

This hard collinear logarithm is not universal, it reads the structure of the B meson and has to be calculated nonperturbatively!

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tree-level decay rate with $m_\gamma = 0$

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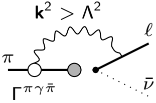

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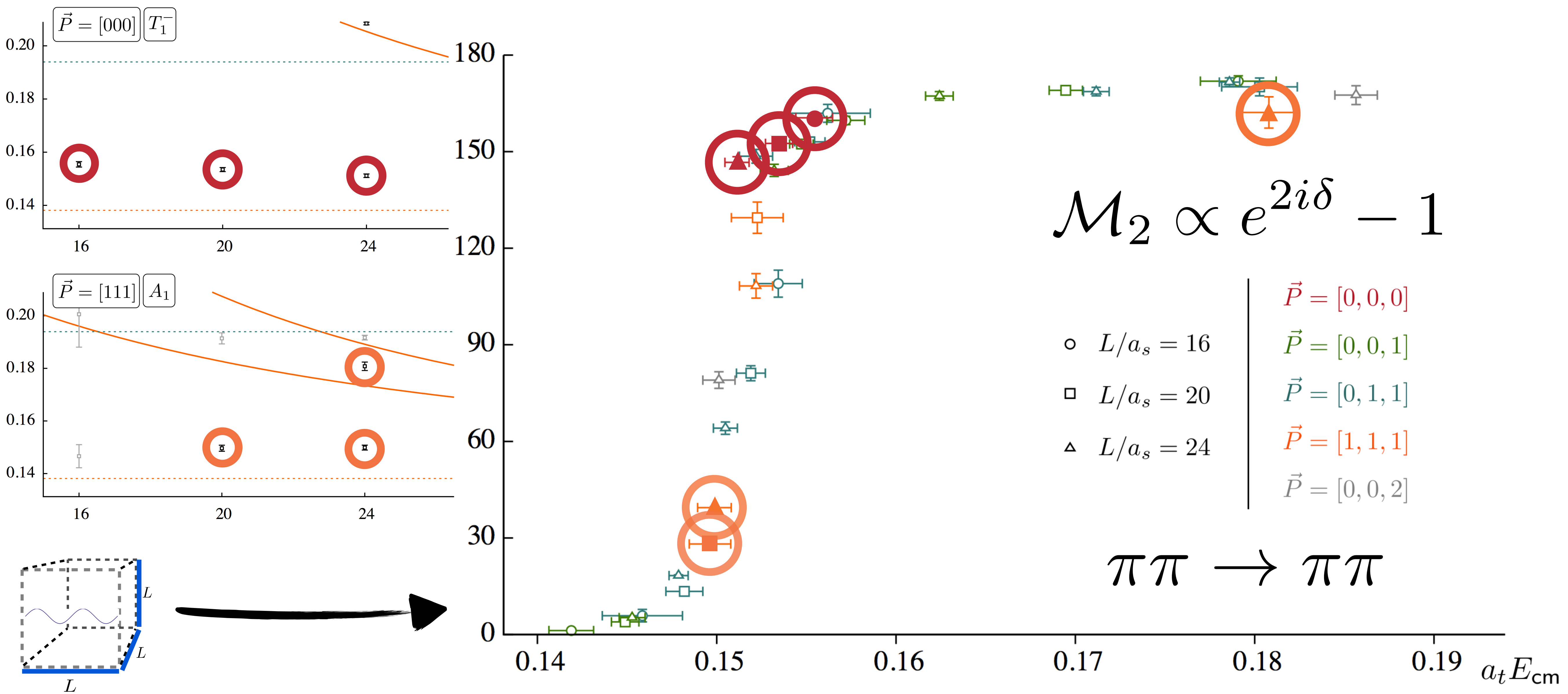
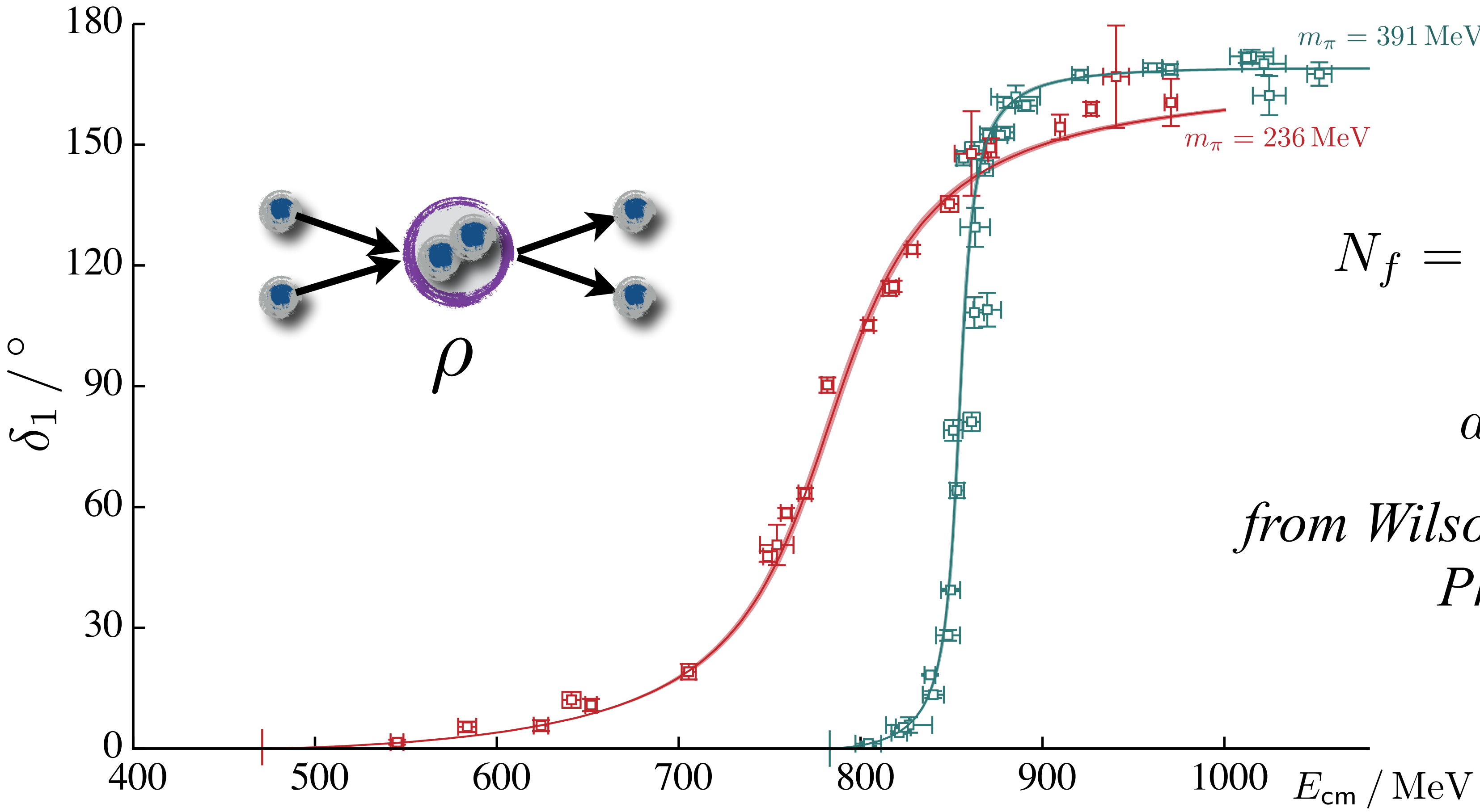
Using the result

- Simplest case is a single channel (e.g. for pions in a p-wave the relation reduces to)

$$\mathcal{M}_2(E_n^*) = -1/F(E_n, \vec{P}, L)$$

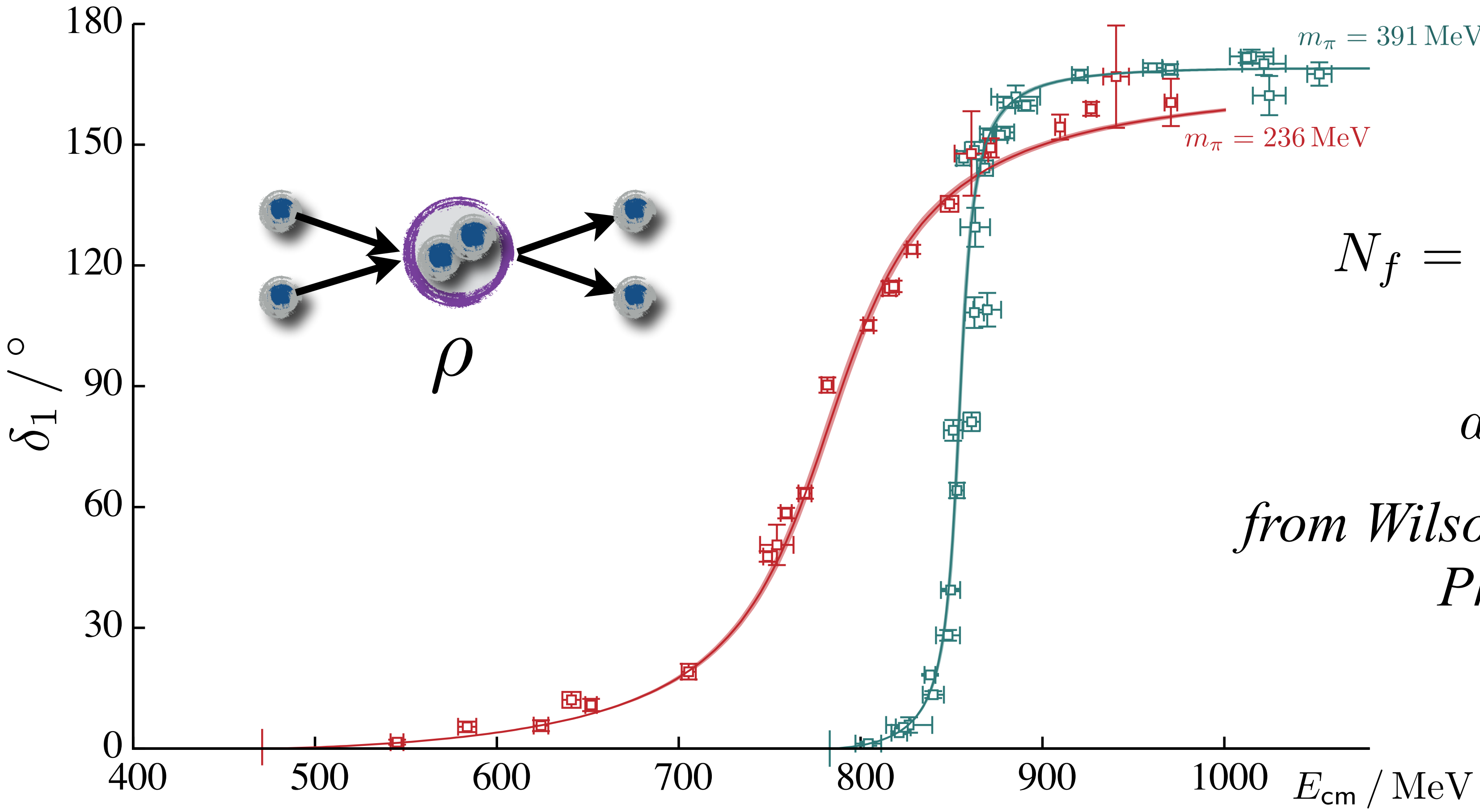


from Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505



$$N_f = 2+1\,, \quad M_\pi \sim 240 \text{ and } 390\,\text{MeV}\,,$$
$$M_\pi L \sim 4-6\,,$$
$$a_s \sim 0.12\,\text{fm}\,, \quad a_t \sim 0.035\,\text{fm}$$

from Wilson, Briceño, Dudek, Edwards, Thomas,
Phys. Rev. D 92, 094502 (2015)



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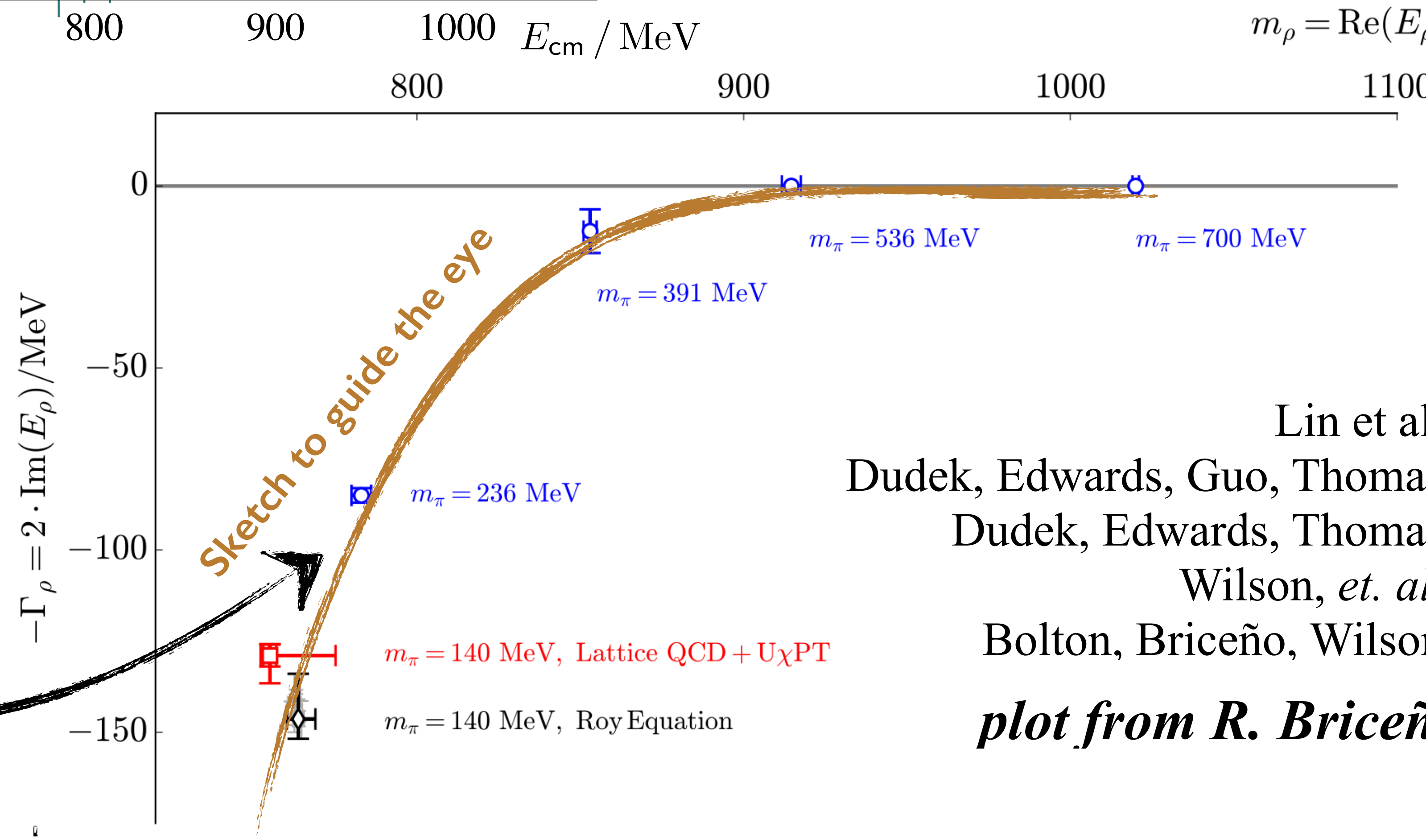
Quantitatively trace the pole position in the complex plane



Lin et al. (2009)
Dudek, Edwards, Guo, Thomas (2013)
Dudek, Edwards, Thomas (2012)
Wilson, *et. al.* (2015)
Bolton, Briceño, Wilson (2015)

plot from R. Briceño