# FRW bulk viscous cosmology with modified Chaplygin gas in flat space in (2+1)-dimensional spacetime

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1. Photo of Praveen Kumar
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Kumar.jpg

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Khadekar.jpg 

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Islam.jpg 

# ABSTRACT :

In this paper, we study the FRW bulk viscous cosmology in presence of modified Chaplygin gas in $(2+1)$-dimensional spacetime. The modified Friedmann equations due to bulk viscosity and Chaplygin gas are derived. For a particular choice of constant $\zeta$ in the energy-momentum conservation equation, we find that the energy density is dependent on the scale factor $a(t)$. The variation of energy density with time is plotted. The Hubble expansion and deceleration parameters are studied. In this work, we consider Chaplygin gas and bulk viscous effect as a linear combination of two terms, one constant and other is a linear combination of the Hubble parameter '$H$'. In this framework, we obtain the time-dependent energy density and also discourse the stability of the model in the $(2+1)$- dimension spacetime.

# Outline :

- FRW cosmological Model in (2+1)-dimensional spacetime
- Time-dependent density
- Hubble and deceleration parameters
- Stability
- Concluding Remarks
- References

# Motivation about the (2+1)-dimension theory :

The general cosmological solutions of (3+1)-dimensional Einstein equations are intractably complicated and likely dominated by non-integrability,the structure of the theory in (2+1)-dimensions offers possibility of making considerable progress towards finding the general solution in several interesting situations.This fact,together with the current perception that quantum field theory fits more naturally in three dimension rather than four dimensions, has motivated me to study of Einstein's theory in (2+1)-dimensional space time.

# FRW cosmological Model in (2+1)-dimensional spacetime

We consider (2+1)-dimensional Friedmann-Robertson-walker (FRW) line elements .

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2\right], \qquad (1)$$

where $a(t)$ is a scale factor with cosmic time $t$.
The Einstein field equations in (2+1)-dimensions for flat space, $k = 0$ is,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} + g_{\mu\nu}\Lambda, \qquad (2)$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor , $g_{\mu\nu}$ is metric tensor and $T_{\mu\nu}$ is the energy momentum tensor.
We consider $8\pi G = c = 1$. The energy momentum tensor corresponding to the bulk viscous fluid and modified Chaplygin gas is given by

# FRW cosmological Model in (2+1)-dimensional spacetime

$$T_{\mu\nu} = (\rho + \bar{p})u_\mu u_\nu - \bar{p}g_{\mu\nu}, \tag{3}$$

where $\rho$ is the energy density and $u^\mu$ is the velocity four vector with $u^\mu u_\nu = -1$.

The total pressure and the proper pressure involve bulk viscosity coefficient $\zeta$ and Hubble expansion parameter $H = \dot{a}/a$ are given by in eqn(4) and (5),

$$\bar{p} = p - 2\zeta H, \tag{4}$$

and

$$p = \gamma\rho - \frac{B}{\rho^\alpha}, \tag{5}$$

with $B > 0$ and $0 < \alpha \leq 1$. Here $\gamma$ and $B$ describes the features of dark energy models and Chaplygin gas respectively whereas $\zeta$ represents bulk viscosity.

# FRW cosmological Model in (2+1)-dimensional spacetime

We derive the Friedmann equations in $(2+1)$ dimensions as,

$$(\frac{\dot{a}}{a})^2 + \frac{k}{a^2} = \frac{\rho}{2}, \tag{6}$$

and

$$\frac{\ddot{a}}{a} = -\bar{p}, \tag{7}$$

where "." denotes derivative with respect to cosmic time t.
The energy-momentum conservation law is hereby deduced as,

$$\dot{\rho} + 2\frac{\dot{a}}{a}(\rho + \bar{p}) = 0. \tag{8}$$

# Time-dependent density when $\zeta$ is a constant

By using eqns.(4), (5) and (6) the above conservation eqn.(8) reduce as,

$$\dot{\rho} + \sqrt{2}(\gamma + 1)\rho^{3/2} - 2\zeta\rho - \sqrt{2}B = 0. \tag{9}$$

**Case-a(i):** $\zeta = 0$

We set $\zeta = 0$, to derive the energy density depend on scale factor as,

$$\rho(a) = [\frac{1}{(\gamma + 1)}(B - \frac{c}{a^{3(\gamma+1)}})]^{2/3}, \tag{10}$$

where c is a constant.

# Time-dependent density when $\zeta$ is a constant

It is plotted below,

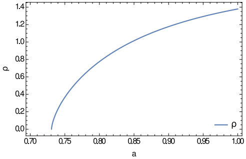


Figure: The energy density is shown against the scale factor $a$ for values of the constants $\gamma = 0.3, B = 3.4, c = 1$.

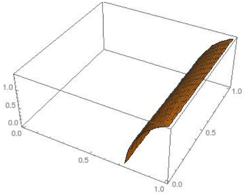


Figure: The energy density (3D) is shown against the scale factor $a$.

# Time-dependent density when $\zeta$ is a constant

**Case-a(ii):** $\zeta \neq 0$

We now obtain the time dependent energy density and solve eqn. (9) using the ansatz,

$$\rho = \frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}, \tag{11}$$

where A, E, h, C and b are arbitrary constants, whose values are derived from eqns.(9) and (11) as,

$$h = \sqrt{2}B, \tag{12}$$

$$A = \frac{2}{(\gamma+1)^2}, \tag{13}$$

$$E = \frac{2\zeta}{(\gamma+1)^2}, \tag{14}$$

# Time-dependent density when $\zeta$ is a constant

$$C=\frac{(\gamma+1)^2}{6}\left[\frac{8\sqrt{2}\zeta^2}{(\gamma+1)^3}-\frac{3}{16}\frac{(\gamma+1)^4}{\zeta^2}\right], \tag{15}$$

$$\begin{aligned} b=&\frac{2}{3}\zeta\left[\frac{2\sqrt{2}}{3}\zeta(\gamma+1)\left(\frac{2}{3}B(\gamma+1)-\frac{8}{3}\zeta^3\right)+\right.\\ &\left.\frac{9\sqrt{3}}{16\sqrt{2}}\left((\gamma+1)+\frac{7}{8}\right)+\frac{1}{9}\zeta^4+o(\gamma^n)\right]\\ &\left(\frac{8\sqrt{2}}{\sqrt{3}}(\gamma+1)\left(\frac{16}{9\sqrt{3}}\zeta^4-\frac{1}{72\sqrt{2}}(\gamma+1)^7\right)\right)^{-1}, \end{aligned} \tag{16}$$

where,

$$\begin{aligned} O(\gamma^n)=&\frac{189}{64}\gamma^2+\frac{189}{32}\gamma^3+\frac{945}{128}\gamma^4+\frac{189}{32}\gamma^5\\ &+\frac{189}{64}\gamma^6+\frac{27}{32}\gamma^7+\frac{27}{256}\gamma^8, \end{aligned} \tag{17}$$

# Time-dependent density when $\zeta$ is a constant

In the absence of both bulk viscosity and Chaplygin gas however,

$$\rho = \frac{2}{(\gamma+1)^2 t^2}, \tag{18}$$

which confirms with results(21) and (22). where $\rho \propto t^{-2}$. However large bulk viscosity coefficient gives $b < 0$ and $\rho \propto \zeta/t$. Constant negative energy density is obtained as $\zeta \to \infty$. With increase in time we observe that the last term of eqn.(11) is dominant which implies $Ce^{bt}$. It is indicative of the fact that the energy density is decreasing function of time. Such behavior is dependent on the Hubble expansion parameter as discussed in the next section.

# Hubble and deceleration parameters when $\zeta$ is a constant

Since $H^2 = \frac{\rho}{2}$, we get from eqn.(11) the Hubble parameter as,

$$H = \frac{1}{\sqrt{2}}\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right)^{\frac{1}{2}} \tag{19}$$

and the deceleration parameter as,

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right)$$
$$= -\left\{1 + \frac{\left(-\frac{2A}{t^3} - \frac{E}{t^2} + h + Cbe^{bt}\right)}{\sqrt{2}\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right)^{\frac{3}{2}}}\right\} \tag{20}$$

# Hubble and deceleration parameters when $\zeta$ is a constant

The above parameters are plotted below,

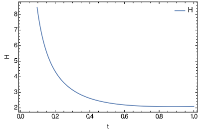


Figure: The Hubble parameter is shown against time for the values of the constants $A = 1.1834, E = 1.1834, h = 4.8083, C = 1.2996, b = 0.1997$.

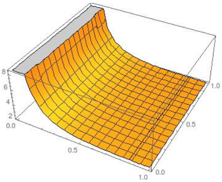


Figure: The Hubble parameter (3D) is shown against time.

# Stability when $\zeta$ is a constant

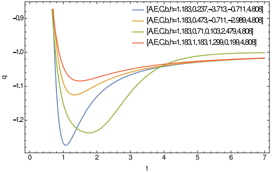


Figure: The deceleration parameter is plotted against time for various values of the constants $A, E, C, b, h$.

We investigate the stability of the system and find the sound speed $C_s^2$ in viscous medium. Using eqns.(4),(5) and (11) we get,

$$C_s^2 = \frac{d\bar{p}}{d\rho} = \gamma + \frac{B\alpha}{(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt})^{\alpha+1}} - \frac{\zeta}{\sqrt{2}(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt})^{\alpha}} \tag{21}$$

# Stability when $\zeta$ is a constant

For a particular choice of parameters $A = 0.788954, \gamma = 0.3, B = 3.4, h = 5.889$, the stability of the medium is graphically represented as below,

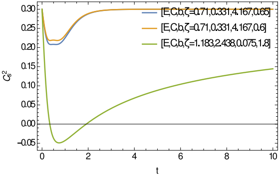


Figure: The stability of the system is shown against time for various values of the constants $E, C, b, \zeta$.

According to stability (21), $C_s^2 > 0$ indicates the stability region. We observe that the fluid is completely stable for $\zeta < 1.65$ and has an unstable region for $\zeta > 1.65$ as is evident from the above figure.

# Time-dependent density when $\zeta$ is linear combination of Hubble parameter'H'

We now study the above case; when $\zeta \neq 0$ then $\zeta = \zeta_0 + \zeta_1 H$ and $H = \alpha_0 \rho^{1/2}$. Hence using eqns.(4), (5) and (6) the above conservation eqn.(8) reduce as,

$$\dot{\rho} + \rho_0 \rho^{3/2} - 2\zeta_0 \rho - \sqrt{2}B = 0. \tag{22}$$

where $\rho_0 = \sqrt{2}(\gamma + 1) - 2\zeta_1 \alpha_0$

**Case-b(i):** $B = 0$

We set $B = 0$, to derive the energy density depend on scale factor (22) as,

$$\rho(a) = \frac{4\zeta_0^2 e^{2\zeta_0 t}}{(\rho_0 e^{\zeta_0 t} + c_1)^2}, \tag{23}$$

where $c_1$ is a constant.

# Time-dependent density when $\zeta$ is linear combination of Hubble parameter'H'

**Case-b(i):** $B \neq 0$ We now obtain the time dependent energy density and solve eqn. (22) using the ansatz,

$$\rho = \frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}, \quad (24)$$

where A, E, h, C and b are arbitrary constants, whose values are derived from eqns.(22) and (24) as,

$$h = \sqrt{2}B, \quad (25)$$

$$A = \frac{4}{{\rho_0}^2} = \frac{4}{\sqrt{2}(\gamma + 1) - 2\zeta_1{\alpha_0}^2} \quad (26)$$

$$E = \frac{4\zeta_0}{{\rho_0}^2} = \frac{4\zeta_0}{\sqrt{2}(\gamma + 1) - 2\zeta_1{\alpha_0}^2} \quad (27)$$

# Time-dependent density when $\zeta$ is linear combination of Hubble parameter'H'

$$C = \frac{{\rho_0}^2}{12}[\frac{32{\zeta_0}^2}{{\rho_0}^3} - \frac{3}{64}\frac{{\rho_0}^4}{{\zeta_0}^2}], \tag{28}$$

$$b = \frac{2}{3}\zeta_0[\frac{2}{9}\zeta_0\rho_0(\sqrt{2}B\rho_0 - 4{\zeta_0}^3) + \frac{9\sqrt{3}}{256}(8\rho_0 - 7\sqrt{3}) + \frac{1}{9}{\zeta_0}^4 + o(\gamma^n)] (8\rho_0)(\frac{16}{81}{\zeta_0}^4 - \frac{1}{128}\frac{{\rho_0}^7}{27})^{-1}, \tag{29}$$

where,

$$O(\gamma^n) = \frac{189}{64}\gamma^2 + \frac{189}{32}\gamma^3 + \frac{945}{128}\gamma^4 + \frac{189}{32}\gamma^5 + \frac{189}{64}\gamma^6 + \frac{27}{32}\gamma^7 + \frac{27}{256}\gamma^8 \tag{30}$$

# Hubble and deceleration parameters when $\zeta$ is linear combination of Hubble parameter'H'

The corresponding Hubble parameters are shown in Figures below,

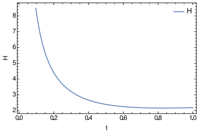


Figure: The Hubble parameter is shown against time for the values of the constants $A = 1.2096, E = 1.2096, h = 4.8083, C = 1.32517, b = 0.5625$.

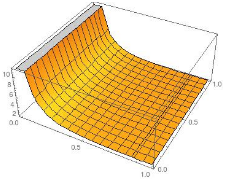


Figure: The Hubble parameter (3D) is shown against time.

# Stability criterion when $\zeta$ is linear combination of Hubble parameter'H'

The deceleration parameter is shown below. We observe the minor changes in both these parameters as compared to our earlier case.

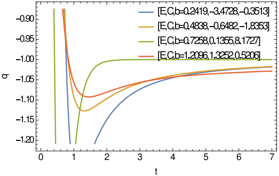


Figure: The deceleration parameter is plotted against time for various values of the constants $E, C, b$ with $A = 1.2096, h = 4.8038$.

For a particular choice of parameters $A = 1.2096, \gamma = 0.3, B = 3.4, h = 4.8083$, the stability of the medium is graphically represented in figure below. According to stability criteria, $C_s^2 > 0$ indicates the stability region.

# Stability criterion when $\zeta$ is linear combination of Hubble parameter'H'

We observe that the fluid is completely stable for $\zeta < 1.62$ and has an unstable region for $\zeta > 1.62$ as is evident from the figure below,

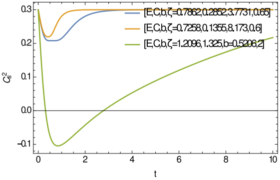


Figure: The stability of the system is shown against time for various values of the constants $E, C, b, \zeta$.

# Conclusion

In this work we studied the FRW bulk viscous cosmology with modified Chaplygin gas as the matter contained in $(2+1)$-dimensions. We obtain the time-dependent energy density for the special case of flat space using a particular ansatz. We have found time-dependent energy density and extracted Hubble expansion and deceleration parameters too. We also studied the stability theory. It confirms from our study that at the late time theory speed of sound has constant real value.

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**“Creativity is intelligence having fun.”...... Albert Einstein.**


Thank you . . .