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A fine-tuned interpretation of the charged lepton mass hierarchy in a microscopic cosmological model

Vo Van Thuan

Duy Tan University (DTU)
3 Quang Trung street, Hai Chau district, Danang, Vietnam

Vietnam Atomic Energy Institute (VINATOM-Hanoi)
Email: vvthuan@vinatom.gov.vn

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Contents

1- Time-Space Symmetry (1): Problem

- **Problems:** ***The origin of the Mass Hierarchy of elementary fermions?***
- **Fermion masses are induced in interaction of genetic particles with Higgs field. All Yukawa couplings are determined independently.**

→ Interpretation of mass hierarchy within the Standard Model (SM) or even in moderate extension of SM are mostly phenomenological with qualitative predictions (e.g. models with flavor democracy, quark-lepton correspondences etc.).

1/ Quark mass hierarchy is parametrized by CKM mixing matrices being in consistency with QCD within SM;

2/ Neutrino mass hierarchy and their masses are parametrized by MNSP mixing matrices, being solved by extension of SM or by Beyond SM approaches;

3/ Mass hierarchy of charged leptons is as large as of both up-type and down-type quarks. Probably, this is a Beyond SM problem !

→ Exception: an excellent prediction by the Koide empirical formula, the origin of which is interpreted by Sumino in correlation with vacuum expectations of heavier family gauge bosons at 10E2-10E3 TeV.

1- Time-Space Symmetry (2): Motivation

- **Objectives:** ***To solve the mass hierarchy problem of charged leptons and to predict tauon mass with high accuracy.***
- **Motivations of the present research: Looking for an alternative Beyond SM ≡ Higher-dimensional space-time approach.**

1/ Searching *for a link between:* GR and QM,

→ To formulate a “microscopic” cosmological model (MicroCoM) to describe masses and mass hierarchy of leptons.

2/ Searching for a link between:

The number of dimensions = The number of lepton generations.

→ To add time-like EDs up to 3D-time: Time-Space Symmetry={3T-3X}:

→ 3 orders of time-like curvatures induce masses of 03 charged leptons (electron, muon, tauon).

→ To predict the mass ratios and to calculate the absolute masses of charged leptons.

1- Time-Space Symmetry (3): References

- **Phenomenological interpretations within SM or by moderate extension of SM.** See, e.g. reviews:

[1] H. Fritzsch and Z.Z.Xing, Phys.Rev.D61(2000)073016.

[2] W.G.Hollik, U.J.S.Salazar, Nucl.Phys.B892(2015)364.

- **Koide empirical formula (an excellent prediction of tauon mass):**

[3] Y.Koide, Phys.Lett.B 120(1983)161.

[4] Y. Sumino, Phys.Lett.B671(2009)477 **(Effective field theory interpretation of Koide formula by family gauge symmetry U(3) broken at $10^3 - 10^3$TeV).**

- **Beyond SM- an alternative: ED geometrical dynamics** - Modern Kaluza-Klein Theories: **5D Space-Time-Matter (STM) theory is applied for QM**:

[5] P.S. Wesson, Phys.Lett.B 701(2011)379; Phys.Lett.B 706(2011)1;

[6] Phys.Lett.B 722(2013)1; IJMPD 24(2015)1530001.

- **Following induced matter approach we proposed a semi-phenomenological model with {3T-3X} Time-Space Symmetry (TSS):**

Bi-cylindrical GR Equation → Formulation of the Microscopic Cosmology in 3T sub-space → Charged lepton mass hierarchy → Tauon Mass:

[7] Thuan Vo Van, Foundations of Physics 47(2017)1559.

[8] Vo Van Thuan, arXiv:1711.08346 [physics.gen-ph] to be published.

1- Time-Space Symmetry (4)- Cylindrical Geometry

- In an ideal ***6D flat time-space*** $\{t_1, t_2, t_3 | x_1, x_2, x_3\}$ considering orthonormal sub-spaces 3D-time and 3D-space:

$$\boldsymbol{dS^2 = dt_k^2 - dx_l^2} \text{ ; summation: } k, l = 1 \div 3. \qquad (1.1)$$

- Our physics works on its ***symmetrical "light-cone"***:

$$\boldsymbol{d\vec{k}^2 = d\vec{l}^2} \quad \text{or} \quad dt_k{}^2 = dx_l{}^2\text{; summation: } k, l = 1 \div 3 \qquad (1.2)$$

Natural units ($\hbar = c = 1$) used unless it needs an explicit quantum dimension.

→ It is equivalent to a 6D-vacuum with: $\boldsymbol{dS}$=0.

- **Introducing a 6D isotropic plane wave equation**:

$$\frac{\partial^2 \psi_0}{\partial t_k^2} = \frac{\partial^2 \psi_0}{\partial x_l^2} \quad ; \qquad (1.3)$$

- Where $\boldsymbol{\psi}_0(\boldsymbol{t_k}, \boldsymbol{x_l})$ is a harmonic correlation of dt and dx, containing only linear variables $\{\boldsymbol{t_k}, \boldsymbol{x_l}\}$, serving a primitive source of quantum fluctuations in space-time. All chaos of displacements dt and dx can form averaged time-like and space-like global potentials $\boldsymbol{V_T}$ and $\boldsymbol{V_X}$.

1- Time-Space Symmetry (5)- Bi-Cylindrical Geometry

- Suggesting that the global potentials, originally, accelerating linear space-time into curved time-space which describes 3D spinning $\vec{\tau}$ and $\vec{s}$ in symmetrical orthonormal subspaces of 3D-time and 3D-space.

- **For a kinetic state** (curved rotation + linear translation): $\{t_{3,} x_3\}$ are accepted as longitudinal central axes of a symmetrical **bi-cylindrical geometry** $(\boldsymbol{\psi},\boldsymbol{\varphi},t_k|\boldsymbol{\psi},\boldsymbol{\varphi},x_l)$;

The curved coordinates for 3D-space: $\{x_j\} \equiv \{x_1, x_2, z\}$ with $\boldsymbol{dz^2 = dx_n}^2 + \boldsymbol{dx_3}^2$;

Similarly, for 3D-time there are $\{t_i\} \equiv \{t_1, t_2, t\}$ with $\boldsymbol{dt^2 = dt_0}^2 + \boldsymbol{dt_3}^2$.

→ t_0 and x_n are local affine parameters in 3D-time and 3D-space, respectively.

EDs turn into dynamical functions of other space-time dimensions:

$\boldsymbol{\psi = \psi(t_0, t_3, x_n, x_l)}$ and $\boldsymbol{\varphi = \Omega t - k_j x_j = \Omega_0 t_0 + \Omega_3\, t_3 - k_n x_n - k_l x_l}$; (1.4)

Here $\psi = \psi$(T).ψ (X) is variable-separable and φ is linear dependent.

→ Possible to express them in a bi-cylindrical geometry.

1- Time-Space Symmetry (6): Bi-Cylindrical Metrics

- In observation of an individual lepton (τ_n , $s_n = \pm 1/2$), due to interaction of a Higgs-like potential V_T the time-space symmetry being spontaneously broken for forming energy-momentum ($ds_0 \gg d\sigma_{spin} \gg d\sigma_{PNC} \gg ds_{CPV}$) leads to an **Asymmetrical Bi-cylindrical Geometry**:

$$d\Sigma_A^2 \equiv dS^2 + d\sigma^2 = ({ds_0}^2 + {ds_{CPV}}^2) - ({d\sigma_{spin}}^2 + {d\sigma_{PNC}}^2) = dt^2 - dz^2 =$$

$$= [d\psi(t_0, t_3)^2 + \psi(t_0, t_3)^2 d\varphi(t_0, t_3)^2 + {dt_3}^2] -$$
$$-[d\psi(x_n, x_3)^2 + \psi(x_n, x_3)^2 d\varphi(x_n, x_3)^2 + {dx_3}^2] . \quad \textbf{(1.5)}$$

- **The Asymmetrical Geometry (5) for charged leptons :**

$${dS_e}^2 \approx {ds_0}^2 - {d\sigma_{Spin}}^2 = dt^2 - dz^2 \equiv dt^2 - {dx_j}^2 ; \quad \textbf{(1.6)}$$

→ It resembles the special relativity (SR), however, here **coordinates {t,z} are curved.**

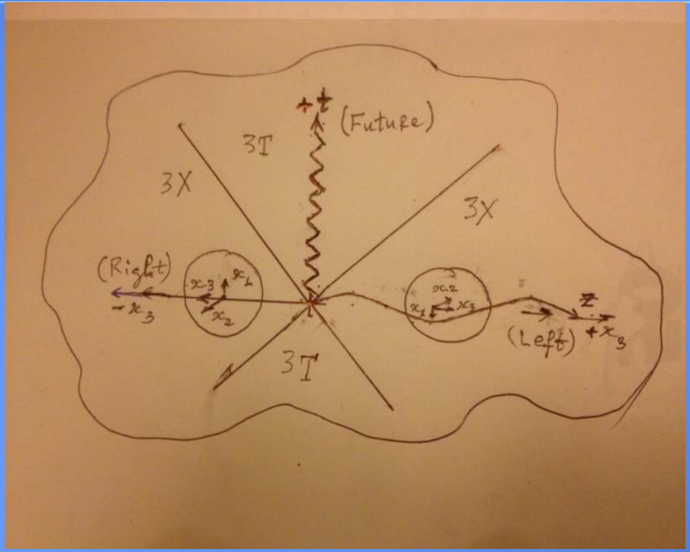

Fig.1.1. **Asymmetrical bi-cylindrical geometry**:

-Time-like curvature ds_0: odd term, being strong and almost absolute;

-Space-like curvature $d\sigma_{spin}$: even term being not absolute (quasi-curvature)

1- Time-Space Symmetry (7): Breaking Symmetry
Phenomenological assumptions

dS and $d\sigma$ are time-like and space-like intervals introduced for compensating the curvatures to maintain a conservation of linear translation (CLT) in a relation to (1) and (2). From a semi-phenomenological view:

$$dS^2 = ds_{odd}{}^2 - ds_{even}{}^2 = ds_0{}^2 - ds_{CPV}{}^2$$
$$d\sigma^2 = d\sigma_{odd}{}^2 - d\sigma_{even}{}^2 = d\sigma_{PNC}{}^2 - d\sigma_{Spin}{}^2.$$

Table 1. Semi-phenomenological based Data for curved geometries of leptons:

Dynamics source	Higgs-like potential	Weak interaction	CPV potential	Spatial spinning
Corresponding Interval	ds_{odd}	$d\sigma_{odd}$	ds_{even}	$d\sigma_{even}$
For heavy leptons (e, μ, τ)	Major	Weak	Super weak	Minor
Corresponding Rotational character	h_τ Iso-Helicity	h_s Helicity	τ *(time-like)* Pseudo-spin	s *(space-like)* Spin

The Asymmetrical Geometry (5) for charged leptons :

$$dS_e{}^2 \approx ds_0{}^2 - d\sigma_{Spin}{}^2 = dt^2 - dz^2 \equiv dt^2 - dx_j{}^2; \qquad (1.6)$$

It resembles the special relativity, however, here **coordinates {t,z} are curved.**

2- A duality of higher-dimensional gravitational equation (1)

- Applying **Geometry (1.6) with signatures {- - - +++} of bi-cylindrical curvatures**, **the gravitational equation in vacuum** (T_i^m=0) reads:

$$R_i^m - \frac{1}{2}\delta_i^m R = 0; \qquad \textbf{(2.1)}$$

- In an apparent vacuum $\Lambda = 0$, Eq. (2.1) leads to {3T,3X}-**Ricci vacuum equation**:

$$R_\alpha^\beta(T) + R_\gamma^\sigma(X) = 0. \qquad \textbf{(2.2)}$$

Where Ricci tensors with $\alpha, \beta \in 3D$-time = 3T and ones with $\gamma, \sigma \in 3D$-space = 3X.

Recalling $\psi = \psi(y)$ and $\varphi = \varphi(y)$ are functional, where: $y \equiv \{t, z\} \in \{t, x_j\} \equiv \{t_0, t_3, x_n, x_l\} \in \{t_i, x_j\}$; $y_3 \equiv \{t_3, x_3\}$ being implicitly embedded in 3D-time: $t_3 \in \{t_k\}$ and in 3D-space: $x_3 \in \{x_l\}$, respectively.

→ Prior assuming the **Hubble law of the cosmological expansion is applied to the Bi-Cylindrical Geometry (1.6)** of microscopic space-time:

$$\frac{\partial \psi}{\partial y} = v_y = H_y \psi \quad \text{and therefore} \quad \left[\frac{\partial y}{\partial \psi}\right] = \frac{1}{H_y \psi}; \qquad \textbf{(2.3)}$$

Where H_y is the "micro-Hubble constant", then the expansion rate v_y increases proportional to the "micro-scale factor" ψ;

2- A duality of higher-dimensional gravitational equation (2)

- **GR equation (2.2) with only diagonal terms leads to two independent sub-equations**:

$$R_3^3(T) + R_3^3(X) = 0\ ; \qquad \textbf{(2.4)}$$

$$R_\psi^\psi(T) + R_\psi^\psi(X) = 0\ ; \qquad \textbf{(2.5)}$$

- **Sub-equation (2.4)** defines conservation of linear translation (**CLT**) of Eq. **(1.3)**:

$$-\frac{\partial^2\psi}{\partial {t_3}^2} + \frac{\partial^2\psi}{\partial {x_3}^2} = 0. \quad \textbf{(2.6)}$$

which leads to a **Lorentz-like condition** for compensating longitudinal fluctuations:

$$\left({\omega_3}^2 - {k_3}^2\right)\psi = 0. \quad \textbf{(2.7)}$$

- **The sub-equation (2.5):** in a time-space symmetrical representation, accounting Lorentz-like condition (**2.7)** reads:

$$\frac{\partial^2\psi}{\partial t^2} - \left(\frac{\partial\varphi}{\partial t_0}\right)^2 \psi = \frac{\partial^2\psi}{\partial {x_j}^2} - \left(\frac{\partial\varphi}{\partial x_n}\right)^2 \psi, \quad \textbf{(2.8)}$$

Where $y = \{t_i, x_j\}$ as for summation of time-like and space-like variables; due to the 3D-local orthogonality: $\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\psi}{\partial {t_0}^2} + \frac{\partial^2\psi}{\partial {t_3}^2}$ and $\frac{\partial^2\psi}{\partial {x_j}^2} = \frac{\partial^2\psi}{\partial {x_n}^2} + \frac{\partial^2\psi}{\partial {x_l}^2}$. (2.9)

2- A duality of higher-dimensional gravitational equation (3)

- For a homogeneity condition → Eq.**(2.8)** is getting a symmetrical equation of bi-geodesic acceleration of deviation ψ :

$$\frac{\partial^2 \psi}{\partial {t_0}^2} - \left(\frac{\partial \varphi}{\partial t_0}\right)^2 \psi = \frac{\partial^2 \psi}{\partial {x_n}^2} - \left(\frac{\partial \varphi}{\partial x_n}\right)^2 \psi \,. \quad \textbf{(2.10)}$$

Due to **3D local geodesic conditions** in 3D-time and in 3D-space, both sides in (**2.10**) are getting independent:

In 3D-time: $$\frac{\partial^2 \psi}{\partial {t_0}^2} - \left(\frac{\partial \varphi}{\partial t_0}\right)^2 \psi = 0. \quad \textbf{(2.11-T)}$$

In 3D-space: $$\frac{\partial^2 \psi}{\partial {x_n}^2} - \left(\frac{\partial \varphi}{\partial x_n}\right)^2 \psi = 0 \,. \quad \textbf{(2.11-X)}$$

Then it leads to de Sitter-like exponential sub-solutions being able to describe Hubble-like expansion in microscopic 3D-time or 3D-space, correspondingly.

→ Those conditions will be applied in **a so-called "microscopic" cosmological model (MicroCoM) for lepton mass hierarchy**.

→ The separated geodesic conditions in 3D sub-spaces ensure that **due to a symmetry-breaking**, their scales are able to renormalized independently without violation of an invariant formalism. In a result→ **Non-zero mass terms appeared**.

2- A duality of higher-dimensional gravitational equation (4)

- Recall that **Geometry (1.6)** is formulated **due to a Higgs-like potential**, producing **time-like polarization** $V_T P \Rightarrow \mathrm{P}^+$ and **a week space-like PNC:**

$$(V_T \boldsymbol{P})^2 = \left[V_T\left(\frac{\partial\varphi}{\partial t_0^-} + \frac{\partial\varphi}{\partial t_0^+}\right)\right]^2 \psi \equiv [\boldsymbol{f_e}(\chi + \boldsymbol{\phi_0})]^2\psi \Rightarrow (P^+)^2 = \left(\frac{\partial\varphi}{\partial t_0^+}\right)^2 \psi \equiv (\boldsymbol{f_e \phi_0})^2\psi = m_0^2\psi\;; \qquad \textbf{(2.12)}$$

where χ is Higgs field and ϕ_0 is Higgs vacuum; $\boldsymbol{f_e}$ is Higgs-electron interaction coupling .

→ **Transformation from 6D time-space to 4D space-time** is performed.

→ Then Geodesics (**2.8**) turns to a formal 4D Asymmetrical equation:

$$\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial {x_j}^2} = \left[\boldsymbol{\Lambda_T} - \left(\frac{\partial\varphi}{\partial x_n}\right)^2_{even} - \boldsymbol{\Lambda_L}\right]\boldsymbol{\psi}; \qquad \textbf{(2.13)}$$

where : Effective strong potentials $\mathbf{V_T}$ of a time-like "cosmological constant" $\boldsymbol{\Lambda_T}$ with a residue of P-odd component $\boldsymbol{\Lambda_L}$ fulfilled breaking symmetry: $[\boldsymbol{\Lambda_T} - \boldsymbol{\Lambda_L}]\boldsymbol{\psi} \equiv \left[\left(\frac{\partial\varphi}{\partial t_0^+}\right)^2 - \left(\frac{\partial\varphi}{\partial x_n^L}\right)^2\right]\boldsymbol{\psi}$.

- A transformation to imaginary variables $\boldsymbol{y} \rightarrow \boldsymbol{i.y}$ leads (**2.8**) to a wave-like solution. Accordingly, with Lorentz-like condition (2.7), the wave-like solution reads:

$$-\frac{\partial^2\psi}{\partial t^2} + \frac{\partial^2\psi}{\partial {x_j}^2} = \left[\left(\frac{\partial\varphi}{\partial t_0^+}\right)^2 - B_e(\boldsymbol{k_n}.\mu_e)^2_{even} - \left(\frac{\partial\varphi}{\partial x_n^L}\right)^2\right]\boldsymbol{\psi}; \qquad \textbf{(2.14)}$$

where B_e is a calibration factor and μ_e is magnetic dipole moment of electron; its orientation is in correlation with spin vector $\vec{s}$ and being P-even.

2- A duality of higher-dimensional gravitational equation (5)

- **R**e-scaling (**2.14**) with Planck constant and Compton length, then adopting the quantum operators, as a rule for transformation from the superluminal phase frame to the subluminal realistic frame: $\frac{\partial}{\partial t} \rightarrow \boldsymbol{i}.\hbar \frac{\partial}{\partial t} = \widehat{\boldsymbol{E}}$ and $\frac{\partial}{\partial x_j} \rightarrow -\boldsymbol{i}.\hbar \frac{\partial}{\partial x_j} = \widehat{\boldsymbol{p}}_j$ → Equation (**2.14**) leads to **a generalized KGF equation** with a wave-like $\boldsymbol{\psi} \equiv \boldsymbol{\psi_w}$:

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} + \hbar^2 \frac{\partial^2 \psi}{\partial {x_j}^2} - \boldsymbol{m}^2 \boldsymbol{\psi} = \boldsymbol{0} \quad ; \qquad \textbf{(2.15)}$$

Or in momentum representation ($\boldsymbol{\psi} \rightarrow \boldsymbol{\psi} \equiv \boldsymbol{\psi_p}$): $(E^2 - \boldsymbol{p_j}^2)\,\boldsymbol{\psi_p} = \boldsymbol{m}^2 \boldsymbol{\psi_p}$; **(2.16)**

where : $\boldsymbol{m}^2 = \boldsymbol{m}_0^2 - \delta \boldsymbol{m}^2 = \boldsymbol{m}_0^2 - \boldsymbol{m}_S^2 - \boldsymbol{m}_L^2$

- $\boldsymbol{m_0} = \hbar\Omega_0$ is the conventional rest mass, defined by $\boldsymbol{\Lambda}_T$; $\boldsymbol{m_S}$ as a P-even contribution links with an external rotational curvature in 3D-space; $\boldsymbol{m_L} \ll \boldsymbol{m_S}$ is a tiny mass factor generated by $\boldsymbol{\Lambda}_L$, related to a P-odd effect of parity non-conservation (PNC).

- For depolarized fields, applying (**2.11**) and ignoring $\boldsymbol{\Lambda}_L$, i.e. $\boldsymbol{m} \rightarrow \boldsymbol{m_0}$, and $x_j \rightarrow x_l$, Equation (**2.15**) is identical to **the traditional *KGF equation (for spin-zero particles)***:

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} + \hbar^2 \frac{\partial^2 \psi}{\partial {x_l}^2} - \boldsymbol{m_0}^2 \boldsymbol{\psi} = \boldsymbol{0} \quad ; \qquad (2.17)$$

→ Generalized KGF Equation (**2.15**) serves as a **QM motion equation of spinning particle**, in particular, **as the squared Dirac equation of electron**.

→ The curved geometry (1.5) is more general → embedding the flat 4D Minkowski space-time for accommodation of the SM of Quantum Field theories.

2- A duality of higher-dimensional gravitational equation (6)

- Interpretation of QM in 4D space-time proves that the extended space-time can serve for accommodation of QM as well as SM of elementary particles.

- In a duality to solution of Gravitational General Relativity Equation **the 3D-time-like local geodesic solution** (**2.11-T**):

$$\frac{\partial^2\psi}{\partial {t_0}^2} - \left(\frac{\partial\varphi}{\partial t_0}\right)^2 \psi = \frac{\partial^2\psi}{\partial {t_0}^2} - \Lambda_T\psi = 0. \quad \textbf{(2.11-T)}$$

→ in a homogeneity and isotropic condition **leads to Hubble-like expansion in the microscopic time-space** and formulates a **Microscopic Cosmological** Model (**MicroCoM**), **in analogue to the standard model** (SM) of macroscopic cosmology.

- In **MicroCoM** the **curvatures from 1D to 3D are formulated in a time-like micro-volume** which are **evolving toward the future along a linearly imitated longitudinal time** axis *dt*.

→ The **proposed MicroCoM** is able to solve the **mass hierarchy problem of elementary particles,** in particular, of **charged leptons by linking the time-like curvatures with proper masses**.

3- Microscopic cosmological model (MicroCoM) for lepton hierarchy (1)

- In 4D space-time assuming that all leptons, as a material points, are to involve in a common basic time-like cylindrical geodesic evolution with a internal 1D circular curvature of the time-like circle $\boldsymbol{S_1}(\varphi^+)$, where $\varphi^+$is azimuth rotation in the plane $\{t_1, t_2\}$ about t_3 and its sign "+" means a fixed time-like polarization from the past to the future;
- Developing more generalized 3D spherical system, described by **nautical angles** $\{\varphi^+, \theta_T, \gamma_T\}$, where θ_T is a zenith in the plane $\{t_1, t_3\}$ and γ_T is another zenith in the orthogonal plane $\{t_2, t_3\}$.

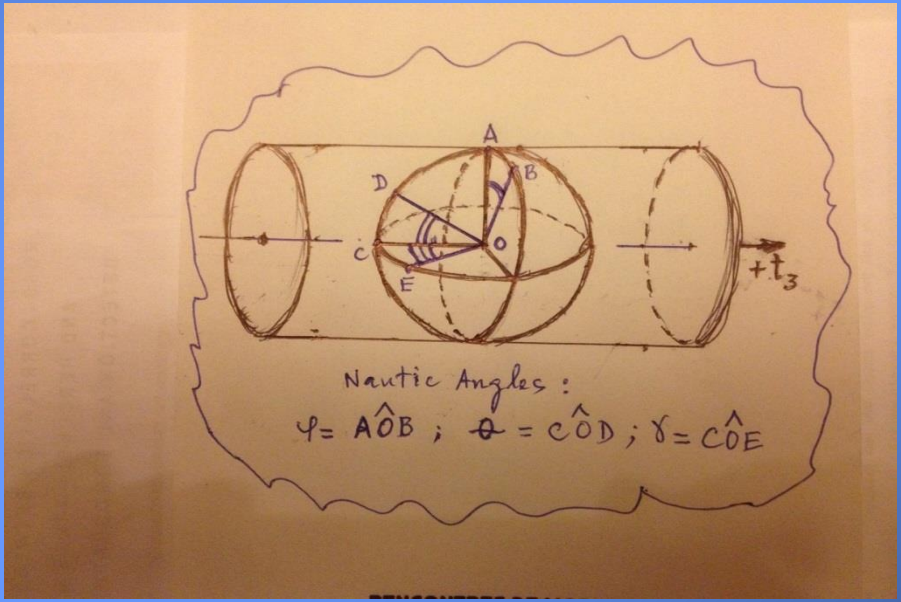

Fig 3.1. Nautical angles to a time-like cylinder.

3- MicroCoM for lepton hierarchy (2)

- For a *n*-hyper spherical surface: its intrinsic curvature C_n is a product of all its principle sectional curvatures (n extrinsic S_1):

$$C_n = C_1 . C_{n-1} = \psi^{-1} . C_{n-1} = \psi^{-n} \quad ; \quad n = 1,3 \quad ; \quad (3.1)$$

→ In according to general relativity, the energy density ρ_n correlates with its curvature and the density ρ_1 of lightest lepton as:

$$\rho_n = \frac{\epsilon_0}{\psi^n} = \frac{\epsilon_0}{\psi} \frac{1}{\psi^{n-1}} = \rho_1 \frac{1}{\psi^{n-1}} \quad ; \qquad (3.2)$$

Where the factor ϵ_0 is assumed a universal lepton energy factor

(universal, because all 3 generations are involved in cylindrical condition and having the same lepton energy factor ϵ_0).

3- MicroCoM for lepton hierarchy (3)

- Electron oscillating on a fixed line-segment of the time-like amplitude $\boldsymbol{\Phi}$, formulating 1D proper (or co-moving) "volume": $\boldsymbol{V_1}(\varphi^+) = \boldsymbol{\Phi} = \boldsymbol{\psi T}$;

  where $\boldsymbol{T}$ is the 1D time-like Lagrange radius.

- For instance, $\boldsymbol{\Phi}$ plays a role of the time-like microscopic Hubble radius and the wave function $\boldsymbol{\psi}$ plays a role of the time-like scale factor.

- The mass of electron defined by 1D Lagrange "volume" will be:

$$\boldsymbol{m_1} = \boldsymbol{\rho_1 V_1} = \boldsymbol{\rho_1 \Phi} = \frac{\boldsymbol{\epsilon_0}}{\boldsymbol{\psi}} \boldsymbol{\psi T} = \boldsymbol{\epsilon_0 T} = \boldsymbol{\epsilon_0 W_1}; \quad \textbf{(3.3)}$$

where $\boldsymbol{W_1}$ is the dimensionless Lagrange volume of electron.

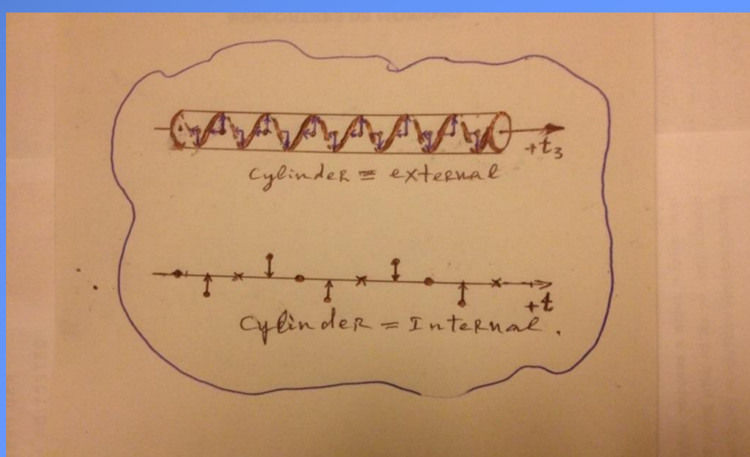

Fig 3.2. Linearization of time axis of electron

For muon and tauon except the basic time-like cylindrical curved evolution φ^+, it needs to add ED curvatures made by evolution in simplest configurations of hyper-spherical "surfaces":

i/ $\boldsymbol{S_1}\ (\theta_T)$ *and* $\boldsymbol{S_1}\ (\gamma_T)$ or ii/ $\boldsymbol{S_2}(\theta_T, \gamma_T)$.

Those curvatures are seen with fixed $\boldsymbol{\Phi}$ from the cylindrical basis.

3- MicroCoM for lepton hierarchy (4)

- The "co-moving volumes" $V_n(\Phi)$ with fixed $S_{n-1}(\Phi)$ are calculated as:

$$V_n(\Phi) = \int_0^{\Phi} S_{n-1}(v)dv = S_{n-1}(\Phi)\int_0^{\Phi} dv = S_{n-1}\Phi = V_1 S_{n-1} \qquad \textbf{(3.4)}$$

(Φ is" fixed" from the 4D-spacetime observation due to not being able to see the additional curvatures, instead of this, observing only their flat footprints at the same maximal level Φ).

- For homogeneity condition the simplest "2D-rotational co-moving volume" is:

$$V_2(\varphi^+, "\theta_T + \gamma_T") \equiv V_2(\varphi^+, \theta_T) + V_2(\varphi^+, \gamma_T) =$$
$$= 2.V_2 = V_1(\varphi^+)[S_1(\theta_T) + S_1(\gamma_T)] = \Phi.2S_1 = 4\pi\Phi^2$$

- Accordingly, the lepton mass of 2D time-like curved particle (muon) is:

$$m_2 = \rho_2 V_2 = \rho_1 \frac{1}{\psi}\Phi.2S_1 = \frac{\epsilon_0}{\psi^2}4\pi\Phi^2 = \epsilon_0 4\pi T^2 = \epsilon_0 W_2; \qquad \textbf{(3.5)}$$

- The next simplest 3D-rotational co-moving volume is:

$$V_3(\varphi^+, "\theta_T \otimes \gamma_T") = V_1(\varphi^+)S_2(\theta_T, \gamma_T) = \Phi.S_2 = 4\pi\Phi^3$$

- Accordingly, the lepton mass of 3D time-like curved particle (tauon) is:

$$m_3 = \rho_3 V_3 = \rho_1 \frac{1}{\psi^2}\Phi.S_2 = \frac{\epsilon_0}{\psi^3}4\pi\Phi^3 = \epsilon_0 4\pi T^3 = \epsilon_0 W_3; \qquad \textbf{(3.6)}$$

We could use the precise experimental data of electron and muon masses to determine ϵ_0 and T in according to (**3.3**) and (**3.5**) as two free parameters, and then to calculate the tauon mass by (**3.6**), as a prediction.

3- MicroCoM for lepton hierarchy (5)

- Using $\boldsymbol{T = 16.454}$, and the lepton energy factor $\boldsymbol{\epsilon_0 = 31.056\ keV}$ calibrated to experimental values of $\boldsymbol{m_e}$ and $\boldsymbol{m_\mu}$ we can **predict the mass of tauon $\boldsymbol{m_\tau}$** then **to come to mass ratios of all three charged lepton generations**:

$\boldsymbol{m_e : m_\mu : m_\tau = m_1 : m_2 : m_3}$=1:206.8:3402.2=**0.511:105.7:1738.5** (*MeV*); **(3.7)**

→ Let's compare with experimental: C.Patrignani et al., Particle Data Group,Chin.Phys. C40 (2016)

The result (as for the 1rst order of approximation) is resumed in **the Table 2**:

n-Lepton	1-electron	2-muon	3-tau lepton
Density, ρ_n	$\frac{\epsilon_0}{\psi}$	$\frac{\epsilon_0}{\psi^2}$	$\frac{\epsilon_0}{\psi^3}$
Comoving volume, $\boldsymbol{V_n}$	$\boldsymbol{\Phi}$	$4\pi\boldsymbol{\Phi}^2$	$4\pi\boldsymbol{\Phi}^3$
Formulas of mass, $\boldsymbol{m_n}$	$\epsilon_0 T$	$\epsilon_0 4\pi T^2$	$\epsilon_0 4\pi T^3$
Calculated mass ratio $\boldsymbol{T \approx 16.454}$; $\boldsymbol{\epsilon_0 = 31.056\ keV}$	**1**	**206.77**	**3402.18**
Experimental leton mass, $\boldsymbol{m_n}$ (MeV)	0.5109989461(31)	105.6583745(24)	1776.86(12)
Calculated lepton mass, $\boldsymbol{m_n}$ (MeV)	**0.5110***	**105.66***	**1738.51**

) Same experimental values $\boldsymbol{m_e}$ **and $\boldsymbol{m_\mu}$ for calibration.*

- The deviation of prediction from the experimental tau-lepton mass is - 2,16%.

4– 2nd approximation of tauon mass by minor curvatures (1)

- Fine-tuning by contribution from minor curvatures C_k to the major curvature C_n where *k<n* producing lepton mass m_n.

Namely: i/ S_1 is added to S_2 major curvature ; ii/ S_1 and S_2 are added to S_3.

- Electron mass is rewritten in 2nd order approximation as:

$$\boldsymbol{m_1(2) = m_1(T_2) = \varepsilon_2 . T_2 .} \qquad (4.1)$$

- Formula of muon mass is upgraded as:

$$m_2(2) = m_2(T_2)\left[1 + \delta\left(\frac{C_1}{C_2}\right)\right] ; \qquad (4.2)$$

where $m_2(T_2) = \varepsilon_2 . 4\pi T_2^2$; $\delta\left(\frac{a}{b}\right)$ is a symbolized scale of the order of ratio $\frac{a}{b}$.
As C_1 and C_2 are of different dimensions, they are re-normalized by their corresponding co-moving volumes: $\boldsymbol{\delta\left(\frac{C_1}{C_2}\right) \equiv \frac{[V_1 . C_1]}{[2V_2 . C_2]} = \frac{W_1}{W_2}}$, which leads to a ratio of dimensionless Lagrange volumes for comparison.

4– 2nd approximation of tauon mass by minor curvatures (2)

- In general, C_k and C_n of different dimensions are re-normalized by corresponding dimensionless Lagrange volumes as follows:

$$\boldsymbol{m_2(2) = m_2(T_2)\left[1 + \frac{W_1}{W_2}\right] = m_2(T_2)\left[1 + \frac{m_1(T_2)}{m_2(T_2)}\right] =}$$

$$\boldsymbol{= m_2(T_2) + m_1(T_2).} \qquad (4.3)$$

- Tauon mass is corrected up to C_2 as:

$$m_3(2) = m_3(T_2)\left[1 + \delta\left(\frac{C_1}{C_3}\right) + \delta\left(\frac{C_2}{C_3}\right)\right] \qquad (4.4)$$

where in particular: $\delta\left(\frac{C_2}{C_3}\right) \equiv \frac{[V_2.C_2]}{[V_3.C_3]} = \frac{1}{2}\frac{W_2}{W_3}$, which leads to:

$$\boldsymbol{m_3(2) = m_3(T_2)\left[1 + \frac{m_1(T_2)}{m_3(T_2)} + \frac{1}{2}\frac{m_2(T_2)}{m_3(T_2)}\right] =}$$

$$\boldsymbol{= m_3\ (T_2) + m_1(T_2) + \frac{1}{2}m_2(T_2);} \qquad (4.5)$$

where: $m_3(T_2) = \varepsilon_2.\,4\pi T_2^3$.

4– 2nd approximation of tauon mass by minor curvatures (3)

- The factor of $\frac{1}{2}m_2(T_2)$ in Equation (4.5) of $m_3(2)$ implies that because the principal muon mass consists of double V_2 co-moving volume as:

$$m_2(T_2) = W_1\rho_1[S_1(\theta_T) + S_1(\gamma_T)] \sim C_2.[\,V_2(\varphi^+, \theta_T)\; + V_2(\varphi^+, \gamma_T)]$$

→different factors of C_2 contribution for muon and tauon mean that in **Equation (4.3) C_2 refers to muon mass (~ double V_2),** while in **Equation (4.5) C_2 relates to a correction to tauon mass, taking a single V_2** only.

→In the result, both corrected configurations of muon in (4.3) and of tauon in (4.5) **contain equally a structural term $m_1(T_2)$** to meet the requirement that **they are involved in the same basic time-like cylindrical geodesic evolution** like electron.

4– 2nd approximation of tauon mass by minor curvatures (4)

- Two new free parameters T_2 and ε_2 are determined based on experimental electron and muon masses as:

$$T_2 = \frac{1}{4\pi}(R_{21} - 1) = 16.37451965.$$

$$\varepsilon_2 = 31.20695794 \text{ (keV)}$$

where R_{21} is the experimental mass ratio of muon to electron.

- Now Equation (4.5) for calculation of tauon mass in the second approximation leads to: $\boldsymbol{m_3(2) = 1774.82}$ **(MeV).**

→The uncertainty of this theoretical prediction is ignorable, because it depends only on experimental errors of electron and muon masses.

→The calculation in the second order approximation deviates from the experimental tauon mass by 0.11% which is by 18.8 times better than the prediction in the first approximation (2.16%).

5- 3rd approximation by perturbative fine-tuning minor curvatures (1)

- The next **infinite perturbative orders of minor curvatures C_k to the major curvature C_n**.

- Electron mass is modified as:

$$m_1(\infty) = m_1(T_\infty) = \varepsilon_\infty . T_\infty . \qquad (5.1)$$

- Formula of muon mass is upgraded as:

$$m_2(\infty) = m_2(T_\infty)\left\langle 1 + \sum_{q=1}^{\infty}\left[\delta\left(\frac{C_1}{C_2}\right)\right]^q\right\rangle = m_2(T_\infty)\sum_{q=0}^{\infty}\left[\delta\left(\frac{C_1}{C_2}\right)\right]^q; \qquad (5.2)$$

→After re-normalization it leads to:

$$m_2(\infty) = m_2(T_\infty)\sum_{q=0}^{\infty}\left[\frac{m_1(T_\infty)}{m_2(T_\infty)}\right]^q = m_2(T_\infty) + m_1(T_\infty)\frac{\rho_{21}}{\rho_{21}-1}; \qquad (5.3)$$

where $m_2(T_\infty) = \varepsilon_\infty . 4\pi T_\infty^2$.

5- 3rd approximation by perturbative fine-tuning minor curvatures (2)

- The summations converge in infinity to finite quantities as:

$$\sum_{q=0}^{\infty} \frac{1}{{\rho_{ij}}^{q}} = \frac{\rho_{ij}}{\rho_{ij}-1}; \qquad (5.4)$$

where for *i* > *j*: $\rho_{ij} = \frac{m_i(T\infty)}{m_j(T\infty)} > 1$

- Tauon mass is corrected in infinity perturbative orders as:

$$m_3(\infty) = m_3(T\infty) + m_1(T\infty) \sum_{p=0}^{\infty} \left[\delta\left(\frac{C_1}{C_2}\right)\right]^p . \sum_{q=0}^{\infty} \left[\delta\left(\frac{C_1}{C_3}\right)\right]^q +$$

$$+\frac{1}{2} m_2(T\infty) \sum_{q=0}^{\infty} \left[\delta\left(\frac{C_2}{C_3}\right)\right]^q; \qquad (5.5)$$

→which leads to:

$$\boldsymbol{m_3(\infty) = m_3(T\infty) + m_1(T\infty) \frac{\rho_{21}}{\rho_{21}-1} * \frac{\rho_{31}}{\rho_{31}-1} +}$$

$$\boldsymbol{+\frac{1}{2} m_2(T\infty) \frac{2.\rho_{32}}{2.\rho_{32}-1};} \qquad (5.6)$$

where: $m_3(T\infty) = \varepsilon\infty . 4\pi {T\infty}^3$.

5- 3rd approximation by perturbative fine-tuning minor curvatures (3)

- Two new free parameters T_2 and ε_2 are determined based on experimental electron and muon masses as:

$T\infty = \frac{1}{4\pi}.\rho_{21} = 16.37413102,$ {where $\rho_{21} = \mathrm{f}(R_{21})$ is determined from the experimental ratio $R_{21} = \frac{m_2(exp)}{m_1(exp)} = \frac{m_2(\infty)}{m_1(\infty)} = \rho_{21} + \frac{\rho_{21}}{\rho_{21}-1}$ }

and: $\varepsilon\infty = 31.20769862$ (keV).

- By Equation (5.6), in the third approximation: $\boldsymbol{m_3(\infty)}$ =**1776.40 (MeV).**

→This theoretical prediction has also ignorable uncertainty due to high precision of the experimental electron and muon masses.

- The fine-tuning approximation in infinite perturbation is 83.4 times better than the prediction in the first approximation.

- In accordance with the notion of the curved 3D-time, it is noticed that all ratios ρ_{ij} are enough large which make all summations $\sum_{q=0}^{\infty}\frac{1}{{\rho_{ij}}^q}$

fast converged at powers of a perturbative order not higher than the major curvature order in each formula, i.e. $\boldsymbol{q \le n \le 3}$.

6- Discussion (1)

- **Similar to STM theory (Wesson et al.), our time-space symmetrical (TSS) model shows that 4D-Quantum Mechanics originates from the Higher-dimensional General Relativity:**

TSS geometrical dynamical approach clarifies QM phenomena (meaning of quantum operators, derivation of KGF equation, Heisenberg inequalities, wave-particle duality, origin of Bohm quantum potential, Schrodinger Zitterbewegung)…

→ The extended space-time can accommodate the 4D-SM of QFT.

→ also serves a basis for a Microscopic Cosmological model: Hubble expansion mechanism is applied in microscopic 3D-time subspace that leads to different time-like configurations with hyper-spherical curvatures.

→ Applying the model with a maximal time-like dimension (3D) :

By extending cylindrical curvature to additional 2D and 3D time-like hyper-spherical configurations:

- ***mass ratios of charged leptons*** **are estimated satisfactory.**

6- Discussion (2)

- The only quantitative prediction of tauon mass has been achieved by Koide empirical formula based on electron and muon masses:

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$$

→ which leads to the quantity $\boldsymbol{m_\tau(Koide) = 1776.97}$ **MeV** being in an excellent agreement **within 1.σ** with **experimental tauon mass**

$\boldsymbol{m_\tau(exp) = 1776.86 \pm 0.12}$ MeV.

- Some geometrical interpretation of Koide formula was proposed by Kocik (arXiv: 1201.2067v1[physics.gen-ph]) where mass correlations are expressed through Descartes-like circles or with their corresponding squared curvatures.

→However, **no more physics could be developed after this point.**

- Sumino assumed the family gauge symmetry U(3) with new gauge bosons at 10E2-10E3 TeV scale to maintain the Koide formula, which due to breaking leads to the SM as an effective field (EFT).

→ The problem: **it seems to require higher symmetries at very high energies, which takes time for the next accelerator generation.**

6- Discussion (3)

- In opposite, our TSS based **MicroCoM demonstrates an explicit physical interpretation**, which serves a solution to the problem of charged **lepton mass hierarchy :**
- **the 3D of time-like sub-space is a constraint of the number of lepton generations (exacting number 3);**
- **the basic common time-like cylindrical evolution ensures the causality by one-directional evolution (toward the future) and together with two universal free parameters (T_∞ and ε_∞) explains why three lepton generations have similar properties.**
- A **theoretical calculation by TSS-MicroCoM in perturbative approximation leads to prediction :**

$$m_3(\infty) = 1776.40 \text{ MeV};$$

→ which is **a fairly passable consistency with experimental tauon mass** : $m_\tau(exp) = 1776.86 \pm 0.12$ MeV.

→ From another perspective, as the deviation of calculation is still **3.83σ,** It needs **further research** for any new hyper-fine adjustment of the present theoretical calculation.

The Literature Pagoda in Hanoi

Thank You for Attention!

Appendix 1: Why Bi-Cylindrical Geometry ?

- **The Bi-cylindrical Formalism is implemented in the following steps**:
- Formulation of bi-cylindrical geometry of {3T-3X} time-space symmetry where the two 3D sub-spaces are orthonormal to each other.
- ***Vacuum solutions of general relativity equation*** in such geometry
- Transformation of the bi-cylindrical variables in to the functions $\psi = \psi(y)$ and $\varphi = \varphi(y)$ of $y \equiv \{t_0, t_3, x_n, x_3\}$ in {3T-3X} time-space symmetrical geometry.

→ ***Most convenient functions are exponential for imitation of both Hubble-like expansion and quantum waves***. Those functions are naturally separable for their variables.

- **A Higgs-like interaction for violating the time-space symmetry**:
- A Lorentz-like condition is introduced for cancelation of all longitudinal fluctuations, which conserves the linear translational equation (CLT) in transformation from a higher dimensional (6D-) geometry to a lower (4D-) realistic geometry.
- Accordingly, the separated geodesic conditions in 3D sub-spaces ensure that **due to a symmetry-breaking**, their scales are able to renormalized independently following an invariant formalism.

→ ***Non-zero mass terms appeared***.

Appendix 2 - Why Dual Solution ?

- **Duality of the solution of 6D-Bi-Cylindrical GR Equation :**
- Bi-cylindrical geodesic equation **(2.13);**
- Wave-like solution **(2.14):**
- → **Dual sub-solutions describe the same physical substance**.

- **Serving for Quantum Mechanical Interpretations:**
- From Wave-like Equation → a generalized QM equation is derived: KGF;
- From separated 3D-local geodesic conditions (in 3T and 3X):

  Heisenberg inequalities are derived.
- Qualitative explanation of QM phenomena: i/ Physical meaning of the QM energy-momentum operators; ii/ Wave-particle duality; iii/ Bohm quantum potential; iv/ Schrodinger ZBW (*Zitterbewegung*).

- **Then, for Formation of a microscopic cosmological model** with a cylindrical basis from 3D-local geodesic equation in 3T → the Hubble-like expansion is in homogeneous and isotropic conditions.

Appendix-3: *Calculation of curvature tensors*

- **Christoffel symbols**: by applying (9) following are found valid:

$$\Gamma^{\psi}_{\varphi\varphi} = -\frac{g^{\psi\psi}}{2}\frac{\partial g_{\varphi\varphi}}{\partial\psi} = -\frac{1}{H_y}\frac{\partial\psi}{\partial y}\ ; \quad \Gamma^{\varphi}_{\psi\varphi} = \Gamma^{\varphi}_{\varphi\psi} = \frac{g^{\varphi\varphi}}{2}\frac{\partial g_{\varphi\varphi}}{\partial\psi} = \frac{1}{\psi^2 H_y}\frac{\partial\psi}{\partial y}$$

$$\Gamma^{3}_{\varphi\varphi} = -\frac{g^{33}}{2}\frac{\partial g_{\varphi\varphi}}{\partial y_3} = -\frac{1}{2}\frac{\partial(\psi^2)}{\partial y_3} = -\psi\frac{\partial\psi}{\partial y_3}\ ; \quad \Gamma^{\varphi}_{3\varphi} = \Gamma^{\varphi}_{\varphi 3} = \frac{g^{\varphi\varphi}}{2}\frac{\partial g_{\varphi\varphi}}{\partial y_3} = \frac{1}{2\psi^2}\frac{\partial(\psi^2)}{\partial y_3} = \frac{1}{\psi}\frac{\partial\psi}{\partial y_3} \ .$$

- **Ricci tensors** for the bi-3D cylindrical geometry:

$$R_{\psi\psi} = -\frac{\partial\Gamma^{\varphi}_{\varphi\psi}}{\partial y}\left[\frac{\partial y}{\partial\psi}\right] - \Gamma^{\varphi}_{\psi\varphi}\Gamma^{\varphi}_{\psi\varphi} = -\frac{1}{\psi^3 {H_y}^2}\frac{\partial^2\psi}{\partial y^2} + \frac{1}{\psi^4 {H_y}^2}\left(\frac{\partial\psi}{\partial y}\right)^2 ;$$

$$R_{33} = -\frac{\partial\Gamma^{\varphi}_{\varphi 3}}{\partial y_3} - \Gamma^{\varphi}_{3\varphi}\Gamma^{\varphi}_{3\varphi} = -\frac{1}{\psi}\frac{\partial^2\psi}{\partial {y_3}^2} ;$$

$$R_{\varphi\varphi} = \frac{\partial\Gamma^{\psi}_{\varphi\varphi}}{\partial y}\left[\frac{\partial y}{\partial\psi}\right] + \frac{\partial\Gamma^{3}_{\varphi\varphi}}{\partial y_3} - \Gamma^{\varphi}_{\varphi\psi}\Gamma^{\psi}_{\varphi\varphi} - \Gamma^{\varphi}_{\varphi 3}\Gamma^{3}_{\varphi\varphi} = -\frac{1}{\psi {H_y}^2}\frac{\partial^2\psi}{\partial y^2} + \frac{1}{\psi^2 {H_y}^2}\left(\frac{\partial\psi}{\partial y}\right)^2 - \psi\frac{\partial^2\psi}{\partial {y_3}^2} \ .$$

Obviously, $\boldsymbol{R = g^{im}R_{im} = \delta^m_i R^m_i = -\frac{2}{\psi^3 {H_y}^2}\frac{\partial^2\psi}{\partial y^2} + \frac{2}{\psi^4 {H_y}^2}\left(\frac{\partial\psi}{\partial y}\right)^2 - \frac{2}{\psi}\frac{\partial^2\psi}{\partial {y_3}^2}}$.

Its space-time representation reads: $R = \delta^{\sigma}_{\gamma}R^{\sigma}_{\gamma}(X) - \delta^{\beta}_{\alpha}R^{\beta}_{\alpha}(T)$.

Appendix–4: Outputs from the dual solutions of GR (1)

- **The meaning of quantum energy momentum operators:**

Mathematical transformation from the geodesic equation (**2.13**) with an exponential solution to a wave-like solution (**2.14**) is performed by **transformation of variables: $t \rightarrow -it$ and $x_j \rightarrow ix_j$** in **similar to that of quantum dynamic operators**. This is not only **a mathematical formalism**, but also a significant physical operation, **equivalent to transformation from external to internal investigation**. Indeed, for the phase $\varphi = \Omega t - k_j x_j = const$ in the **internal phase continuum**: the phase velocity is superluminal, i.e. $v_{phase} = \frac{dx_j}{dt} = \frac{\Omega}{k_j} > c$. It is equivalent to converting the role of space ←→ time in the **internal superluminal frame** comparing with the **external subluminal space-time**.

- ***Wave-particle duality:***

Subjecting the same microscopic substance:

i/ the monotonic exponential solution describes motion/evolution of a material point, as a localized particle; while

ii/ the wave-like solution transforming into QM equation (KGF) describes the motion/evolution of same particle, but as a wave-like substance.

Appendix–4: Outputs from the dual solutions of GR (2)

Based on **3D-local geodesic deviation acceleration conditions (2.11-T) and (2.11-X)*, we can understand some important QM phenomena:***

- ***Bohm quantum Potential***:

$$\left(\frac{\partial s}{\partial x_n}\right)^2 = B_e(\hbar.\boldsymbol{k_n}.\mu_e)^2_{even} = \frac{\hbar^2}{\psi}\frac{\partial^2\psi}{\partial {x_n}^2} = -2mQ_B; \qquad (2.11\text{-B})$$

which is ***proportional to Bohm's quantum potential*** Q_B.

- ***Schrödinger's Zitterbewegung***:

  - The existence of ***the spin term*** in Generalized QM Klein-Gordon-Fock equation (2.15) is reminiscent of ***ZBW of free electron***.

→ When we ***describe a linear translation*** of the freely moving particle by Equation (2.17), the ***ZBW term is almost compensated by the geodesic condition (2.11-X)*** except a tiny P-odd term (However the latter is hard to observe).

Appendix-5: Heisenberg Indeterminism (1)

A. Coordinate-momentum inequality:

- The local geodesic condition (2.10) leads to: $\frac{1}{\psi} d\left(\frac{\partial\psi}{\partial x_n}\right).dx_n = d\varphi^2 \geq 0$; (H1)

→ $|\Delta p|.|\Delta x| \geq |\Delta p_n|.|\Delta x_n| > \psi^{-1}\left|d\left(i.\hbar\frac{\partial\psi}{\partial x_n}\right)\right|.|dx_n| = |i.\hbar|.d\varphi^2 \geq 0$; (H2)

Accepting the conditions: i/ Quantization of azimuth: $\varphi = n.2\pi$;

ii/ For Poisson/Gaussian distribution of quantum statistics: $< \varphi >_{min} = 2\pi$ and $d\varphi \approx \sigma_\varphi = \sqrt{2\pi}$ = standard deviation.

→ Then, from (H2): $|\Delta p|.|\Delta x| > 2\pi\,\hbar$. (H3)

***B. Time-energy inequality*:**

Following 3D-time local geodesic condition (2.10) : $\frac{1}{\psi} d\left(\frac{\partial\psi}{\partial t_0}\right).dt_0 = d\varphi^2 \geq 0$; (H4)

→ $|\Delta E|.|\Delta t| \geq |\Delta E_0|.|\Delta t_0| > \psi^{-1}\left|d\left(i.\hbar\frac{\partial\psi}{\partial t_0}\right)\right|.|dt_0| = |i.\hbar|.d\varphi^2 \geq 0$; (H5)

→ With the same conditions (i) and (ii): $|\Delta E|.|\Delta t| > 2\pi\,\hbar$. (H6)

The inequalities (H3) and (H6) show that **the QM indeterminism takes origin from the curvatures of space and time**.

Appendix-5: Heisenberg indeterminism (2)

- For a local geodesic in closed 3D-time:

$$\frac{1}{\psi} d\left(\frac{\partial \psi}{\partial t_0}\right). dt_0 = d\varphi^2 \geq 0 \ ;$$

- Multiplying both sides on the quantum scale unit $i.\hbar$, and turning to finite differentials we get the time-energy indetermination:

$$|\Delta E|.|\Delta t| \geq |\Delta E_0|.|\Delta t_0| > |dE_0|.|dt_0| =$$

$$= \psi^{-1}|d(E_0.\psi)dt_0| \ = \psi^{-1}\left|d\left(i.\hbar\frac{\partial \psi}{\partial t_0}\right)dt_0\right| =$$

$$= |i.\hbar|.d\varphi^2 \geq 0 \ ;$$

where due to involving in the internal curvature $E_0(n) = m_0\ (n) = \frac{A_n}{\psi^n}$

then, in average:

$$\langle|d(E_0.\psi)|\rangle = \langle|\psi d(E_0) + E_0 d(\psi)|\rangle = \left\langle|\psi dE_0| \pm \frac{1}{n}|(\psi dE_0)|\right\rangle = |\psi dE_0|.$$

- Similarly we can get the space-momentum indetermination.

Appendix-6: *A scenario similar to the Standard cosmological model (A hypothesis)*

During the Big-Bang inflation, we suggest the following ***scenario of MicroCoM, similar to the Standard Cosmological model of the Universe***:

The micro-scale factor $\boldsymbol{\psi}$ increases exponentially (time-like Hubble constant $\boldsymbol{H_T} = \sqrt{\Lambda_{\boldsymbol{T}}}$ = 7.764*10^{20} sec^{-1} and the instant of inflation $\Delta t_1 = 1.926 * 10^{-20}$ sec after 1 sec from the Big-Bang).

For the next time-life of the Universe =13.7 *10^9 years, **based on the idea of time-space symmetry**, it is assumed for 3D-time (exactly as in 3D-space): $\boldsymbol{\psi}$ **~** $t^{1/2}$ for radiation dominant era and $\boldsymbol{\psi}$ **~**$t^{2/3}$ for matter dominant era.

In a result, the time-like Lagrange radius $\boldsymbol{T}$ decreases from $\boldsymbol{T}_0 = \frac{\boldsymbol{\Phi}}{\boldsymbol{\psi_0}} = \mathbf{1}$ for Δt_1 then steps up to the present value $\boldsymbol{T} = \frac{\boldsymbol{\Phi}}{\boldsymbol{\psi}} \approx 16.5$.

For leptons born after the inflation era, assuming following anthropic principle (very *qualitatively)* that the Hubble radius of any quantum fluctuations should adapt the contemporary value $\boldsymbol{\Phi}$, while the scale factor $\boldsymbol{\psi}$ being governed by a contemporary chaotic Higgs-like potential in such a way, that is to meet the contemporary time-like Lagrange radius $\boldsymbol{T}$ (for today, $\boldsymbol{T}$ =16.5).