

SUSY before/around Big Bang

(Nonlinear-supersymmetric general relativity theory)

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OUTLINE

1. Motivation
2. Nonlinear-supersymmetric general relativity theory(**NLSUSYGR**)
3. Evolution of **NLSUSYGR** and the vacuum structure
4. Cosmology and low energy particle physics of **NLSUSYGR**
5. Rarita-Schwinger NLSUSYGR
6. Summary

1. Motivation

@ The success of **Two SMs**, i.e. **GR** and **GWS model**.

However, **many unsolved fundamental problems** in SMs: e.g.,

- Unification of two SMs.
- Space-time dimension **four**,
- Three generations of quarks and leptons,
- Tiny Neutrino mass M_ν , proton stability in GUT
- Dark Matter, Dark energy; $\rho_{D.E.} \sim (M_\nu)^4 \Leftrightarrow \Lambda(\text{cosmological term})?$

\Rightarrow **SUGRA!?**, Origin of SUSY breaking ?, ... etc.

@ GR describes geometry of space-time.

However, **unpleasant differences** between **GR** and **SUGRA**:

- **GR** \Leftrightarrow Geometry of Riemann space-time (**Physical**: $[x^\mu]$, $GL(4, \mathbb{R})$)
- **SUGRA** \Leftrightarrow Geometry of superspace (**Mathematical**: $[x^\mu, \theta_\alpha]$, sPoincaré)

\Rightarrow New **SUSY** paradigm on specific **physical space-time!**.

@ As for the **three-generations structure**:

Among all **SO(N) sP**, $[N = 8 \text{ by M. Gell-Mann}]$

SM with just 3 generations emerges from **one** irreducible rep. of **only SO(10) sP**.

• **10 supercharges** $Q^I, (I = 1, 2, \dots, 10)$ are assigned as follows:

$$\underline{10}_{SO(10)} = \underline{5}_{SU(5)} + \underline{5}^*_{SU(5)}$$

$$\underline{5}_{SU(5)} = [\{Q_a (a = 1, 2, 3); \underline{3}^{*c}, \underline{1}^{ew}, (\frac{e}{3}, \frac{e}{3}, \frac{e}{3})\}, \{Q_m (m = 4, 5); \underline{1}^c, \underline{2}^{ew}, (-e, 0)\}] .$$

\Leftrightarrow Supercharge $\underline{5}_{SU(5)}$ has the same quantum numbers as $\underline{5}$ of **SU(5) GUT**.

• **Massless helicity states of gravity multiplet** of **SO(10) sP** with **CPT conjugation** are specified by the helicity $h = (2 - \frac{n}{2})$ and the dimension $\underline{d}_{[n]} = \frac{10!}{n!(10-n)!}$:

$$|h \rangle = Q^n Q^{n-1} \dots Q^2 Q^1 |2 \rangle, \quad Q^n \quad (n = \{a, m, a^*, m^*\}): \text{supercharge}$$

$ h $	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
			$\underline{1}_{[0]}$	$\underline{10}_{[1]}$	$\underline{45}_{[2]}$	$\underline{120}_{[3]}$	$\underline{210}_{[4]}$
$\underline{d}_{[n]}$	$\underline{1}_{[10]}$	$\underline{10}_{[9]}$	$\underline{45}_{[8]}$	$\underline{120}_{[7]}$	$\underline{210}_{[6]}$	$\underline{252}_{[5]}$	$\underline{210}_{[4]}$

@ Spin $\frac{1}{2}$ Dirac survivors after superHiggs ($SU(2)$: L-R symm. case)

$SU(3)$	Q_e	$SU(2) \otimes U(1)$
<u>1</u>	0 -1 -2	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ (E)
<u>3</u>	5/3 2/3 -1/3 -4/3	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} h \\ o \end{pmatrix} \begin{pmatrix} a \\ f \end{pmatrix} \begin{pmatrix} g \\ m \end{pmatrix} \begin{pmatrix} r \\ i \\ n \end{pmatrix}$
<u>6</u>	4/3 1/3 -2/3	$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$
<u>8</u>	0 -1	$\begin{pmatrix} N_1 \\ E_1 \end{pmatrix} \begin{pmatrix} N_2 \\ E_2 \end{pmatrix}$

@ SM Higgs doublet state survives in $h = 0$ state.

- How to construct N=10 SUSY with gravity beyond **No-Go** theorem in **S-matrix** ?
- To circumvent the No-Go theorem degeneracy of space-time is considered.

We show in this talk:

- N=10 SUSY with gravity is obtained by the geometric description of General Relativity principle on **specific unstable *physical* (Riemann) space-time** whose tangent space possesses NLSUSY structure.



A quick review of NLSUSY:

- Take flat space-time specified by x^a for $SO(1,3)$ and ψ_α for $SL(2,C)$.
- Consider one form $\omega^a = dx^a + \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$,
 κ is an **arbitrary** constant with the dimension l^{+2} .

- $\delta\omega^a = 0$ under $\delta x^a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$ and $\delta\psi = \zeta$ with a **global** spinor parameter ζ .

- An invariant action (\sim **invariant volume**) is obtained:

$$S = -\frac{1}{2\kappa^2} \int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA},$$

L_{VA} is **N=1 Volkov-Akulov model of NLSUSY** given by

$$L_{VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2} \left[1 - t^a_a + \frac{1}{2}(t^a_a t^b_b - t^a_b t^b_a) + \dots \right],$$

$$|w_{VA}| = \det w^a_b = \det(\delta_b^a + t^a_b),$$

$$t^a_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \bar{\psi}\gamma^a\partial_b\psi),$$

which is invariant under **N=1 NLSUSY** transformation:

$$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)\partial_a\psi, \quad [\delta_1, \delta_2] = \delta_P.$$

- ψ is **Nambu-Goldstone(NG) fermion** (the coset space coordinate) of $\frac{\text{superPoincare}}{\text{Poincare}}$.

- ψ is quantized **canonically** in compatible with SUSY algebra.

2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

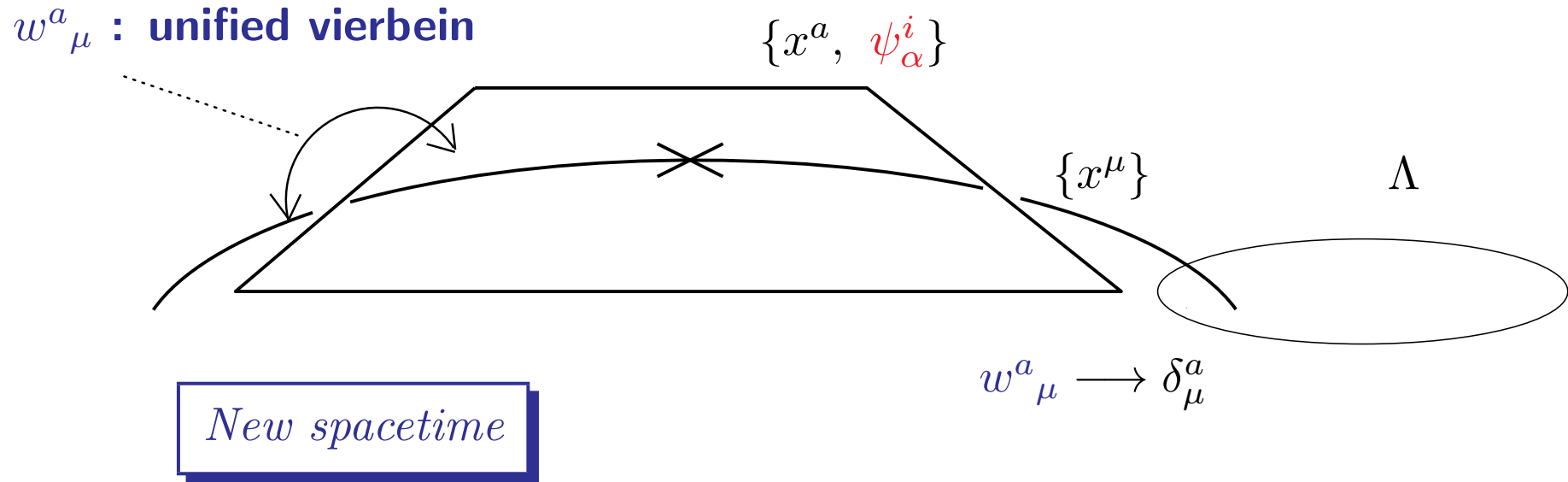
2.1. New Space-time as Ultimate Shape of Nature

We consider **new (unstable) physical space-time** inspired by **nonlinear(NL) SUSY:**

The tangent space of new space-time is specified by **Grassmann coordinates** ψ_α for $SL(2,C)$ besides the ordinary Minkowski coordinates x^a for $SO(1,3)$,

i.e., the coordinate ψ_α of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ turning to the **NLSUSY NG fermion** (called *superon* hereafter) are attached at every curved space-time point besides x^a .

- New (empty) unstable space-time:



(Non-compact groups $SO(1,3)$ and $SL(2,C)$ for space-time symmetry are analogous to compact groups $SO(3)$ and $SU(2)$ for gauge symmetry of 't Hooft-Polyakov monopole, though **$SL(2,C)$ is realized nonlinearly.**)

- Note that $SO(1,3) \cong SL(2,C)$ is crucial for NLSUSYGR scenario.

4 dimensional space-time is singled out.

2.2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

The geometrical arguments of Einstein general relativity (GR) can be extended to new (unstable) space-time.

- Unified vierbein $w^a{}_\mu(x)$ (*ulvierbein*) of new space-time:

$$w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi) \equiv w^a{}_b e^b{}_\mu,$$

$$w^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a \equiv (w^a{}_b e^b{}_\mu)^{-1},$$

$$w^a{}_\mu(x) w^\mu{}_b(x) = \delta^a{}_b$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i} (\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), (I = 1, 2, \dots, N)$$

(Note that Grassmann d.o.f. induces the imaginary part of $w^a{}_\mu(x)$.)

- N -extended NLSUSYGR action of Einstein-Hilbert (EH)-type for new space-time. \implies

N -extended NLSUSYGR action:

(*Phys.Lett.B501,237(2001), B507,260(2001)*)

$$L_{\text{NLSUSYGR}}(w) = -\frac{c^4}{16\pi G} |w| \{ \Omega(w) + \Lambda \}, \quad (1)$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \quad (2)$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i} (\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), \quad (I = 1, 2, \dots, N) \quad (3)$$

- $w^a{}_\mu(x) (= e^a{}_\mu + t^a{}_\mu(\psi))$: **the unified vierbein of new space-time** (*ulvierbein*)
 - $e^a{}_\mu(x)$: **the ordinary vierbein for the local SO(1,3) d.o.f. of GR,**
 - $t^a{}_\mu(\psi(x))$: **the mimic vierbein for the local SL(2,C) d.o.f. composed of the stress-energy-momentum of NG fermion $\psi(x)^I$ (called *superons*),**
 - $\Omega(w)$: **Ricci scalar curvature of new space-time computed in terms of $w^a{}_\mu$,**
 - $s_{\mu\nu} \equiv w^a{}_\mu \eta_{ab} w^b{}_\nu$, $s^{\mu\nu}(x) \equiv w^\mu{}_a(x) \eta^{ab} w^\nu{}_a(x)$: **metric tensors of new space-time.**
 - G : **the Newton gravitational constant.**
 - $\Lambda > 0$: **cosmological constant required**
- by the correct NLSUSY structure of tangent space.**

- **NLSUSYGR scenario fixes the arbitrary constant κ^2 to**

$$\kappa^2 = \left(\frac{c^4 \Lambda}{8\pi G}\right)^{-1},$$

with the dimension $(length)^4 \sim (energy)^{-4}$.

- $\Lambda > 0$ in the action L_{NLSUSYGR} allows **negative dark energy density interpretation of $\frac{\Lambda}{G}$ in the Einstein equation.** → Sec.4.

- **No-go theorem for $N > 8$ SUSY has been circumvented by using NLSUSY, i.e. by the vacuum(flat space) degeneracy.**

- **Note that $SO(1, D - 1) \cong SL(d, C)$, i.e. $\frac{D(D-1)}{2} = 2(d^2 - 1)$ holds for **only $D = 4, d = 2$.****

NLSUSYGR scenario predicts 4 dimensional space-time.

2.3. Symmetries of NLSUSYGR(N-extended action)

- **Space-time symmetries** ($\sim sP$):

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \mathbb{R})] \otimes [\text{local Lorentz}] \quad (4)$$

- **Internal symmetries** for N-extended NLSUSY GR (N-superons ψ^I ($I = 1, 2, \dots, N$)):

$$[\text{global SO}(N)] \otimes [\text{local U}(1)^N] \otimes [\text{chiral}]. \quad (5)$$

For example:

- Invariance under the new NLSUSY transformation;

$$\delta_{\zeta}\psi^I = \frac{1}{\kappa}\zeta^I - i\kappa\bar{\zeta}^J\gamma^{\rho}\psi^J\partial_{\rho}\psi^I, \quad \delta_{\zeta}e^a_{\mu} = i\kappa\bar{\zeta}^J\gamma^{\rho}\psi^J\partial_{[\mu}e^a_{\rho]}. \quad (6)$$

induce **GL(4,R) transformations** on w^a_{μ} and the unified metric $s_{\mu\nu}$

$$\delta_{\zeta}w^a_{\mu} = \xi^{\nu}\partial_{\nu}w^a_{\mu} + \partial_{\mu}\xi^{\nu}w^a_{\nu}, \quad \delta_{\zeta}s_{\mu\nu} = \xi^{\kappa}\partial_{\kappa}s_{\mu\nu} + \partial_{\mu}\xi^{\kappa}s_{\kappa\nu} + \partial_{\nu}\xi^{\kappa}s_{\mu\kappa}, \quad (7)$$

where ζ is a constant spinor parameter, $\partial_{[\rho}e^a_{\mu]} = \partial_{\rho}e^a_{\mu} - \partial_{\mu}e^a_{\rho}$ and $\xi^{\rho} = -i\kappa\bar{\zeta}^I\gamma^{\rho}\psi^I$.

- Commutators of two new NLSUSY transformations (6) on ψ^I and e^a_{μ} close to **GL(4,R)**,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^I = \Xi^{\mu}\partial_{\mu}\psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a_{\mu} = \Xi^{\rho}\partial_{\rho}e^a_{\mu} + e^a_{\rho}\partial_{\mu}\Xi^{\rho}, \quad (8)$$

where $\Xi^{\mu} = 2i\bar{\zeta}_1^I\gamma^{\mu}\zeta_2^I - \xi_1^{\rho}\xi_2^{\sigma}e_a^{\mu}\partial_{[\rho}e^a_{\sigma]}$.
q.e.d.

- New NLSUSY (6) is the square-root of GL(4,R);

$$[\delta_1, \delta_2] = \delta_{\text{GL}(4,\mathbb{R})}, \quad \text{i.e.} \quad \delta \sim \sqrt{\delta_{\text{GL}(4,\mathbb{R})}}.$$

c.f. SUGRA (LSUSY)

$$[\delta_1, \delta_2] = \delta_P + \delta_L + \delta_g$$

- The ordinary local GL(4,R) invariance is manifest by the construction.

- Invariance under new local Lorentz transformation;

$$\delta_L \psi^I = -\frac{i}{2} \epsilon_{ab} \sigma^{ab} \psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^I \gamma_5 \gamma_d \psi^I (\partial_\mu \epsilon_{bc}) \quad (9)$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$.

(9) induce the familiar local Lorentz transformation on $w^a{}_\mu$:

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \quad (10)$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra,
e.g., **the new form** on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}] e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^j \gamma_5 \gamma_d \psi^j (\partial_\mu \beta_{bc}), \quad (11)$$

where $\beta_{ab} = -\beta_{ba}$ is given by $\beta_{ab} = \epsilon_{2ac} \epsilon_1^c{}_b - \epsilon_{2bc} \epsilon_1^c{}_a$.
q.e.d.

3. Evolution of New Space-time:

3.1. Big Collapse of new space-time toward true vacuum

@ $\Lambda > 0$ allows $L_{\text{NLSUSYGR}}(w)$ breaks down spontaneously (**Big Collapse**) to ordinary Riemann space-time (graviton) and NG fermion (superon): $L_{\text{SGM}}(e, \psi)$.

$$L_{\text{NLSUSYGR}}(w) = L_{\text{SGM}}(e, \psi) = -\frac{c^4}{16\pi G} |e| \{ R(e) + |w(\psi^I)| \Lambda + \tilde{T}(e, \psi^I) \}. \quad (12)$$

- $R(e)$: the Ricci scalar curvature of ordinary Riemann space-time
- Λ : the cosmological constant
- $|w(\psi^I)| = \det w^a_b = \det \{ \delta^a_b + t^a_b(\psi^I) \}$: NLSUSY action for superon
- $\tilde{T}(e, \psi^I)$: the gravitational interaction of superon

@ **Big Collapse** may induce the **rapid spacial expansion** of space-time..

@ $L_{\text{SGM}}(e, \psi^I)$ evolves toward the true vacuum by constituting **gravitational composite (massless) eigenstates of broken LSUSY SO(N) sP**, which is the **ignition of the Big Bang SMs scenario**. \implies

- The Noether's theorem gives the conserved supercurrent:

$$S^{I\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}} e_a{}^\mu \gamma^a \psi^I + \dots \quad (13)$$

- The supercurrent couples the graviton and the superon(NG fermion) to the vacuum with the strength $\frac{c^4\Lambda}{8\pi G}$:

$$\langle e_b{}^\nu \psi_\beta{}^J | S_\alpha{}^{I\mu} | 0 \rangle = i\sqrt{\frac{c^4\Lambda}{8\pi G}} \delta^{\mu\nu} \delta^{IJ} (\gamma_b)_{\alpha\beta} \quad (14)$$

with the strength $g_{sv} = \sqrt{\frac{c^4\Lambda}{8\pi G}} = \kappa^{-1}$.

3.2. Linearization of NLSUSY and vacuum of $L_{\text{SGM}}(e, \psi)$

The graviton is the universal attractive force and creates all possible gravitational composites of superons, which are helicity eigenstates of LSUSY sP algebra ($(Q^I)^n \sim (\psi^I + \dots)^n$).

@ The vacuum structure of $L_{\text{SGM}}(e, \psi)$ is obtained by linearizing NLSUSY into LSUSY based on NLSUSY algebra and sP algebra.

- **Linearized broken LSUSY theory**

$L_{\text{brLSUSY}}(\tilde{e}_\mu^a, \psi_\nu(e, \psi), v^a(e, \psi), \lambda(e, \psi), \phi(e, \psi), M(e, \psi), N(e, \psi), \dots)$
is obtained as composites of NG fermions and satisfies:

$$L_{\text{NLSUSYGR}}(w) = L_{\text{SGM}}(e, \psi) = L_{\text{brLSUSY}}(e^a_\mu, \psi_\nu, v^a, \lambda, \phi, M, N, \dots)$$

\iff **NL/L SUSY relation (equivalence)**

- **The phase transition of NLSUSY $L_{\text{SGM}}(e, \psi)$ towards the vacuum achieved by gravitational composite particle states of LSUSY.**

3.3. NL/L SUSY relation (equivalence)

We demonstrate **NL/L SUSY relation for $N=2$ SUSY** in *flat space*:

$$L_{NLSUSYGR}(w) = L_{SGM}(e, \psi) \rightarrow L_{NLSUSY}(\psi) = L_{LSUSY}(v^a(\psi), \phi(\psi), \dots)$$

($N = 2$ **SGM** reduces to $N = 2$ **NLSUSY** of Λ term of **NLSUSYGR**)

@ **N=2, d=2 NLSUSY model:**

$$L_{NLSUSY} = -\frac{1}{2\kappa^2} |w_{NLSUSY}| = -\frac{1}{2\kappa^2} \left[1 + t^a_a + \frac{1}{2}(t^a_a t^b_b - t^a_b t^b_a) + \dots \right], \quad (15)$$

$$|w_{NLSUSY}| = \det w^a_b = \det(\delta^a_b + t^a_b), \quad t^a_b = -i\kappa^2(\bar{\psi}_j \gamma^a \partial_b \psi^j - \bar{\psi}_j \gamma^a \partial_b \psi^j), \quad (j = 1, 2),$$

which is invariant under $N=2$ NLSUSY transformation,

$$\delta_\zeta \psi^j = \frac{1}{\kappa} \zeta^j - i\kappa(\bar{\zeta}_k \gamma^a \psi^k - \bar{\zeta}_k \gamma^a \psi^k) \partial_a \psi^j, \quad (j = 1, 2).$$

N=2, d=2 LSUSY Theory (SUSY QED):

- Helicity states of N=2 vector supermultiplet:

$$\left(\begin{array}{c} +1 \\ +\frac{1}{2}, +\frac{1}{2} \\ 0 \end{array} \right) + [\text{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell **minimal** vector supermultiplet: $(v^a, \lambda^i, A, \phi, D; i=1,2)$ in *WZ gauge*. (A and ϕ are two singlets, 0^+ and 0^- , scalar fields.)

- Helicity states of N=2 scalar supermultiplet:

$$\left(\begin{array}{c} +\frac{1}{2} \\ 0, 0 \\ -\frac{1}{2} \end{array} \right) + [\text{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets: $(\chi, B^i, \nu, F^i; i = 1, 2)$, B^i and F^i are complex.

- The most general $N = 2, d = 2$ SUSYQED action ($m = 0$ case) :

$$L_{N=2\text{SUSYQED}} = L_{V0} + L'_{\Phi0} + L_e + L_{Vf}, \quad (16)$$

$$L_{V0} = -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{\partial} \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D,$$

$$L'_{\Phi0} = \frac{i}{2}\bar{\chi} \not{\partial} \chi + \frac{1}{2}|\partial_a B^i|^2 + \frac{i}{2}\bar{\nu} \not{\partial} \nu + \frac{1}{2}|F^i|^2,$$

$$L_e = e \left\{ i v_a \bar{\chi} \gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi \bar{\chi} \gamma_5 \nu \right. \\ \left. + B^i (\bar{\lambda}^i \chi - \epsilon^{ij} \bar{\lambda}^j \nu) - \frac{1}{2}|B^i|^2 D \right\} + \{h.c.\} + \frac{1}{2}e^2(v_a^2 - A^2 - \phi^2)|B^i|^2,$$

$$L_{Vf} = f \{ A \bar{\lambda}^i \lambda^i + \epsilon^{ij} \phi \bar{\lambda}^i \gamma_5 \lambda^j + (A^2 - \phi^2)D - \epsilon^{ab} A \phi F_{ab} \} \quad (17)$$

- **Note that**

$J = 0$ states in the vector multiplet for $N \geq 2$ SUSY induce Yukawa coupling.

$L_{N=2\text{SUSYQED}}$ is invariant under $N = 2$ **LSUSY** transformation:

- For the **minimal vector off-shell supermultiplet**:

$$\begin{aligned}
 \delta_\zeta v^a &= -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j, \\
 \delta_\zeta\lambda^i &= (D - i\cancel{\partial}A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\cancel{\partial}\phi\zeta^j, \\
 \delta_\zeta A &= \bar{\zeta}^i\lambda^i, \\
 \delta_\zeta\phi &= -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
 \delta_\zeta D &= -i\bar{\zeta}^i\cancel{\partial}\lambda^i.
 \end{aligned} \tag{18}$$

$$[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{19}$$

where $\zeta^i, i = 1, 2$ are constant spinors and $\delta_g(\theta)$ is the $U(1)$ gauge transformation for only v^a with $\Xi^\mu = 2i\bar{\zeta}_1^I\gamma^\mu\zeta_2^I$, $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- For the **two scalar off-shell supermultiplets**:

$$\begin{aligned}
\delta_\zeta \chi &= (F^i - i\rlap{/}\partial B^i)\zeta^i - e\epsilon^{ij}V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij}\bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\rlap{/}\partial B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \rlap{/}\partial \chi - i\epsilon^{ij}\bar{\zeta}^j \rlap{/}\partial \nu \\
&\quad - e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i)B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}]\chi &= \Xi^a \partial_a \chi - e\theta \nu, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}]B^i &= \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}]\nu &= \Xi^a \partial_a \nu + e\theta \chi, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}]F^i &= \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{20}$$

with $V^i = iv_a \gamma^a \zeta^i - \epsilon^{ij} A \zeta^j - \phi \gamma_5 \zeta^i$ **and the U(1) gauge parameter** θ .

$N = 2$ NL/L SUSY relation:

$$L_{\text{brN=2SUSYQED}} = L_{V0} + L'_{\Phi0} + L_e + L_{Vf} = L_{\text{N=2NLSUSY}} + [\text{surface terms}], \quad (21)$$

is established by

(I) commutator-based (heuristic aspect) linearization

or

(II) superfield-based (systematic aspect) linearization.

For example,

(I) Commutator-based linearization:

@The product of **Lorentz tensors composed of ψ^i** and $|w|$ play a basic role.

• For such Lorentz tensors (of current) of ψ^i multiplied by $|w|$,

$$b^i_A{}^{jk}{}_B{}^{l\dots m}{}_C{}^n \left((\psi^i)^{2(n-1)} |w| \right) = \kappa^{2n-3} \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n |w|, \quad (22)$$

$$f^{ij}{}_A{}^{kl}{}_B{}^{m\dots n}{}_C{}^p \left((\psi^i)^{2n-1} |w| \right) = \kappa^{2(n-1)} \psi^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p |w|, \quad (23)$$

e. g.,

$$b^i_A{}^{jk}{}_B{}^l = \kappa^3 \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l |w|, \quad f^i = \psi^i |w|, \quad f^{ij}{}_A{}^k = \kappa^2 \psi^i \bar{\psi}^j \gamma_A \psi^k |w|, \quad \dots, \quad (24)$$

the variations under the NLSUSY transformations become

$$\begin{aligned} \delta_\zeta b^i_A{}^{jk}{}_B{}^{l\dots m}{}_C{}^n &= \kappa^{2(n-1)} \left[\left\{ (\bar{\zeta}^i \gamma_A \psi^j + \bar{\psi}^i \gamma_A \zeta^j) \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n + \dots \right\} |w| \right. \\ &\quad \left. + \kappa \partial_a (\xi^a \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n |w|) \right], \end{aligned} \quad (25)$$

$$\begin{aligned} \delta_\zeta f^{ij}{}_A{}^{kl}{}_B{}^{m\dots n}{}_C{}^p &= \kappa^{2n-1} \left[\left\{ \zeta^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p \right. \right. \\ &\quad \left. \left. + \psi^i (\bar{\zeta}^j \gamma_A \psi^k + \bar{\psi}^j \gamma_A \zeta^k) \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p + \dots \right\} |w| \right. \\ &\quad \left. + \kappa \partial_a (\xi^a \psi^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p |w|) \right], \end{aligned} \quad (26)$$

where $\xi^a = i\kappa \bar{\zeta}^i \gamma^a \psi^i$.

- These show that **Lorentz tensors (of currents) of ψ^i multiplied by $|w|$ give a finite representation of NLSUSY algebra.**

- They satisfy the commutator

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \quad (27)$$

where $\delta_P(v)$ is a translation with a parameter $v^a = 2i(\bar{\zeta}_{1L}^i \gamma^a \zeta_{2L}^i - \bar{\zeta}_{1R}^i \gamma^a \zeta_{2R}^i)$

- These results show that the commutator-based linearization **closes on the all possible Lorentz tensors composed of ψ^i** and gives a **finite dimensional representation of sP algebra**.

- Assign each composite Lorentz tensor to the component field of the LSUSY supermultiplet including the auxiliary field (**susy compositeness**), which reproduces the familiar LSUSY transformation among the supermultiplet under the NLSUSY transformations of constituents ψ^i .

- Substituting **SUSY compositeness** into $L_{N=2LSUSYQED}$, we obtain $L_{N=2NLSUSY}$, i .e. the **NL/L SUSY relation(equivalence)**.

- **SUSY compositeness** for the vector off-shell **minimal** supermultiplet:

$$v^a = -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|,$$

$$\lambda^i = \xi\psi^i|w|,$$

$$A = \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|,$$

$$\phi = -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|,$$

$$D = \frac{\xi}{\kappa}|w|, \tag{28}$$

where ξ is a VEV factor of the auxiliary field D .

- Note that ψ^i is the leading term of the supercharge Q^i and exact at low energy.

- **SUSY compositeness** for scalar off-shell **minimal** supermultiplets:

$$\begin{aligned}
\chi &= \xi^i \left[\psi^i |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j |w| \} \right] \\
B^i &= -\kappa \left(\frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|, \\
\nu &= \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k |w| \} \right], \\
F^i &= \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8} \kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\} - i \kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|) \\
&\quad - \frac{1}{4} e \kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|. \tag{29}
\end{aligned}$$

- The **quartic fermion self-interaction term** in F^i is the origin of the local $U(1)$ gauge symmetry of **LSUSY**.
- ξ^i is the **VEV factor** of the auxiliary field F^i .

- **SUSY compositeness** produces under **NLSUSY** transformation

a **new off-shell commutator algebra which closes on only a translation:**

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \quad (30)$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}^i \gamma^a \zeta_{2L}^i - \bar{\zeta}_{1R}^i \gamma^a \zeta_{2R}^i) \quad (31)$$

- Note that the commutator does not induce the **U(1)** gauge transformation, which is **different from the ordinary LSUSY**.

- Substituting **SUSY copositeness** into $L_{N=2LSUSYQED}$, we find **NL/L SUSY relation for the minimal supermultiplet**:

$$L_{N=2LSUSYQED} = f(\xi, \xi^i) L_{N=2NLSUSY} + [\text{suface terms}], \quad (32)$$

$$f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1. \quad (33)$$

⇒ **LSUSY** may be regarded as composite eigenstates of **space-time** symmetries.

- **NL/L SUSY relation** bridges naturally **the cosmology and the low energy particle physics in NLSUSYGR**. (⇒ Sec. 4).

- **The direct linearization** of highly nonlinear **SGM action (12)** in curved space **remains to be carried out**.

→

**In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the cosmological term Λ of SGM and materializes the true vacuum of SGM
(as gravitational composites of NG fermion)**

SM can be a low energy effective theory of SGM/NLSUSYGR.

(II) Linearization of NLSUSY by the superfield formulation ($d = 2$)

- **General superfields** are given for the $N = 2$ vector supermultiplet by

$$\begin{aligned} \mathcal{V}(x, \theta^i) = & C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2} \bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2} \bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4} \epsilon^{ij} \bar{\theta}^i \gamma_5 \theta^j \phi(x) \\ & - \frac{i}{4} \epsilon^{ij} \bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2} \bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8} \bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x), \end{aligned} \quad (34)$$

and for the $N = 2$ scalar supermultiplet by

$$\begin{aligned} \Phi^i(x, \theta^i) = & B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij} \bar{\theta}^j \nu(x) - \frac{1}{2} \bar{\theta}^j \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i \bar{\theta}^i \not{\partial} B^j(x) \theta^j \\ & + \frac{i}{2} \bar{\theta}^j \theta^j (\bar{\theta}^i \not{\partial} \chi(x) - \epsilon^{ik} \bar{\theta}^k \not{\partial} \nu(x)) + \frac{1}{8} \bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \partial_a \partial^a B^i(x). \end{aligned} \quad (35)$$

- Extend the superspace to the following superspace (x'^a, θ') with $-\kappa\psi(x)$,

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \quad (36)$$

and denote the extended superfields on (x'^a, θ'^i) and their θ -expansions as

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \quad (37)$$

- **Extended** global SUSY transformations $\tilde{\delta} = \delta^L(x.\theta) + \delta^{NL}(\psi)$ on (x'^a, θ'^i) give:

$$\tilde{\delta}\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \tilde{\delta}\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \quad (38)$$

- Therefore, the conditions (**SUSY invariant constraints**):

$$\tilde{\varphi}_{\mathcal{V}}^I(x) = \xi_{\mathcal{V}}^I(\text{constant}) \quad \tilde{\varphi}_{\Phi}^I(x) = \xi_{\Phi}^I(\text{constant}), \quad (39)$$

make extended superfields invariant under the extended SUSY transformations. which provide **SUSY compositeness**.

- Putting constants in the most general case as follows:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi_\Lambda^i, \quad \tilde{M}^{ij} = \xi_M^{ij}, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi_v^a, \quad \tilde{\lambda}^i = \xi_\lambda^i, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad (40)$$

$$\tilde{B}^i = \xi_B^i, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (41)$$

- We obtain straightforwardly **SUSY compositeness** $\varphi_V^I = \varphi_V^I(\psi)$ for the vector supermultiplet

$$C = \xi_c + \kappa \bar{\psi}^i \xi_\Lambda^i + \frac{1}{2} \kappa^2 (\xi_M^{ij} \bar{\psi}^i \psi^j - \xi_M^{ii} \bar{\psi}^j \psi^j) + \frac{1}{4} \xi_\phi \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - \frac{i}{4} \xi_v^a \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \\ - \frac{1}{2} \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \xi_\lambda^j - \frac{1}{8} \xi \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j,$$

$$\Lambda^i = \xi_\Lambda^i + \kappa (\xi_M^{ij} \psi^j - \xi_M^{jj} \psi^i) + \frac{1}{2} \xi_\phi \kappa \epsilon^{ij} \gamma_5 \psi^j - \frac{i}{2} \xi_v^a \kappa \epsilon^{ij} \gamma_a \psi^j \\ - \frac{1}{2} \xi_\lambda^i \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2} \kappa^2 (\psi^j \bar{\psi}^i \xi_\lambda^j - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi_\lambda^j - \gamma_a \psi^j \bar{\psi}^i \gamma_a \xi_\lambda^j)$$

$$-\frac{1}{2}\xi\kappa^2\psi^i\bar{\psi}^j\psi^j - i\kappa\partial C(\psi)\psi^i,$$

$$M^{ij} = \xi_M^{ij} + \kappa\bar{\psi}^{(i}\xi_\lambda^{j)} + \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^j + i\kappa\epsilon^{(i|k|\epsilon^{j)l}}\bar{\psi}^k\partial\Lambda^l(\psi) - \frac{1}{2}\kappa^2\epsilon^{ik}\epsilon^{jl}\bar{\psi}^k\psi^l\partial^2C(\psi),$$

$$\phi = \xi_\phi - \kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\xi_\lambda^j - \frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\partial\Lambda^j(\psi) + \frac{1}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2C(\psi),$$

$$v^a = \xi_v^a - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\xi_\lambda^j - \frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j - \kappa\epsilon^{ij}\bar{\psi}^i\partial\gamma^a\Lambda^j(\psi) + \frac{i}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j\partial^2C(\psi) \\ - i\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^b\psi^j\partial^a\partial_bC(\psi),$$

$$\lambda^i = \xi_\lambda^i + \xi\psi^i - i\kappa\partial M^{ij}(\psi)\psi^j + \frac{i}{2}\kappa\epsilon^{ab}\epsilon^{ij}\gamma_a\psi^j\partial_b\phi(\psi)$$

$$-\frac{1}{2}\kappa\epsilon^{ij} \left\{ \psi^j\partial_a v^a(\psi) - \frac{1}{2}\epsilon^{ab}\gamma_5\psi^j F_{ab}(\psi) \right\}$$

$$-\frac{1}{2}\kappa^2 \{ \partial^2\Lambda^i(\psi)\bar{\psi}^j\psi^j - \partial^2\Lambda^j(\psi)\bar{\psi}^i\psi^j - \gamma_5\partial^2\Lambda^j(\psi)\bar{\psi}^i\gamma_5\psi^j$$

$$-\gamma_a\partial^2\Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j + 2\partial\partial_a\Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j \} - \frac{i}{2}\kappa^3\partial\partial^2C(\psi)\psi^i\bar{\psi}^j\psi^j,$$

$$\begin{aligned}
D = & \frac{\xi}{\kappa} - i\kappa\bar{\psi}^i\partial\lambda^i(\psi) \\
& + \frac{1}{2}\kappa^2 \left\{ \bar{\psi}^i\psi^j\partial^2 M^{ij}(\psi) - \frac{1}{2}\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2\phi(\psi) \right. \\
& \left. + \frac{i}{2}\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial^2 v^a(\psi) - i\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial_a\partial_b v^b(\psi) \right\} \\
& - \frac{i}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\partial\partial^2\Lambda^j(\psi) + \frac{1}{8}\kappa^4\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j\partial^4 C(\psi), \tag{42}
\end{aligned}$$

and **SUSY compositeness for the scalar multiplet** $\varphi_{\Phi}^I = \varphi_{\Phi}^I(\psi)$:

$$\begin{aligned}
B^i = & \xi_B^i + \kappa(\bar{\psi}^i\xi_{\chi} - \epsilon^{ij}\bar{\psi}^j\xi_{\nu}) - \frac{1}{2}\kappa^2\{\bar{\psi}^j\psi^j F^i(\psi) - 2\bar{\psi}^i\psi^j F^j(\psi) + 2i\bar{\psi}^i\partial B^j(\psi)\psi^j\} \\
& - i\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial\chi(\psi) - \epsilon^{ik}\bar{\psi}^k\partial\nu(\psi)\} + \frac{3}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 B^i(\psi), \\
\chi = & \xi_{\chi} + \kappa\{\psi^i F^i(\psi) - i\partial B^i(\psi)\psi^i\} \\
& - \frac{i}{2}\kappa^2[\partial\chi(\psi)\bar{\psi}^i\psi^i - \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\nu(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\nu(\psi)\}]
\end{aligned}$$

$$\begin{aligned}
& +\frac{1}{2}\kappa^3\psi^i\bar{\psi}^j\psi^j\partial^2B^i(\psi) + \frac{i}{2}\kappa^3\partial F^i(\psi)\psi^i\bar{\psi}^j\psi^j + \frac{1}{8}\kappa^4\partial^2\chi(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
\nu = & \xi_\nu - \kappa\epsilon^{ij}\{\psi^iF^j(\psi) - i\partial B^i(\psi)\psi^j\} \\
& -\frac{i}{2}\kappa^2[\partial\nu(\psi)\bar{\psi}^i\psi^i + \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\chi(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\chi(\psi)\}] \\
& +\frac{1}{2}\kappa^3\epsilon^{ij}\psi^i\bar{\psi}^k\psi^k\partial^2B^j(\psi) + \frac{i}{2}\kappa^3\epsilon^{ij}\partial F^i(\psi)\psi^j\bar{\psi}^k\psi^k + \frac{1}{8}\kappa^4\partial^2\nu(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
F^i = & \frac{\xi^i}{\kappa} - i\kappa\{\bar{\psi}^i\partial\chi(\psi) + \epsilon^{ij}\bar{\psi}^j\partial\nu(\psi)\} \\
& -\frac{1}{2}\kappa^2\bar{\psi}^j\psi^j\partial^2B^i(\psi) + \kappa^2\bar{\psi}^i\psi^j\partial^2B^j(\psi) + i\kappa^2\bar{\psi}^i\partial F^j(\psi)\psi^j \\
& +\frac{1}{2}\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial^2\chi(\psi) + \epsilon^{ik}\bar{\psi}^k\partial^2\nu(\psi)\} - \frac{1}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2F^i(\psi). \tag{43}
\end{aligned}$$

- Choosing the following **Lorentz invariant** and **SUSY invariant** constraints of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \quad (44)$$

give **previous SUSY compositeness** for the minimal supermultiplet.

Therefore,

•

The $N = 2$ **NLSUSY** action $S_{N=2\text{NLSUSY}}$ is related to $N = 2$ **SUSY QED** action:

$$f(\xi, \xi^i) S_{N=2\text{NLSUSY}} = S_{N=2\text{SUSYQED}} \equiv S_{\mathcal{V}\text{free}} + S_{\mathcal{V}f} + S_{\text{gauge}} \quad (45)$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$.

- **NL/L SUSY relation** bridges **the cosmology** and **the low energy particle physics** in **NLSUSYGR** scenario \implies **Sec. 4**.

- SGM scenario predicts the magnitude of the bare gauge coupling constant.

For more general SUSY invariant constraints, i.e. for NLSUSY vev of 0^+ auxiliary fields

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (46)$$

NL/L SUSY relation gives

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \quad e = \frac{\ln\left(\frac{\xi^{i2}}{\xi^2 - 1}\right)}{4\xi_c}, \quad (47)$$

where e is the bare gauge coupling constant.

- This mechanism is natural and favorable for SGM scenario as a theory of everything.

Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.

3.3. $N = 3$ NL/L SUSY relation and SUSY Yang-Mills theory

- Physical helicity states of $N = 3$ LSUSY vector supermultiplet:

$$\left[\underline{1}(+1), \underline{3}\left(+\frac{1}{2}\right), \underline{3}(0), \underline{1}\left(-\frac{1}{2}\right) \right] + [\text{CPT conjugate}], \quad (48)$$

where $\underline{n}(\lambda)$ means the dimension \underline{n} and the helicity λ , are accommodated in $N = 3$ off-shell vector supermultiplet ($d = 2$):

- $N = 3$ superYang-Mills(SUSYYM) minimal off-shell gauge multiplet,

$$\{v^{aI}(x), \lambda^{iI}(x), A^{iI}(x), \chi_{\alpha}^I(x), \phi^I(x), D^{iI}(x)\}, \quad (I = 1, 2, \dots, \dim.G) \quad (49)$$

Each component field belongs to the adjoint representation of the YM gauge group G :

$[T^I, T^J] = if^{IJK}T^K$ and denoted as $\varphi^i = \varphi^{iI}T^I$, etc..

- $N = 3$ (pure) SUSYM action:

$$\begin{aligned}
S_{\text{SYM}} = \int d^2x \operatorname{tr} \left\{ & -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{D} \lambda^i + \frac{1}{2}(D_a A^i)^2 + \frac{i}{2}\bar{\chi} \not{D} \chi + \frac{1}{2}(D_a \phi)^2 + \frac{1}{2}(D^i)^2 \right. \\
& -ig\{\epsilon^{ijk} A^i \bar{\lambda}^j \lambda^k - [A^i, \bar{\lambda}^i] \chi + \phi(\bar{\lambda}^i \gamma_5 \lambda^i + \bar{\chi} \gamma_5 \chi)\} \\
& \left. + \frac{1}{4}g^2([A^i, A^j]^2 + 2[A^i, \phi]^2) \right\}, \tag{50}
\end{aligned}$$

where g is the gauge coupling constant, D_a and F_{ab} are the covariant derivative and the YM gauge field strength defined as

$$\begin{aligned}
D_a \varphi &= \partial_a \varphi - ig[v_a, \varphi], \\
F_{ab} &= \partial_a v_b - \partial_b v_a - ig[v_a, v_b]. \tag{51}
\end{aligned}$$

• **SUSY YM action is invariant under $N = 3$ LSUSY transformations:**

$$\begin{aligned}
\delta_\zeta v^a &= i\bar{\zeta}^i \gamma^a \lambda^i, \\
\delta_\zeta \lambda^i &= \epsilon^{ijk} (D^j - i\mathcal{D}A^j) \zeta^k + \frac{1}{2} \epsilon^{ab} F_{ab} \gamma_5 \zeta^i - i\gamma_5 \mathcal{D}\phi \zeta^i \\
&\quad + ig([A^i, A^j] \zeta^j + \epsilon^{ijk} [A^j, \phi] \gamma_5 \zeta^k), \\
\delta_\zeta A^i &= \epsilon^{ijk} \bar{\zeta}^j \lambda^k - \bar{\zeta}^i \chi, \\
\delta_\zeta \chi &= (D^i + i\mathcal{D}A^i) \zeta^i + ig(\epsilon^{ijk} A^i A^j \zeta^k - [A^i, \phi] \gamma_5 \zeta^i), \\
\delta_\zeta \phi &= \bar{\zeta}^i \gamma_5 \lambda^i, \\
\delta_\zeta D^i &= -i\epsilon^{ijk} \bar{\zeta}^j \mathcal{D}\lambda^k - i\bar{\zeta}^i \mathcal{D}\chi + ig(\bar{\zeta}^i [\lambda^j, A^j] + \bar{\zeta}^j [\lambda^i, A^j] - \bar{\zeta}^j [\lambda^j, A^i] \\
&\quad - \epsilon^{ijk} \bar{\zeta}^j [\chi, A^k] + \epsilon^{ijk} \bar{\zeta}^j \gamma_5 [\lambda^k, \phi] + \bar{\zeta}^i \gamma_5 [\chi, \phi]), \tag{52}
\end{aligned}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P(\Xi^a) + \delta_G(\theta) + \delta_g(\theta), \tag{53}$$

where $\delta_G(\theta)$ means $\delta_G(\theta)\varphi = ig[\theta, \varphi]$ and $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i \gamma^a \zeta_2^i v_a - \epsilon^{ijk} \bar{\zeta}_1^i \zeta_2^j A^k - \bar{\zeta}_1^i \gamma_5 \zeta_2^i \phi)$.

- **SUSY invariant(composite) relations for $N = 3$ YM off-shell gauge supermultiplet**

$$\begin{aligned}
v^{aI} &= -\frac{i}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma^a\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) + \frac{1}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_b(\bar{\psi}^j\gamma_5\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
\lambda^{iI} &= \epsilon^{ijk}\xi^{jI}\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) \\
&\quad + \frac{i}{2}\kappa^2\xi^{jI}\partial_a\{\epsilon^{ijk}\gamma^a\psi^k\bar{\psi}^l\psi^l + \epsilon^{ab}\epsilon^{jkl}(\gamma_b\psi^i\bar{\psi}^k\gamma_5\psi^l - \gamma_5\psi^i\bar{\psi}^k\gamma_b\psi^l)\} + \mathcal{O}(\kappa^4), \\
A^{iI} &= \kappa\left(\frac{1}{2}\xi^{iI}\bar{\psi}^j\psi^j - \xi^{jI}\bar{\psi}^i\psi^j\right)(1 - i\kappa^2\bar{\psi}^k\partial\psi^k) - \frac{i}{2}\kappa^3\xi^{iI}\partial_a(\bar{\psi}^i\gamma^a\psi^j\bar{\psi}^k\psi^k) + \mathcal{O}(\kappa^5), \\
\chi^I &= \xi^{iI}\psi^i(1 - i\kappa^2\bar{\psi}^j\partial\psi^j) + \frac{i}{2}\kappa^2\xi^{iI}\partial_a(\gamma^a\psi^i\bar{\psi}^j\psi^j) + \mathcal{O}(\kappa^4), \\
\phi^I &= -\frac{1}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma_5\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) - \frac{i}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_a(\bar{\psi}^j\gamma_b\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
D^{iI} &= \frac{1}{\kappa}\xi^{iI}|w| - i\kappa\xi^{jI}\partial_a\{\bar{\psi}^i\gamma^a\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\psi^k)\} \\
&\quad - \frac{1}{8}\kappa^3\partial_a\partial^a\{(\xi^{iI}\bar{\psi}^j\psi^j - 4\xi^{jI}\bar{\psi}^i\psi^j)\bar{\psi}^k\psi^k\} + \mathcal{O}(\kappa^5), \tag{54}
\end{aligned}$$

- Arbitrary real constants ξ^{iI} of auxiliary fields D^{iI} bridge $N = 3$ SUSY and the YM gauge group G .

- Substituting (54) into the SYM action (50), we can show the NL/L SUSY relation for $N = 3$ SUSY:

$$S_{\text{SUSY YM}}(\psi) = -(\xi^{iI})^2 S_{\text{NLSUSY}} + [\text{surface terms}]. \quad (55)$$

4. Low energy particle physics and Cosmology of NLSUSYGR

4.1. Low Energy Particle Physics of NLSUSYGR :

@ As we have seen that

$N = 2$ SGM is essentially $N=2$ NLSUSY action in tangent(flat)) space-time, we focus on $N=2$ NLSUSY action for extracting physical implications of SGM.

- The low energy theorem for NLSUSY gives the following **superon(massless NG fermion)-vacuum coupling**

$$\langle \psi^j_\alpha(x) | J^{k\mu}_\beta | 0 \rangle = i \sqrt{\frac{c^4 \Lambda}{8\pi G}} (\gamma^\mu)_{\alpha\beta} \delta^{jk} + \dots, \quad (56)$$

where $J^{k\mu} = i \sqrt{\frac{c^4 \Lambda}{8\pi G}} \gamma^\mu \psi^k + \dots$ is the conserved supercurrent.

$\sqrt{\frac{c^4 \Lambda}{8\pi G}} = \frac{1}{\sqrt{2\kappa}}$ is the coupling constant (g_{sv}) of superon with the vacuum.

- How to extract the vacuum configuration of SGM.

In Riemann-flat space-time, **NL/L SUSY relation(equivalence)** gives:

$$L_{N=2\text{SGM}} \longrightarrow L_{N=2\text{NLSUSY}} + [\text{surface terms}] = L_{N=2\text{SUSYQED}}. \quad (57)$$

- We study vacuum structures of $N = 2$ **LSUSY QED** action in stead of $N = 2$ SGM.

The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\text{LSUSYQED}}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e|B^i|^2 \right\} D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2. \quad (58)$$

- Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V(A, \phi, B^i) = \frac{1}{2}f^2 \left\{ A^2 - \phi^2 - \frac{e}{2f}|B^i|^2 - \frac{\xi}{f\kappa} \right\}^2 + \frac{1}{2}e^2(A^2 + \phi^2)|B^i|^2 \geq 0. \quad (59)$$

- Two different types of vacua $V = 0$ exist in (A, ϕ, B^i) -space:

$$(I) \quad A = \phi = 0, \quad |\tilde{B}^i|^2 = -k^2 \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i, \quad k^2 = \frac{\xi}{f\kappa} \right) \quad (60)$$

and

$$(II) \quad |\tilde{B}^i|^2 = 0, \quad A^2 - \phi^2 = k^2. \quad \left(k^2 = \frac{\xi}{f\kappa} \right) \quad (61)$$

- Expansions of A, ϕ, \tilde{B}^i around vacuum values give low energy particles $\hat{A}, \hat{\phi}, \hat{B}^i$ in the true vacuum.

- For the type (I) vacuum with $SO(2)$ symmetry for $(\tilde{B}^1, \tilde{B}^2)$, $e\xi < 0$,

$$\begin{aligned}
L_{N=2\text{SUSYQED}} = & \frac{1}{2}\{(\partial_a\rho)^2 - 2(-ef)k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a\hat{A})^2 + (\partial_a\hat{\phi})^2 - 2(-ef)k^2(\hat{A}^2 + \hat{\phi}^2)\} \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i\partial\lambda^i + \frac{i}{2}\bar{\chi}\partial\chi + \frac{i}{2}\bar{\nu}\partial\nu + \sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) + \dots,
\end{aligned}
\tag{62}$$

and following mass spectra

$$\begin{aligned} m_\rho^2 = m_{\hat{A}}^2 = m_{\hat{\phi}}^2 = m_{v_a}^2 &= 2(-ef)k^2 = -\frac{2\xi e}{\kappa}, \\ m_{\lambda^i} = m_\chi = m_\nu &= 0. \end{aligned} \tag{63}$$

- The **vacuum** breaks **both SUSY and the local $U(1)(O(2))$ spontaneously** and Higgs-Kibble mechanism works.
- All bosons have the same mass and **all fermions remain massless**.
- λ^i are **NG fermions**.

- For the type (II) vacuum with $SO(1, 1)$ symmetry for (A, ϕ) , e.g. $f\xi > 0$,

$$\begin{aligned}
L_{N=2\text{SUSYQED}} = & \frac{1}{2}\{(\partial_a \hat{A})^2 - 4f^2 k^2 \hat{A}^2\} \\
& + \frac{1}{2}\{|\partial_a \hat{B}^1|^2 + |\partial_a \hat{B}^2|^2 - e^2 k^2 (|\hat{B}^1|^2 + |\hat{B}^2|^2)\} \\
& + \frac{1}{2}(\partial_a \hat{\phi})^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i \not{\partial} \lambda^i - 2fk\bar{\lambda}^i \lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi} \not{\partial} \chi + \bar{\nu} \not{\partial} \nu) - ek(\bar{\chi} \chi + \bar{\nu} \nu)\} + \dots
\end{aligned} \tag{64}$$

and following mass spectra:

$$\begin{aligned}
 m_{\hat{A}}^2 &= m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa}, \\
 m_{\hat{B}^1}^2 &= m_{\hat{B}^2}^2 = m_{\chi}^2 = m_{\nu}^2 = e^2k^2 = \frac{\xi e^2}{\kappa f}, \\
 m_{\nu_a} &= m_{\hat{\phi}} = 0,
 \end{aligned}
 \tag{65}$$

which produces mass hierarchy by the factor $\frac{e}{f}$ independent of κ . ($\kappa^{-2} = \frac{c^4 \Lambda}{16\pi G}$)

• The vacuum breaks both SUSY and $SO(1,1)$ for (A, ϕ)
 and restores(maintains) $SO(2)(U(1))$ for $(\tilde{B}^1, \tilde{B}^2)$, spontaneously,

which produces NG-Boson $\hat{\phi}$ and massless photon ν_a
 and gives soft masses $\langle A \rangle$ to λ^i .

- We have shown explicitly that

N=2 LSUSY QED, i.e. the matter sector(Λ term) of $N = 2$ SGM (in flat-space), possesses a true vacuum type (II).

- The resulting model describes

lepton-Higgs-U(1) sector analogue of SM:

one massive charged Dirac fermion ($\psi_D^c \sim \chi + i\nu$),

one massive neutral Dirac fermion ($\lambda_D^0 \sim \lambda^1 - i\lambda^2$),

one massless vector (a photon) (v_a),

one charged scalar ($\hat{B}^1 + i\hat{B}^2$),

one neutral complex scalar ($\hat{A} + i\hat{\phi}$),

which are **composites of superons.**

**In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
as composites of NG fermion
from
the NLSUSY cosmological term of SGM.**

4.2 Cosmological implications of SGM scenario

The variation of SGM action $L_{\text{SGM}}(e, \psi)$ with respect to $e^a{}_\mu$ yields **Einstein equation equipping with matter and cosmological term:**

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4} \left\{ \tilde{T}_{\mu\nu}(e, \psi) - g_{\mu\nu} \frac{c^4 \Lambda}{8\pi G} \right\}. \quad (66)$$

where $\tilde{T}_{\mu\nu}(e, \psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

- Note that the cosmological term $-\frac{c^4 \Lambda}{8\pi G}$ can be interpreted as **the negative energy density of space-time**, i.e. **the dark energy density ρ_D** .

- **Big collapse** may induce **3 dimensional expansion** of space-time by **Pauli principle**:

$$ds^2 = s_{\mu\nu}(x)dx^\mu dx^\nu = \{g_{\mu\nu} + \Phi_{\mu\nu}(e, \psi)\}dx^\mu dx^\nu.$$

$$\{\psi(x), \bar{\psi}(y)\} = 0 \Rightarrow \{\psi(x), \bar{\psi}(y)\} = \delta^{(3)}(\mathbf{x} - \mathbf{y})$$

- **Big Collapse** produces **composite (massless) eigenstates of SO(N) sP algebra** due to the **universal gravitational force**,

which is **the ignition of Big Bang(BB) SM scenario**.

- In the composite **SGM** view of $N = 2$ **LSUSY QED**, the vacuum (II) may explain naturally **observed mysterious (numerical) relations**:

$$\text{dark energy density } \rho_D \sim O(\kappa^{-2}) \sim m_\nu^4 \sim (10^{-12} \text{GeV})^4 \sim g_{sv}^2,$$

provided λ_D^0 is identified with neutrino and $f\xi \sim O(1)$.

- In this case, **one neutral scalar new particle with mass $\sim O(m_\nu)$ exists.**
A candidate of dark matter.

5. Rarita-Schwinger NLSUSYGR

- New SUSY algebra containing **spinor-vector generators** Q_α^μ :

$$\{Q_\alpha^\mu, Q_\beta^\nu\} = \varepsilon^{\mu\nu\lambda\rho} P_\lambda (\gamma_\rho \gamma_5 C)_{\alpha\beta}, \quad (67)$$

$$[Q_\alpha^\mu, P^\nu] = 0, \quad (68)$$

$$[Q_\alpha^\mu, J^{\lambda\rho}] = \frac{1}{2}(\sigma^{\lambda\rho} Q^\mu)_\alpha + i\eta^{\lambda\mu} Q_\alpha^\rho - i\eta^{\rho\mu} Q_\alpha^\lambda, \quad (69)$$

where Q_α^μ are vector-spinor generators satisfying Majorana condition $Q_\alpha^\mu = C_{\alpha\beta} \bar{Q}_\alpha^\mu$.

- Consider the following global (3/2 super)translations:

$$\psi_\alpha^a \longrightarrow \psi_\alpha^a + \zeta_\alpha^a. \quad (70)$$

$$x_a \longrightarrow x_a + i\kappa \varepsilon_{abcd} \bar{\psi}^b \gamma^c \gamma_5 \zeta^d, \quad (71)$$

where ζ_α^a is a constant Majorana tensor-spinor parameter.

- The invariant differential forms become:

$$\omega_a = dx_a + i\kappa \varepsilon_{abcd} \bar{\psi}^b \gamma^c \gamma_5 d\psi^d. \quad (72)$$

- **Invariant action of nonlinear representation of vector-spinor SUSY:**

$$S = \frac{1}{\kappa} \int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \frac{1}{\kappa} \int \det w_{ab} d^4x, \quad (73)$$

$$w_{ab} = \delta_{ab} + t_{ab}, \quad t_{ab} = i\kappa \varepsilon_{acde} \bar{\psi}^c \gamma^d \gamma_5 \partial_b \psi^e, \quad (74)$$

- **By similar geometrical arguments to SGM we obtain vector-spinor NLSUSY GR:**

$$L_{vsNLSUSYGR} = -\frac{c^3}{16\pi G} |w| \{ \Omega(w^a{}_\mu) + \Lambda \}, \quad (75)$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu), \quad (76)$$

- **Unified vierbeins are:**

$$w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x), \quad t^a{}_\mu(x) = i\kappa \varepsilon^{abcd} \bar{\psi}_b \gamma_c \gamma_5 \partial_\mu \psi_d, \quad (77)$$

- **$L_{vsNLSUSYGR}$ possesses similar symmetry properties as SGM.**

6. Summary

NLSUSYGR(SGM) for unity of nature:

- **Ultimate entity; Unstable $d = 4$ space-time:** $[x^a, \psi_\alpha^N; x^\mu]$ described by $[L_{\text{NLSUSYGR}}(w^a{}_\mu)]$: **NLSUSYGR on New space-time with $\Lambda > 0$**

\implies **Big Collapse (BC)**

- **The creation of Riemann space-time [graviton $e^a{}_\mu$] and massless fermionic matter [superon ψ_α^N]**
 $[L_{\text{SGM}} = L_{\text{EH}}(e) - \Lambda + T(\psi.e)]$: **Einstein GR with $\Lambda > 0$ and N superon**

- **The universal attractive force graviton dictates the phase transition by forming gravitational composite LSUSY supermultiplet corresponding to (massless) eigenstates of space-time symmetry.**

\implies **Ignition of Big Bang (BB)** \implies (MS)SM

- **In flat space-time, broken N -LSUSY theory emerges from the N -NLSUSY cosmological term of $L_{\text{SGM}}(e, \psi)$ [NL/L SUSY relation]. \longleftrightarrow BCS vs GL**

The cosmological constant is the origin of everything!

Predictions and Speculations:

@SO(10) sP algebra with $\underline{10} = \underline{5}_{\text{SU}(5)\text{GUT}} + \underline{5}^*_{\text{SU}(5)\text{GUT}}$ **SGM:**

- **New 1^C states** besides SM particles:

double-charge spin 1/2 fermion $E^{2\pm}$: $\epsilon^{abc} Q_a Q_b Q_c \epsilon^{mn} Q^*_m Q^*_n$

One neutral massive vector boson S : $\delta^{ab} Q_a Q^*_b$

- Proton decay diagrams of SU(5) GUT **in SGM view** are forbidden by **superon selection rule**. \Rightarrow **stable proton**

@Field theory via Linearization:

- NLSUSYGR(SGM) scenario **predicts** 4 dimensional space-time.
- The bare gauge coupling constant is determined.
- N-L/NL **SUSY relation** in general \iff **superon-quintet hypothesis**

cosmological term \leftrightarrow dark energy density \leftrightarrow SUSY Br. $\rightarrow m_\nu$

Many Open Questions ! e.g.,

- Direct linearization of SGM action, i.e. **NL/L SUSY relation in curved space-time.**
(Find the broken SUGRA-like(?) theory.)

@The products of $|w|$ and Lorentz tensors composed of $\psi^i, \partial\psi, e^a{}_\mu, \partial e^a{}_\mu$ play basic roles. \implies (i)

@ Superfield systematics of NL/L SUSY relation for SGM action. \implies (ii)

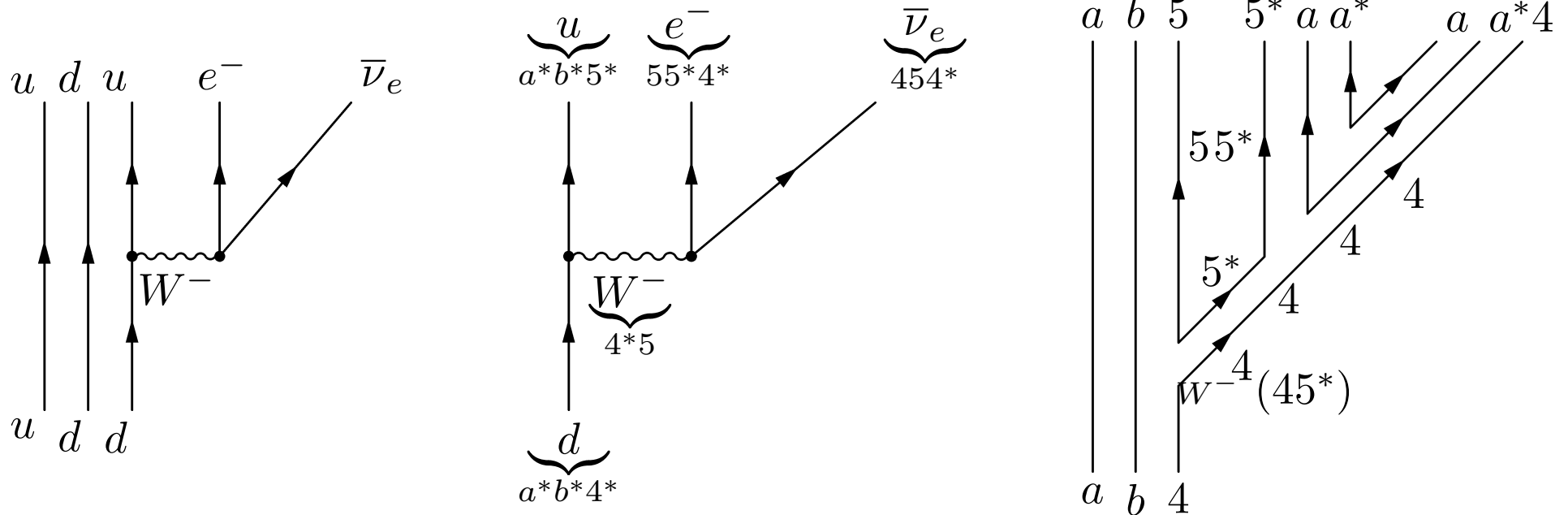
• Revisit unsolved problems of SMs and GUT from SQM composite viewpoints.

e.g.,

$$(e, \nu_e): \delta^{ab} Q_a Q_b^* Q_m, \quad (\mu, \nu_\mu): \delta^{ab} Q_a Q_b^* \epsilon^{lm} Q_l Q_m Q_n^*, \quad (\tau, \nu_\tau): \epsilon^{abc} Q_b Q_c \epsilon^{abc} Q_b^* Q_c^* Q_m$$

$$(u, d): \epsilon^{abc} Q_b Q_c Q_m, \quad (c, s): \epsilon^{abc} Q_a Q_b Q_c Q^*_d Q_m, \quad (t, b): \epsilon^{lm} Q_l Q_m \epsilon^{abc} Q_b Q_c Q^*_n,$$

$$\text{Higgs: } \delta^{ab} Q_a Q_b^* Q_l Q^*_m, \quad \dots$$



- **Physical consequences of spin $\frac{3}{2}$ NLSUSYGR.**

**[Ref.] K. Shima, Plenary talk at *Conference on Cosmology, Gravitational Waves and Particles*, 6-10, January, 2017, NTU, Singapore.
Proceeding of *CCGWP*, ed. Harald Fritzsch, (World Scientific, Singapore, 2017), 301.**