Is the cosmic no-hair conjecture valid in conformal-violating Maxwell models?

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Based on PRD84(2011)123009; CQG33(2016)085009; PRD96(2017)023529; EPJC78(2018)360 (all done with W. F. Kao)

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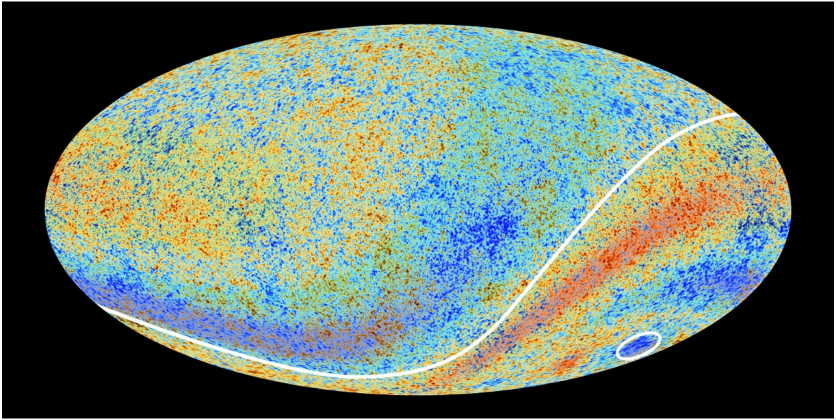
Contents

Motivations

- Inflation = A rapid expansion in a very short time.
- Cosmic inflation was firstly proposed by Guth [PRD23(1981)347] as a solution to several important problems in cosmology such as flatness, horizon, and magnetic monopole problems, due to its rapid expansion. [See Pryke's talk for more details].
- Cosmic inflation also predicts many properties of early universe through the cosmic microwave background (CMB), which have been well confirmed by the recent high-tech satellites of the WMAP and Planck collaborations.
- The other pioneers of the cosmic inflation paradigm are Starobinsky, PLB91(1980)99; Linde, PLB108(1982)389, PLB129(1983)177; Albrecht & Steinhardt, PRL48(1982)1220, and many others.

The 2014 Kavli Prize Laureates in Astrophysics: A. Guth, A. Linde, and A. Starobinsky for pioneering the theory of cosmic inflation (Source: Kavliprize.org).

Motivations



Two CMB anomalous features, the hemispherical asymmetry and the Cold Spot, hinted by Planck's predecessor, NASA's WMAP, are confirmed in the new high precision data from Planck, **both are not predicted by standard inflationary models**. (Information source and picture credit: ESA and the Planck Collaboration).

Cosmic no-hair conjecture: basic ideas

- Anomalies imply that the early universe might be slightly anisotropic. What is the state of our current universe ? Is it isotropic or still slightly anisotropic ?
- It has been widely assumed that the current universe is just homogeneous and isotropic such as the flat FLRW (or de Sitter) spacetime:

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right).$$

- If this assumption is the case, how did the universe transform from an anisotropic state in the early time to an isotropic state in the late time ?
- A cosmic no-hair conjecture proposed by Hawking and his colleagues might provide an important hint to this question. **It claims that all classical hairs of the early universe [anisotropy and/or homogeneity] will disappear at the late time** [Gibbons & Hawking, PRD15(1977)2738; Hawking & Moss, PLB110(1982)35].

From left to right: S. W. Hawking, G. W. Gibbons, and I. G. Moss (Source: Internet).

Cosmic no-hair conjecture: (incomplete) proofs

- This conjecture was partially proven by Wald [PRD28(1983)2118] for the Bianchi spacetimes, which are homogeneous but anisotropic, using energy conditions approach.
- Kleban & Senatore, JCAP10(2016)022; East, Kleban, Linde & Senatore, JCAP09(2016)010: try to extend the Wald's proof to inhomogeneous and anisotropic spacetimes.
- Carroll & Chatwin-Davies, PRD97(2018)046012: try to prove the conjecture in a difference approach using the idea of maximum entropy of de Sitter spacetime.
- There are several claimed (Bianchi) counterexamples to the cosmic no-hair conjecture, e.g., Kaloper, PRD44(1991)2380; Barrow & Hervik, PRD73(2006)023007, PRD81(2010)023513; Kanno, Soda & Watanabe (KSW), PRL102(2009)191302, JCAP12(2010)024.
- Some claimed counterexamples have been shown to be unstable by stability analysis, e.g., Kao & Lin, JCAP01(2009)022, PRD79(2009)043001, PRD83(2011)063004; Chang, Kao & Lin, PRD84(2011)063014, meaning that they do not really violate the cosmic no-hair conjecture.

Kanno-Soda-Watanabe model

- It seems to be the first counterexample to the cosmic no-hair conjecture [Kanno, Soda & Watanabe, PRL102(2009)191302, JCAP12(2010)024].

$$S_{\mathrm{KSW}} = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right],$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the field strength of the electromagnetic (Maxwell) field A_μ.

- The Bianchi type I metric (BI):

$$ds^2 = -dt^2 + \exp\left[2\alpha(t) - 4\sigma(t)\right]dx^2 + \exp\left[2\alpha(t) + 2\sigma(t)\right]\left(dy^2 + dz^2\right).$$

- $\sigma(t)$ stands for a deviation from the isotropy determined by $\alpha(t)$, i.e., $\sigma(t) \ll \alpha(t)$.
- The vector and scalar fields: $A_\mu = (0, A_x(t), 0, 0)$ and $\phi = \phi(t)$.
- Solution of $A_x(t)$: $\dot{A}_x(t) = f^{-2}(\phi)\exp\left[-\alpha - 4\sigma\right]p_A$.

KSW model: few main points

- The following set of field equations:

$$\dot{\alpha}^2 = \dot{\sigma}^2 + \frac{1}{3M_p^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}f^{-2}(\phi)\exp\left[-4\alpha - 4\sigma\right]p_A^2\right],$$

$$\ddot{\alpha} = -3\dot{\alpha}^2 + \frac{1}{M_p^2}V(\phi) + \frac{1}{6M_p^2}f^{-2}(\phi)\exp\left[-4\alpha - 4\sigma\right]p_A^2,$$

$$\ddot{\sigma} = -3\dot{\alpha}\dot{\sigma} + \frac{1}{3M_p^2}f^{-2}(\phi)\exp\left[-4\alpha - 4\sigma\right]p_A^2,$$

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - \frac{\partial V(\phi)}{\partial \phi} + f^{-3}(\phi)\frac{\partial f(\phi)}{\partial \phi}\exp\left[-4\alpha - 4\sigma\right]p_A^2.$$

- Choose the potentials of the exponential forms:

$$V(\phi) = V_0 \exp\left[\frac{\lambda}{M_p}\phi\right];\; f(\phi) = f_0 \exp\left[\frac{\rho}{M_p}\phi\right].$$

along with the following forms of scale factors and scalar field:

$$\alpha = \zeta \log(t);\, \sigma = \eta \log(t);\, \frac{\phi}{M_p} = \xi \log(t) + \phi_0.$$

KSW model: few main points

- The following solution is

$$\zeta = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda + 2\rho)}; \ \eta = \frac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda + 2\rho)}.$$

- For an inflationary universe, $\alpha \gg \sigma \to \zeta \gg \eta$. If $\rho \gg \lambda$ then $\zeta \simeq \rho/\lambda \gg \eta \simeq 1/3$.
- This solution can be shown to be stable and attractive by converting the field equations into the autonomous equations of dynamical variables:

$$X = \frac{\dot{\sigma}}{\dot{\alpha}}; \ Y = \frac{\dot{\phi}}{M_p \dot{\alpha}}; \ Z = \frac{1}{f_0 M_p \dot{\alpha}} \exp\left[-\frac{\rho}{M_p}\phi - 2\alpha - 2\sigma\right] p_A.$$

- Autonomous equations:

$$\frac{dX}{d\alpha} = \frac{1}{3}Z^2(X+1) + X\left\{3(X^2-1) + \frac{1}{2}Y^2\right\},$$

$$\frac{dY}{d\alpha} = (Y+\lambda)\left\{3(X^2-1) + \frac{1}{2}Y^2\right\} + \frac{1}{3}YZ^2 + \left(\rho + \frac{\lambda}{2}\right)Z^2,$$

$$\frac{dZ}{d\alpha} = Z\left[3(X^2-1) + \frac{1}{2}Y^2 - \rho Y + 1 - 2X + \frac{1}{3}Z^2\right].$$

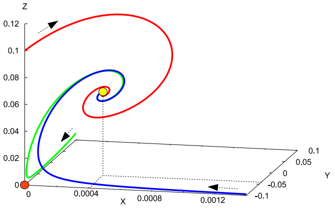
KSW model: few main points

- Anisotropic fixed point as solutions of $dX/d\alpha = dY/d\alpha = dZ/d\alpha = 0$:

$$X = \frac{2\left(\lambda^2 + 2\rho\lambda - 4\right)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}; \quad Y = -\frac{12\left(\lambda + 2\rho\right)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8},$$

$$Z^2 = \frac{18\left(\lambda^2 + 2\rho\lambda - 4\right)\left(-\lambda^2 + 4\rho\lambda + 12\rho^2 + 8\right)}{\left(\lambda^2 + 8\rho\lambda + 12\rho^2 + 8\right)^2}.$$

- This fixed point is equivalent to the anisotropic power-law solution.
- Taking exponential perturbations: $\delta X,\ \delta Y,\ \delta Z \sim \exp[\omega\alpha]$. Can show that all $\omega < 0$, e.g., the fixed point is stable. It can also shown to be attractive.



Attractor behavior of the anisotropic fixed point with $\rho = 50,\ \lambda = 0.1$ [taken from JCAP12(2010)024].

Noncanonical extensions of KSW model: DBI model

- Action of Dirac-Born-Infeld model [Silverstein & Tong, PRD70(2004)103505; Alishahiha, Silverstein & Tong, PRD70(2004)123505; Do & Kao PRD84(2011)123009]:

$$S_{\mathrm{DBI}} = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{\tilde{f}(\phi)}\frac{\gamma-1}{\gamma} - V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right].$$

- The Lorentz factor $\gamma = 1/\sqrt{1+\tilde{f}(\phi)\,\partial_\mu\phi\partial^\mu\phi} \geq 1$.
- Power-law solution with the choice $\tilde{f}(\phi) \sim \exp[-\lambda\phi]$:

$$\zeta = \frac{\lambda^2+8\rho\lambda+12\rho^2+8\gamma_0}{6\lambda(\lambda+2\rho)};\ \eta = \frac{\lambda^2+2\rho\lambda-4\gamma_0}{3\lambda(\lambda+2\rho)}.$$

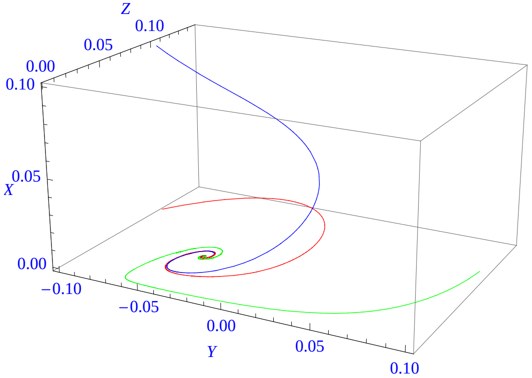
- The corresponding fixed point:

$$X = \frac{2\left[\hat{\gamma}_0\lambda(\lambda+2\rho)-4\right]}{\hat{\gamma}_0(\lambda^2+8\lambda\rho+12\rho^2)+8};\ Y = -\frac{12\hat{\gamma}_0(\lambda+2\rho)}{\hat{\gamma}_0(\lambda^2+8\lambda\rho+12\rho^2)+8};$$

$$Z^2 = \frac{18\left[\hat{\gamma}_0\lambda(\lambda+2\rho)-4\right]\left[\hat{\gamma}_0\left(-\lambda^2+4\lambda\rho+12\rho^2\right)+8\right]}{\left[\hat{\gamma}_0(\lambda^2+8\lambda\rho+12\rho^2)+8\right]^2};\ \hat{\gamma}_0 = \gamma_0^{-1}.$$

Non-canonical extensions of KSW model: DBI model

The attractor behavior of anisotropic fixed point with $\rho = 50,\ \lambda = 0.1$:



Noncanonical extensions of KSW model: SDBI model

- Action of supersymmetric DBI (SDBI) model [Sasaki, Yamaguchi & Yokoyama, PLB718(2012)1; Do & Kao, CQG33(2016)085009]:

$$S_{\mathrm{SDBI}} = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{\tilde{f}(\phi)}\frac{\gamma-1}{\gamma} - \Sigma_0^2\, V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right],$$

$$\Sigma_0(\gamma) = [(\gamma+1)/(2\gamma)]^{1/3} \leq 1.$$

- Power-law solution:

$$\zeta = \frac{N - \sqrt{N^2 - 4MP}}{2M};\ \eta = -\zeta + \frac{\rho}{\lambda} + \frac{1}{2},$$

$$M = 18\lambda^2\left(\gamma_0^2 - 1\right) \geq 0,$$

$$N = 3\lambda\left(\gamma_0+1\right)\left[\lambda\left(5\gamma_0+1\right) + 6\rho\left(\gamma_0+1\right)\right] \geq 0,$$

$$P = \left(\gamma_0+1\right)\left[\lambda^2\left(2\gamma_0+1\right) + 2\lambda\rho\left(5\gamma_0+7\right) + 12\rho^2\left(\gamma_0+2\right)\right] + 8\gamma_0\left(5\gamma_0+1\right) \geq 0.$$

Noncanonical extensions of KSW model: SDBI model

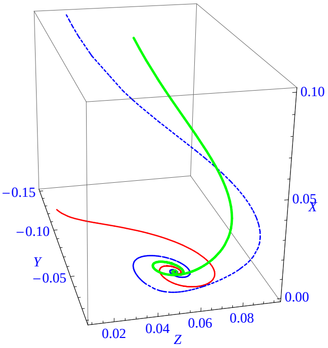
- During the inflationary phase with $\rho \gg \lambda$:

$$\zeta \simeq (1+\delta)\frac{\rho}{\lambda};\ \eta \simeq \frac{1}{2}-\frac{\rho}{\lambda}\delta;\ \gamma_0 = 1+3\delta.$$

- Constraint for δ (or for γ_0) (related to the positivity of potential) :

$$\delta < \frac{\lambda}{3\rho} \to \gamma_0 - 1 = 3\delta < \frac{\lambda}{\rho} \ll 1.$$

- Note that γ_0 can arbitrarily be larger than 1 in DBI model. This is a main difference between DBI and SDBI models.

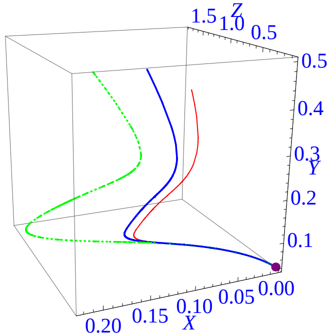


Attractor behavior of the anisotropic fixed point in SDBI model ($\rho = 50$, $\lambda = 0.1$, $\gamma = 1.0001$).

Noncanonical extensions of KSW model: covariant Galileon model

Action of covariant Galileon model [Deffayet, Esposito-Farese & Vikman, PRD79(2009)084003; Kobayashi, Yamaguchi & Yokoyama, PRL105(2010)231302; Do & Kao, PRD96(2017)023529]:

$$S_{\rm G} = \int d^4x\sqrt{g}\left\{\frac{1}{2}R + k_0 \exp\left[\tau\phi\right] X - g_0 \exp\left[\lambda\phi\right] X\Box\phi - \frac{f_0^2}{4}\exp\left[-2\rho\phi\right] F_{\mu\nu}F^{\mu\nu}\right\}.$$



Attractor behavior of the anisotropic fixed point (purple) in the Galileon model ($\rho = 50,\ \lambda = 0.1,\ k_0 = -3\rho^2/2$).

The conformal-violating Maxwell theory

- The conformal-violating Maxwell theory [Dolgov, PRD48(1993)2499; Bamba & Sasaki, JCAP02(2007)030; Demozzi, Mukhanov & Rubinstein, JCAP08(2009)025]:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + X - V(\phi) - \frac{1}{4}I(\phi, R, X, \ldots)F_{\mu\nu}F^{\mu\nu}\right].$$

- $I(\phi, R, X, \ldots)$ a function of any field of interest.
- The Maxwell theory with $I = 1$ is conformally invariant:

$$x \to x' : \; g'_{\mu\nu}(x') = \Omega(x)g_{\mu\nu}(x)$$
$$\to \int d^4x' F'_{\mu\nu}(x')F'^{\mu\nu}(x') = \int d^4x F_{\mu\nu}(x)F^{\mu\nu}(x) \sim \text{invariant !}.$$

- It is known that the conformal invariance must be broken to generate non-trivial large-scale galactic electromagnetic fields [Drummond & Hathrell, PRD22(1980)343; Turner & Widrow, PRD37(1988)2743; Ratra, AJ391(1992)L1; Dolgov, PRD48(1993)2499].
- The appearance of non-trivial function I does break the conformal invariance since $I'[\phi(x'), R(x'), X(x'), \ldots] \neq I[\phi(x), R(x), X(x), \ldots]$.
- For example, $I = \exp[\phi]$ was proposed by Ratra AJ391(1992)L1 as a natural origin of large-scale galactic electromagnetic field in the present universe.

Quick Observations

- The KSW model acts as a subclass of the conformal-violating Maxwell theory with $I = f^2(\phi)$.
- The cosmic no-hair seems to be violated generally in the KSW model, even when the scalar field ϕ is non-canonical, due to the existence of the unusual coupling $f^2(\phi)F^{\mu\nu}F_{\mu\nu}$.
- A close relation exists between the stable spatial anisotropy of inflationary universe and the broken conformal invariance? The mechanism of the broken conformal invariance should induce both non-trivial magnetic field and a stable spatial anisotropy of spacetime during an inflationary phase?

A proposed model: $J^2(X)F^2$ model

- Propose a subclass of conformal-violating Maxwell theory with $I = J^2(X)$ [Do & Kao, EPJC78(2018)360; see also Holland, Kanno & Zavala, PRD97(2018)103534]

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + X - V(\phi) - \frac{1}{4}J^2(X)F_{\mu\nu}F^{\mu\nu}\right].$$

with $X \equiv -\partial_\mu\phi\partial^\mu\phi/2$.

- Potentials:

$$V(\phi) = V_0 \exp[\lambda\phi]; \; J(X) = J_0 X^n.$$

- Power-law solution:

$$\zeta = \frac{6n^2+3n-1}{12n} + \frac{\sqrt{\lambda^2(36n^4+36n^3+5n^2-2n+1)+32n}}{12\lambda n},$$

$$\eta = \frac{6n^2+3n+1}{12n} - \frac{\sqrt{\lambda^2(36n^4+36n^3+5n^2-2n+1)+32n}}{12\lambda n}.$$

- For expanding solutions with $\zeta+\eta>0$ and $\zeta-2\eta>0$:

$$n > (\sqrt{5}-1)/4 \approx 0.309.$$

- For inflationary solutions with $\zeta+\eta \gg 1$ and $\zeta - 2\eta \gg 1$:

$$n \gg 1 \to \zeta \simeq n \gg 1.$$

$J^2(X)F^2$ model: comparison with $f^2(\phi)F_{\mu\nu}F^{\mu\nu}$ model

- Choose $n = 40$ and $\lambda = 1$ *vs* $\rho = 40$ and $\lambda = 1$

$$\zeta \simeq 40.4974, \ \eta \simeq 0.0026 \ \textit{vs} \ \zeta_{\rm KSW} = 40.1831, \ \eta_{\rm KSW} = 0.3169.$$

- While $\zeta \simeq \zeta_{\rm KSW}$, the *anisotropy parameter in the proposed model is much smaller than that of the KSW model*

$$\frac{\eta_{\rm KSW}}{\eta} \simeq 122.$$

- The anisotropic inflation model with the coupling $J^2(X)F^2$ seems to be more consistent with the observational data than the KSW model with the coupling $f^2(\phi)F^2$. [See also Holland, Kanno & Zavala, PRD97(2018)103534]

$J^2(X)F^2$ model: Stability during inflationary phase

- Consider the power-law perturbations:

$$\delta\alpha = A_\alpha t^m, \ \delta\sigma = A_\sigma t^m, \ \delta\phi = A_\phi t^m.$$

- Perturbation equations:

$$\mathcal{D}\begin{pmatrix} A_\alpha \\ A_\sigma \\ A_\phi \end{pmatrix} \equiv \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{pmatrix} A_\alpha \\ A_\sigma \\ A_\phi \end{pmatrix} = 0,$$

with

$$A_{11} = \left(\lambda n v - \frac{6}{\lambda}\right) m - 4\lambda n v\left(\zeta + 4\eta - 4n - 1\right); \ A_{12} = 4\lambda n v m - 4\lambda n v\left(\zeta + 4\eta - 4n - 1\right),$$

$$A_{13} = -\left[\frac{\lambda^2 v n}{2}(4n+1) - 1\right] m^2$$
$$-\lambda^2 v\left[8n^3 - 2\left(\zeta + 4\eta - 2\right) n^2 - \frac{1}{2}\left(\zeta + 4\eta - 1\right) n - \frac{1}{\lambda^2 v}\left(3\zeta - 1\right)\right] m + \lambda^2 u,$$

$$A_{21} = m^2 + \left(6\zeta - 1\right) m + \frac{2v}{3}\left(6n+1\right); \ A_{22} = \frac{2v}{3}\left(6n+1\right); \ A_{23} = -\frac{\lambda n v}{3}\left(6n+1\right) m - \lambda u,$$

$$A_{31} = 3\eta m + \frac{4}{3}v; \ A_{32} = m^2 + \left(3\zeta - 1\right) m + \frac{4}{3}v; \ A_{33} = -\frac{2\lambda}{3} n v m.$$

$J^2(X)F^2$ model: Stability during inflationary phase

- Nontrivial solutions exist only when $\det \mathcal{D} = \mathbf{0}$, or equivalently,

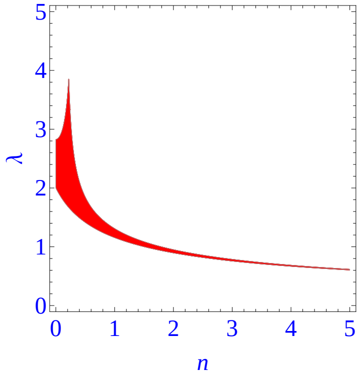
$$mf(m) \equiv m\left(a_6 m^5 + a_5 m^4 + a_4 m^3 + a_3 m^2 + a_2 m + a_1\right) = 0.$$

with

$$\begin{aligned} a_6 &= -\frac{1}{2}\left(4\lambda^2 v n^2 + \lambda^2 v n - 2\right), \\ a_1 &= 2\lambda^2 u v \Big[8\left(3\zeta - 3\eta - 1\right) n^2 - 2\left(3\zeta^2 + 9\zeta\eta - 12\eta^2 - 7\zeta + 2\eta + v + 2\right) n \\ &\quad +5\zeta - \eta - 1 - \frac{4}{\lambda^2}\Big]. \end{aligned}$$

- During the inflationary phase, $a_6 < 0$ and $a_1 > 0 \rightarrow f(m) = 0$ has at least one positive root $\rightarrow$ *the anisotropic inflationary solution is unstable* $\rightarrow$ consistent with the cosmic no-hair conjecture (in contrast to the KSW model).

$J^2(X)F^2$ model: Stability during slowly expanding phase



The red region represents the λ-n domain for the existence of stable slowly expanding solutions, in which all real parts of the five roots of $f(m) = 0$ are non-positive, i.e., $\mathrm{Re}(m_i) \leq 0$.

$J^2(X)F^2$ model: Stability during slowly expanding phase

- Dynamical variables:

$$\hat{X} = \frac{\dot{\sigma}}{\dot{\alpha}};\ Y = \frac{\dot{\phi}}{\dot{\alpha}};\ Z = \frac{J_0^{-1} p_A}{\dot{\alpha}} \left(\frac{\dot{\phi}^2}{2} \right)^{-n} \exp\left[-2\alpha - 2\sigma\right].$$

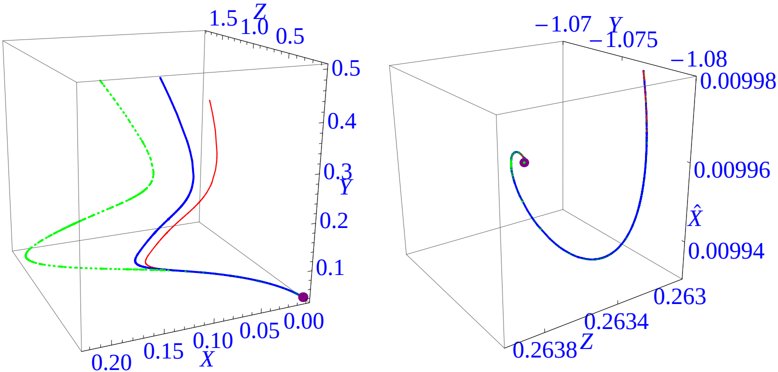
- Fixed point:

$$\hat{X} = -\frac{\lambda^2 (2n+1)\left(18n^2 + 9n - 1\right) + 16}{2\left[\lambda^2(2n+1) + 8\right]} - \frac{3\lambda (2n+1)\sqrt{\lambda^2 \left(36n^4 + 36n^3 + 5n^2 - 2n + 1\right) + 32n}}{2\left[\lambda^2(2n+1) + 8\right]},$$

$$Y = -\frac{4\left(\hat{X} + 1\right)}{\lambda(2n+1)},$$

$$Z^2 = -3\hat{X}\left[\frac{2(\hat{X}+1)}{2n+1} - 3\right].$$

$J^2(X)F^2$ model: Stability during slowly expanding phase



The attractor behavior of the anisotropic fixed point with the field parameters chosen as $\lambda = 1.25$ and $n = 1$.

Conclusions

- **Q**: Is the cosmic no-hair conjecture valid in conformal-violating Maxwell models?
  A: NO. It seems that the cosmic no-hair conjecture is generally violated in conformal-violating Maxwell models.
- A close relation exists between the stable spatial anisotropy of inflationary universe and the broken conformal invariance. The mechanism of the broken conformal invariance should induce both non-trivial magnetic field and a stable spatial anisotropy of spacetime during an inflationary phase.
- What if the cosmic no-hair conjecture is violated in other sub-classes of the conformal-violating Maxwell theory, e.g., $I = I(R)$, $I = I(R_{\mu\nu})$, $I = I(G)$?
- Which subclass of the conformal-violating Maxwell theory is mostly consistent with observational data of Planck and WMAP?
- Observational signatures of anisotropic inflation in CMB, e.g., TB and EB correlations? [Watanabe, Kanno, & Soda, arXiv:1003.0056; Chen, Emami, Firouzjahi, & Wang, arXiv:1404.4083]

Thank you all for your attention !