Open problems of the Skyrme model and the proton radius puzzle

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Overview

- Baryons in the framework of the Skyrme model
- Suggested corrections
- Proton radius puzzle
- Suggested solutions

The Skyrme model

QCD: For low energies effectively described by a theory of SU(2)-fields

$$S[U] = \int d^4x \; \frac{f_\pi^2}{4}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + m_\pi^2 f_\pi^2 \left(\mathrm{tr}(U) - 2\right) + \\ + \frac{1}{32e^2}\mathrm{tr}\left([\partial_\mu U, \partial_\nu U^\dagger][\partial^\mu U, \partial^\nu U^\dagger]\right)$$

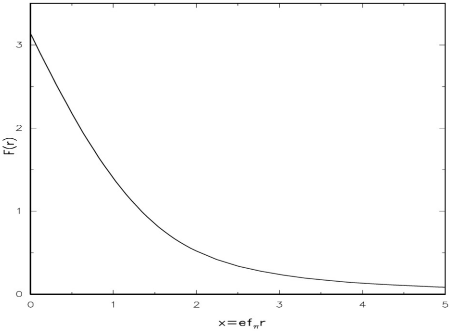
with $U(x) = \exp\left(\frac{i}{f_\pi}\pi^a(x)\tau_a\right)$

Skyrmions

Soliton of the Skyrme model ("Skyrmion"):

$$U_S(x) = \exp\left(i \frac{x^i}{|x|} \tau_i \cdot F(r)\right)$$

$\rightarrow$ localized by function $F(r)$, minimizing the energy



Rotating Skyrmion

Baryons correspond to "rotating" Skyrmions:

$$U_S(x) \to U_{rot}(t,x) = A^\dagger(t) U_S(x) A(t) = U_S(R_A(t)x)$$

With Cayley-Klein parameters $A(t) = a^0(t)\mathbb{1} + ia^i(t)\tau_i$ with $\sum_{b=0}^3 (a^b)^2 = 1$, the classical kinetic action becomes

$$S_{kin}[a] = \int dt\ 2\lambda \cdot \sum_{b=0}^{3} (\dot{a}^b)^2, \quad \lambda \approx \frac{53}{e^3 f_\pi}$$

Quantization

$\rightarrow$ Hamiltonian $\hat{H} = M_S - \frac{1}{8\lambda}\Delta_{S^3}$ with eigenfunctions $|i,j;n\rangle \doteq (a^i + ia^j)^n$

$$\Rightarrow E_{ijn} = \langle i,j;n|\hat{H}|i,j;n\rangle = M_S + \frac{n(n+2)}{8\lambda}$$

n=1 corresponds to nucleons:

$$|p\uparrow\rangle = \frac{1}{\pi}(a^1 + ia^2) \qquad |p\downarrow\rangle = -\frac{1}{\pi}(a^0 - ia^3)$$

$$|n\uparrow\rangle = \frac{1}{\pi}(a^0 + ia^3) \qquad |n\downarrow\rangle = -\frac{1}{\pi}(a^1 - ia^2)$$

Experimental data

Quantity	Prediction (this model)	Experiment
M_N	938.9 MeV (input)	938.9 MeV
M_Δ	1232 MeV (input)	1232 MeV
m_π	138 MeV (input)	138 MeV
F_π	108 MeV	186 MeV
$\langle r^2 \rangle^{1/2}_{I=0}$	0.68 fm	0.72 fm
$\langle r^2 \rangle^{1/2}_{I=1}$	1.04 fm	0.88 fm
$\langle r^2 \rangle^{1/2}_{M,I=0}$	0.95 fm	0.81 fm
$\langle r^2 \rangle^{1/2}_{M,I=1}$	1.04 fm	0.80 fm
μ_p	1.97	2.79
μ_n	−1.24	−1.91
$\left\lvert \frac{\mu_p}{\mu_n} \right\rvert$	1.59	1.46
g_A	0.65	1.23
$g_{\pi NN}$	11.9	13.5
$g_{\pi N\Delta}$	17.8	20.3
$\mu_{N\Delta}$	2.3	3.3
σ	49 MeV	36 ± 20 MeV

Source: *G. S. Adkins, C. R. Nappi, Nucl. Phys. B 233, 109 (1984)*

Suggested corrections

Deformation of the Skyrmion

- due to relativistic effects caused by the rotation
- as a result of "centrifugal forces"

Correction of the mass

- due to quantum effects

Relativistic rotation: idea

H. Hata and T. Kikuchi (2010): Equations of motion of the rotation ensure those of the original field theory
$\Rightarrow$ higher order terms in $R(t)$:

$$U_{rot}(x,t) = U_S(y(t,x))$$
$$y = (1 + A(r)(\dot{R}R^{-1}x)^2 + r^2 B(r)\,Tr(R^{-1}\dot{R})^2)R^{-1}x +$$
$$+ r^2 C(r)(R^{-1}\dot{R})^2 R^{-1}x + \ldots$$

Relativistic rotation: numerical results

	Prediction (this paper)	Prediction (Ref. [3])	Experiment
f_π	125 MeV	108 MeV	186 MeV
$\langle r^2 \rangle^{1/2}_{I=0}$	0.59 fm	0.68 fm	0.81 fm
$\langle r^2 \rangle^{1/2}_{I=1}$	1.17 fm	1.04 fm	0.94 fm
$\langle r^2 \rangle^{1/2}_{M,I=0}$	0.85 fm	0.95 fm	0.82 fm
$\langle r^2 \rangle^{1/2}_{M,I=1}$	1.17 fm	1.04 fm	0.86 fm
μ_p	1.65	1.97	2.79
μ_n	−0.99	−1.24	−1.91
$\lvert\mu_p/\mu_n\rvert$	1.67	1.59	1.46
g_A	0.58	0.65	1.24

Remark: [3] refers to the results of Adkins & Nappi
Source: *H. Hata, T. Kikuchi, Phys. Rev. D 82, 025017 (2010) [arXiv:hep-th/1002.2464]*

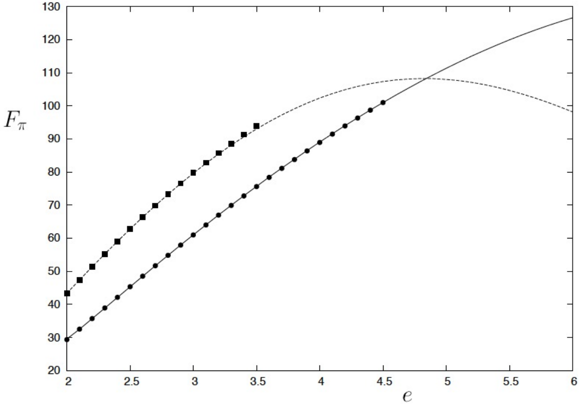
Spinning Skyrmion: idea

Battye, Krusch, Sutcliffe (2005): Allow the Skyrmion to deform and break spherical symmetry by the ansatz

$$U_S(x) \to U_{rot}(t,r,\phi,z) = \psi_3 \mathbb{I} + i\psi_2\tau_3 + \\ + i\psi_1(\cos(\phi+\omega t)\tau_1 + \sin(\phi+\omega t)\tau_2)$$

with $\psi_i = \psi_i(\rho, z)$, $|\vec{\psi}| = 1$ and $\vec{\psi} \to (0,0,1)$ as $\rho^2 + z^2 \to \infty$
Minimizing energy $\to$ condition $\frac{e^2 F_\pi^2}{4}\omega^2 \leq m_\pi^2$ (production of pions)

Spinning Skyrmion: numerical results



$$m_\pi = 138 MeV$$

Source: *R. Battye, S.Krusch, P. Sutcliffe, Phys.Lett. B626 (2005) 120-126*

Mass Correction due to Casimir energy

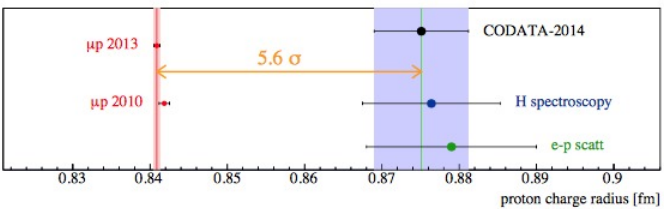
Idea: Change in mode functions by presence of Skyrmion $\Rightarrow$ change in vacuum energy $\Rightarrow$ change of rest mass
Moussallam (1992): Taking into account 6^{th}-order terms one finds

$$M_N = M_{cl} + M_{Cas} + M_{ct} = 1103 MeV$$

Remark: Here F_π was used as input parameter

Proton radius puzzle

Problem: Value of the proton charge radius obtained by measurements involving muonic hydrogen differ significantly from earlier results
$\rightarrow$ cannot be explained by previous corrections



Source: *J. Krauth, K.Schuhmann et al., arXiv:1706.00696v2 (2017)*

Influence of extra dimensions

Dahia, Lemos (2016): n small extra dimensions $\rightarrow$ small scale gravitational potential $\propto \frac{1}{R^{n+1}} >> \frac{1}{R}$ for small enough R

$\Rightarrow$ energy level of muonic hydrogen shifted by

$$\delta E_S^g \approx -\gamma_n \frac{G_n m_\rho m_\mu}{\sigma_{n-2}} |\psi(0)|^2 (1 - \frac{3r_p}{2a_o})$$

$\rightarrow$ number and size of extra dimensions can be chosen in agreement with the data and upper limits from different experiments

Electrophobic scalar: idea

D. Tucker-Smith, I. Yavin (2010): add new field ϕ interacting with the fermions via

$$\mathcal{L}_{int} \propto g_f \phi \bar{f} f$$

$\Rightarrow$ change in the energy of the $2S$-$2P$-transition of muonic hydrogen $\delta E_\phi \propto \frac{\alpha}{a_\mu^3} \frac{g_\mu g_p}{e^2 m_\phi^2} f(a_\mu m_\phi) \leftrightarrow \delta E_p \propto \frac{\alpha}{a_\mu^3} \langle r_p^2 \rangle$

Side effect: Also capable of explaining the $(g-2)_\mu$ puzzle!

Electrophobic scalar: numerical results

Y. Liu, D. McKeen, G. Miller (2016): Taking into account numerous experiments constrains m_ϕ as well as corresponding coupling contants:

m_ϕ (MeV)	$\lvert\epsilon_e\rvert$	ϵ_μ	ϵ_p	ϵ_n
0.13	$< 2.0 \times 10^{-6}$	$1.29(18) \times 10^{-3}$	3.0×10^{-3}	-2.0×10^{-3} to 2.8×10^{-7}
1	$< 2.6 \times 10^{-6}$	$1.30(18) \times 10^{-3}$	$1.60(37) \times 10^{-3}$	-1.7×10^{-3} to 2.0×10^{-4}
10	$< 7.6 \times 10^{-8}$	$1.40(20) \times 10^{-3}$	$2.37(54) \times 10^{-2}$	-2.9×10^{-2} to 9.1×10^{-3}
73	$< 9.1 \times 10^{-8}$ 3.3×10^{-6} to 1.8×10^{-3}	$1.96(27) \times 10^{-3}$	0.39	-0.29 to 5.6×10^{-4}

Remark: Here the convention $\epsilon_f = \frac{g_f}{e}$ is used
Source: *Y. Liu, D. McKeen, G. Miller Phys. Rev. Lett. 117, 101801 (2016) [arXiv:hep-th/1002.2464]*

Gravitoweak unification

R. Onofrio (2013): Weak interactions as short-distance manifestations of gravity?
→ main idea: coupling strength $G_F \propto G_N^{ren}$ on small enough scales, effectve potential interpolating these regimes:

$$V_{eff}(r) = \frac{-G_N m_1 m_2}{r}\left[1 + \left(\frac{G_N^{ren}}{G_N} - 1\right)e^{-r/\Lambda_P^{ren}}\right]$$

→ shift in the energy levels of the muonic hydrogen estimated to be of right order of magnitude