Gauged two Higgs doublet model: theoretical and phenomenological constraints

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Overview

- Motivation
- The Gauged 2 Higgs Doublet Model
- Matter content of the G2HDM
- The Scalar Potential
- Theoretical constraints
- Higgs phenomenology
- Final Remarks

Motivation

A few years after the discovery of the Higgs boson, we can still ask some questions

- is it *"the"* Higgs or is it *"a"* Higgs?
- What about dark matter candidates?
- And neutrino masses (oscillation)?
- Are there any extra symmetries?

A popular class of models that aims to solve some of these problems is the two Higgs doublet model.

Motivation: Two Higgs Doublet models

Simple extensions that explain BSM physics

- MSSM: 2 Higgs doublets due to superpotential.
- 2HDM: the prototype, extra CP phases.
- IHDM: dark matter candidate, no FCNC at tree level.
- G2HDM: more details ahead...

The G2HDM

Main feature: the two Higgs doublets are embedded in a doublet of an extra $SU(2)_H$

- ✓ New gauge group $SU(2)_H \times U(1)_X$
- ✓ This approach simplifies the potential for the doublets.
- ✓ The additional complex vector fields are neutral
- ✓ Anomaly free through the addition of heavy fermions.
- ✓ One of the Higgs can be inert $\longrightarrow$ DM candidate.
- ! The cost is the introduction of new scalars: An $SU(2)_H$ triplet and a doublet.

Matter content of the G2HDM

Matter Fields	$SU(3)_C$	$SU(2)_L$	$SU(2)_H$	$U(1)_Y$	$U(1)_X$
$Q_L = (u_L \;\; d_L)^T$	3	2	1	1/6	0
$U_R = \left(u_R \;\; u_R^H\right)^T$	3	1	2	2/3	1
$D_R = \left(d_R^H \;\; d_R\right)^T$	3	1	2	$-1/3$	-1
u_L^H	3	1	1	2/3	0
d_L^H	3	1	1	$-1/3$	0
$L_L = (\nu_L \;\; e_L)^T$	1	2	1	$-1/2$	0
$N_R = \left(\nu_R \;\; \nu_R^H\right)^T$	1	1	2	0	1
$E_R = \left(e_R^H \;\; e_R\right)^T$	1	1	2	-1	-1
ν_L^H	1	1	1	0	0
e_L^H	1	1	1	-1	0

NOT relevant for today! (for more information check 1512.00229)

Scalar content of the G2HDM

Relevant for today:

Matter Fields	$SU(3)_C$	$SU(2)_L$	$SU(2)_H$	$U(1)_Y$	$U(1)_X$
$H = (H_1 \quad H_2)^T$	1	2	2	1/2	1
$\Delta_H = \begin{pmatrix} \frac{\Delta_3}{2} & \frac{\Delta_p}{\sqrt{2}} \\ \frac{\Delta_m}{\sqrt{2}} & -\frac{\Delta_3}{2} \end{pmatrix}$	1	1	3	0	0
$\Phi_H = (\Phi_1 \quad \Phi_2)^T$	1	1	2	0	1

The Scalar Potential

$$V\left(H,\Delta_H,\Phi_H\right) = V(H) + V(\Phi_H) + V(\Delta_H) + V_{\mathrm{mix}}\left(H,\Delta_H,\Phi_H\right) \;,$$

where

$$\begin{aligned}
V(H) &= \mu_H^2\left(H^{\dagger\alpha i}H_{\alpha i}\right) + \lambda_H\left(H^{\dagger\alpha i}H_{\alpha i}\right)^2 \\
&\quad + \frac{1}{2}\lambda'_H\epsilon_{\alpha\beta}\epsilon^{\gamma\delta}\left(H^{\dagger\alpha i}H_{\gamma i}\right)\left(H^{\dagger\beta j}H_{\delta j}\right) \;, \\
V(\Phi_H) &= \mu_\Phi^2\Phi_H^\dagger\Phi_H + \lambda_\Phi\left(\Phi_H^\dagger\Phi_H\right)^2, \\
V(\Delta_H) &= \; -\mu_\Delta^2\mathrm{tr}\left(\Delta_H^\dagger\Delta_H\right) \; + \lambda_\Delta\left[\mathrm{tr}\left(\Delta_H^\dagger\Delta_H\right)\right]^2 \;,
\end{aligned}$$

The Scalar Potential

$$
\begin{aligned}
V_{\text{mix}}(H, \Delta_H, \Phi_H) = \; & + M_{H\Delta}\left(H^\dagger \Delta_H H\right) - M_{\Phi\Delta}\left(\Phi_H^\dagger \Delta_H \Phi_H\right) \\
& + \lambda_{H\Phi}\left(H^\dagger H\right)\left(\Phi_H^\dagger \Phi_H\right) + \lambda'_{H\Phi}\left(H^\dagger \Phi_H\right)\left(\Phi_H^\dagger H\right) \\
& + \lambda_{H\Delta}\left(H^\dagger H\right) \operatorname{tr}\left(\Delta_H^\dagger \Delta_H\right) + \lambda_{\Phi\Delta}\left(\Phi_H^\dagger \Phi_H\right) \operatorname{tr}\left(\Delta_H^\dagger \Delta_H\right) \; ,
\end{aligned}
$$

Theoretical constraints: Vacuum stability (VS)

Consider all the quartic terms in the scalar potential

$$
\begin{aligned}
V_4 \quad = \quad & +\lambda_H \left(H^\dagger H\right)^2 + \lambda'_H \left(-H_1^\dagger H_1 H_2^\dagger H_2 + H_1^\dagger H_2 H_2^\dagger H_1\right) \\
& +\lambda_\Phi \left(\Phi_H^\dagger \Phi_H\right)^2 + \lambda_\Delta \left(\mathrm{Tr}\left(\Delta_H^\dagger \Delta_H\right)\right)^2 \\
& +\lambda_{H\Phi} \left(H^\dagger H\right)\left(\Phi_H^\dagger \Phi_H\right) + \lambda'_{H\Phi} \left(H^\dagger \Phi_H\right)\left(\Phi_H^\dagger H\right) \\
& +\lambda_{H\Delta} \left(H^\dagger H\right) \mathrm{Tr}\left(\Delta_H^\dagger \Delta_H\right) + \lambda_{\Phi\Delta} \left(\Phi_H^\dagger \Phi_H\right) \mathrm{Tr}\left(\Delta_H^\dagger \Delta_H\right) \; .
\end{aligned}
$$

The challenge: Write V_4 as a matrix

Theoretical constraints: Vacuum stability (VS)

We introduce the following basis x, y, z and two ratios ξ and η,

$$
\begin{aligned}
x &\equiv H^\dagger H\,, \\
y &\equiv \Phi_H^\dagger \Phi_H\,, \\
z &\equiv \mathrm{Tr}\left(\Delta_H^\dagger \Delta_H\right)\,,
\end{aligned}
$$

$$
\xi \equiv \frac{\left(H^\dagger \Phi_H\right)\left(\Phi_H^\dagger H\right)}{\left(H^\dagger H\right)\left(\Phi_H^\dagger \Phi_H\right)}\,, \qquad 0 < \xi < 1
$$

$$
\eta \equiv \frac{\left(-H_1^\dagger H_1 H_2^\dagger H_2 + H_1^\dagger H_2 H_2^\dagger H_1\right)}{\left(H^\dagger H\right)^2}\,, \qquad -1 < \eta < 0.
$$

Theoretical constraints: Vacuum stability (VS)

Write V_4 as a quadratic form of x, y, z with parameters ξ and η

$$V_4 = \begin{pmatrix} x & y & z \end{pmatrix} \cdot \mathbf{Q}(\xi,\eta) \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} ,$$

with

$$\mathbf{Q}(\xi,\eta) = \begin{pmatrix} \widetilde{\lambda}_H(\eta) & \frac{1}{2}\widetilde{\lambda}_{H\Phi}(\xi) & \frac{1}{2}\lambda_{H\Delta} \\ \frac{1}{2}\widetilde{\lambda}_{H\Phi}(\xi) & \lambda_\Phi & \frac{1}{2}\lambda_{\Phi\Delta} \\ \frac{1}{2}\lambda_{H\Delta} & \frac{1}{2}\lambda_{\Phi\Delta} & \lambda_\Delta \end{pmatrix} ,$$

where $\quad \widetilde{\lambda}_H(\eta) \equiv \lambda_H + \eta\lambda'_H, \quad \widetilde{\lambda}_{H\Phi}(\xi) \equiv \lambda_{H\Phi} + \xi\lambda'_{H\Phi}.$

Theoretical constraints: Vacuum stability (VS)

x, y and z are always positive. From copositive criteria (K. Kannike, 1205.3781) the resulting conditions are

$$\widetilde{\lambda}_H(\eta) \geq 0\,, \quad \lambda_\Phi \geq 0\,, \quad \lambda_\Delta \geq 0\,,$$

$$\begin{aligned}
\Lambda_{H\Phi}(\xi,\eta) &\equiv \widetilde{\lambda}_{H\Phi}(\xi) + 2\sqrt{\widetilde{\lambda}_H(\eta)\lambda_\Phi} \geq 0\,, \\
\Lambda_{H\Delta}(\eta) &\equiv \lambda_{H\Delta} + 2\sqrt{\widetilde{\lambda}_H(\eta)\lambda_\Delta} \geq 0\,, \\
\Lambda_{\Phi\Delta} &\equiv \lambda_{\Phi\Delta} + 2\sqrt{\lambda_\Phi\lambda_\Delta} \geq 0\,,
\end{aligned}$$

$$\sqrt{\widetilde{\lambda}_H(\eta)\lambda_\Phi\lambda_\Delta} + \frac{1}{2}\left(\widetilde{\lambda}_{H\Phi}(\xi)\sqrt{\lambda_\Delta} + \lambda_{H\Delta}\sqrt{\lambda_\Phi} + \lambda_{\Phi\Delta}\sqrt{\widetilde{\lambda}_H(\eta)}\right) + \frac{1}{2}\sqrt{\Lambda_{H\Phi}(\xi,\eta)\Lambda_{H\Delta}(\eta)\Lambda_{\Phi\Delta}} \geq 0\,.$$

Theor. constraints: Perturbative Unitarity (PU)

The scattering amplitudes for the $2 \to 2$ processes have initial and final states belonging to

$$\left\{ \frac{hh}{\sqrt{2}}, \frac{G^0 G^0}{\sqrt{2}}, G^+ G^-, H_2^{0*} H_2^0, H^+ H^-, \frac{\phi_2 \phi_2}{\sqrt{2}}, \frac{G_H^0 G_H^0}{\sqrt{2}}, G_H^p G_H^m, \frac{\delta_3 \delta_3}{\sqrt{2}}, \Delta_p \Delta_m \right\}$$

The matrix of scattering amplitudes $\mathcal{M}_1$ is

$$\begin{pmatrix}
3\lambda_H & \lambda_H & \frac{2}{\sqrt{2}}\lambda_H & \frac{2}{\sqrt{2}}\lambda_H & \frac{2}{\sqrt{2}}\widetilde{\lambda}_H & \frac{1}{2}\lambda_{H\Phi} & \frac{1}{2}\lambda_{H\Phi} & \frac{1}{\sqrt{2}}\widetilde{\lambda}_{H\Phi} & \frac{1}{2}\lambda_{H\Delta} & \frac{1}{\sqrt{2}}\lambda_{H\Delta} \\
- & 3\lambda_H & \frac{2}{\sqrt{2}}\lambda_H & \frac{2}{\sqrt{2}}\lambda_H & \frac{2}{\sqrt{2}}\widetilde{\lambda}_H & \frac{1}{2}\lambda_{H\Phi} & \frac{1}{2}\lambda_{H\Phi} & \frac{1}{\sqrt{2}}\widetilde{\lambda}_{H\Phi} & \frac{1}{2}\lambda_{H\Delta} & \frac{1}{\sqrt{2}}\lambda_{H\Delta} \\
- & - & 4\lambda_H & 2\widetilde{\lambda}_H & 2\lambda_H & \frac{1}{\sqrt{2}}\lambda_{H\Phi} & \frac{1}{\sqrt{2}}\lambda_{H\Phi} & \widetilde{\lambda}_{H\Phi} & \frac{1}{\sqrt{2}}\lambda_{H\Delta} & \lambda_{H\Delta} \\
- & - & - & 4\lambda_H & 2\lambda_H & \frac{1}{\sqrt{2}}\widetilde{\lambda}_{H\Phi} & \frac{1}{\sqrt{2}}\widetilde{\lambda}_{H\Phi} & \lambda_{H\Phi} & \frac{1}{\sqrt{2}}\lambda_{H\Delta} & \lambda_{H\Delta} \\
- & - & - & - & 4\lambda_H & \frac{1}{\sqrt{2}}\widetilde{\lambda}_{H\Phi} & \frac{1}{\sqrt{2}}\widetilde{\lambda}_{H\Phi} & \lambda_{H\Phi} & \frac{1}{\sqrt{2}}\lambda_{H\Delta} & \lambda_{H\Delta} \\
- & - & - & - & - & 3\lambda_\Phi & \lambda_\Phi & \frac{2}{\sqrt{2}}\lambda_\Phi & \frac{1}{2}\lambda_{\Phi\Delta} & \frac{1}{\sqrt{2}}\lambda_{\Phi\Delta} \\
- & - & - & - & - & - & 3\lambda_\Phi & \frac{2}{\sqrt{2}}\lambda_\Phi & \frac{1}{2}\lambda_{\Phi\Delta} & \frac{1}{\sqrt{2}}\lambda_{\Phi\Delta} \\
- & - & - & - & - & - & - & 4\lambda_\Phi & \frac{1}{\sqrt{2}}\lambda_{\Phi\Delta} & \lambda_{\Phi\Delta} \\
- & - & - & - & - & - & - & - & 3\lambda_\Delta & \frac{2}{\sqrt{2}}\lambda_\Delta \\
- & - & - & - & - & - & - & - & - & 4\lambda_\Delta
\end{pmatrix}$$

Theor. constraints: Perturbative Unitarity (PU)

In both basis $\left\{hH_2^{0*}, G^0 H_2^{0*}, G^+ H^-\right\}$ and $\left\{hG^+, H_2^{0*} H^+, G^0 G^+\right\}$ we find the matrix

$$\mathcal{M}_2 = \begin{pmatrix} 2\lambda_H & 0 & \frac{\sqrt{2}}{2}\lambda'_H \\ 0 & 2\lambda_H & -\frac{i\sqrt{2}}{2}\lambda'_H \\ \frac{\sqrt{2}}{2}\lambda'_H & +\frac{i\sqrt{2}}{2}\lambda'_H & 2\lambda_H \end{pmatrix}$$

with eigenvalues $2\lambda_H$, $2\lambda_H \pm \lambda'_H$.

Theor. constraints: Perturbative Unitarity (PU)

And we have a series of individual amplitudes:

$$2\lambda_H : \quad hG^0 \longleftrightarrow hG^0 \,, \; G^+H^+ \longleftrightarrow G^+H^+ \,,$$

$$2\widetilde{\lambda}_H : \quad \begin{array}{c} hH^+ \longleftrightarrow hH^+ \\ G^0H^+ \longleftrightarrow G^0H^+ \\ H_2^{0*}G^+ \longleftrightarrow H_2^{0*}G^+ \end{array}$$

$$\mathcal{M}\left(G^+H_2^0 \longleftrightarrow hH^+\right) = \frac{1}{\sqrt{2}}\lambda'_H$$
$$\mathcal{M}\left(G^+H_2^0 \longleftrightarrow G^0H^+\right) = -\frac{i}{\sqrt{2}}\lambda'_H$$

$$2\lambda_\Phi : \quad \phi_2 G_H^0 \longleftrightarrow \phi_2 G_H^0 \,, \quad \begin{array}{c} \phi_2 G_H^p \longleftrightarrow \phi_2 G_H^p \,, \\ G_H^0 G_H^p \longleftrightarrow G_H^0 G_H^p \,, \end{array}$$

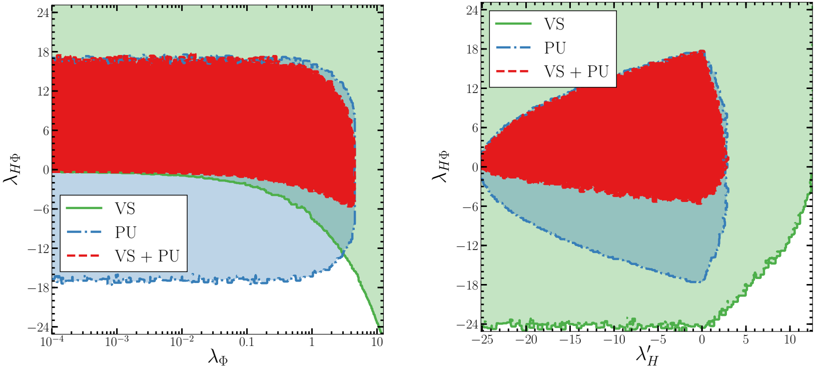
$$\vdots$$

Theor. constraints: Perturbative Unitarity (PU)

In summary, perturbative unitarity limits the parameters to the following ranges:

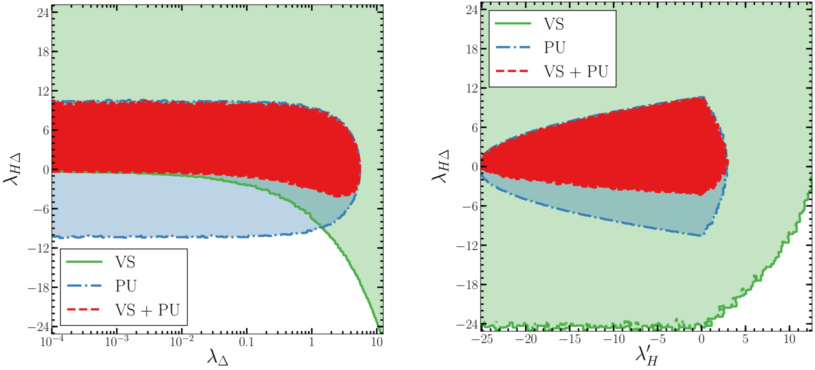
1. Between -4π and $+4\pi$:
   - the parameters λ_H, λ_Φ and λ_Δ
2. Between -8π and $+8\pi$:
   - the eigenvalues $\lambda_i(\mathcal{M}_1)$, for $i = 1, \ldots, 10$
   - the combinations $2\lambda_H \pm \lambda'_H$ and $\lambda_{H\Phi} + \lambda'_{H\Phi}$
   - the parameters $\lambda_{H\Phi}$, $\lambda_{H\Delta}$ and $\lambda_{\Phi\Delta}$
3. Between $-8\sqrt{2}\pi$ and $+8\sqrt{2}\pi$:
   - the parameters λ'_H and $\lambda'_{H\Phi}$

Theoretical constraints: Results



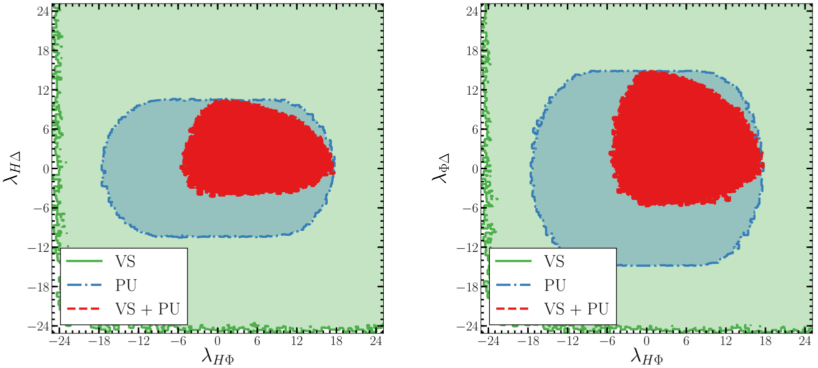
Green lower bound controlled by: $\widetilde{\lambda}_{H\Phi}(\xi) + 2\sqrt{\widetilde{\lambda}_H(\eta)\lambda_\Phi} \geq 0$

Theoretical constraints: Results



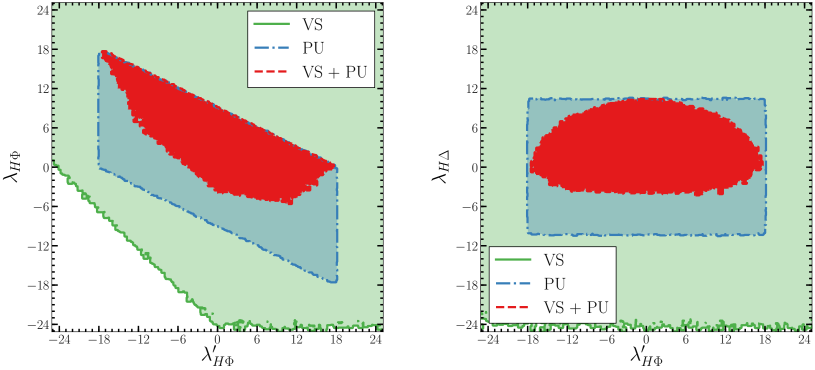
Green lower bound controlled by: $\lambda_{H\Delta} + 2\sqrt{\widetilde{\lambda}_H(\eta)\lambda_\Delta} \geq 0$

Theoretical constraints: Results



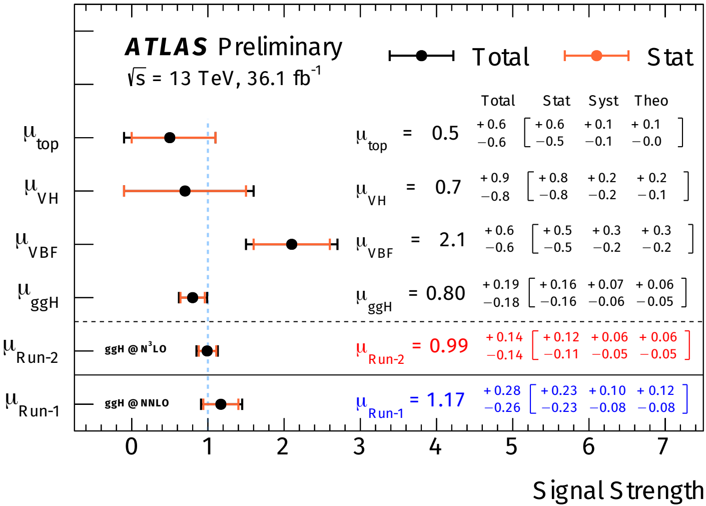
Non-diagonal couplings limited mostly by perturbative unitarity

Theoretical constraints: Results



Non-diagonal couplings limited mostly by perturbative unitarity

Higgs phenomenology



Taken from: arXiv:1802.04146, ATLAS Collaboration

Higgs phenomenology

The 125 GeV Higgs boson h_1 is a linear combination of h, ϕ_2 and δ_3:

$$h_1 = O_{11} h + O_{21} \phi_2 + O_{31} \delta_3,$$

Narrow Higgs decay width $\rightarrow$ Higgs production dominated by the resonance region $\rightarrow$ Approximate $\sigma(pp \rightarrow h_1 \rightarrow \gamma\gamma)$ by

$$\sigma\left(gg \rightarrow h_1 \rightarrow \gamma\gamma\right) = \frac{\pi^2}{8s\, m_{h_1}\, \Gamma_{h_1}} f_{gg}\left(\frac{m_{h_1}}{\sqrt{s}}\right) \Gamma\left(h_1 \rightarrow gg\right) \Gamma\left(h_1 \rightarrow \gamma\gamma\right)$$

Higgs phenomenology

$$\Gamma\,(h_1 \to \gamma\gamma) = \frac{G_F\,\alpha^2\,m_{h_1}^3\,O_{11}^2}{128\,\sqrt{2}\,\pi^3}\left|A_1(\tau_W) + \sum_f N_c Q_f^2 A_{1/2}(\tau_f)\right.$$

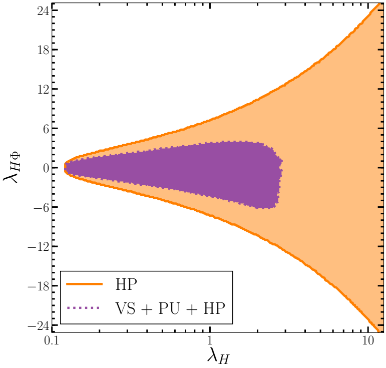
$$\left. + \; C_h \frac{\lambda_H v^2}{m_{H^\pm}^2} A_0(\tau_{H^\pm}) + \frac{O_{21}}{O_{11}}\frac{v}{v_\Phi}\sum_F N_c Q_F^2 A_{1/2}(\tau_F)\right|^2 ,$$

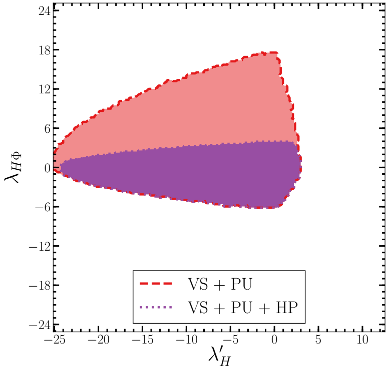
$$C_h = 1 + \frac{O_{21}}{O_{11}}\frac{\lambda_{H\Phi} v_\Phi}{2\lambda_H v} - \frac{O_{31}}{O_{11}}\frac{2\lambda_{H\Delta} v_\Delta + M_{H\Delta}}{4\lambda_H v} \; .$$

$$\Gamma(h_1 \to gg) = \frac{\alpha_s^2\,m_{h_1}^3\,O_{11}^2}{72\,v^2\,\pi^3}\left|\sum_f \frac{3}{4} A_{1/2}(\tau_f) + \frac{O_{21}}{O_{11}}\frac{v}{v_\Phi}\sum_F \frac{3}{4} A_{1/2}(\tau_F)\right|^2 .$$

Higgs phenomenology results

$v = 246$ TeV, $v_\Phi = 10$ TeV, $v_\Delta = [0.5, 20]$ TeV

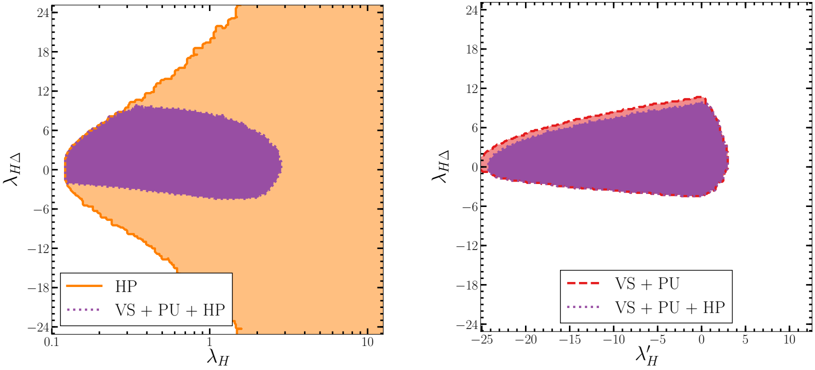




λ_H lower bound corresponds to the SM limit.

Higgs phenomenology results

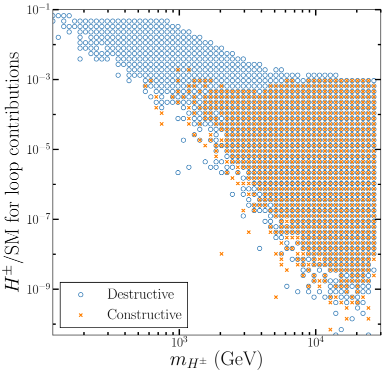
$$v = 246 \text{ TeV}, \quad v_\Phi = 10 \text{ TeV}, \quad v_\Delta = [0.5, 20] \text{ TeV}$$

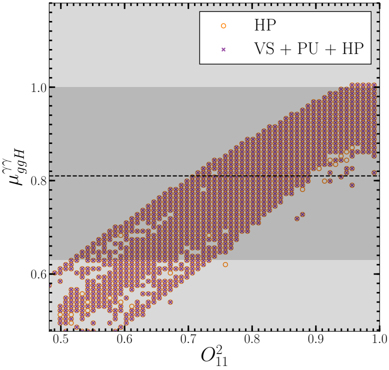


Higgs phenomenology constrains non-diagonal couplings symmetrically around the zero.

Higgs phenomenology results

$$v = 246 \text{ TeV}, \quad v_\Phi = 10 \text{ TeV}, \quad v_\Delta = [0.5, 20] \text{ TeV}$$

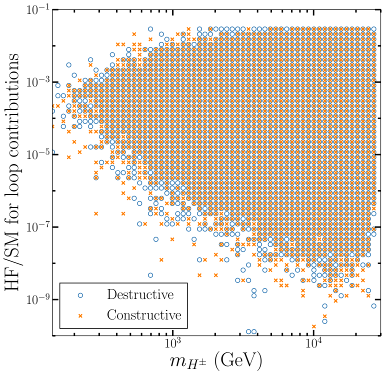


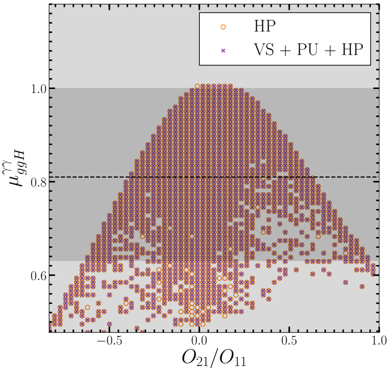


Light charged Higgs has the most (new physics) relevant contribution

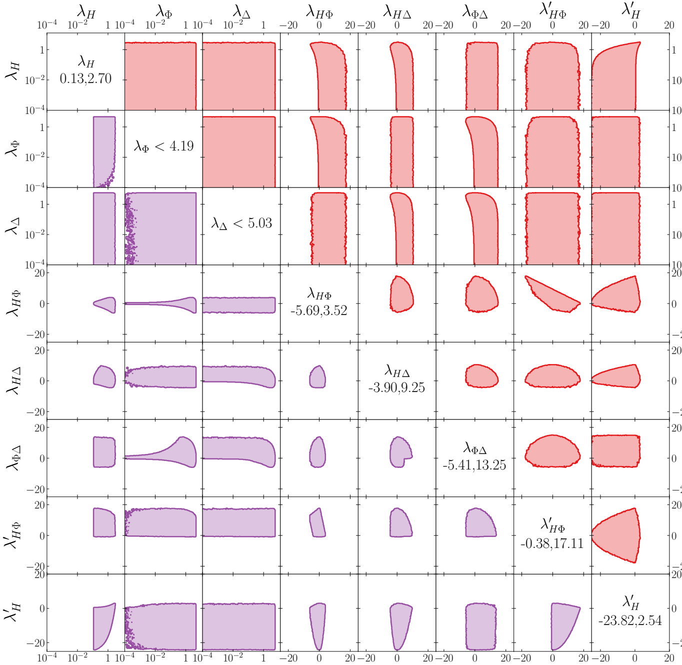
Higgs phenomenology results

$v = 246$ TeV, $v_\Phi = 10$ TeV, $v_\Delta = [0.5, 20]$ TeV





Small heavy fermion contribution. Shape controlled by overall O_{11}^2 and O_{21}/O_{11} factor from gluon side.



λ_H
λ_Φ
λ_Δ
$\lambda_{H\Phi}$
$\lambda_{H\Delta}$
$\lambda_{\Phi\Delta}$
$\lambda'_{H\Phi}$
λ'_H
λ_H 0.13,2.70
$\lambda_\Phi < 4.19$
$\lambda_\Delta < 5.03$
$\lambda_{H\Phi}$ -5.69,3.52
$\lambda_{H\Delta}$ -3.90,9.25
$\lambda_{\Phi\Delta}$ -5.41,13.25
$\lambda'_{H\Phi}$ -0.38,17.11
λ'_H -23.82,2.54

Final Remarks

- 2HDM represent a popular extension to the SM.
- Each variant has its own merits and failures.
- The G2HDM represents a new setup still requiring exploration.
- We found scalar parameter regions of this model that follows standard constraints.
- In the future we plan to extend this analysis to dark matter, the muon g−2, electroweak precision test and so on.

BACKUP

Symmetry Breaking

If $-\mu_\Delta^2 < 0$, then $SU(2)_H$ is spontaneously broken by the vev

$$\langle \Delta_3 \rangle = -v_\Delta \neq 0, \quad \langle \Delta_{p,m} \rangle = 0$$

Shift the triplet diagonal components

$$\Delta_H = \begin{pmatrix} \frac{-v_\Delta + \delta_3}{2} & \frac{1}{\sqrt{2}}\Delta_p \\ \frac{1}{\sqrt{2}}\Delta_m & \frac{v_\Delta - \delta_3}{2} \end{pmatrix}.$$

Symmetry Breaking

The quadratic terms for H_1 and H_2 have the following coefficients

$$|H_1|^2: \quad \mu_H^2 - \frac{1}{2}M_{H\Delta} \cdot v_\Delta + \frac{1}{2}\lambda_{H\Delta} \cdot v_\Delta^2 + \frac{1}{2}\lambda_{H\Phi} \cdot v_\Phi^2 \,,$$

$$|H_2|^2: \quad \mu_H^2 + \frac{1}{2}M_{H\Delta} \cdot v_\Delta + \frac{1}{2}\lambda_{H\Delta} \cdot v_\Delta^2 + \frac{1}{2}(\lambda_{H\Phi} + \lambda'_{H\Phi}) \cdot v_\Phi^2 \,,$$

Similarly, for Φ_1 and Φ_2

$$|\Phi_1|^2: \quad \mu_\Phi^2 + \frac{1}{2}M_{\Phi\Delta} \cdot v_\Delta + \frac{1}{2}\lambda_{\Phi\Delta} \cdot v_\Delta^2 + \frac{1}{2}(\lambda_{H\Phi} + \lambda'_{H\Phi}) \cdot v^2 \,,$$

$$|\Phi_2|^2: \quad \mu_\Phi^2 - \frac{1}{2}M_{\Phi\Delta} \cdot v_\Delta + \frac{1}{2}\lambda_{\Phi\Delta} \cdot v_\Delta^2 + \frac{1}{2}\lambda_{H\Phi} \cdot v^2 \,,$$

Symmetry Breaking

Shift the rest of the scalars

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+h}{\sqrt{2}} + i\frac{G^0}{\sqrt{2}} \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H^+ \\ H_2^0 \end{pmatrix},$$

$$\Phi_H = \begin{pmatrix} G_H^p \\ \frac{v_\Phi+\phi_2}{\sqrt{2}} + i\frac{G_H^0}{\sqrt{2}} \end{pmatrix}.$$

Symmetry Breaking

Substituting the VEVs in the potential V leads to

$$\begin{aligned} V(v, v_\Delta, v_\Phi) &= \frac{1}{4}\Big[\lambda_H v^4 + \lambda_\Phi v_\Phi^4 + \lambda_\Delta v_\Delta^4 + 2\left(\mu_H^2 v^2 + \mu_\Phi^2 v_\Phi^2 - \mu_\Delta^2 v_\Delta^2\right) \\ &\quad - \left(M_{H\Delta} v^2 + M_{\Phi\Delta} v_\Phi^2\right) v_\Delta + \lambda_{H\Phi} v^2 v_\Phi^2 + \lambda_{H\Delta} v^2 v_\Delta^2 + \lambda_{\Phi\Delta} v_\Phi^2 v_\Delta^2\Big] \end{aligned}$$

Minimization of the potential leads to the conditions:

$$\begin{aligned} v \cdot \left(2\lambda_H v^2 + 2\mu_H^2 - M_{H\Delta} v_\Delta + \lambda_{H\Phi} v_\Phi^2 + \lambda_{H\Delta} v_\Delta^2\right) &= 0\,, \\ v_\Phi \cdot \left(2\lambda_\Phi v_\Phi^2 + 2\mu_\Phi^2 - M_{\Phi\Delta} v_\Delta + \lambda_{H\Phi} v^2 + \lambda_{\Phi\Delta} v_\Delta^2\right) &= 0\,, \\ 4\lambda_\Delta v_\Delta^3 - 4\mu_\Delta^2 v_\Delta - M_{H\Delta} v^2 - M_{\Phi\Delta} v_\Phi^2 + 2 v_\Delta \left(\lambda_{H\Delta} v^2 + \lambda_{\Phi\Delta} v_\Phi^2\right) &= 0\,. \end{aligned}$$

Scalar Mass Spectrum

In the basis of $S = \{h, \phi_2, \delta_3\}$ it is given by

$$\mathcal{M}_0^2 = \begin{pmatrix} 2\lambda_H v^2 & \lambda_{H\Phi} v v_\Phi & \frac{v}{2}\left(M_{H\Delta} - 2\lambda_{H\Delta} v_\Delta\right) \\ - & 2\lambda_\Phi v_\Phi^2 & \frac{v_\Phi}{2}\left(M_{\Phi\Delta} - 2\lambda_{\Phi\Delta} v_\Delta\right) \\ - & - & \frac{1}{4v_\Delta}\left(8\lambda_\Delta v_\Delta^3 + M_{H\Delta} v^2 + M_{\Phi\Delta} v_\Phi^2\right) \end{pmatrix} .$$

This matrix can be diagonalized by a rotation with an orthogonal matrix O

$$O^T \cdot \mathcal{M}_0^2 \cdot O = \mathrm{Diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2) ,$$

Scalar Mass Spectrum

The second block is also 3×3. In the basis of $G = \{G_H^p, H_2^{0*}, \Delta_p\}$, it is given by

$$\mathcal{M}_0'^2 = \begin{pmatrix} M_{\Phi\Delta} v_\Delta + \frac{1}{2}\lambda'_{H\Phi} v^2 & \frac{1}{2}\lambda'_{H\Phi} v v_\Phi & -\frac{1}{2} M_{\Phi\Delta} v_\Phi \\ \frac{1}{2}\lambda'_{H\Phi} v v_\Phi & M_{H\Delta} v_\Delta + \frac{1}{2}\lambda'_{H\Phi} v_\Phi^2 & \frac{1}{2} M_{H\Delta} v \\ -\frac{1}{2} M_{\Phi\Delta} v_\Phi & \frac{1}{2} M_{H\Delta} v & \frac{1}{4 v_\Delta}\left(M_{H\Delta} v^2 + M_{\Phi\Delta} v_\Phi^2 \right) \end{pmatrix} .$$

One eigenvalue vanishes, corresponding to the physical Goldstone boson $\widetilde{G}_H^p$

Scalar Mass Spectrum

The other two eigenvalues are the masses of two physical fields $\widetilde{\Delta}$ and D. They are given by

$$M^2_{\widetilde{\Delta},D} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A} ,$$

with

$$\begin{aligned} A &= 8 v_\Delta , \\ B &= 2 \left[M_{H\Delta} \left(v^2 + 4 v_\Delta^2 \right) + M_{\Phi\Delta} \left(4 v_\Delta^2 + v_\Phi^2 \right) + 2 \lambda'_{H\Phi} v_\Delta \left(v^2 + v_\Phi^2 \right) \right] , \\ C &= \left(v^2 + v_\Phi^2 + 4 v_\Delta^2 \right) \left[M_{H\Delta} \left(\lambda'_{H\Phi} v^2 + 2 M_{\Phi\Delta} v_\Delta \right) + \lambda'_{H\Phi} M_{\Phi\Delta} v_\Phi^2 \right] . \end{aligned}$$

Scalar Mass Spectrum

The final block is 4×4 diagonal, giving

$$m^2_{H_2^\pm} = M_{H\Delta} v_\Delta - \frac{1}{2}\lambda'_H v^2 + \frac{1}{2}\lambda'_{H\Phi} v_\Phi^2 \, ,$$

for the physical charged Higgs $H_2^\pm$, and

$$m^2_{G^\pm} = m^2_{G^0} = m^2_{G^0_H} = 0 \, ,$$

for the three Goldstone boson fields $G^\pm$, G^0 and G^0_H.

Theoretical constraints: Results

Remember that this analysis uses an 8-dim parameter space

$\big(x, y, \{z_1\}\big)$ VS OK on green,

$\big(x, y, \{z_2\}\big)$ PU OK on blue,

$\big(x, y\big)$ on red only if there is $\{z\}$ that is both VS+PU OK.

For example:

$(\lambda_\Delta = 1,\ \lambda_{H\Delta} = -4,\ \lambda_H = 4\pi)$ is VS OK but no PU,

$(\lambda_\Delta = 1,\ \lambda_{H\Delta} = -4,\ \lambda_H = 0.13)$ is PU OK but no VS,