# FERMION MASS AND MIXING IN THE STANDARD MODEL EXTENSION WITH DISCRETE SYMMETRY

V. V. Vien

Tay Nguyen University, Buon Ma Thuot, Dak Lak, Vietnam,

Quy Nhon, August 5-11, 2018

# Outline

# 1. Motivation

■ **Why the Standard Model needs to be extended?**

The SM leaves many striking features:

✓ Why are neutrino masses so tiny?

✓ The mixing profile of neutrinos with $\theta_{13} \neq 0$ and Dirac CP phase $\delta_{CP}$,

✓ The quark mixings,

✓ The origin of the large mass and mixing hierarchies, ect.

## ■ The recent experimental data:

- Leptons [M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)]:

$$m_e \simeq 0.51\,\mathrm{MeV},\ m_\mu \simeq 105.65\mathrm{MeV},\ m_\tau \simeq 1776.86\mathrm{MeV},$$

$$\sin^2\theta_{12} = 0.307,\ \ \sin^2\theta_{13} = 0.0212,$$

$$\sin^2\theta_{23} = 0.421\,(\mathrm{IH},\mathrm{quad.I}),\ \sin^2\theta_{23} = 0.592\,(\mathrm{IH},\mathrm{quad.II}),$$

$$\sin^2\theta_{23} = 0.417\,(\mathrm{NH},\mathrm{quad.I}), \sin^2\theta_{23} = 0.597\,(\mathrm{NH},\mathrm{quad.II}),$$

$$\Delta m^2_{21} = 7.53\times 10^{-5}\mathrm{eV}^2,$$

$$\Delta m^2_{32} = -2.56\times 10^{-3}\mathrm{eV}^2(\mathrm{IH}),$$

$$\Delta m^2_{32} = 2.51\times 10^{-3}\mathrm{eV}^2\ \ (\mathrm{NH}).$$

- Quarks [M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)]:

$$m_u = 2.2\,\mathrm{MeV},\;\; m_d = 4.7\,\mathrm{MeV},\;\; m_s = 95\,\mathrm{MeV},$$

$$m_c = 1.275\,\mathrm{GeV},\;\; m_b = 4.18\,\mathrm{GeV},\;\; m_t = 173.0\,\mathrm{GeV},$$

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.999105 \end{pmatrix},$$

■ **There are different types of leptonic mixing pattern:**

✓ The bimaximal mixing: $\theta_{12} = 45^\circ, \theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$ in the standard parametrization.

✓ The tri-bimaximal mixing: $\theta_{12} = 35.3^\circ, \theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$.

✓ The democratic mixing: $\theta_{12} = 45^\circ$, $\theta_{23} = 54.7^\circ$, $\theta_{13} = 0^\circ$.

✓ The hexagonal mixing: $\theta_{12} = 30^\circ$, $\theta_{23} = 45^\circ$, and $\theta_{13} = 0^\circ$.

✓ The golden-ratio mixing:

$$\theta_{12} = \arctan\left(\frac{2}{1+\sqrt{5}}\right) \simeq 31.7^\circ,\ \theta_{23} = 45^\circ,\ \text{and}\ \theta_{13} = 0^\circ$$

✓ These mixing patterns can be considered as leading order approximations.

■ **Possibilities to extend the standard model**

✓ $\nu$MSM [T. Asaka, Mikhail Shaposhnikov, Phys.Lett.B 620:17-26, 2005]

✓ Two-Higgs-doublet model (Three, four Higgs,...) [G. C. Branco *et.al.*, Phys. Rep., 516, Iss. 1-2, 2012, 1-102.]

✓ Zee-babu model [A. Zee, Nucl. Phys. B 264, 99 (1986); K. S. Babu, Phys. Lett. B 203, 132 (1988)]

✓ $SU(5)$ model [H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974)]

✓ (Two, Three, Four, Five, Six) Zero Texture Fermion Mass Matrices[C. I. Low, Phys. Rev. D 70 (2004) 073013; H. Serodio, Phys. Rev. B 88 (2013) 056015]

✓ The 3-3-1 models, and so on.

⊛ These extensions do not provide a natural explanation for large mass splitting between neutrinos as well as lepton and quark mixing patterns.

# 2. The choice of the non-Abelian discrete group

- All of the above models can not provide a explanation for large mass splitting between neutrinos as well as lepton and quark mixing patterns.
- These problems can be understood on the basis of the class of discrete symmetries.

- There are three generations according to the Standard Model of particle physics. We should look for symmetry groups contain 1, 2 and 3- irreducible representations, or more than two 1- irreducible representations.

✓ $A_4$ : $\underline{1}$, $\underline{1}'$, $\underline{1}''$, $\underline{3}$.

✓ $S_4$ : $\underline{1}$, $\underline{1}'$, $\underline{2}$, $\underline{3}$, $\underline{3}'$.

✓ $T_7$ : $\underline{1}$, $\underline{1}'$, $\underline{1}''$ , $\underline{3}$, $\underline{3}^*$.

✓ $Q_6$: $\underline{1}$, $\underline{1}'$, $\underline{1}''$, $\underline{1}'''$, $\underline{2}$, $\underline{2}'$.

✓ e.c.t.

✓ $S_3$ : $\underline{1}$, $\underline{1}'$, $\underline{2}$.

✓ $D_4$ : $\underline{1}$, $\underline{1}'$, $\underline{1}''$, $\underline{1}'''$, $\underline{2}$.

✓ $\Delta(27)$: $\underline{1}_i$ $(i = 1, 2, ..., 9)$, $\underline{3}$, $\underline{\bar{3}}$.

✓ $D_5$: $\underline{1}_1$, $\underline{1}_2$, $\underline{2}_1$, $\underline{2}_2$.

# 3. Some models with discrete symmetries

## 3.1 The 3-3-1 models with discrete symmetry

✓ $S_3$: P.V. Dong, H.N. Long, C.H. Nam and V.V. Vien, Phys.Rev. D85 (2012) 053001; V.V. Vien and H.N. Long, J.Exp.Theor.Phys. 118 (2014) 6, 869-890;

✓ $D_4$: V.V. Vien and H.N. Long, Int.J.Mod.Phys. A28 (2013) 1350159;

✓ $S_4$: V.V. Vien and H.N. Long, AHEP 2014 (2014) 192536; V. V. Vien, H. N. Long and D. P. Khoi, Int.J.Mod.Phys. A30 (2015) 17, 1550102;

✓ $T_7$: V. V. Vien, H. N. Long, J. High Energy Phys. 04 (2014) 133; V. V. Vien, Mod. Phys. Lett. A 29, 28 (2014) 1450139;

✓ $A_4$: V. V. Vien and H. N. Long, Int. J. Mod. Phys. A, Vol. 30 (2015) 1550117;

✓ $D_4$: V. V. Vien, Mod.Phys.Lett.A, Vol. 29, No. 23 (2014) 1450122; V.V. Vien and H.N. Long, Int.J.Mod.Phys. A.2013; J.Korean Phys.Soc. 66 (2015) 12, 1809-1815.

✓ $\Delta(27)$: A. E. Carcamo Hernandez, H.N. Long, V.V. Vien, Eur.Phys.J. C76 (2016), 5, 242; V. V. Vien, A. E. Carcamo Hernandez, H.N. Long, Nucl.Phys. B913 (2016) 792-814.

## 3.2 The standard model extension with discrete symmetry

### SM extension with $S_4$

- The Clebsch-Gordan coefficients of $S_4$:

$$
\begin{aligned}
\underline{1} \otimes \underline{3} &= \underline{3}(11, 12, 13),\ \ \underline{2} \otimes \underline{2} = \underline{1}(12 + 21) \oplus .., \\
\underline{2} \otimes \underline{3} &= \underline{3}\left((1 + 2)1, \omega(1 + \omega 2)2, \omega^2(1 + \omega^2 2)3\right) \oplus ... \\
\underline{3}' \otimes \underline{3}' &= \underline{3} \otimes \underline{3} = \underline{1}(11 + 22 + 33) \\
&\oplus \underline{2}(11 + \omega^2 22 + \omega 33, 11 + \omega 22 + \omega^2 33) \\
&\oplus \underline{3}_s(23 + 32, 31 + 13, 12 + 21) \oplus ..., \\
\underline{3} \otimes \underline{3}' &= \underline{1}'(11 + 22 + 33) \\
&\oplus \underline{2}(11 + \omega^2 22 + \omega 33, -11 - \omega 22 - \omega^2 33) \oplus ...
\end{aligned}
$$

- Lepton contents of the model:

| Fields | $\psi_{1,2,3L}$ | $l_{1(2,3)R}$ | $\nu_R$ | $\phi$ | $\phi'$ | $\varphi$ | $\chi$ | $\zeta$ |
|---|---|---|---|---|---|---|---|---|
| $\mathrm{SU}(2)_L$ | 2 | 1 | 1 | 2 | 2 | 2 | 1 | 1 |
| $\mathrm{U}(1)_Y$ | $-1$ | $-2$ | 0 | 1 | 1 | 1 | 0 | 0 |
| $\mathrm{U}(1)_X$ | 1 | 1 | 0 | 0 | 0 | $-1$ | 0 | 0 |
| $S_4$ | $\underline{3}$ | $\underline{1}(\underline{2})$ | $\underline{3}$ | $\underline{3}$ | $\underline{3}'$ | $\underline{1}$ | $\underline{3}$ | $\underline{2}$ |

$$-\mathcal{L}_l = h_1(\bar{\psi}_L\phi)_{\underline{1}}l_{1R} + h_2(\bar{\psi}_L\phi)_{\underline{2}}l_R + h_3(\bar{\psi}_{iL}\phi')_{\underline{2}}l_R + \mathrm{H.c.}$$

$$-\mathcal{L}_\nu = \frac{x}{2}(\bar{\psi}_L\tilde{\varphi})_{\underline{3}}\nu_R + \frac{y}{2}(\bar{\nu}^c_R\chi)_{\underline{3}_s}\nu_R + \frac{M}{2}\bar{\nu}^c_R\nu_R + \frac{z}{2}(\bar{\nu}^c_R\zeta)_{\underline{3}}\nu_R + \mathrm{H.c.}\,,$$

- Main results:

$$m_e = \sqrt{3}h_1 v\,, \quad m_\mu = \sqrt{3}(h_2 v - h_3 v')\,, \quad m_\tau = \sqrt{3}(h_2 v + h_3 v')\,,$$

$$m_1 = \frac{1}{2}\left(B_1 + B_2 + \sqrt{(B_1+B_2)^2 + 4C^2}\right), \quad m_2 = A\,,$$

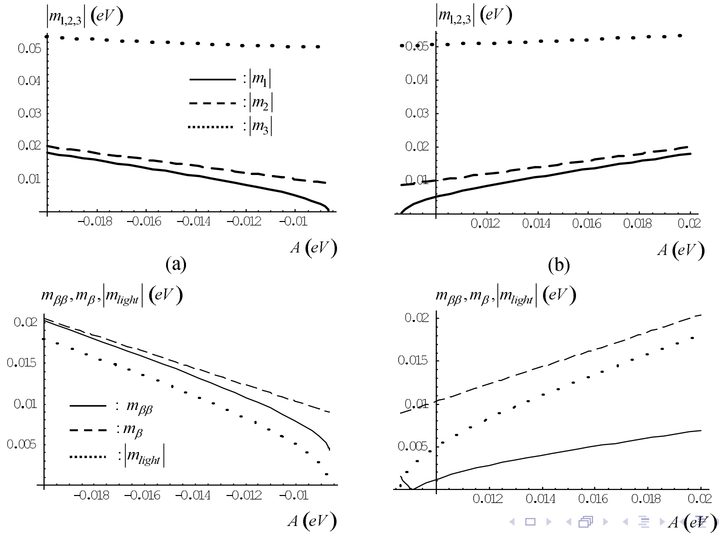
$$m_3 = \frac{1}{2}\left(B_1 + B_2 - \sqrt{(B_1+B_2)^2 + 4C^2}\right),$$

$$U_L = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad U_R = 1\,.$$

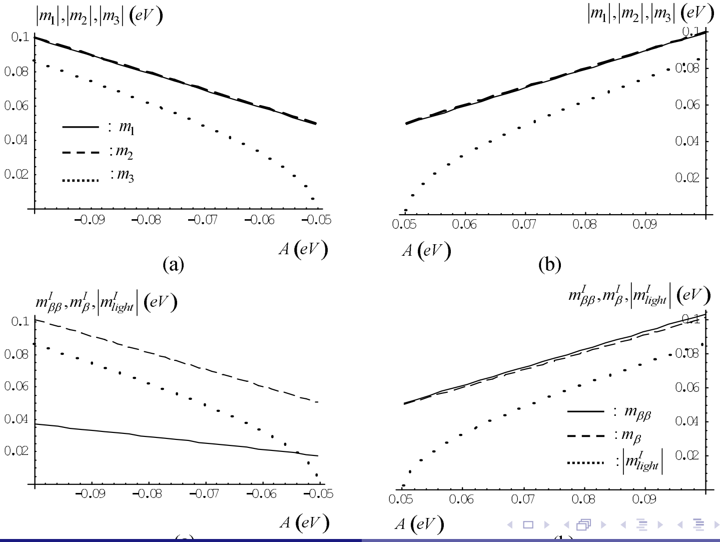
$$U_\nu = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{K^2+1}} & 0 & \frac{K}{\sqrt{K^2+1}} \\ -\frac{K}{\sqrt{K^2+1}} & 0 & \frac{1}{\sqrt{K^2+1}} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix},$$

$$U = U_L^\dagger U_\nu = \frac{1}{\sqrt{3}}\begin{pmatrix} \frac{1-K}{\sqrt{K^2+1}} & 1 & \frac{1+K}{\sqrt{K^2+1}} \\ \frac{\omega(\omega-K)}{\sqrt{K^2+1}} & 1 & \frac{\omega(1+K\omega)}{\sqrt{K^2+1}} \\ \frac{\omega(1-K\omega)}{\sqrt{K^2+1}} & 1 & \frac{\omega(\omega+K)}{\sqrt{K^2+1}} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix},$$

# Normal Hierarchy:



# Inverted Hierarchy:



## SM extension with $D_4$

- The Clebsch-Gordan coefficients of $D_4$:

$$
\begin{aligned}
\underline{1}'(1)\otimes\underline{1}'(1) &= \underline{1}''(1)\otimes\underline{1}''(1)=\underline{1}'''(1)\otimes\underline{1}'''(1)=\underline{1}(11),\\
\underline{1}(1)\otimes\underline{1}'(1) &= \underline{1}'(11),\ \underline{1}(1)\otimes\underline{1}''(1)=\underline{1}''(11),\\
\underline{1}(1)\otimes\underline{1}'''(1) &= \underline{1}'''(11),\ \underline{1}'(1)\otimes\underline{1}''(1)=\underline{1}'''(11),\\
\underline{1}''(1)\otimes\underline{1}'''(1) &= \underline{1}'(11),\ \underline{1}'''(1)\otimes\underline{1}'(1)=\underline{1}''(11),\\
\underline{1}(1)\otimes\underline{2}(1,2) &= \underline{2}(11,12),\ \underline{1}'(1)\otimes\underline{2}(1,2)=\underline{2}(11,-12),\\
\underline{1}''(1)\otimes\underline{2}(1,2) &= \underline{2}(12,11),\ \underline{1}'''(1)\otimes\underline{2}(1,2)=\underline{2}(-12,11),\\
\underline{2}(1,2)\otimes\underline{2}(1,2) &= \underline{1}(11+22)\oplus\underline{1}'(11-22)\\
&\oplus \underline{1}''(12+21)\oplus\underline{1}'''(12-21).
\end{aligned}
$$

- Quark contents of the model:

| | $Q_L(Q_{3L})$ | $u_R(u_{3R})$ | $d_R(d_{3R})$ | $\phi\,(\phi',\phi'')$ | $\phi_i$ |
|---|---|---|---|---|---|
| $\mathrm{SU}(2)_L$ | 2 | 1 | 1 | 2 | 2 |
| $\mathrm{U}(1)_Y$ | 1/3 | 4/3 | −2/3 | 1 | 1 |
| $\mathrm{D}_4$ | $\underline{2}\,(\underline{1})$ | $\underline{2}\,(\underline{1})$ | $\underline{2}\,(\underline{1})$ | $\underline{1}\,(\underline{1}',\underline{1}'')$ | $\underline{2}$ |

$$\begin{aligned}
-\mathcal{L}_q \;=\; & h_3^u\bar{Q}_{3L}\widetilde{\phi}u_{3R}+h^u(\bar{Q}_L\widetilde{\phi})_{\underline{2}}u_R+h'^u(\bar{Q}_L\widetilde{\phi'})_{\underline{2}}u_R\\
+\; & h''^u(\bar{Q}_L\widetilde{\phi''})_{\underline{2}}u_R+k_1^u\bar{Q}_L\widetilde{\phi}_iu_{3R}+k_2^u\bar{Q}_{3L}\widetilde{\phi}_iu_R\\
+\; & h_3^d\bar{Q}_{3L}\phi d_{3R}+h^d(\bar{Q}_L\phi)_{\underline{2}}d_R+h'^d(\bar{Q}_L\phi')_{\underline{2}}d_R\\
+\; & h''^d(\bar{Q}_L\phi'')_{\underline{2}}d_R+k_1^d\bar{Q}_L\phi_id_{3R}+k_2^d\bar{Q}_{3L}\phi_id_R+H.C,
\end{aligned}$$

- Main results:

$$M_u = \begin{pmatrix} h^u v + h'^u v' & h''^u v'' & k_1^u v_1 \\ h''^u v'' & h^u v - h'^u v' & k_1^u v_2 \\ k_2^u v_1 & k_2^u v_2 & h_3^u v \end{pmatrix}, \; M_d = \begin{pmatrix} h^d v + h'^d v' & h''^d v'' & k_1^d v_1 \\ h''^d v'' & h^d v - h'^d v' & k_1^d v_2 \\ k_2^d v_1 & k_2^d v_2 & h_3^d v \end{pmatrix}$$

$$M_u = M_u^0 + \delta M_u, \quad M_d = M_d^0 + \delta M_d,$$

$$\delta M_u = \begin{pmatrix} 0 & 0 & k_1^u v_1 \\ 0 & 0 & k_1^u v_2 \\ k_2^u v_1 & k_2^u v_2 & 0 \end{pmatrix}, \; \delta M_d = \begin{pmatrix} 0 & 0 & k_1^d v_1 \\ 0 & 0 & k_1^d v_2 \\ k_2^d v_1 & k_2^d v_2 & 0 \end{pmatrix}.$$

$$M_u^0 = \begin{pmatrix} h^u v + h'^u v' & h''^u v'' & 0 \\ h''^u v'' & h^u v - h'^u v' & 0 \\ 0 & 0 & h_3^u v \end{pmatrix}, \; M_d^0 = \begin{pmatrix} h^d v + h'^d v' & h''^d v'' & 0 \\ h''^d v'' & h^d v - h'^d v' & 0 \\ 0 & 0 & h_3^d v \end{pmatrix}$$

- At tree level:

$$m_u^0 = h^u v + \sqrt{(h'^u v')^2 + (h''^u v'')^2},$$

$$m_c^0 = h^u v - \sqrt{(h'^u v')^2 + (h''^u v'')^2},\ m_t^0 = h_3^u v$$

$$m_d^0 = h^d v - \sqrt{(h'^d v')^2 + (h''^d v'')^2},$$

$$m_s^0 = h^d v + \sqrt{(h'^d v')^2 + (h''^d v'')^2},\ m_b^0 = h_3^d v,$$

$$V_{0L}^u = V_{0R}^u = \begin{pmatrix} s & -c & 0 \\ c & s & 0 \\ 0 & 0 & 1 \end{pmatrix},\ V_{0L}^d = V_{0R}^d = \begin{pmatrix} s' & -c' & 0 \\ c' & s & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$V_{CKM}^0 = V_{0L}^u V_{0L}^{d+} = \begin{pmatrix} cc' + ss' & -c's + s'c & 0 \\ c's - s'c & cc' + ss' & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

- At the first order of perturbation:

$$V_L^u = \begin{pmatrix} s & -c & -k_1^u(sx_1 + cx_2) \\ c & s & k_1^u(-cx_1 + sx_2) \\ k_2^u x_1 & -k_2^u x_2 & 1 \end{pmatrix},$$

$$V_L^d = \begin{pmatrix} s' & -c' & -k_1^d(s'y_1 + c'y_2) \\ c' & s' & k_1^d(-c'y_1 + s'y_2) \\ k_2^d y_1 & -k_2^d y_2 & 1 \end{pmatrix},$$

$$V_{CKM} = V_L^u V_L^{d+}.$$

## Model and experimental values of CKM matix elements

| Observable | Wolfenstein parametrization[5] | Model value | Experimental value[5] |
|---|---|---|---|
| $\|V_{11}\|$ | 0.974696 | 0.97434 | $0.97434^{+0.00011}_{-0.00012}$ |
| $\|V_{12}\|$ | 0.22496 | 0.22506 | $0.22506 \pm 0.00050$ |
| $\|V_{13}\|$ | 0.003618 | 0.0035 | $0.00357 \pm 0.00015$ |
| $\|V_{21}\|$ | 0.22496 | 0.22477 | $0.22492 \pm 0.00050$ |
| $\|V_{22}\|$ | 0.974696 | 0.97143 | $0.97351 \pm 0.00013$ |
| $\|V_{23}\|$ | 0.04165 | 0.0411 | $0.0411 \pm 0.0013$ |
| $\|V_{31}\|$ | 0.008688 | 0.00484 | $0.00875^{+0.00032}_{-0.00033}$ |
| $\|V_{32}\|$ | 0.04165 | 0.04243 | $0.0403 \pm 0.0013$ |
| $\|V_{33}\|$ | 1.0 | 0.99915 | $0.99915 \pm 0.00005$ |
| $J$ | $3.06378 \times 10^{-5}$ | $2.916 \times 10^{-5}$ | $(3.04^{+0.21}_{-0.20}) \times 10^{-5}$ |

Model and experimental values of quark masses

| | Model value | Experimental value[5] |
|---|---|---|
| $m_u$ (MeV) | 2.2 | $2.2^{+0.6}_{-0.4}$ |
| $m_c$ (GeV) | 1.27 | $1.27 \pm 0.03$ |
| $m_t$ (GeV) | 173.21 | $173.21 \pm 0.51 \pm 0.71$ |
| $m_d$ (MeV) | 4.7 | $4.7^{+0.5}_{-0.4}$ |
| $m_s$ (MeV) | 96 | $96^{+8}_{-4}$ |
| $m_b$ (GeV) | 4.18 | $4.18^{+0.04}_{-0.03}$ |

The main references:

[1] V.V. Vien, Int.J.Mod.Phys. A31 (2016), 09, 1650039.

[2] V. V. Vien and H. N. Long, Phys. Atom. Nucl. Vol. 81, No. 6, 2018.

# 4. Conclusions

(1) The non-Abelian discrete groups play an important role in model building of particle physics. Indeed, they overcome some of the limitations of the previous model such as fermion masses and mixings,...

# 4. Conclusions

(1) The non-Abelian discrete groups play an important role in model building of particle physics. Indeed, they overcome some of the limitations of the previous model such as fermion masses and mixings,...

(2) When SM model is supplemented by a discrete symmetry, it can fit the most recent data on fermion masses and mixing with non-zero $\theta_{13}$ and gives a remarkable prediction of Dirac CP phase and the quarks have consistent masses and a realistic quark mixing matrix.

THANK YOU FOR YOUR ATTENTION!