Curing strong coupling in the Higgs inflation with R^2-term

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'Windows on the Universe'

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Outline

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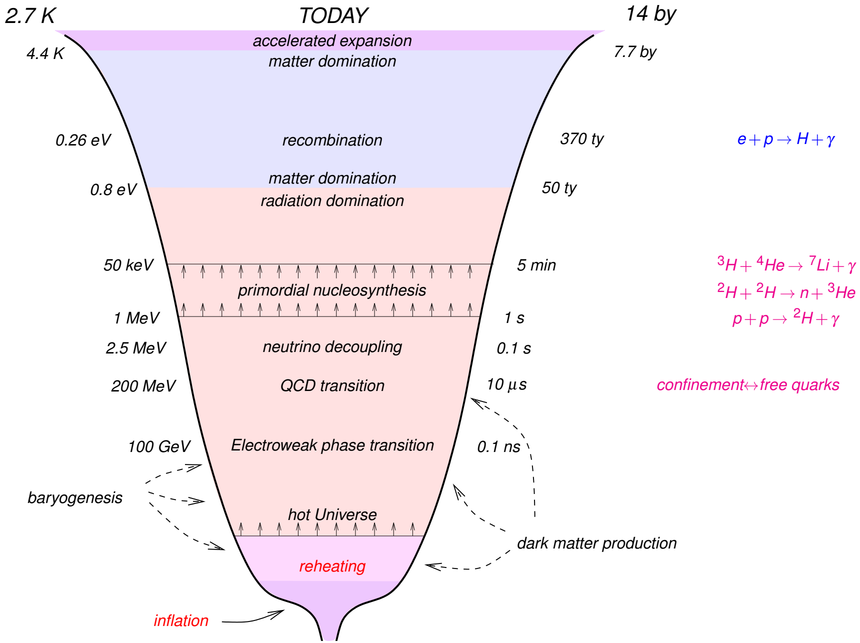
Big Bang within GR and SM: problems

- Dark Matter
- Baryogenesis
- Dark Energy $0 \neq \Lambda \ll M_{Pl}^4,\ M_W^4,\ \Lambda_{QCD}^4,$ etc ?
- Coincidence problems:

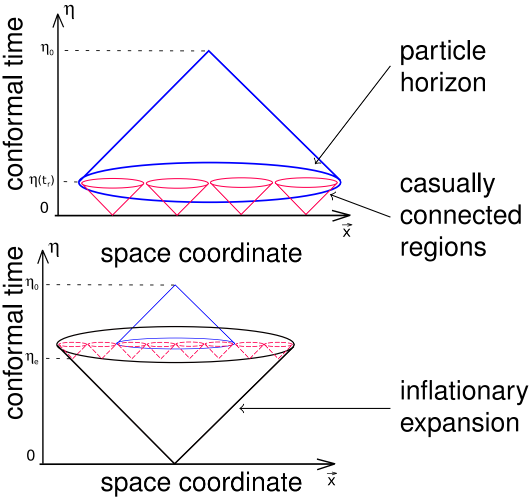
$$\Omega_B \sim \Omega_{DM} \sim \Omega_\Lambda\ ,$$
$$\eta_B = n_B/n_\gamma \sim (\delta T/T)^2\ ,$$
$$T_d^n \sim (m_n - m_p)\ ,$$
$$\ldots$$

- ΛCDM tensions: lack of dwarfs? cusps?
- Horizon, Enthropy, Flatness, ...problems
  $l_{H_0}/l_{\mathrm{H},r}(t_0) \sim \sqrt{1+z_r} \simeq 30$
- Singularity at the beginning
- Heavy relics
- Initial fluctuations $\delta T/T \sim \delta\rho/\rho \sim 10^{-4}$, scale-invariant


2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
matter domination
0.8 eV
50 ty
radiation domination
50 keV
5 min
$^3H + {}^4He \to {}^7Li + \gamma$
primordial nucleosynthesis
$^2H + {}^2H \to n + {}^3He$
1 MeV
1 s
$p + p \to {}^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement$\leftrightarrow$free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Inflationary solution of Hot Big Bang problems

- no initial singularity in dS space
- all scales grow exponentially, including the radius of the 3-sphere
  the Universe becomes exponentially flat
- any two particles are at exponentially large distances
  no heavy relics
  no traces of previous epochs!
- no particles in post-inflationary Universe
  to solve entropy problem we need post-inflationary reheating



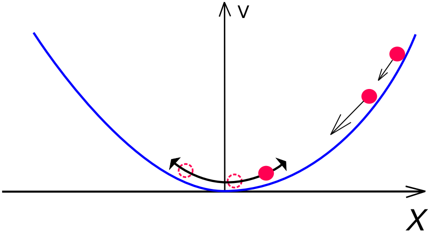
Chaotic inflation at large fields

Chaotic inflation, A.Linde (1983), A.Linde (1984)

in all domains of Planck size

each of the form of inflaton energy fluctuates similarly

$$\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i \phi)^2 \sim V(\phi) \lesssim M_{\rm Pl}^4$$



If $V(\phi)$ dominates by chance

$$\ddot{\phi} - \Delta\phi/a^2 + 3H\dot{\phi} + V'(\phi) = 0$$

for power-law potential at $\phi > M_{Pl}$

$$V \simeq \text{const}$$

"slow roll" solution

$$H^2 = \frac{8\pi}{3M_P^2} V(\phi)\,, \quad a(t) \propto e^{Ht}$$

The idea is great,

but is not verifiable

except the flatness. . .

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Unexpected bonus: generation of perturbations

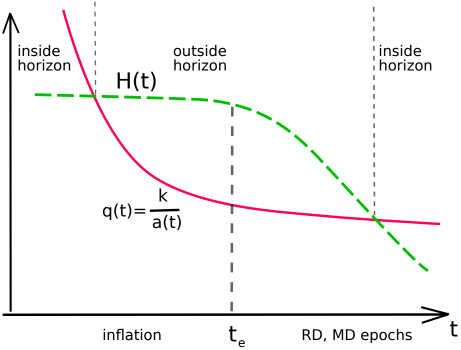
Quantum fluctuations of wavelength λ of a free massless field φ (inflaton and gravitons !!)
have 3-momenta $q \simeq 1/\lambda$ and an amplitudes of $\delta\varphi_\lambda \simeq q$

Evolution at inflation

- inside horizon: $q > H$
  $q \propto 1/a \Rightarrow$ $\delta\varphi_\lambda \propto q \propto 1/a$

- outside horison: $q < H$
  $q \propto a \Rightarrow$ $\delta\varphi_\lambda = \text{const} = H_{infl}/2\pi$!!!

- got "classical" fluctuations:
  $$\delta\varphi_\lambda = \delta\varphi_\lambda^{quantum} \times e^{N_e}$$



scalar modes $\delta\phi_\lambda \sim H_{infl}$

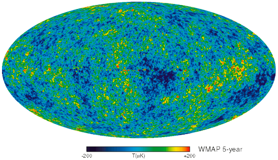
tensor modes $\delta g_{\mu\nu} \sim h \sim H_{infl}/M_{Pl}$

Later at normal stage

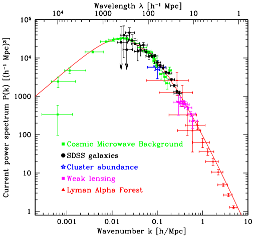
$H \propto 1/t$, $q/H \nearrow$, modes "enter horizon"

Inflationary solution of Hot Big Bang problems

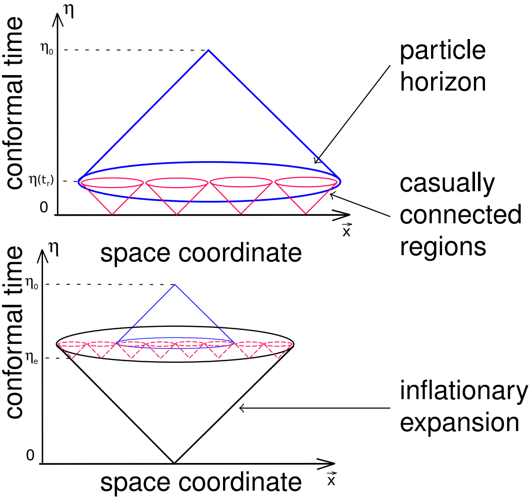
Temperature fluctuations $\delta T/T \sim 10^{-5}$



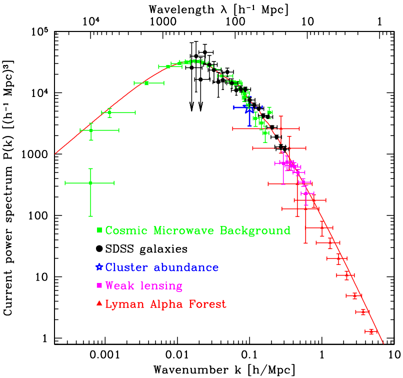
Universe is uniform!

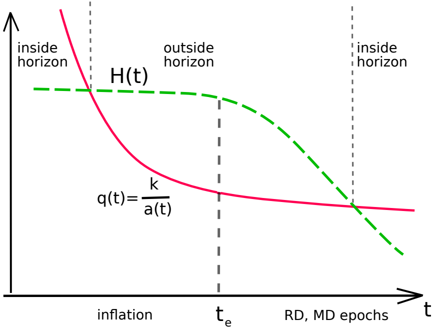


$\delta\rho/\rho \sim 10^{-5}$



Probing the matter power spectrum





$$\delta\phi \to \frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot\phi} \propto \frac{V^{3/2}}{V'}\,, \qquad h \sim \frac{H}{M_{Pl}} \propto V^{1/2}$$

$$A_S \to \frac{V^{3/2}}{V'}\,, \qquad n_S \to \frac{V''}{V}\,, \left(\frac{V'}{V}\right)^2\,, \qquad r \equiv \frac{A_T}{A_S} \to \left(\frac{V'}{V}\right)^2$$

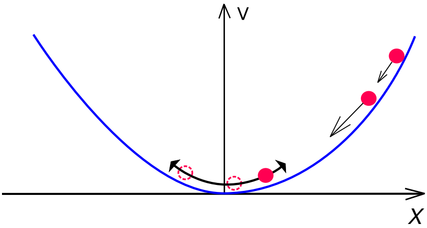
Chaotic inflation: simple realization

$$S = \int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{(\partial_\mu X)^2}{2} - \beta X^4\right)$$

$$\ddot{X} + 3H\dot{X} + V'(X) = 0$$

$$H^2 = \frac{1}{M_P^2}V(X)\;, \qquad a(t) \propto e^{Ht}$$

Chaotic inflation, A.Linde (1983)



slow roll conditions get satisfied at
$X_e > M_{Pl}$ $\qquad M_P^2 = M_{Pl}^2/(8\pi)$

generation of scale-invariant scalar (and tensor) perturbations from exponentially stretched quantum fluctuations of X $\Longrightarrow$

$\delta\rho/\rho \sim 10^{-5}$ requires
$V = \beta X^4 : \beta \sim 10^{-13}$

We have scalar in the SM! The Higgs field!

In a unitary gauge $H^T = \left(0, (h+v)/\sqrt{2}\right)$ (and neglecting $v = 246\,\text{GeV}$) $\lambda \sim 0.01 - 1$

$$S = \int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda h^4}{4}\right)$$

Chaotic inflation: simple realization

$$S=\int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R+\frac{(\partial_\mu X)^2}{2}-\beta X^4\right)$$

$$\ddot{X}+3H\dot{X}+V'(X)=0$$

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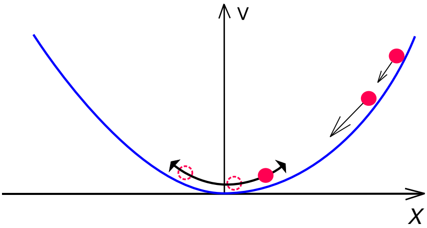

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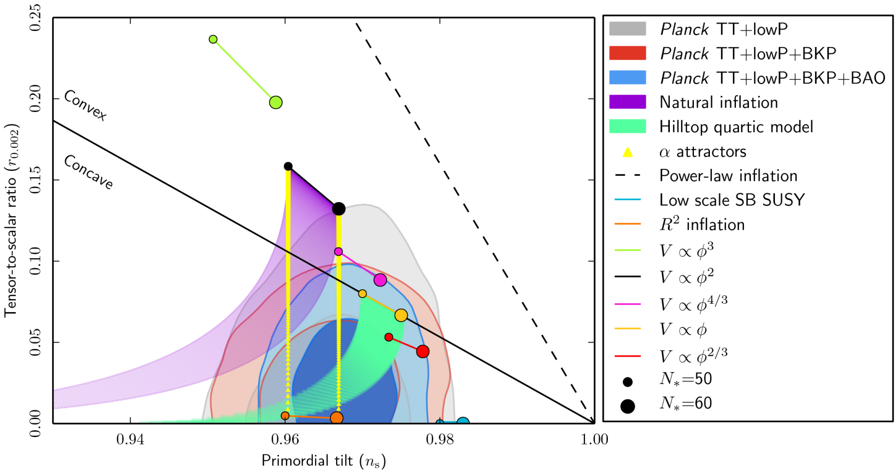
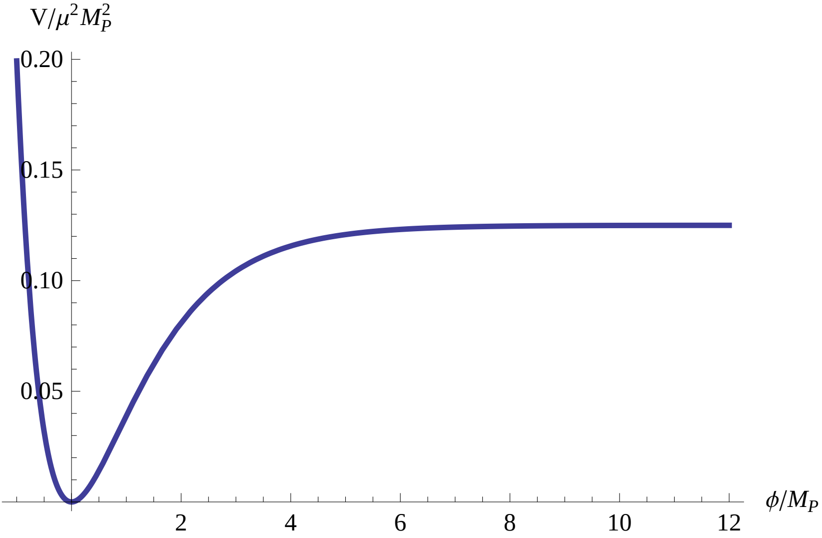
Planck 2015 favors flat inflaton potentials



$$r = \frac{A_T}{A_S} \propto \frac{\dot{\phi}^2}{H^2 M_{Pl}^2} \propto \left(\frac{V'}{V}\right)^2 \ll 1$$

Inflaton potential is apparently concave



Higgs-driven inflation

F.Bezrukov, M.Shaposhnikov (2007)

$$S^{JF} = \int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R - \xi H^\dagger H R + \mathscr{L}_{SM}\right)$$

In a unitary gauge $H^T = \left(0, (h+v)/\sqrt{2}\right)$ (and neglecting $v = 246\,\text{GeV}$)

$$S = \int d^4x\sqrt{-g}\left(-\frac{M_P^2+\xi h^2}{2}R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda h^4}{4}\right)$$

slow roll behavior due to modified kinetic term even for $\lambda \sim 1$

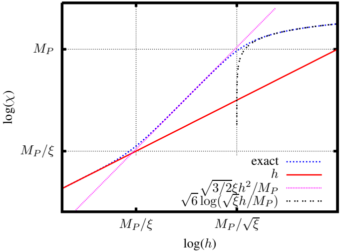
Go to the Einstein frame: $(M_P^2 + \xi h^2)\,R^{JR} \to M_P^2\,R^{EF}$

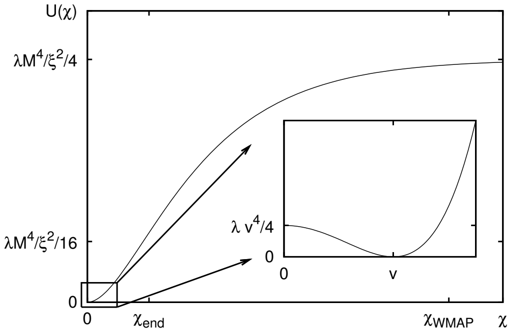
$$g_{\mu\nu}^{JF} = \Omega^{-2}\tilde{g}_{\mu\nu}^{EF}\,, \qquad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

with canonically normalized χ: interval ds^2 changes !

$$\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2+(6\xi+1)\xi h^2}}{M_P^2+\xi h^2}\,,\quad U(\chi) = \frac{\lambda M_P^4 h^4(\chi)}{4(M_P^2+\xi h^2(\chi))^2}\,.$$

we have a flat potential at large fields: $U(\chi) \to \text{const}$ @ $h \gg M_P/\sqrt{\xi}$





Reheating by Higgs field

after inflation: $M_P/\xi < h < M_P/\sqrt{\xi}$

effective dynamics : $h^2 \to \chi$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{6}\frac{M_P^2}{\xi^2}\chi^2$$

Advantage: NO NEW interactions to reheat the Universe

inflaton couples to all SM fields!

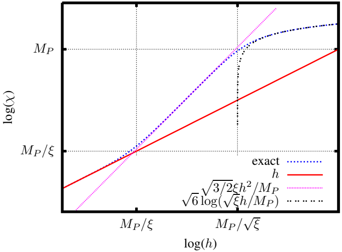
exponentially flat potential! @ $h \gg M_P/\sqrt{\xi}$:

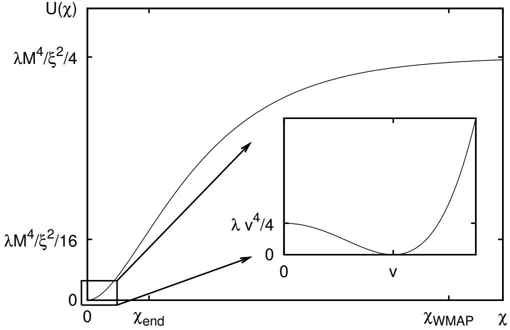
$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - \exp\left(-\frac{\sqrt{2}\chi}{\sqrt{3}M_P}\right)\right)^2$$

NO NEW d.o.f.

0812.3622

from WMAP-normalization: $\xi \approx 47000 \times \sqrt{\lambda}$





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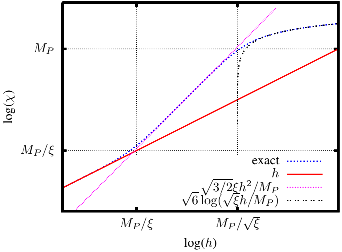
0812.3622

from WMAP-normalization: $\xi \approx 47000 \times \sqrt{\lambda}$

F.Bezrukov, D.G., M.Shaposhnikov, 0812.3622

similar result J.Garcia-Bellido, D.Figueroa, J.Rubio, 0812.4624



$$m_W^2(\chi) = \frac{g^2}{2\sqrt{6}} \frac{M_P|\chi(t)|}{\xi}$$

$$m_t(\chi) = y_t \sqrt{\frac{M_P|\chi(t)|}{\sqrt{6}\xi}} \,\text{sign}\,\chi(t)$$

reheating via W^+W^-, ZZ production at zero crossings

then nonrelativistic gauge bosons scatter to light fermions

$$\chi \to W^+W^- \to f\bar{f}$$

Reheating by Higgs field

after inflation: $M_P/\xi < h < M_P/\sqrt{\xi}$

Hot stage starts almost from $T = M_P/\xi \sim 10^{14}$ GeV:

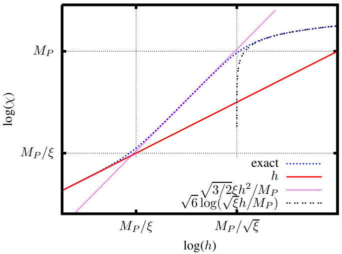
effective dynamics : $h^2 \to \chi$

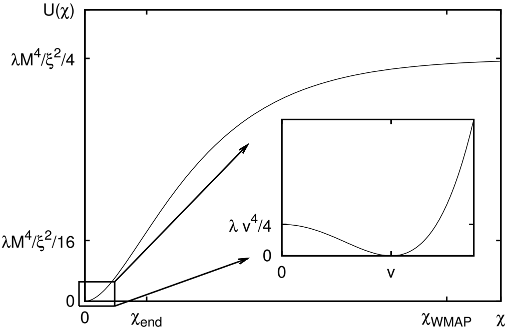
$$\mathscr{L} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{6}\frac{M_P^2}{\xi^2}\chi^2$$

$$3.4 \times 10^{13}\,\text{GeV} < T_r < 9.2 \times 10^{13}\left(\frac{\lambda}{0.125}\right)^{1/4}\text{GeV}$$

Advantage: NO NEW interactions
to reheat the Universe
inflaton couples to all SM fields!

$n_s = 0.967\,, r = 0.0032$ F.Bezrukov, D.G. (2012)





$$\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2 + (6\xi+1)\xi\, h^2}}{M_P^2 + \xi\, h^2},$$

$$U(\chi) = \frac{\lambda M_P^4\, h^4(\chi)}{4(M_P^2 + \xi\, h^2(\chi))^2}.$$

exponentially flat potential! @ $h \gg M_P/\sqrt{\xi}$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - \exp\left(-\frac{\sqrt{2}\chi}{\sqrt{3}M_P}\right)\right)^2$$

- renormalizable except gravity
  frame-dependent renormalization scale
- strong coupling (ϕ-dependent)
  save for inflation
  but reheating is questionable

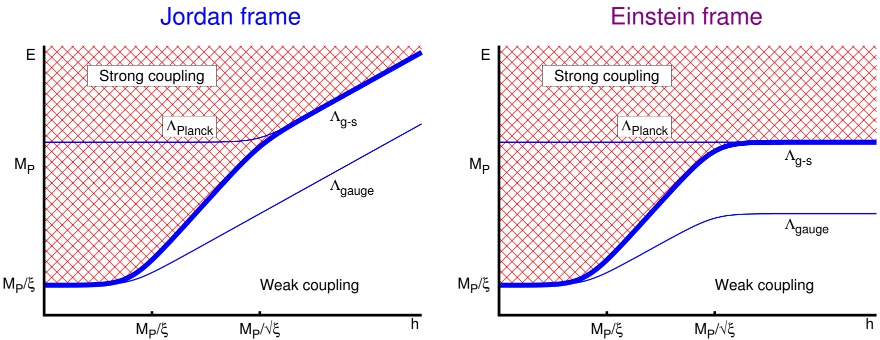
F.Bezrukov et al (2008)

$V_0 \simeq 10^{-12} M_{Pl}^4$

from WMAP-normalization: $\xi \approx 47000 \times \sqrt{\lambda}$

F.Bezrukov, D.G., M.Shaposhnikov (2008,2011)

Strong coupling in Higgs-inflation



gravity-scalar sector:

$$\Lambda_{g-s}(h) \simeq \begin{cases} \frac{M_P}{\xi}, & \text{for } h \lesssim \frac{M_P}{\xi}, \\ \frac{\xi h^2}{M_P}, & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}}, \\ \sqrt{\xi} h, & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}}. \end{cases}$$

gravitons: $\Lambda^2_{\text{Planck}} \simeq M_P^2 + \xi h^2$

gauge interactions:

$$\Lambda_{\text{gauge}}(h) \simeq \begin{cases} \frac{M_P}{\xi}, & \text{for } h \lesssim \frac{M_P}{\xi}, \\ h, & \text{for } \frac{M_P}{\xi} \lesssim h, \end{cases}$$

1008.5157

Then we have problems...

- During inflation and preheating we are always below Λ
- However, both parametrically and numerically

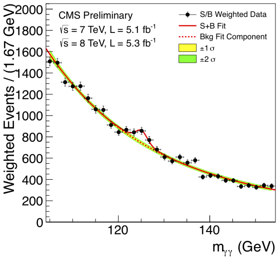
$$T_r \simeq \frac{M_P}{\xi} \simeq \Lambda$$

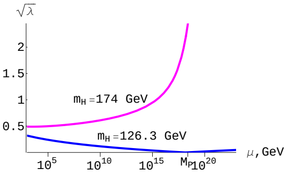
- Recent more detailed studies 1609.05209, 1610.08916
  of reheating in multi-scalar inflationary models indicate amplification of inflaton decays

$$T_r \gg \Lambda$$

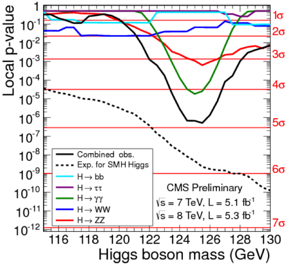
  so the reheating may be out of control
- On top of that: it may be that $\lambda < 0$...

LHC: . . . Higgs of 125 GeV



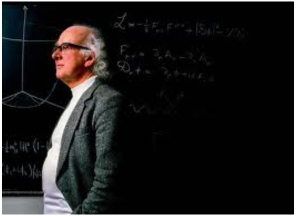


$$m_h = \sqrt{2\lambda}\, v$$



- LEPII: $m_h > 114\,\mathrm{GeV}$
- fit to EW data: $m_h \sim 90 < 114\,\mathrm{GeV}$
- TeVatron: not in $156 < m_h < 177\,\mathrm{GeV}$
- CMS & ATLAS:

$$m_h \approx 125\,\mathrm{GeV}$$



$$\lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2$$

$$\to \frac{\lambda}{4} h^4 + \lambda v^2 h^2$$

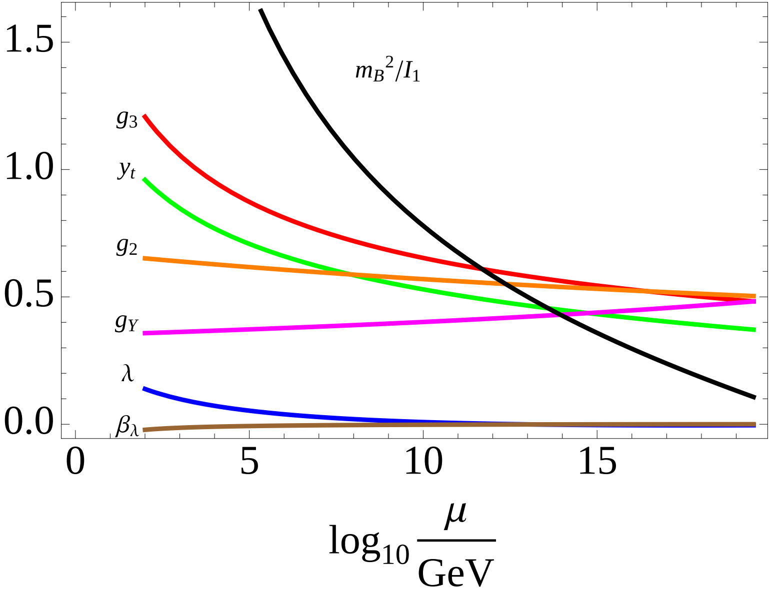
$$\mathscr{L}_Y \propto Y_f h \bar{f} f/\sqrt{2}$$

- renormgroup equation

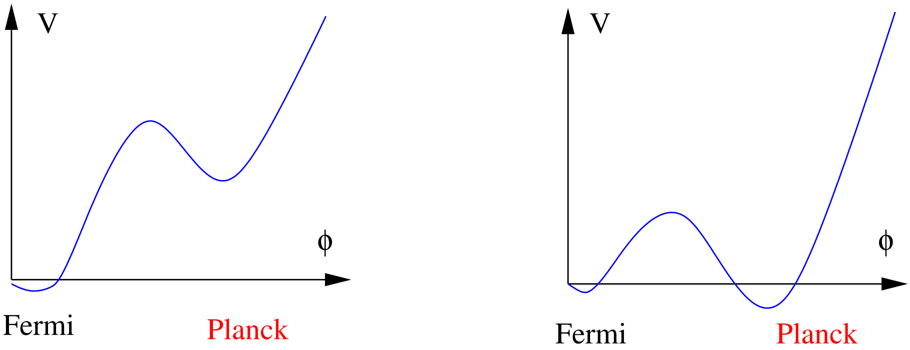
$$\frac{d\lambda}{d\log\mu} \propto +\#\lambda^2 - \#Y_t^4$$

Running of the SM couplings

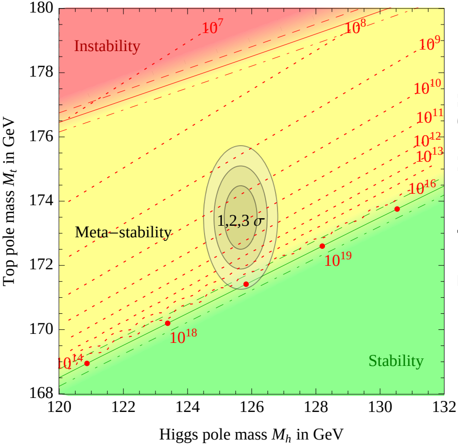
1305.7055

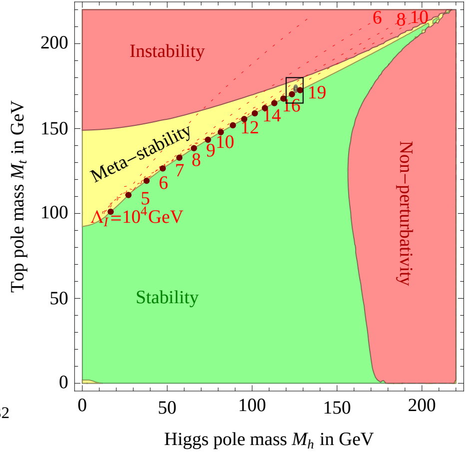


Higgs potential with quantum corrections



How weird to live with 125 GeV Higgs. . .





1307.7879

All in all

It would be nice to modify the model

- Introducing as little new physics as possible
- Keeping cosmological observables determined by the Higgs sector parameters
- Possibly avoiding the negative selfcoupling for the Higgs

Outline

Natural completion with R^2

D.G., A.Tokareva 1807.02392

$\xi h^2 R$ induces R^2-term

hep-th/9510140

$$S_0 = \int d^4x\sqrt{-g}\left(-\frac{M_P^2 + \xi h^2}{2}R + \frac{\beta}{4}R^2 + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2 - v^2)^2\right).$$

introduce a Lagrange multiplier L and auxiliary scalar $\mathscr{R}$

$$S = \int d^4x\sqrt{-g}\left(\frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2 - v^2)^2 - \frac{M_P^2 + \xi h^2}{2}\mathscr{R} + \frac{\beta}{4}\mathscr{R}^2 - L\mathscr{R} + LR\right).$$

integrate out $\mathscr{R}$

$$S = \int d^4x\sqrt{-g}\left(\frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2 - v^2)^2 + LR - \frac{1}{\beta}(L + \frac{1}{2}\xi h^2 + \frac{1}{2}M_P^2)^2\right)$$

$$\xi \to \xi^2/\beta$$

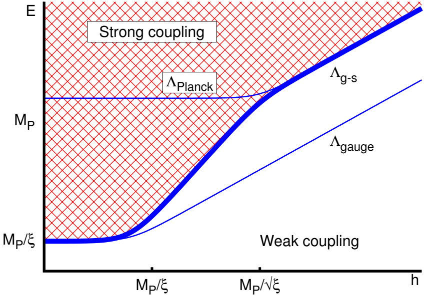
with

$$\beta \gtrsim \frac{\xi^2}{4\pi}$$

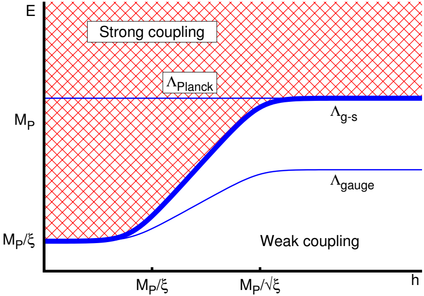
everything here look healthy

Strong coupling: the lowest scale is in the gauge sector

Jordan frame



Einstein frame



gravity-scalar sector:

$$\Lambda_{g-s}(h) \simeq \begin{cases} \frac{M_P}{\xi}, & \text{for } h \lesssim \frac{M_P}{\xi}, \\ \frac{\xi h^2}{M_P}, & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}}, \\ \sqrt{\xi}\, h, & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}}. \end{cases}$$

gravitons: $\Lambda^2_{\text{Planck}} \simeq M_P^2 + \xi h^2$

gauge interactions:

$$\Lambda_{\text{gauge}}(h) \simeq \begin{cases} \frac{M_P}{\xi}, & \text{for } h \lesssim \frac{M_P}{\xi}, \\ h, & \text{for } \frac{M_P}{\xi} \lesssim h, \end{cases}$$

1008.5157

Further transformations...

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introducing scalaron ϕ with $m = M_P/\sqrt{3\beta}$

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}\,, \quad \Omega^2 \equiv \frac{2L}{M_P^2}\,, \; L \to \phi \equiv M_P\sqrt{\frac{2}{3}}\log\Omega^2\,.$$

and setting $M_P = 1/\sqrt{6}$

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{12} + \frac{1}{2}e^{-2\phi}(\partial h)^2 + \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-4\phi}\left(\lambda h^4 + \frac{1}{36\beta}(e^{2\phi} - 1 - 6\xi h^2)^2\right)\right)$$

both gravity and scalar sector are weakly coupled up to M_P with $\beta \gtrsim \xi^2/(4\pi)$

And one more. . .

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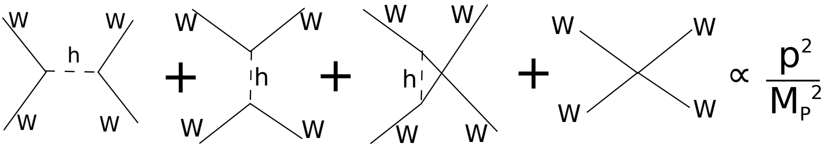
$$h = e^{\Phi} \tanh H, \; \phi = e^{\Phi} / \cosh H,$$

The scalar sector becomes

$$L = \frac{1}{2}\cosh^2 H (\partial\Phi)^2 + \frac{1}{2}(\partial H)^2 - \frac{\lambda}{4}\sinh^4 H - \frac{\lambda}{144\beta}(1 - e^{-2\Phi}\cosh^2 H - 6\xi \sinh^2 H)^2.$$

and the Higgs coupling to gauge bosons, e.g.,

$$L_{gauge} = \frac{g^2 h^2}{4} e^{-2\phi} W^+_\mu W^-_\mu = \frac{g^2}{4}\sinh^2 H \; W^+_\mu W^-_\mu \, .$$



$$\mathscr{A} \sim \frac{g^2 p^2}{m_W^2}\left(\frac{4}{g^2}\left(\frac{dm_W(H)}{dH}\right)^2 - 1\right) \; \to \mathscr{A} \propto \frac{p^2}{M_P^2}$$

Cosmological spectra

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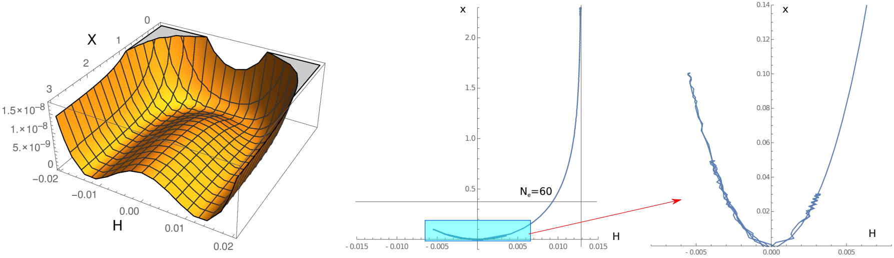


Scalar perturbations:

1701.07665

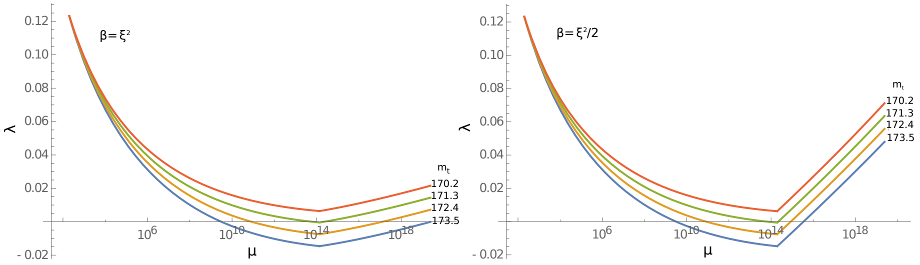
$$\beta + \frac{\xi^2}{\lambda} \simeq 2 \times 10^9$$

At small β like in the Higgs-inflation

heavy scalaron is integrated out

$$\frac{\xi^2}{4\pi} < \beta < \frac{\xi^2}{\lambda} \quad \to \quad 5 \times 10^{13}\,\text{GeV} < m < 1.5 \times 10^{15}\,\text{GeV}$$

Bonus: stable for a bit heavier top-quark



Outline

Conclusions

- Higgs inflation is a viable cosmological model unique in minimality:
  no new d.o.f., no new interactions to reheat the Universe
- however it suffers from the stong coupling problem:
  predictivity $\delta\rho/\rho \leftrightarrow \lambda$ is lost
- R^2-term with heavy scalaron cures the model:
  it seems minimal, natural, Higgs-inflation predictions remain intact
- to refine them we must study the reheating
  and improve the accuracy of Y_t, (m_h, α_s, etc) to convince it works indeed
  ... ILC, FCC, etc

Backup slides

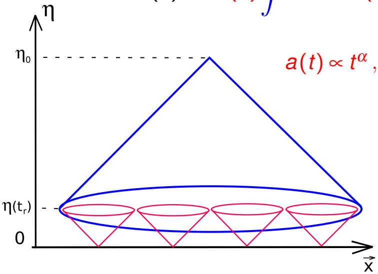
Horizon problem $l_H(t)$

a distance covered by photon emitted at $t = 0$

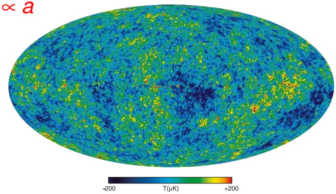
size of the causally connected part, that is the visible part of the Universe ("inside horison")

$ds^2 = dt^2 - a^2(t)\,dx^2 = a^2(\eta)\,(d\eta^2 - dx^2)$ $\quad ds^2 = 0$

$$l_H(t) = a(t)\int dx = a(t)\int d\eta = a(t)\int_0^t \frac{c\,dt'}{a(t')} \propto t \quad \propto 1/H(t)$$



$a(t) \propto t^\alpha\,,\ 0 < \alpha < 1\,,\ L_{phys} \propto a$



$l_{H_0}/l_{H,r}(t_0) \sim l_{H_0}/l_{H,r}(t_r)\,a(t_r)/a_0 \sim H_r/H_0\,a(t_r)/a_0 \sim \sqrt{1+z_r} \simeq 30$

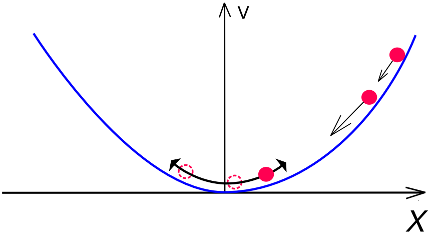
Chaotic inflation at large fields: initial conditions

$$\ddot{X} - \Delta X/a^2 + 3H\dot{X} + V'(X) = 0$$

slow roll: $X_e > M_{Pl}$

$\delta\rho/\rho \sim 10^{-5}$ requires
$V_0 \simeq 10^{-12} \times M_{\rm Pl}^4$

inflation starts in a relatively uniform domain of Planck size



Chaotic inflation, A.Linde (1983), A.Linde (1984)

$$\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i\phi)^2 \sim V(\phi) \sim M_{\rm Pl}^4$$

looks rather natural:
each of the form of inflaton energy fluctuates similarly

Is the Higgs-inflation realy unlikely ??

D.G., A.Panin (2014)

Start with the Jordan frame

Higgs-scalaron mixture

$$S^{JF} = \int d^4x \sqrt{-g^{JF}} \left(-\frac{M_P^2 + \xi h^2}{2} R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda h^4}{4} \right)$$

Go to the Einstein frame:

$$(M_P^2 + \xi h^2) R \to M_P^2 \tilde{R}$$

$$g_{\mu\nu} = \Omega^{-2} \tilde{g}_{\mu\nu} , \qquad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2} , \qquad \frac{d\phi}{dh} = \sqrt{\frac{\Omega^2 + 48\pi\xi^2 h^2 / M_{Pl}^2}{\Omega^4}}$$

"chaotic initial conditions" effective M_{Pl}

Higgs noncanonical kinetic term

Hence in the EF

$$\Omega^2 M_{Pl}^2 R^{JF} \sim \xi \dot{h}^2 \sim \xi (\partial_i h)^2 \sim \lambda h^4 \sim M_{Pl}^4$$

$$\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i \phi)^2 \sim M_{Pl}^4/\Omega^4 \sim 10^{-12} \times M_{Pl}^4$$

hence

$$R^{JF} \sim M_{Pl}^2/\Omega^2$$

CMB-amplitude $V_0^{EF} \sim 10^{-12} \times M_{Pl}^4$ fixes $\Omega^2 \sim 10^6$

all terms in EF
happen to be of the same order !!

Is the Higgs-inflation realy unlikely ??

D.G., A.Panin (2014)

Start with the Jordan frame

Higgs-scalaron mixture

$$S^{JF} = \int d^4x\sqrt{-g^{JF}}\left(-\frac{M_P^2+\xi h^2}{2}R+\frac{(\partial_\mu h)^2}{2}-\frac{\lambda h^4}{4}\right)$$

Go to the Einstein frame:

$(M_P^2+\xi h^2)R \to M_P^2 \tilde{R}$

$$g_{\mu\nu}=\Omega^{-2}\tilde{g}_{\mu\nu}\,,\qquad \Omega^2=1+\frac{\xi h^2}{M_P^2}\,,\qquad \frac{d\phi}{dh}=\sqrt{\frac{\Omega^2+48\pi\xi^2h^2/M_{\rm Pl}^2}{\Omega^4}}$$

"chaotic initial conditions" effective $M_{\rm Pl}$

Higgs noncanonical kinetic term

$$\Omega^2 M_{\rm Pl}^2 R^{JF} \sim \xi \dot{h}^2 \sim \xi(\partial_i h)^2 \sim \lambda h^4 \sim M_{\rm Pl}^4$$

hence

$$R^{JF} \sim M_{\rm Pl}^2/\Omega^2$$

CMB-amplitude $V_0^{EF} \sim 10^{-12}\times M_{\rm Pl}^4$ fixes $\Omega^2 \sim 10^6$

Hence in the EF

$$\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i\phi)^2 \sim M_{\rm Pl}^4/\Omega^4 \sim 10^{-12}\times M_{\rm Pl}^4$$

all terms in EF
happen to be of the same order !!

The first inflationary model: ... from modified gravity!

$$S^{JF} = -\frac{M_P^2}{2}\int\sqrt{-g}\,d^4x\left[R - \frac{R^2}{6\mu^2}\right] \;\to\; \left[\left(1-\frac{Q}{3\mu^2}\right)(R-Q) + \left(Q - \frac{Q^2}{6\mu^2}\right)\right],$$

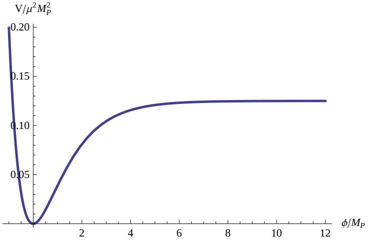
Jordan Frame $\to$ Einstein Frame   get reed $\left(1-\frac{Q}{3\mu^2}\right) = \Omega^2$ A.Starobinsky (1980)

$$g_{\mu\nu}^{JF} \to g_{\mu\nu}^{EF} = \Omega^2\, g_{\mu\nu}^{JF}\,, \qquad \Omega^2 = \exp\left(\sqrt{2/3}\,\phi/M_P\right).$$

$$S^{EF} = \int\sqrt{-g^{EF}}\,d^4x\left[-\frac{M_P^2}{2}R^{EF} + \frac{1}{2}g_{\mu\nu}^{EF}\partial^\mu\phi\,\partial^\nu\phi - \frac{3\mu^2 M_P^2}{4}\left(1-\frac{1}{\Omega^2(\phi)}\right)^2\right],$$

generation of perturbations $\sim 10^{-5}$

requires $V_0 \simeq 10^{-12}\,M_{Pl}^4$



Inflationary models and quantum corrections

- inflationary predictions are robust
- but we cannot test them with low energy particle physics experiment
- including physics at reheating number of e-foldings N
- similar observation for many other models: Higgs-inflation, α-attractor, etc

- large fields: exponentially flat
  protected by the shift invariance
  $\phi \to \phi + const$
- small fields: polynomial potential
  protected by the renormalizability
  $\phi^2 + \phi^4$
- no way to match them at $\phi \sim M_P$