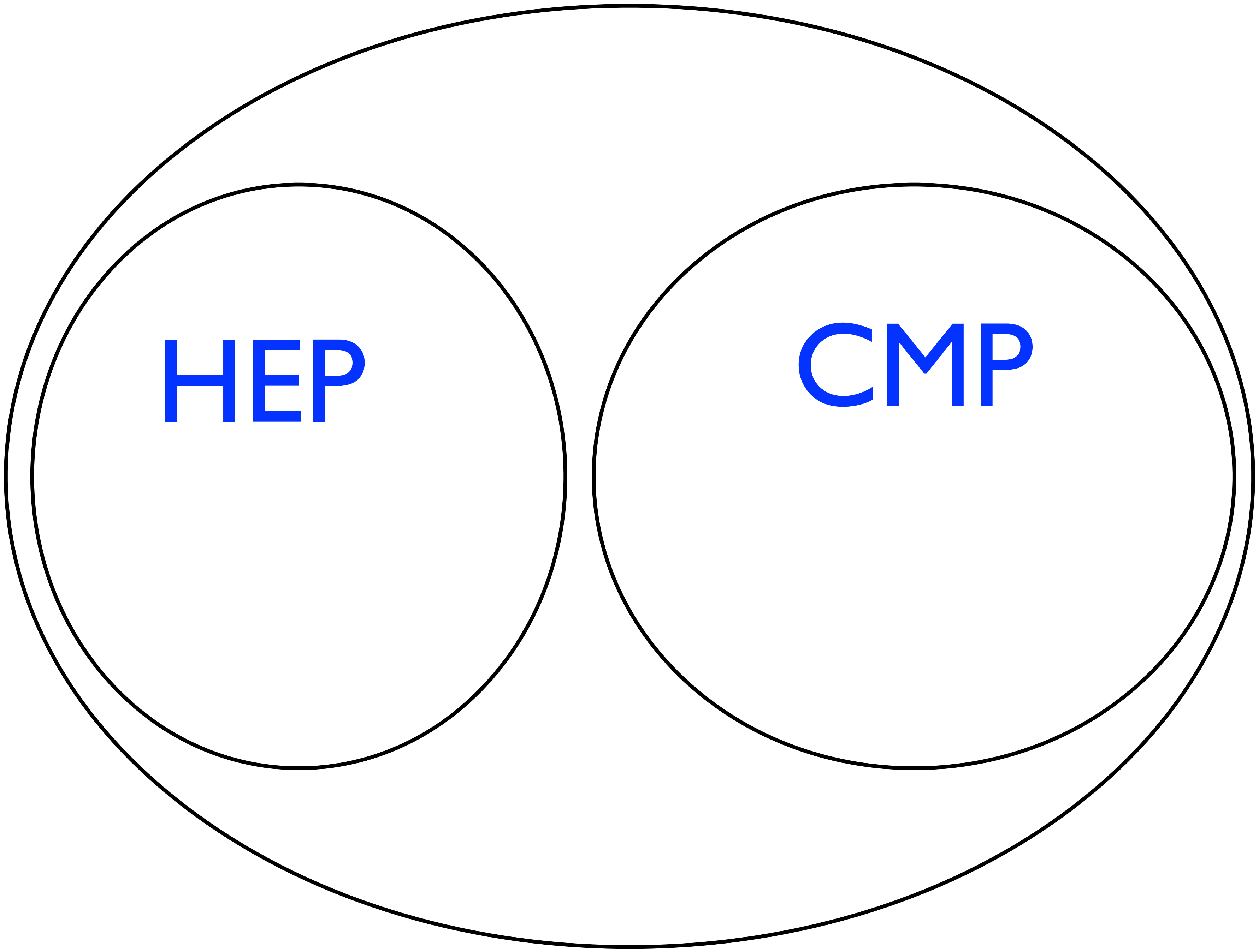
Pushing the emergence frontier: from fractional quantum Hall effect to field-theoretic dualities

Dam Thanh Son (University of Chicago)

Windows to the Universe, Quy Nhon 8/6/2018

Subdivision of physics



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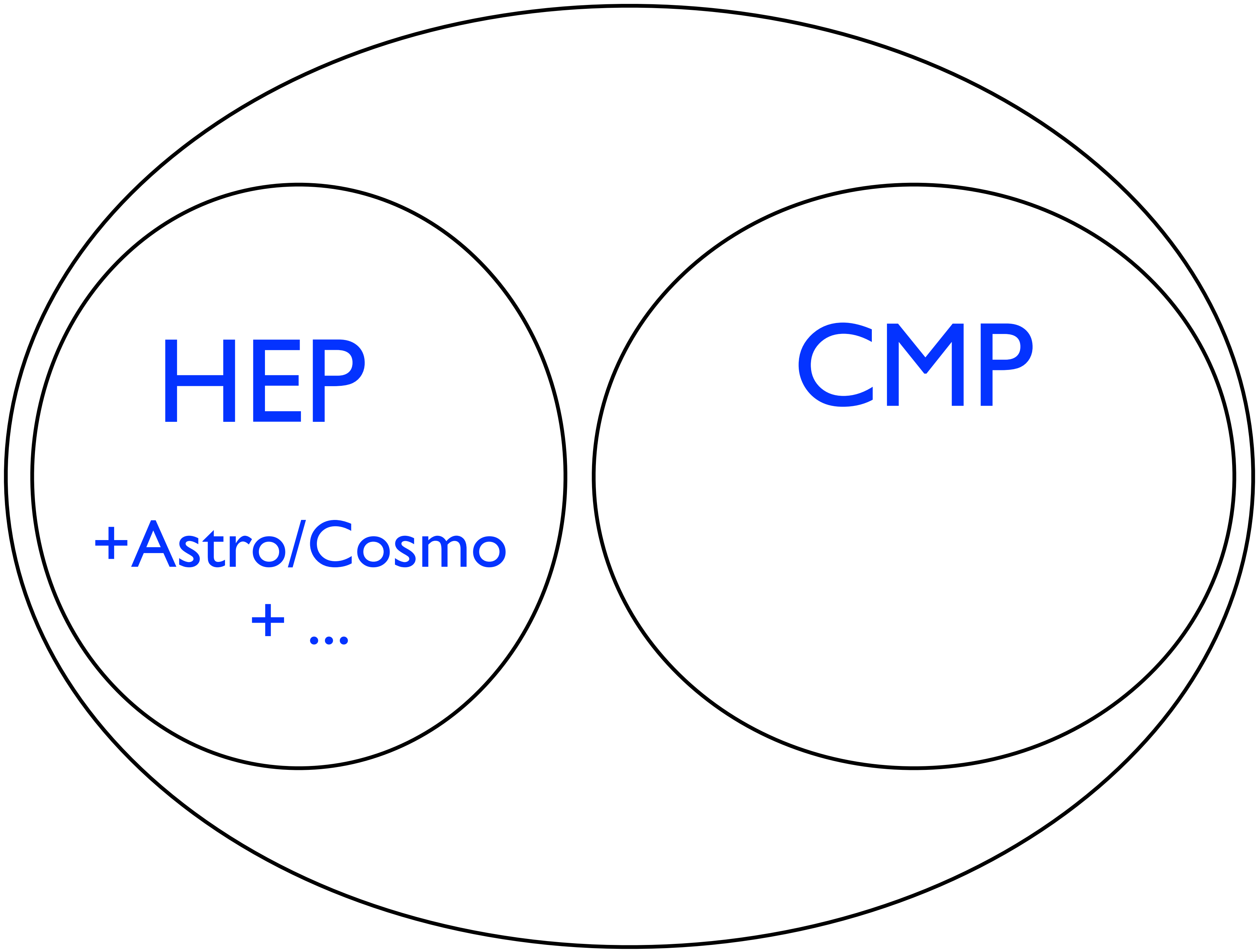

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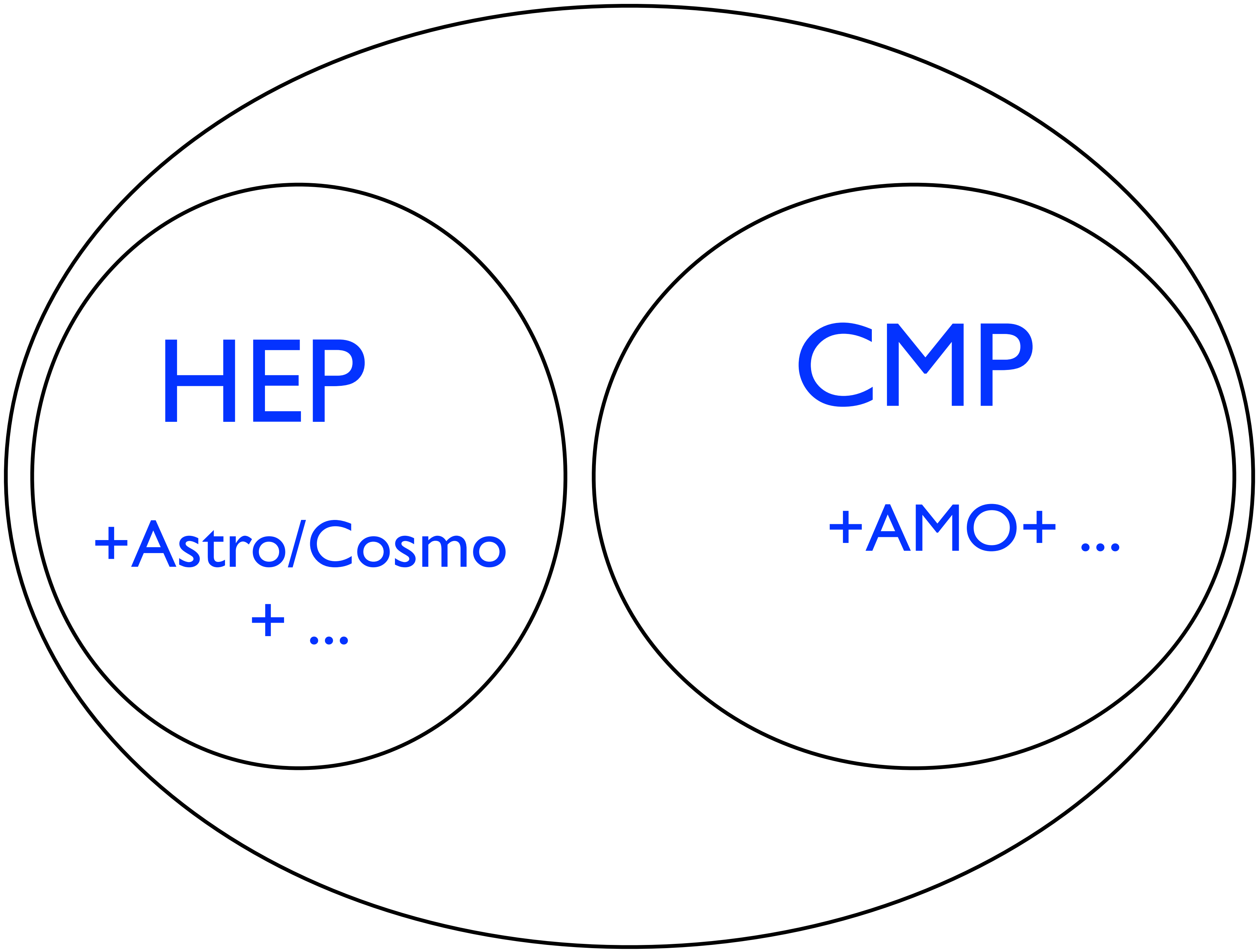

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- clean
- reliable calculations
- highly accurate theoretical predictions

- dirty (“squalid-state physics”)
- approximations
- imprecise (both theory & experiments)

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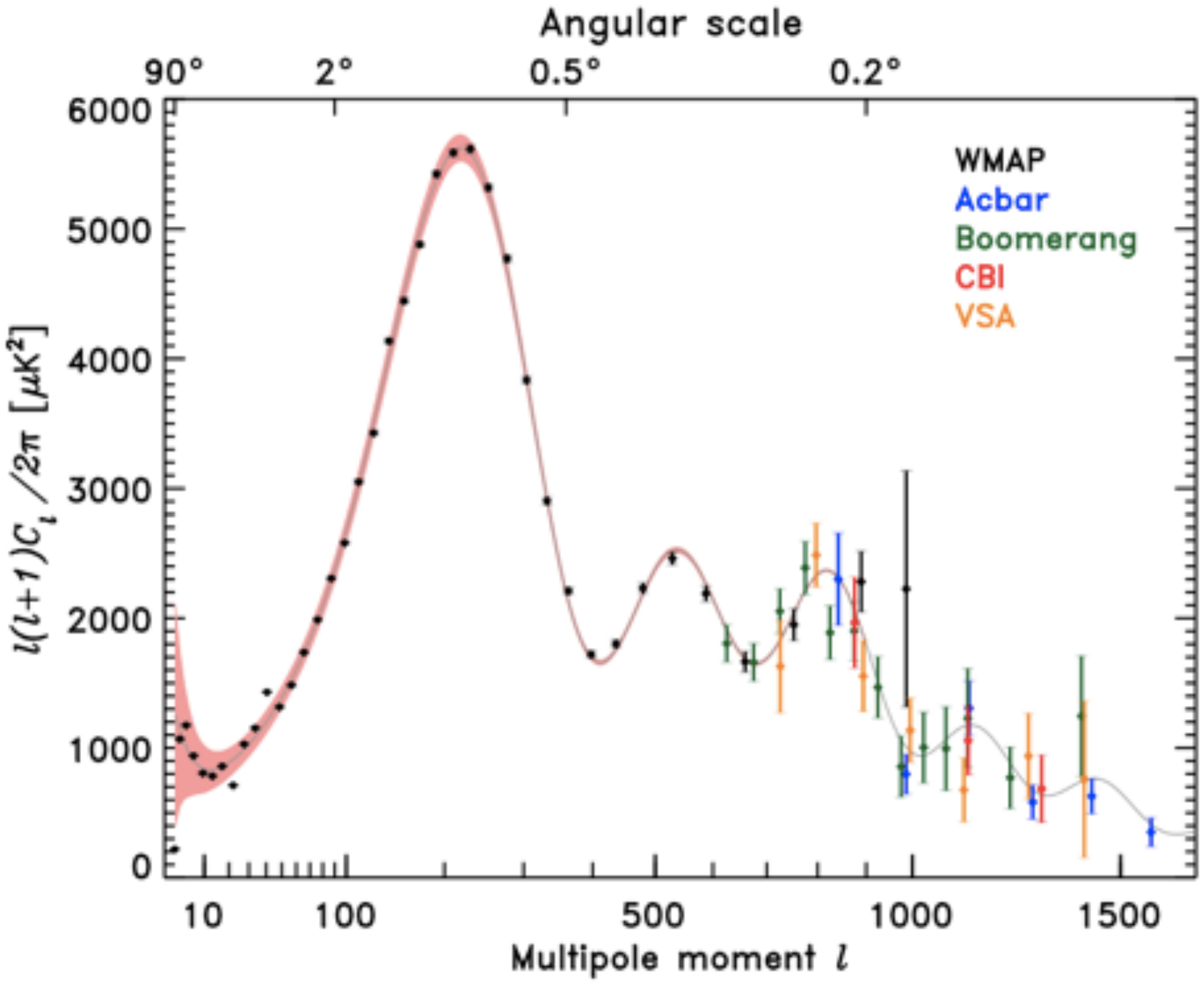

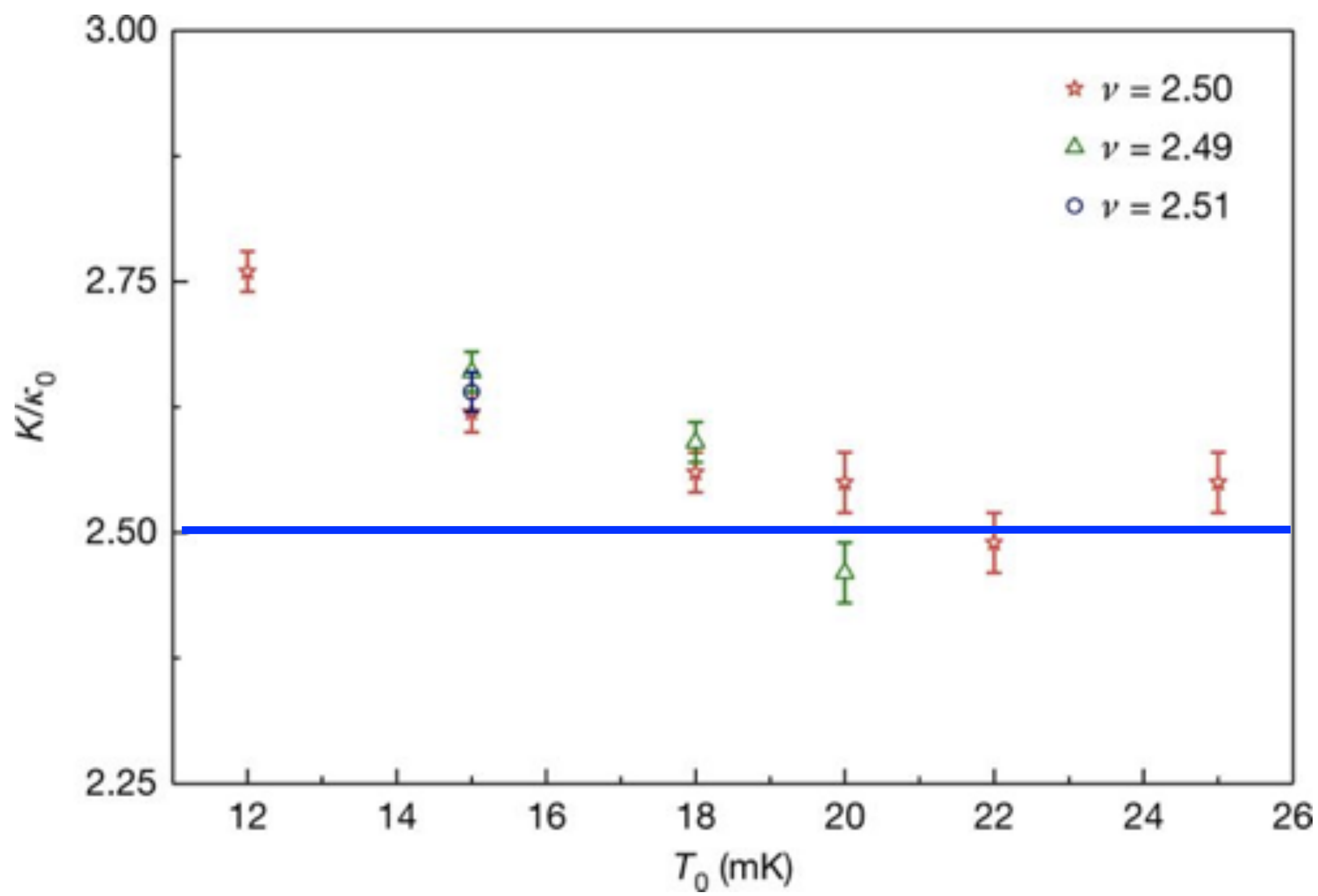

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- CMP: some quantities are determined to extremely high accuracy. Two examples:
  - Josephson effect: protected by gauge invariance
  - Hall conductance in quantum Hall systems is quantized in units of e^2/h up with 10^{-9} precision (protected by topology)

Contacts between HEP & CMP

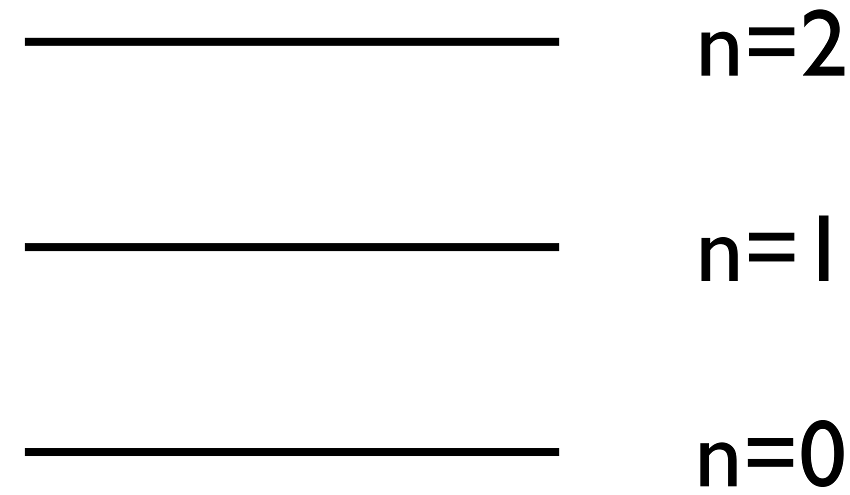
- Spontaneous symmetry breaking (1960s)
- Renormalization group
- Effective field theory (HEP: Weinberg 1979, CMP: Landau's Fermi liquid theory 1957)
- Conformal field theory
- Holography/Duality/Entanglement

HEP and CMP

- are more similar than most people think
- The difference is in the the goal
  - HEP tries to push the reductionist frontier
  - CMP tries to push the emergence frontier

The fractional quantum Hall effect

2D electrons in B field:
Landau levels



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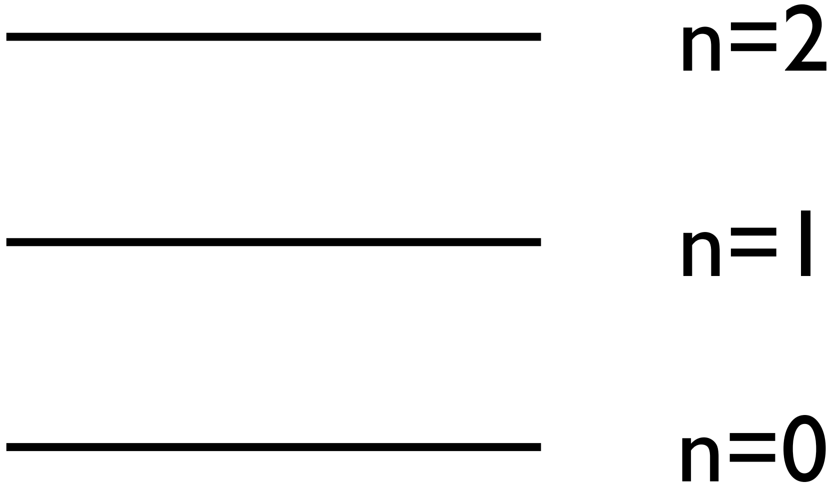
$$\nu = \frac{\text{number of electrons}}{\text{degeneracy of a LL}}$$



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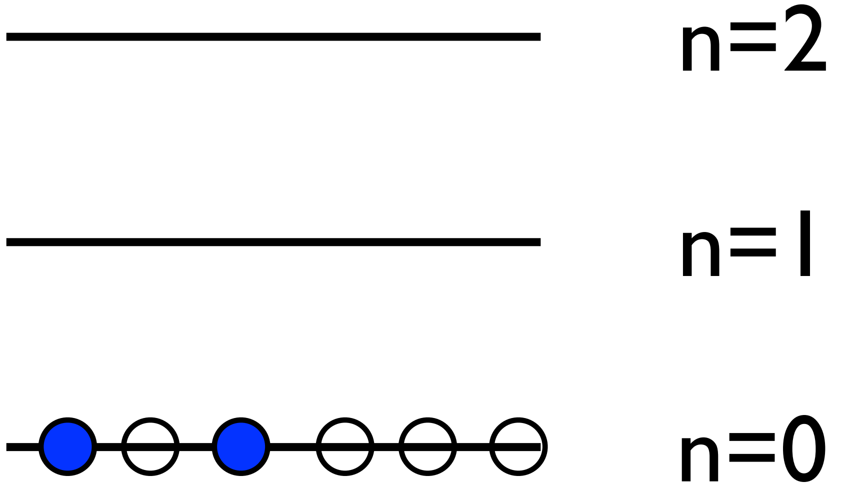

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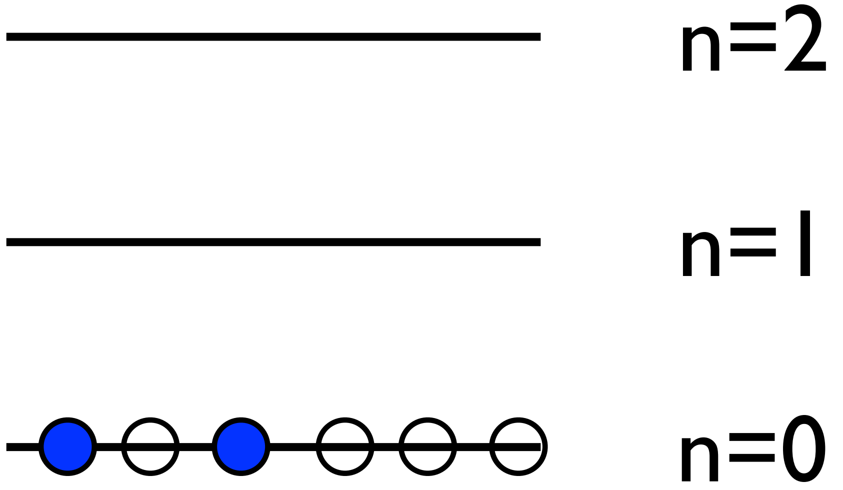

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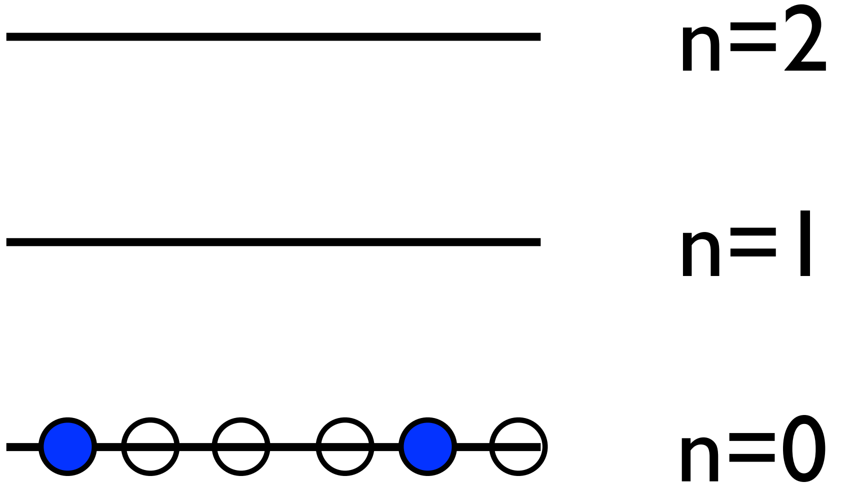

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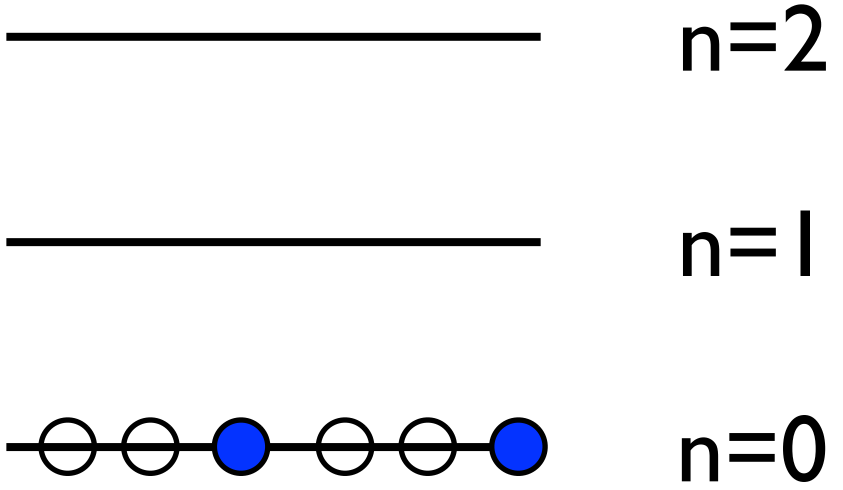

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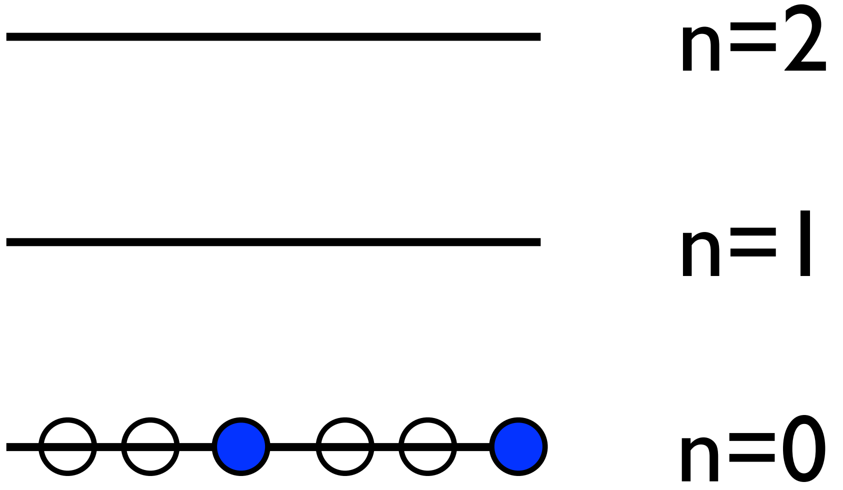
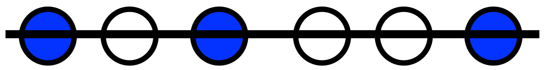
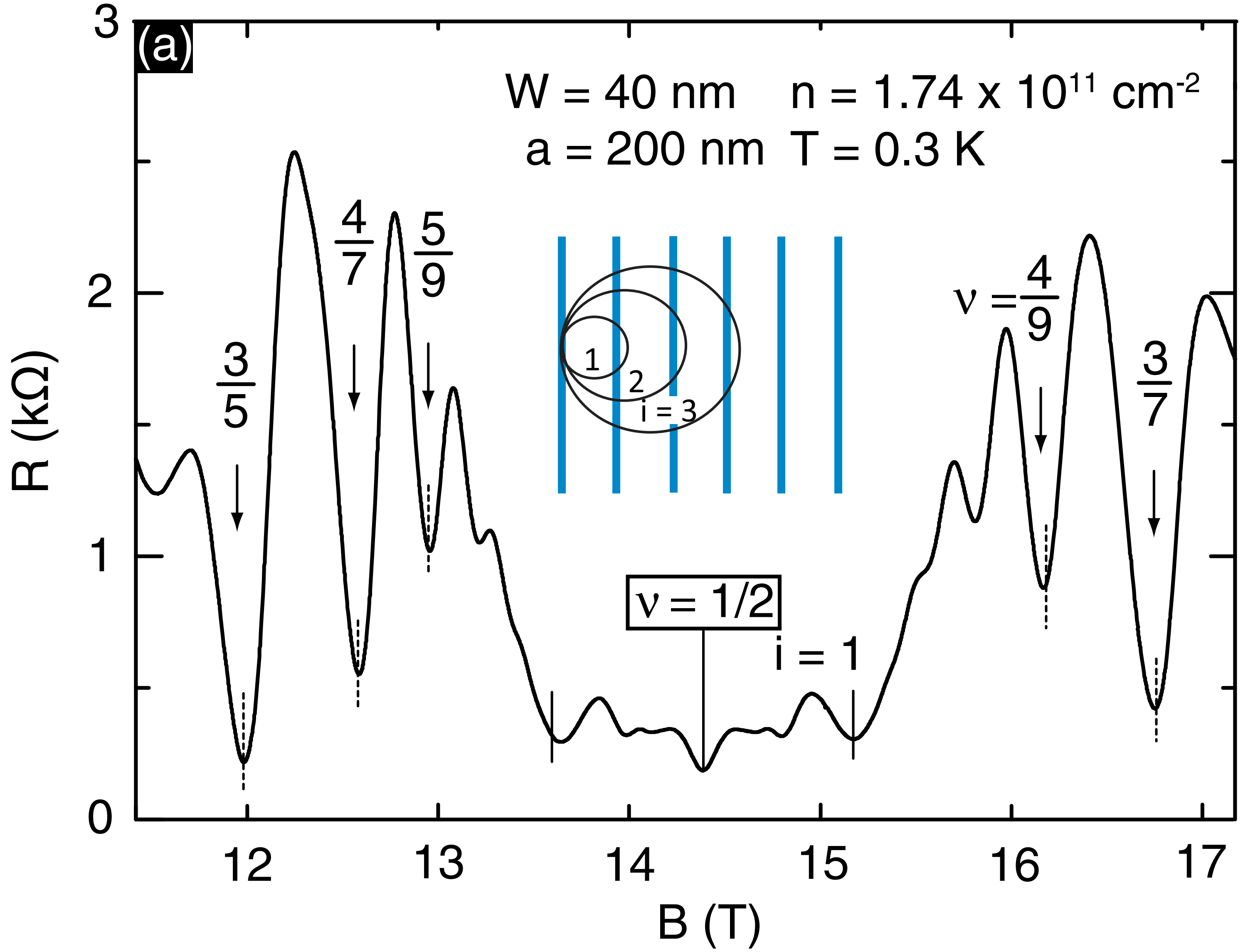


- without interaction: large ground-state degeneracy
- Interactions are essential for determining the ground state
- We will concentrate on filling factor close to 1/2

When ν approaches 1/2, a quasiparticle appears which moves practically in straight line





(Kamburov et al, 2014)

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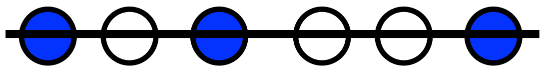

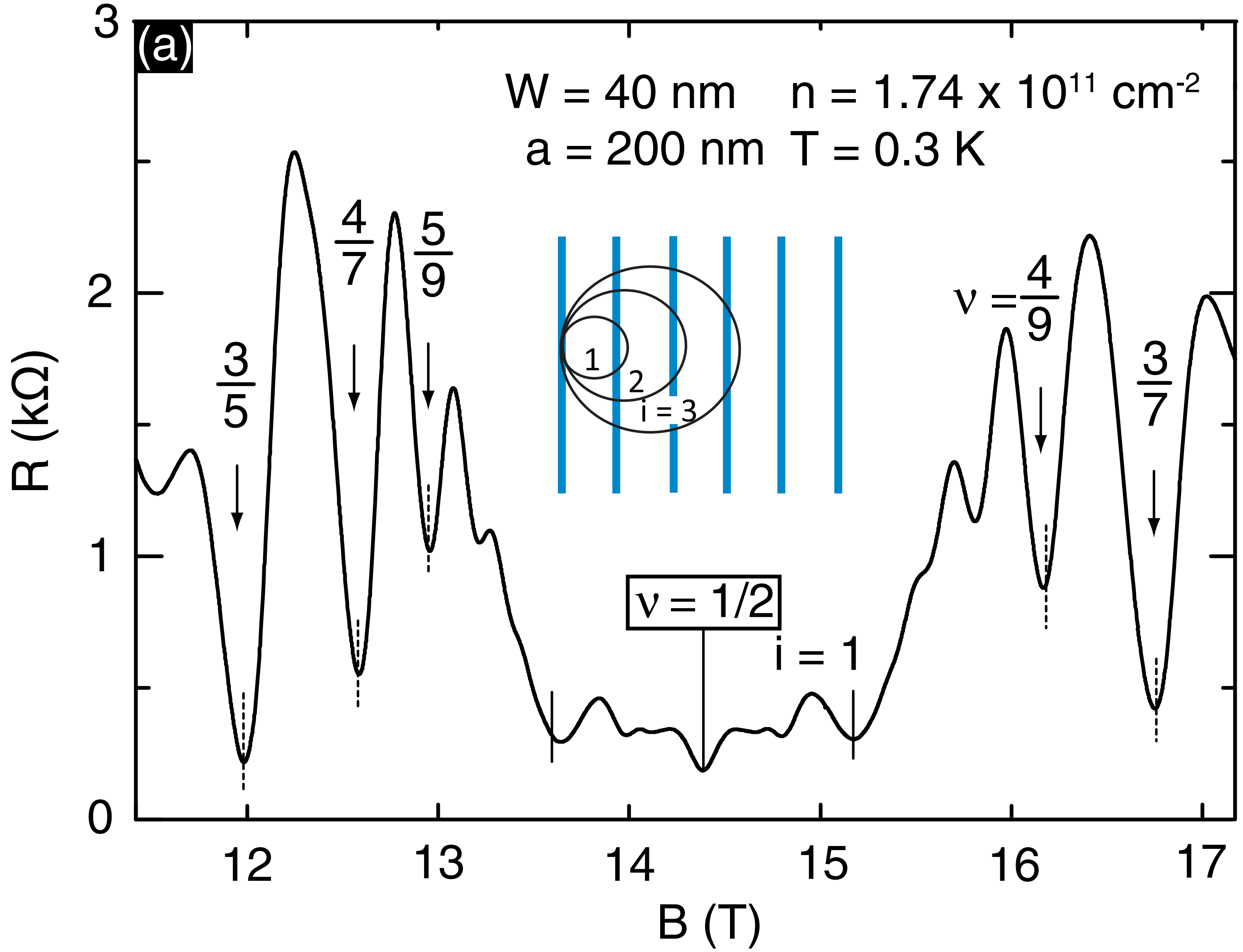

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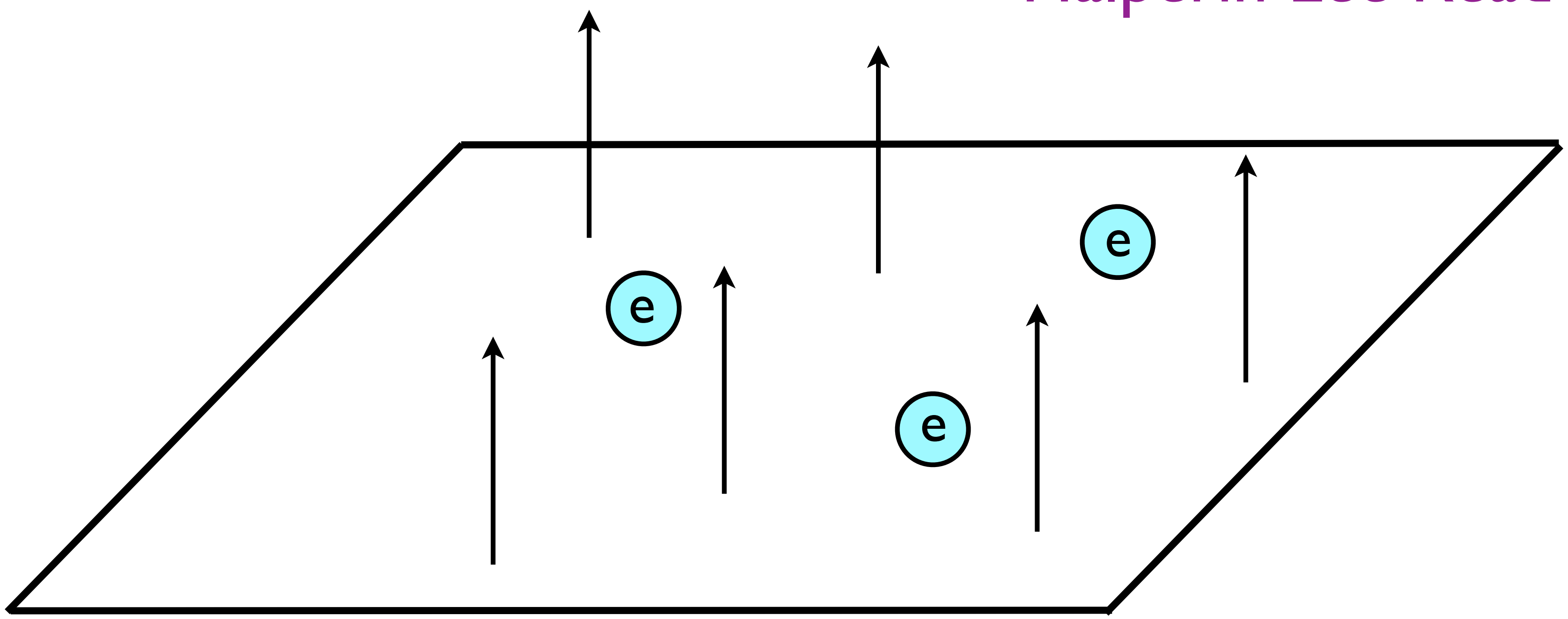
What is the nature of this quasiparticle?

Composite fermion

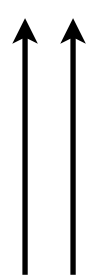
- The standard picture: the quasiparticle is a "composite fermion" = electron + 2 flux quanta

Flux attachment

Jain, Lopez Fradkin, Ovchinnikov,
Halperin Lee Read ~ 1990



$\nu = \frac{1}{2}$



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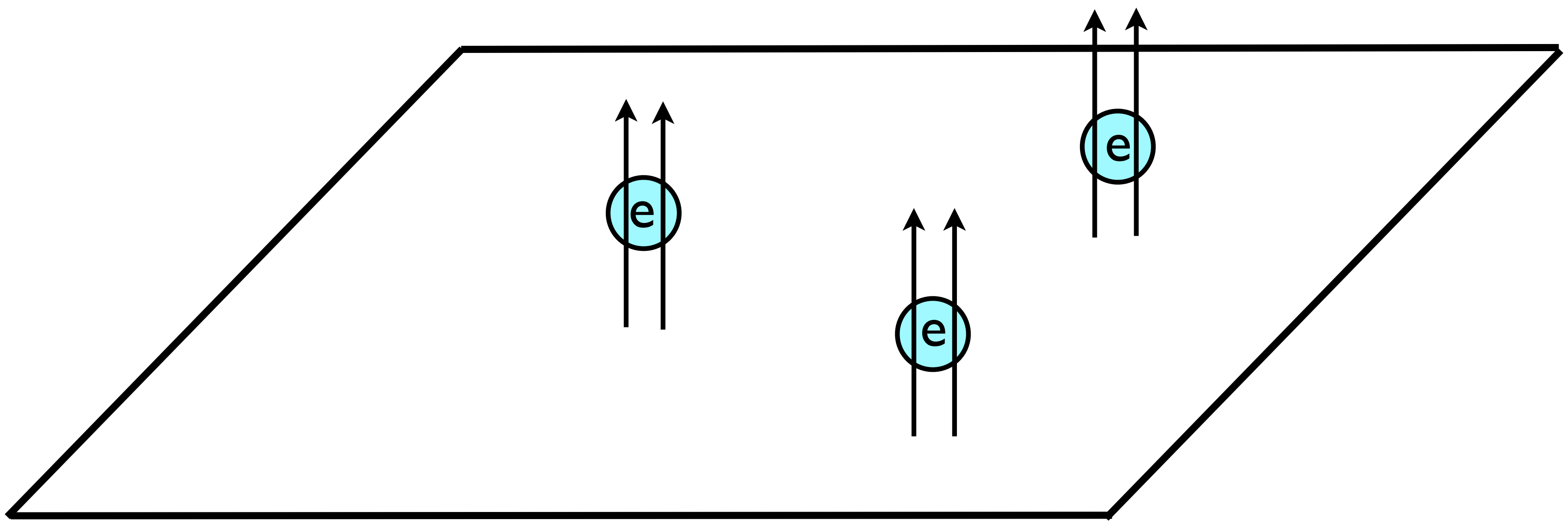

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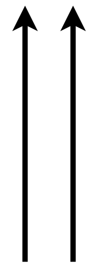

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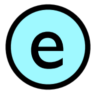

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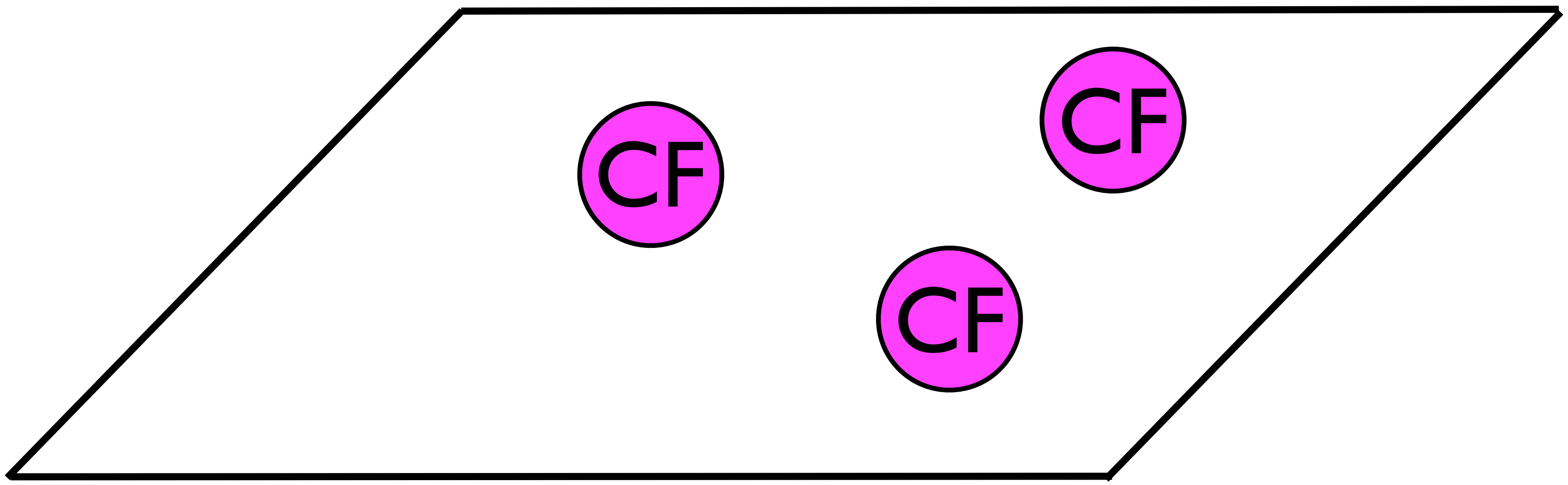

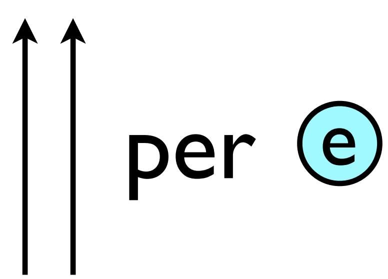

Zero B field for composite fermion (CF)

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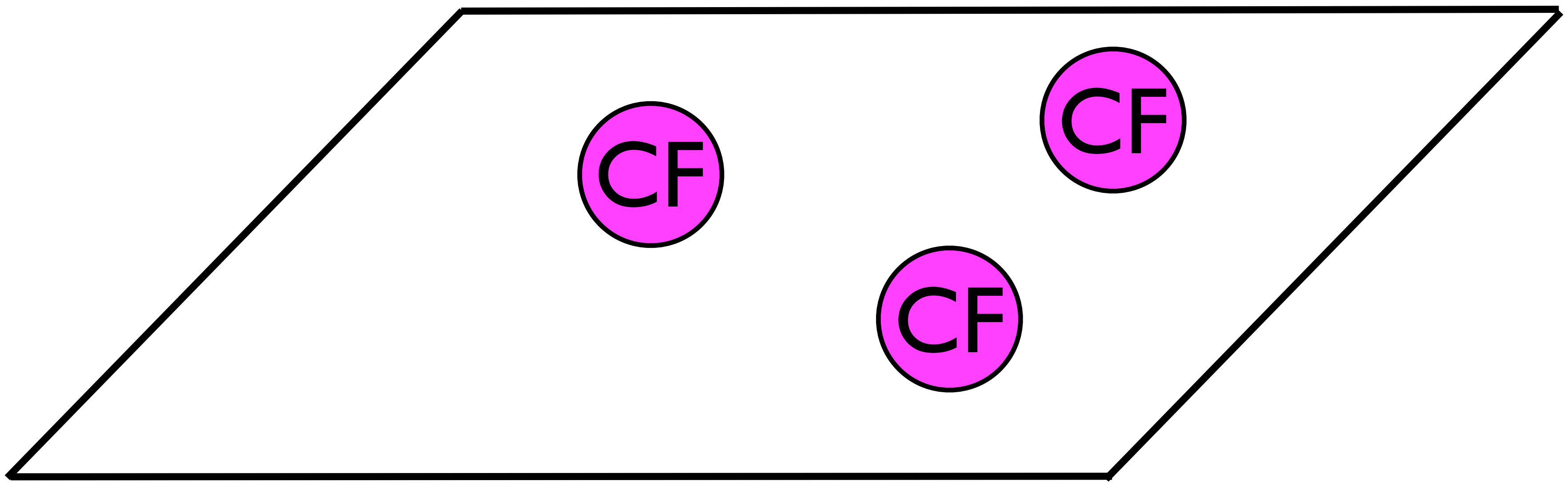
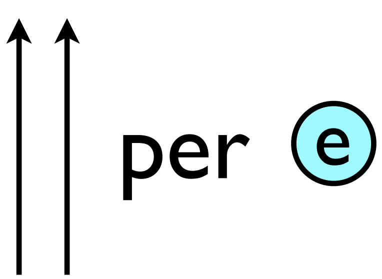


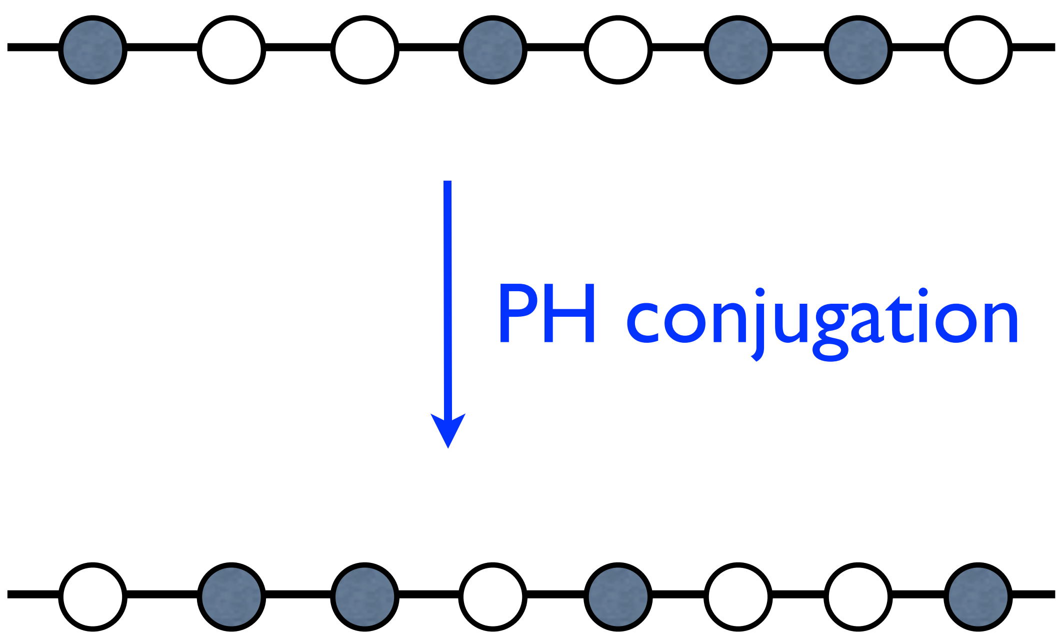


Zero B field for composite fermion (CF)

No left-over magnetic field: CFs move in straight line

Particle-hole symmetry

- But the standard picture does not know about the particle-hole symmetry (known since 1997)
- Roughly speaking, particle-hole conjugation flips the occupation number from 0 to 1 and vice versa



- Flux attachment breaks particle-hole symmetry: flux is attached to particles

Puzzle

- Particle-hole symmetry presents a huge puzzle for CMP
- Composite fermion seen experimentally
- standard picture: CF = a type of “dressed electrons”
- dressed electron = dressed hole?

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- Famous dualities: Coleman-Mandelstam 1975, Seiberg-Witten duality, gauge/gravity duality 1990s
- Most dualities are seen as theorists' toys
- In the context of the quantum Hall effect, a duality proposed ~40 years ago is suddenly relevant

Bosonic particle-vortex duality

Peskin 1978; Dasgupta, Halperin 1981

2+1 dim

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$$\mathcal{L}_1 = |\partial_\mu \phi|^2 - m^2|\phi|^2 - \lambda|\phi|^4$$

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$m^2 < 0$	Goldstone boson	photon

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	magnetic field	charge density

Fermionic particle-vortex duality

DTS; Metlitski, Vishwanath; Wang, Senthil 2015

Roughly: free fermion = "QED" in 2+1 D

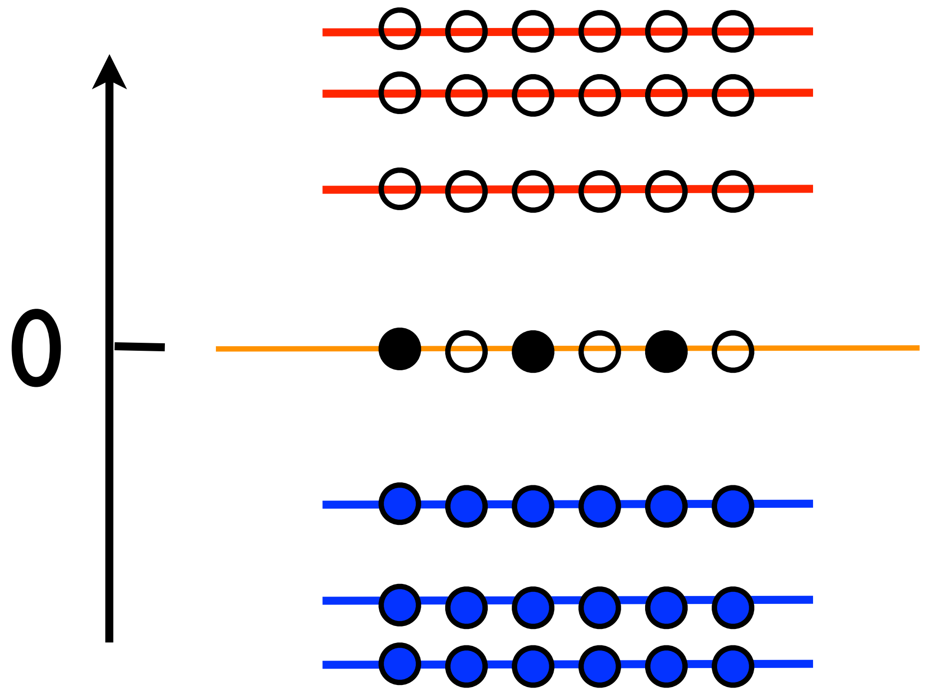
physical EM field

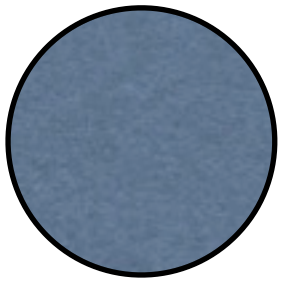
Theory 1: $$\mathcal{L} = i\bar{\psi}_e \gamma^\mu (\partial_\mu - iA_\mu)\psi_e$$

Theory 2: $$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu \partial_\nu a_\lambda$$

emergent U(1) gauge field

Theory 1 ψ_e	Theory 2 ψ
magnetic field	density
density	magnetic field





Theory 1 in magnetic field
zero charge density

Theory 2 at finite density
and zero magnetic field

Half-filled Landau level

Fermi liquid

ψ_e = electrons

ψ = "composite fermion"

- The fermionic particle-vortex duality provides an explanation for the composite fermion near half filling
- The resulting theory, "Dirac composite fermion," has explicit particle-hole symmetry
- Predicts CF has Berry phase of π around the Fermi surface confirmed by numerical simulations
- Suggests a new quantum Hall phase: PH-Pfaffian

PH-Pfaffian

- BCS paired state of composite fermion
- PH-Pfaffian: the simplest pairing pattern

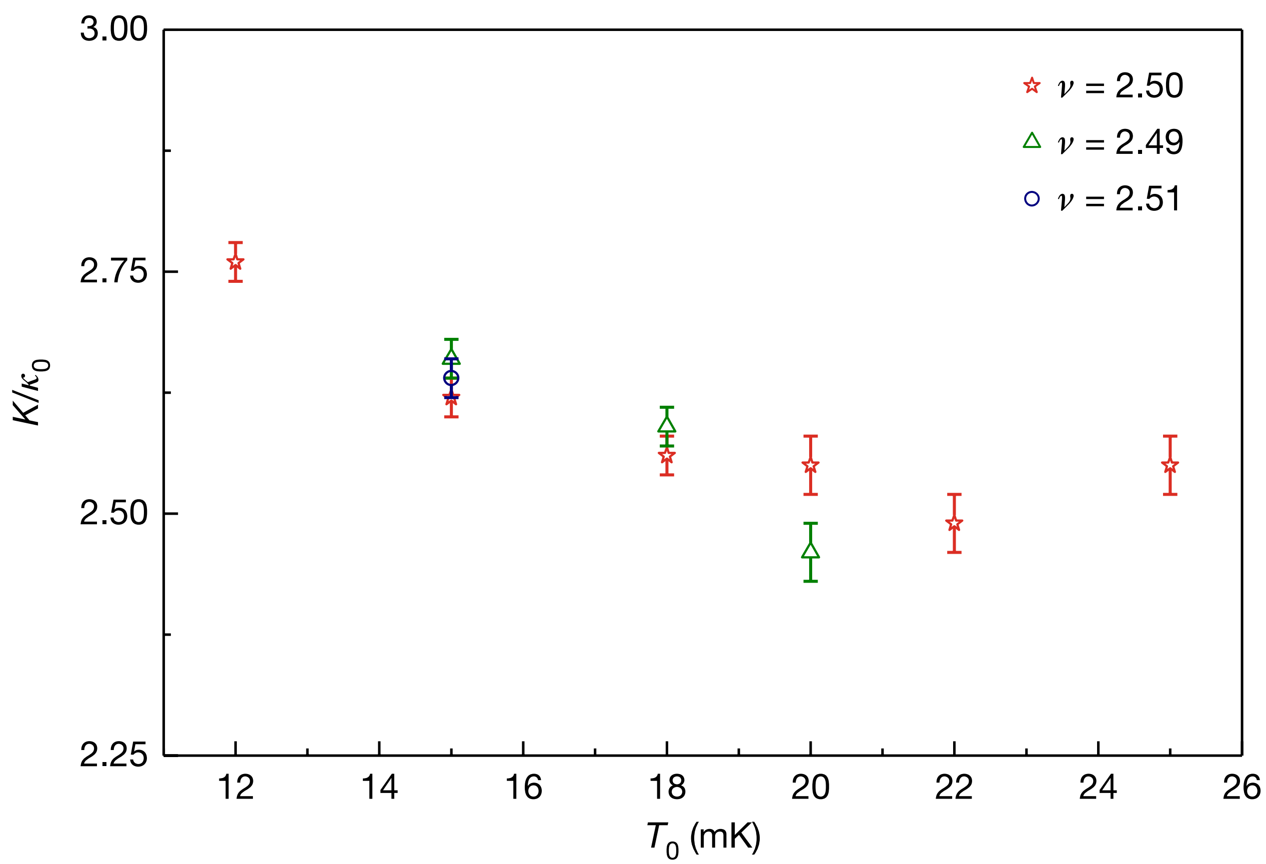
$$\langle \epsilon^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$$

- Is there such a state in nature?
- There is no gapped nu=1/2 state, but there is a gapped nu=5/2=2+1/2 state
  - known to exist from 1987
  - many candidates: Pfaffian Moore-Read 1991 anti-Pfaffian...
  - candidates differ from each other by thermal Hall coeff

$$\mathbf{q} = K\, \boldsymbol{\nabla} T \times \hat{\mathbf{z}}$$

Observation of half-integer thermal Hall conductance

Mitali Banerjee[1], Moty Heiblum[1]*, Vladimir Umansky[1], Dima E. Feldman[2], Yuval Oreg[1] & Ady Stern[1]



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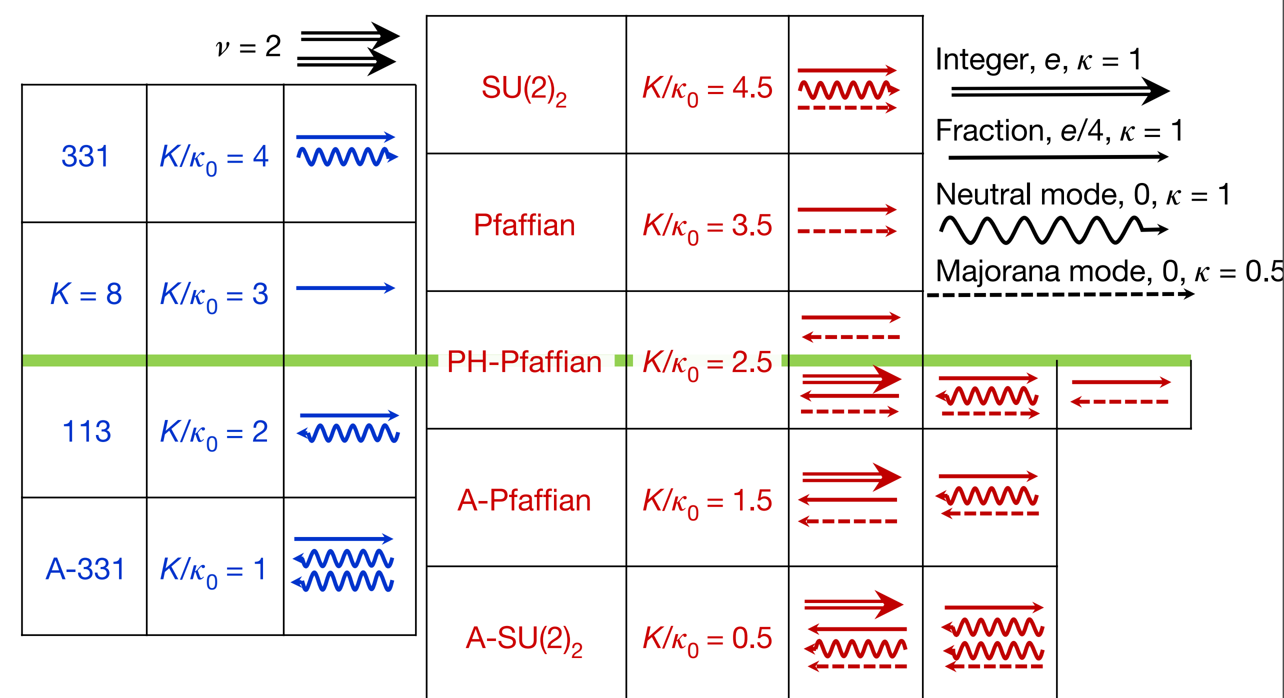

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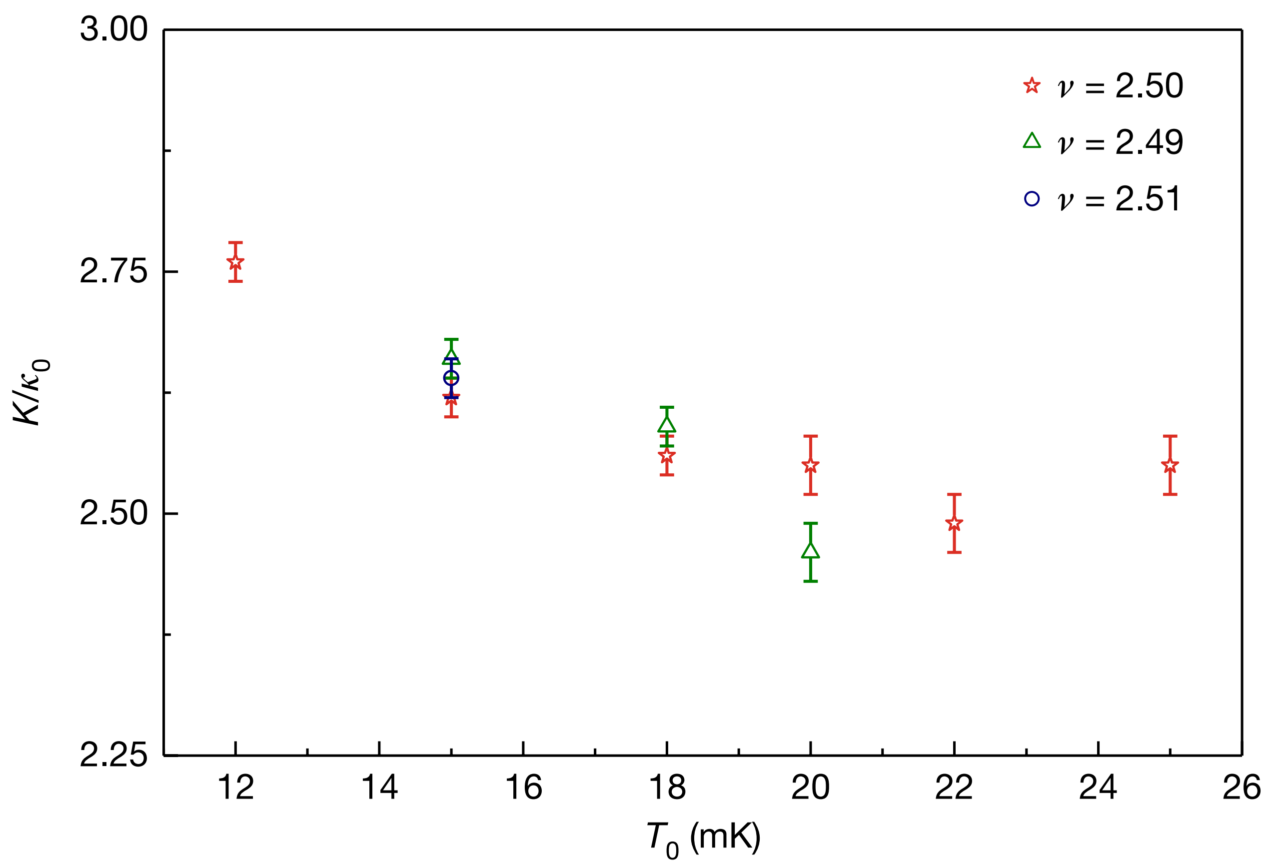

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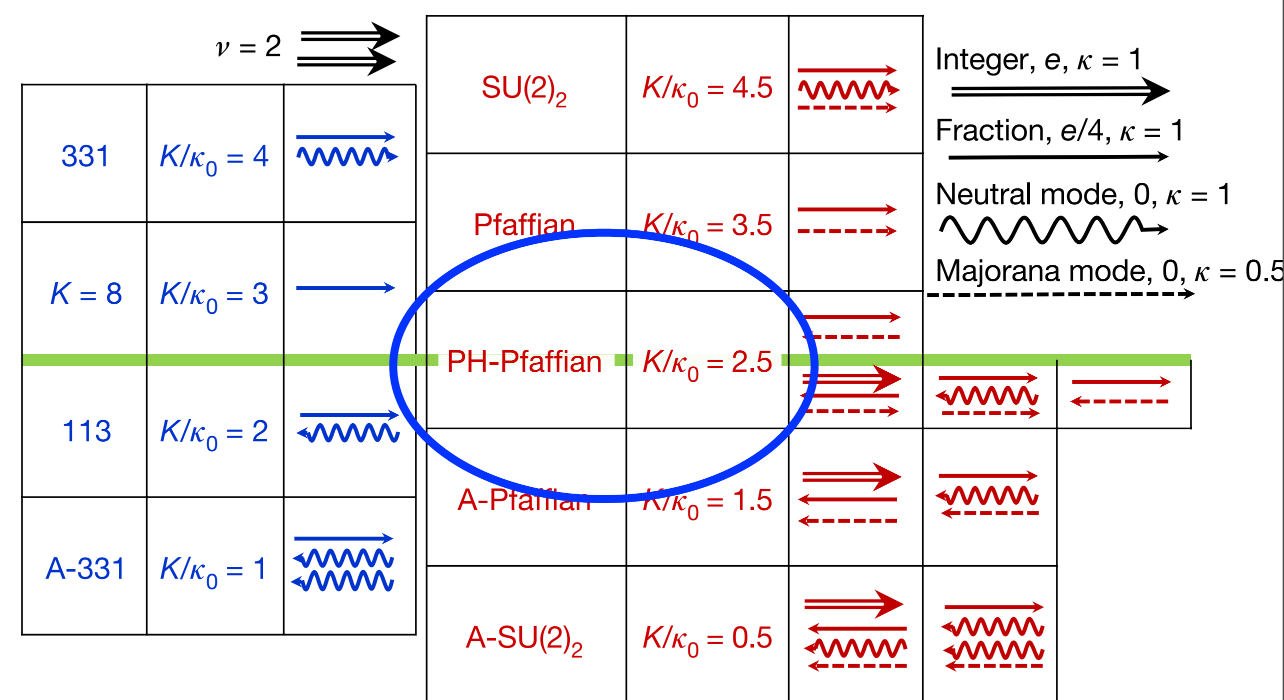

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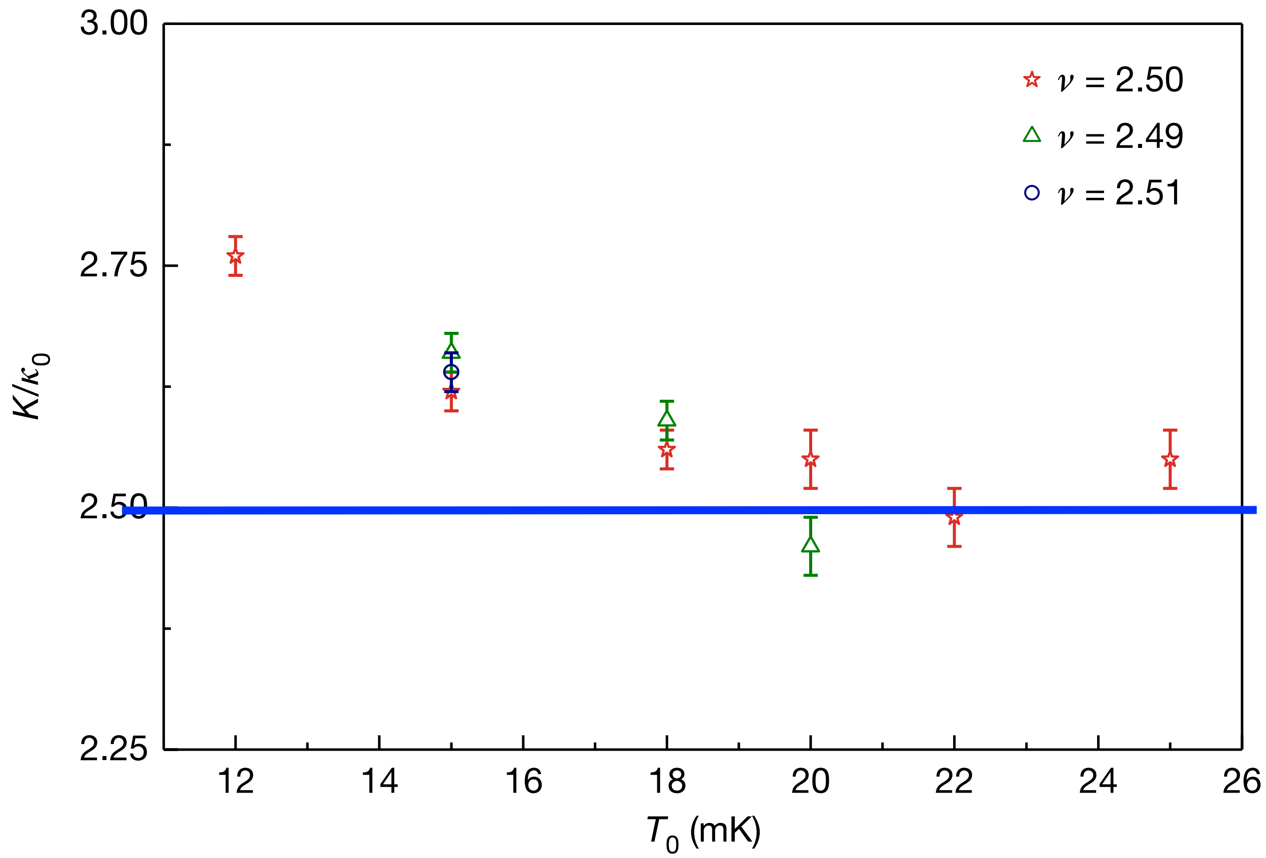

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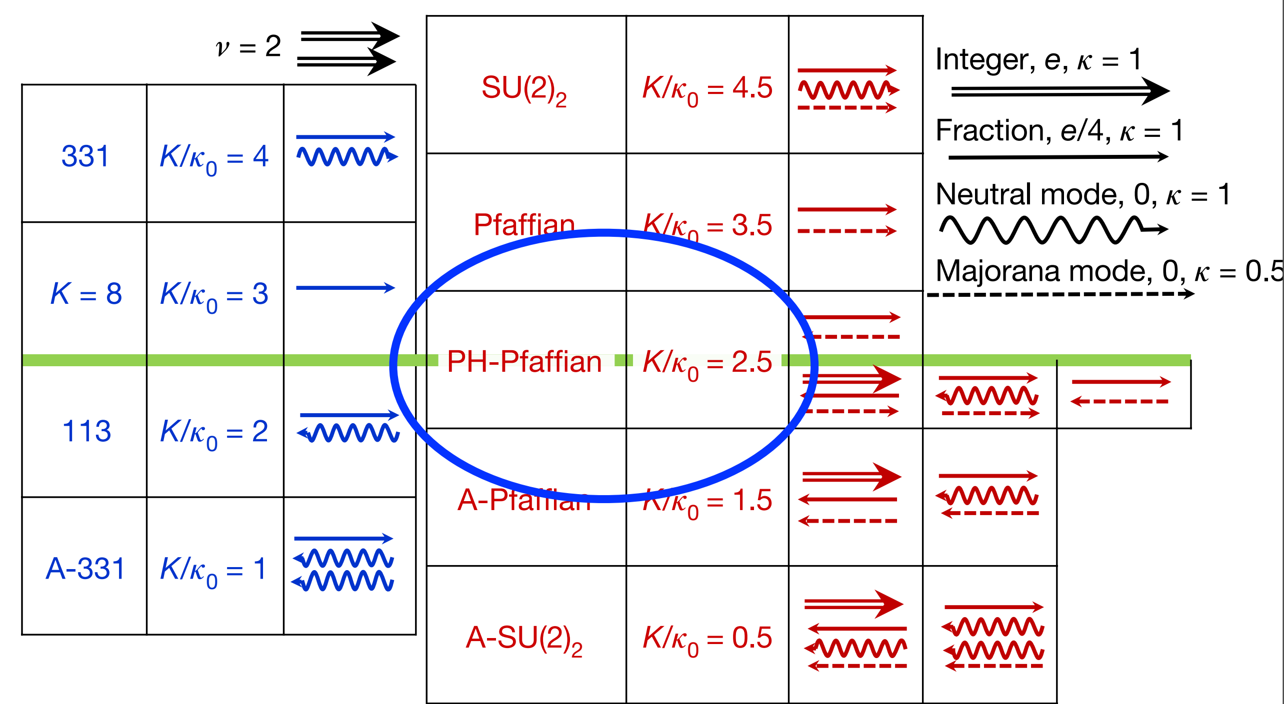

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The “seed duality”

Both the bosonic and fermionic particle-vortex duality can be derived from a “seed duality”

fermion = boson + flux

$$\mathcal{L} = L[\psi, A] - \frac{1}{2}\frac{1}{4\pi}AdA \qquad \mathcal{L} = L[\phi, a] + \frac{1}{4\pi}ada + \frac{1}{2\pi}Ada$$

From this duality, a whole “web” of new dualities can be derived

Karch, Tong; Seiberg, Senthil, Wang, Witten

Extreme small N

- It turns out that in HEP literature there has been suggestions of duality between bosonic and fermionic Chern-Simons theories
- Verified at large N but speculated to be valid also at small N Aharony 2015

Baryons, monopoles and dualities in Chern-Simons-matter theories

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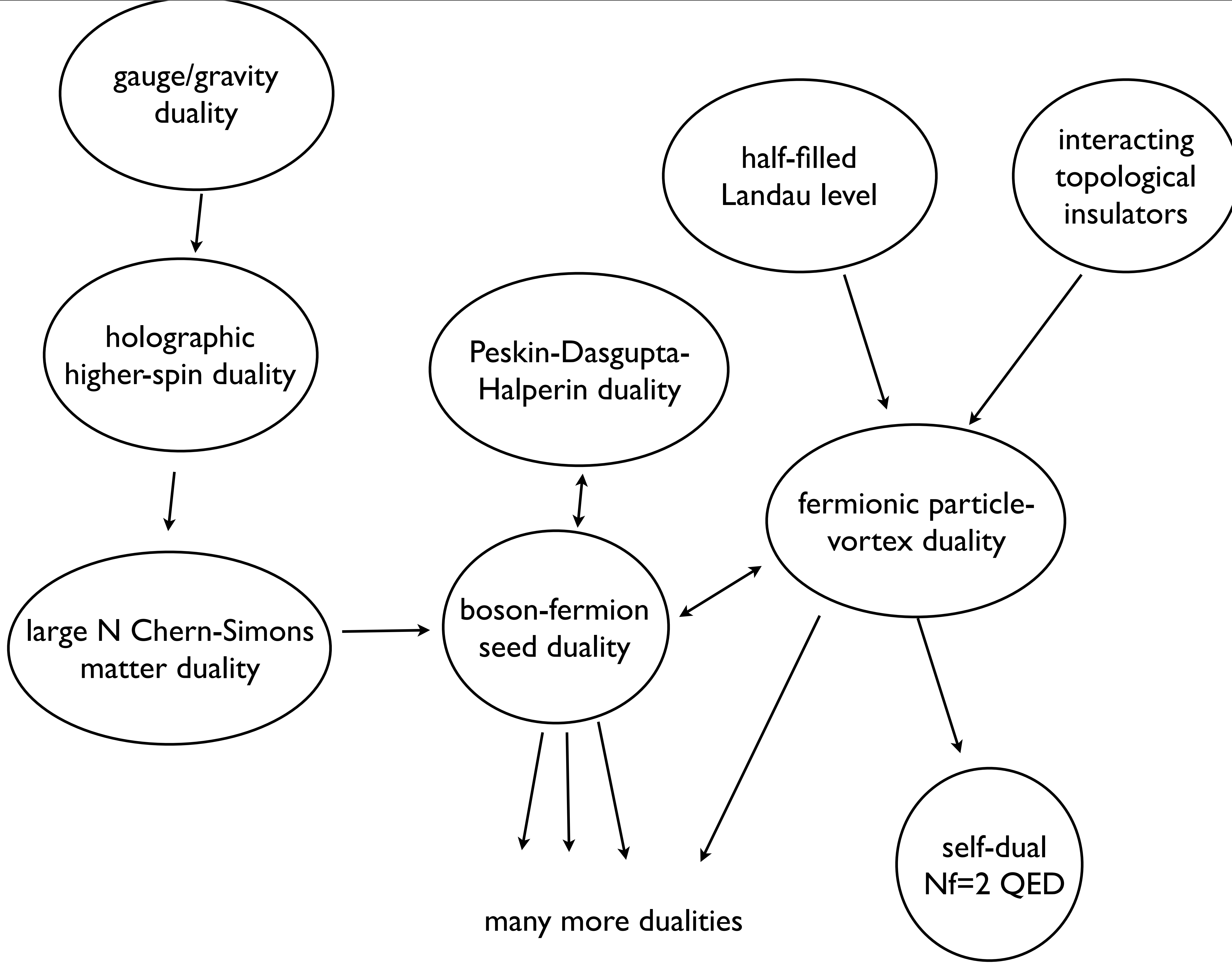
$$U(N)_{k,k} \text{ coupled to scalars} \leftrightarrow SU(k)_{-N+N_f/2} \text{ coupled to fermions,} \tag{2.7}$$

seed duality $\quad N = N_f = k = 1$


gauge/gravity duality
holographic higher-spin duality
large N Chern-Simons matter duality
Peskin-Dasgupta-Halperin duality
boson-fermion seed duality
half-filled Landau level
interacting topological insulators
fermionic particle-vortex duality
self-dual Nf=2 QED
many more dualities

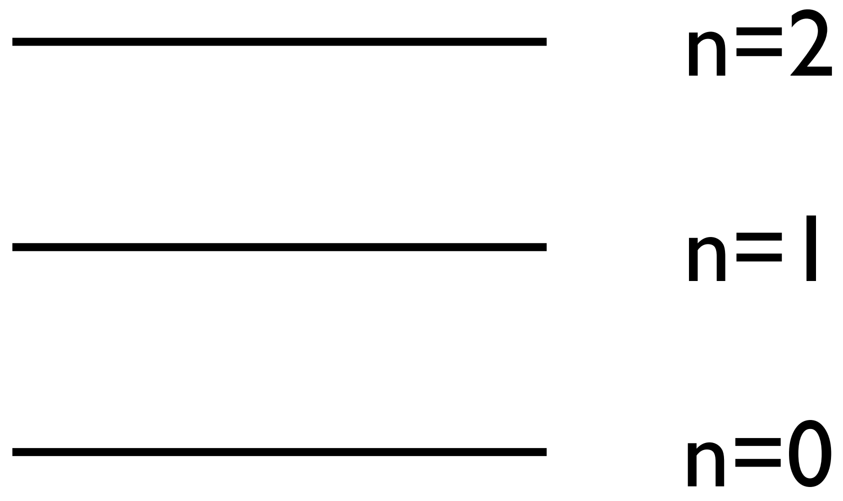
Conclusion

- The half-filled Landau level provides an experimentally realizable example of duality
- Mysterious appearance of a new type of quasiparticle Emergence
- Interaction between high-energy and condensed matter physics is fruitful

Thank you

The fractional quantum Hall effect

2D electrons in B field:
Landau levels

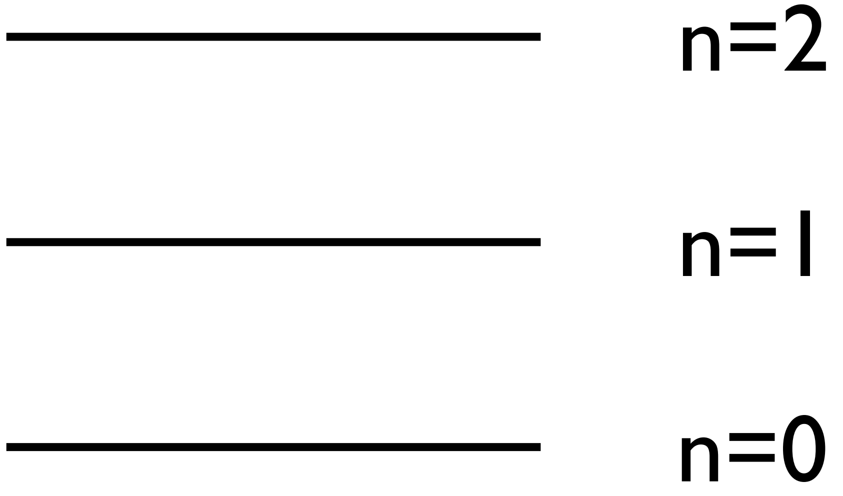


The fractional quantum Hall effect

2D electrons in B field:
Landau levels

filling factor

$$\nu = \frac{\text{number of electrons}}{\text{degeneracy of a LL}}$$



The fractional quantum Hall effect

2D electrons in B field:
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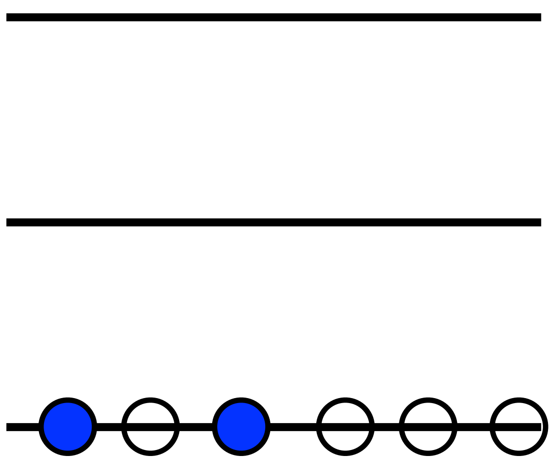
$\nu < 1$



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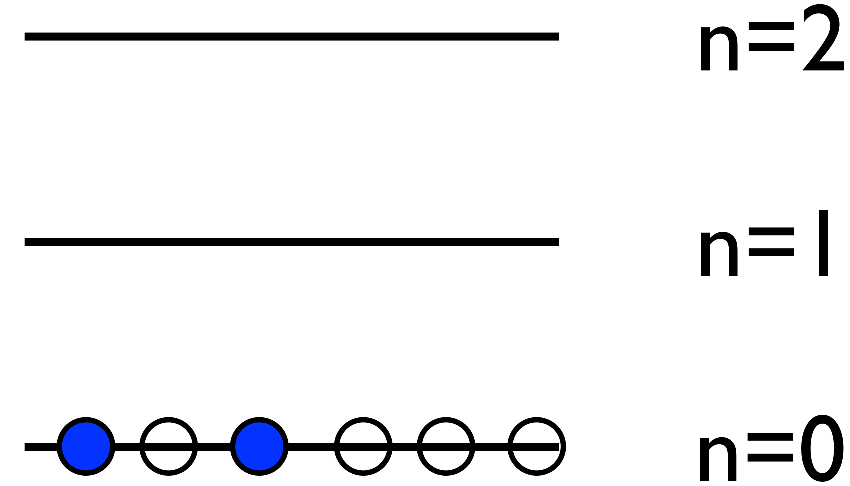

$\nu < 1$

- without interaction: large ground-state degeneracy
- Interactions are essential for determining the ground state
- We will concentrate on filling factor close to 1/2