

Neutrino Mixing and Mass Sum Rules

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Based on collaborations with:

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Outline

- Introduction
- Neutrino Mixing Sum Rules
- Neutrino Mass Sum Rules
- Summary and Conclusions

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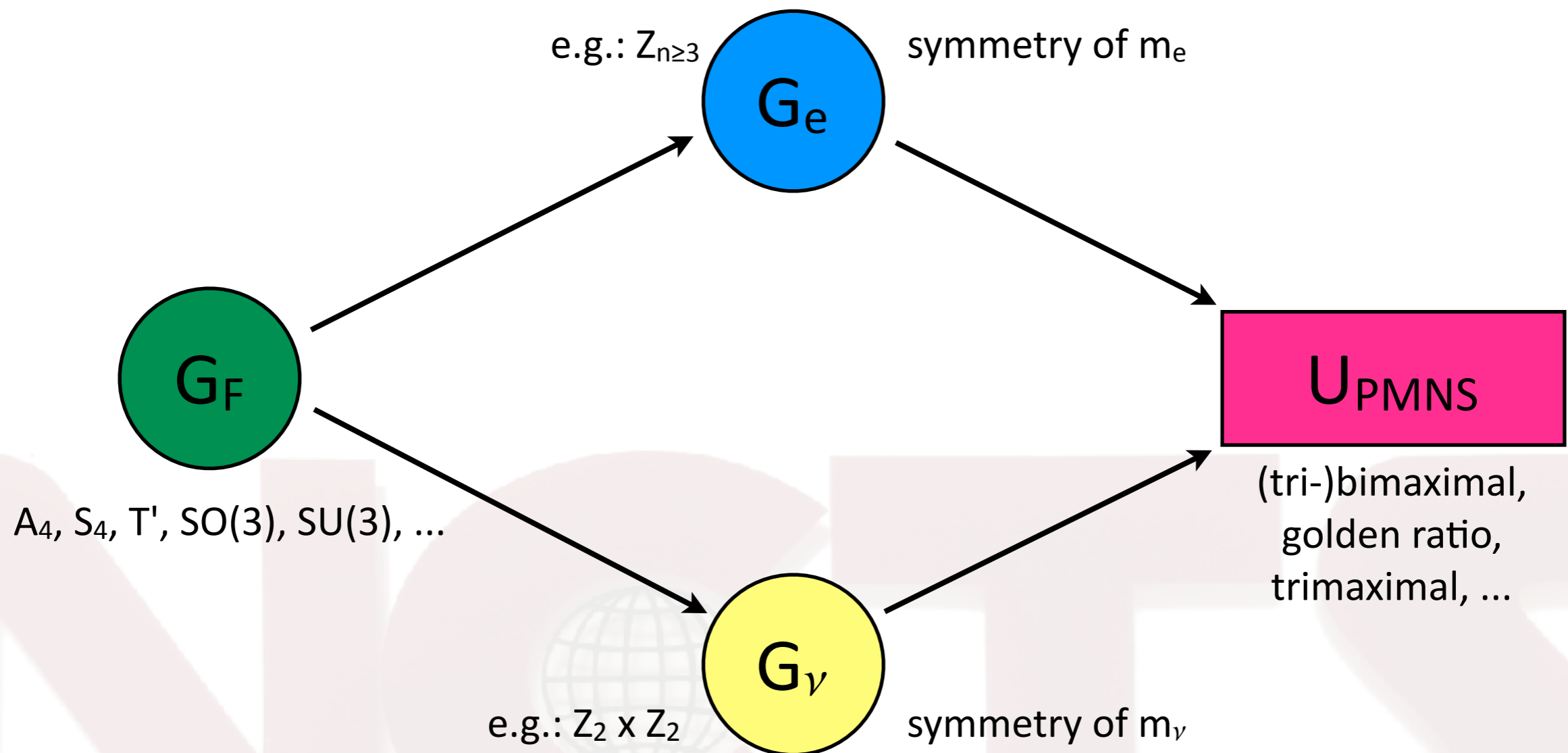
- **Introduction**
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The Flavour Puzzle

- Why do we have three generations?
- Why are the SM fermion masses so vastly different?
- What is the origin of CP violation?
- Why are the quark mixing angles rather small and the leptonic mixing angles rather large?

Non-Abelian (discrete) family symmetries

[for a recent review see King, Luhn 2013]



Sum Rules in the Lepton Sector

- Mixing sum rules, e.g.,

$$\sin^2 \theta_{12} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3} \sin \theta_{13} \cos \delta$$

[Antusch, Aranda, Ballett, Boudjemaa, Damanik, de Medeiros-Varzielas, Ding, Gehrlein, Girardi, Hagedorn, He, Hochmuth, King, Luhn, Malinsky, Marzocca, Masina, Merle, Pascoli, Petcov, Rodejohann, Romanino, Schmidt, Stuart, Tanimoto, Titov, Valle, Volkas,?]

- Mass sum rules, e.g.,

$$m_1 e^{-i\phi_1} + m_2 e^{-i\phi_2} - m_3 = 0$$

[Antusch, Aranda, Ballett, Boudjemaa, Damanik, de Medeiros-Varzielas, Ding, Gehrlein, Girardi, Hagedorn, He, Hochmuth, King, Luhn, Malinsky, Marzocca, Masina, Merle, Pascoli, Petcov, Rodejohann, Romanino, Schmidt, Stuart, Tanimoto, Titov, Valle, Volkas,?]

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Mixing Sum Rules

- Known for a long time and many people worked on them
- Relations between angles and Dirac CP phase

$$s_a(\theta_{ij}, \delta) = 0$$

Mixing Sum Rules

Origin

- The PMNS matrix is a combination

$$U_{\text{PMNS}} = U_e U_\nu^\dagger$$

- Without 1-3 neutrino mixing and only small 1-2 charged lepton mixing

$$\sin \theta_{13} \approx \theta_{12}^e \sin \theta_{23}^\nu$$

- Corrections to symmetry values by charged lepton mixing

Mixing Sum Rules

Examples

[not the first ones, but e.g.: Marzocca, Romanino, Petcov, MS 2011]

- In general to lowest order

$$\sin^2 \theta_{12} \approx \sin^2 \theta_{12}^\nu + \mathcal{O}(\theta_{12}^e)$$

- For Bimaximal mixing:

$$\sin^2 \theta_{12} \approx \frac{1}{2} + \sin \theta_{13} \cos \delta$$

- For Tri-Bimaximal mixing:

$$\sin^2 \theta_{12} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3} \sin \theta_{13} \cos \delta$$

Note on RGE effects

[e.g., Antusch, Kersten, Lindner, Ratz '03]

- Typical size of the corrections

$$\delta\theta_{ij} \sim 10^{-6} (1 + \tan^2 \beta) \frac{m^2}{\Delta m^2}$$

$$\delta\phi_i \sim 10^{-6} (1 + \tan^2 \beta) \frac{m^2}{\Delta m^2}$$

$$\delta m_i \sim (\mathcal{O}(1) + 10^{-6} (1 + \tan^2 \beta)) m_i$$

- Small in Standard Model
- Possibly large in the MSSM

RGE corrected Likelihoods

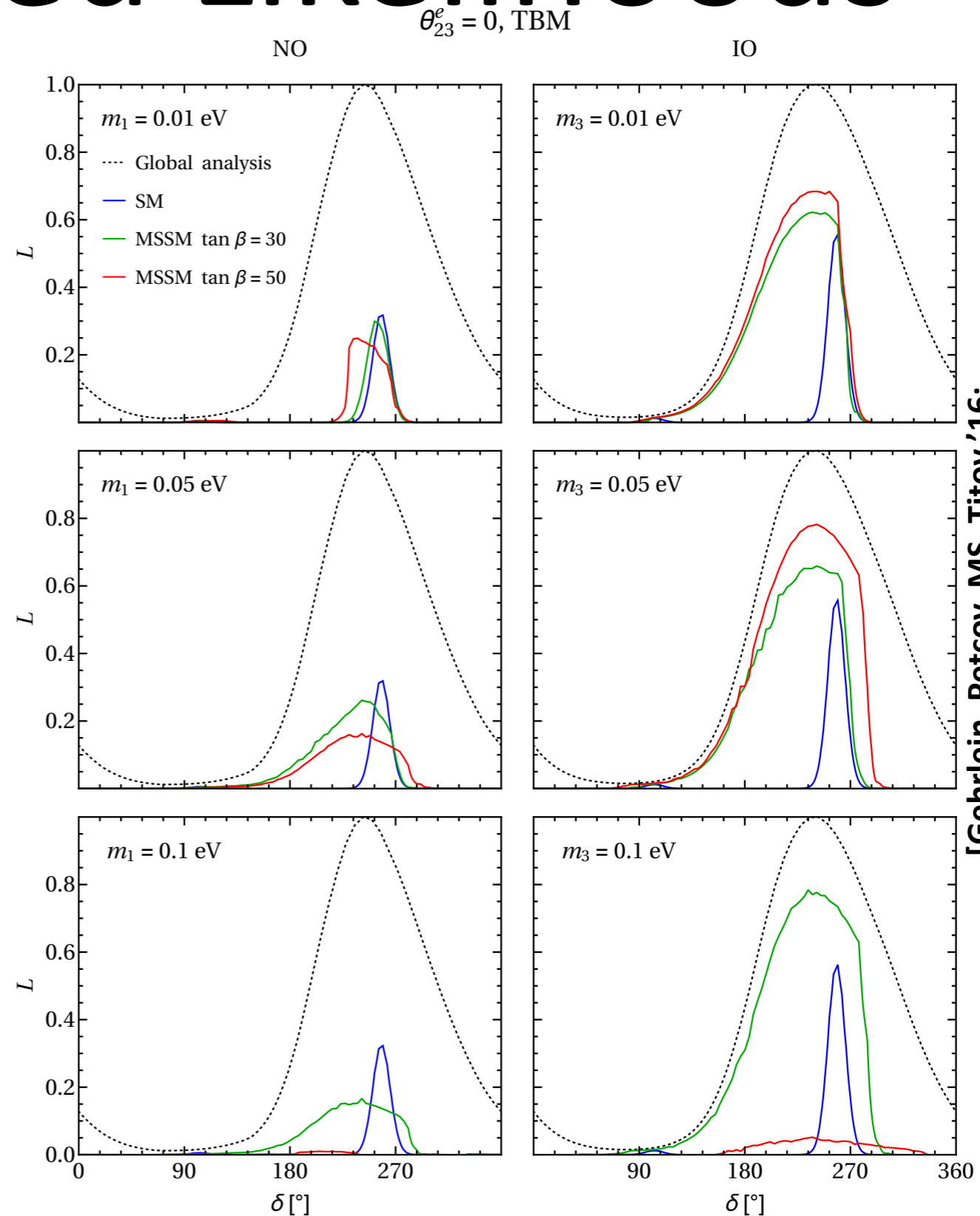
- Systematic study of RGE effects
- Example for tri-bimaximal mixing

$$\sin \theta_{12}^\nu = \frac{1}{\sqrt{3}},$$

$$\sin \theta_{23}^\nu = \frac{1}{\sqrt{2}},$$

$$\sin \theta_{13}^\nu = 0,$$

$$\theta_{13}^e = \theta_{23}^e = 0 \neq \theta_{12}^e$$



[Gehrlein, Petcov, MS, Titov '16; see also Zhang, Zhou '16]

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General remarks

- Not specific to a special family symmetry
- Not related to any residual symmetry [Gehrlein, MS '17]
- Not specific to any mass mechanism
- Only in common: 3 complex neutrino masses depend on 2 complex parameters
→ 2 Relations

One example

[Gehrlein, Oppermann, Schäfer, MS '14]

- $SU(5) \times A_5$ GUT Flavour Model
- Type I seesaw with

$$Y \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } M_{RR} \sim \begin{pmatrix} 2\sqrt{\frac{2}{3}}(v_2 + v_3) & -\sqrt{3}v_2 & -\sqrt{3}v_2 \\ -\sqrt{3}v_2 & \sqrt{6}v_3 & -\sqrt{\frac{2}{3}}(v_2 + v_3) \\ -\sqrt{3}v_2 & -\sqrt{\frac{2}{3}}(v_2 + v_3) & \sqrt{6}v_3 \end{pmatrix}$$

- gives mass sum rule

$$\frac{e^{i\phi_1}}{m_1} + \frac{e^{i\phi_2}}{m_2} = \frac{1}{m_3}$$

The Parametrisation

[King, Stuart, Merle '13; Gehrlein, Merle, MS '15]

$$s \equiv c_1 (m_1 e^{-i\phi_1})^d e^{i\Delta\chi_{13}} + c_2 (m_2 e^{-i\phi_2})^d e^{i\Delta\chi_{23}} + m_3^d \stackrel{!}{=} 0$$

- $c_1, c_2, d, \Delta\chi_{13}$ and $\Delta\chi_{23}$ fixed by the model
- For instance, in the previous example,

$$c_1 = c_2 = 1, d = -1, \Delta\chi_{13} = \Delta\chi_{23} = \pi$$

$$\Rightarrow \frac{e^{i\phi_1}}{m_1} + \frac{e^{i\phi_2}}{m_2} = \frac{1}{m_3}$$

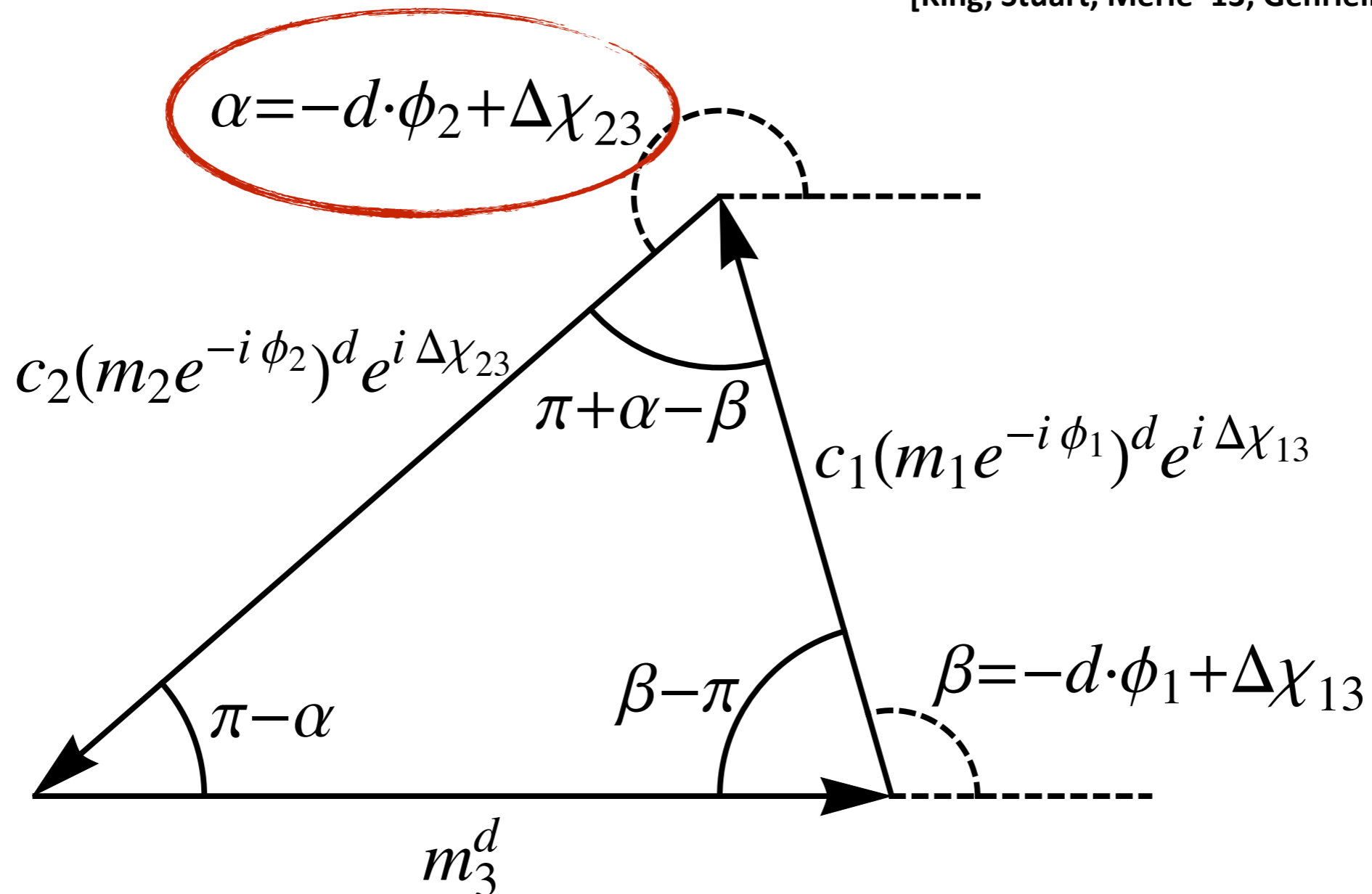
Known Rules

[King, Stuart, Merle '13; Gehrlein, Merle, MS '15]

Sum rule	c_1	c_2	d	$\Delta\chi_{13}$	$\Delta\chi_{23}$
1	1	1	1	π	π
2	1	2	1	π	π
3	1	2	1	π	0
4	1/2	1/2	1	π	π
5	$\frac{2}{\sqrt{3}+1}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	1	0	π
6	1	1	-1	π	π
7	1	2	-1	π	0
8	1	2	-1	0	π
9	1	2	-1	π	$\pi/2, 3\pi/2$
10	1	2	1/2	$\pi, 0, \pi/2$	$0, \pi, \pi/2$
11	1/3	1	1/2	π	0
12	1/2	1/2	-1/2	π	π

Geometrical Interpretation

[King, Stuart, Merle '13; Gehrlein, Merle, MS '15]



Questions

- Can we reconstitute forbidden orderings with small corrections?
- How big is the impact of small corrections on predictions for neutrinoless double beta decay?

Forbidden Orderings

[Gehrlein, Merle, MS '15]

- Sum Rule 2

$$m_1 e^{-i\phi_1} + 2m_2 e^{-i\phi_2} - m_3 = 0$$

- From geometry and inverted ordering

$$\cos \alpha^{\text{tree}} = \frac{m_1^2 - 4m_2^2 - m_3^2}{4m_2 m_3} < -\frac{1}{4} \left(3 \frac{m_2^2}{m_3^2} + 1 \right) < -1$$

- No inverted ordering on tree level

Forbidden Orderings

[Gehrlein, Merle, MS '15]

- RGE corrections in MSSM

$$\delta(\cos \alpha)^{\text{RGE}} \approx - \underbrace{\frac{C y_\tau^2}{192\pi^2}}_{>0} \underbrace{\frac{2.8m_1^2 - 0.4m_2^2 + 0.1m_3^2}{m_2 m_3}}_{>0} \underbrace{\log \frac{M_S}{M_Z}}_{>0}$$

- Forbidden orderings possibly allowed only in very extreme regions of parameter space

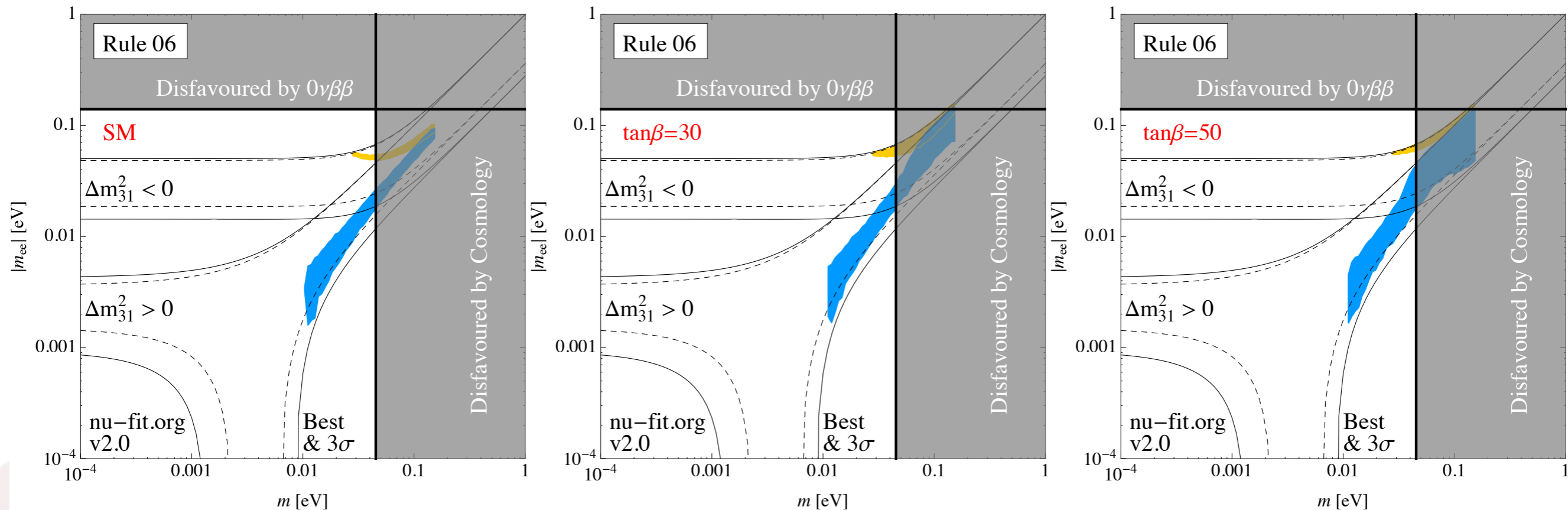
Map of Orderings

Rule	1	2	3	4	5	6	7	8	9	10	11	12
NO	✓	✓	✓	X	X	✓	✓	✓	✓	✓	✓	✓
IO	✓	X	X	✓	✓	✓	✓	✓	✓	X	✓	X

Numerical Results

(RG corrections only)

[Gehrlein, Merle, MS '15]



$$\frac{e^{i\phi_1}}{m_1} + \frac{e^{i\phi_2}}{m_2} = \frac{1}{m_3}$$



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Summary and Conclusions

- Neutrino Flavour Models still interesting
- Constraints on CP violation, neutrino mass scale and ordering
- Mass sum rule predictions are rather robust
- Mixing and mass sum rules combined are very powerful!

Thanks a lot for your
attention!



Backup



RGE corrected Likelihoods

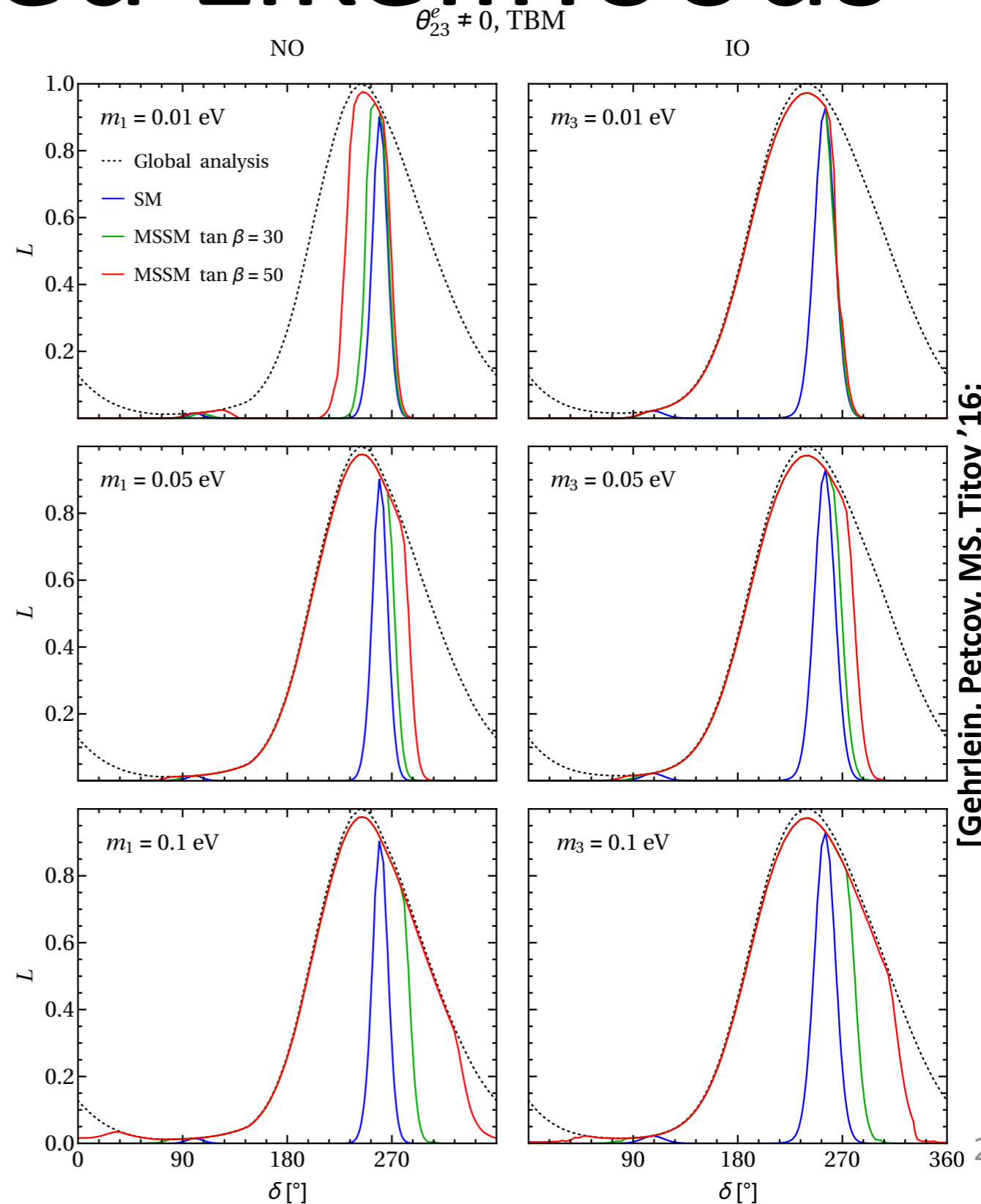
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$$\sin \theta_{12}^\nu = \frac{1}{\sqrt{3}},$$

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$$\sin \theta_{13}^\nu = 0,$$

$$\theta_{13}^e = 0 \neq \theta_{23}^e \text{ and } 0 \neq \theta_{12}^e$$



The Model

[Gehrlein, Oppermann, Schäfer, MS 2014]

- SU(5) x A₅ GUT Flavour Model
- Golden Ratio Neutrino Mixing

$$\tan \theta_{12}^\nu = \frac{2}{1 + \sqrt{5}}, \quad \theta_{23}^\nu = \pi/4, \quad \theta_{13}^\nu = 0$$

- Cabibbo-like 1-2 charged lepton mixing

$$\theta_{12}^e \approx \theta_C \Rightarrow \theta_{13} \approx \frac{\theta_C}{\sqrt{2}} \approx 9^\circ$$

Model Constraints

[Gehrlein, Oppermann, Schäfer, MS 2014]

- Mixing sum rule

$$\theta_{12}^{\text{PMNS}} \approx \theta_{12}^{\nu} + \theta_C \cos \delta$$

- Mass sum rule

$$\frac{e^{i\alpha_1}}{m_1} + \frac{e^{i\alpha_2}}{m_2} = \frac{1}{m_3}$$

- From fermion mass ratios

$$\tan \beta \gtrsim 30$$

Implications

[Gehrlein, Oppermann, Schäfer, MS 2014]

- From the mixing sum rule

$$24^\circ \lesssim \theta_{12}^{\text{PMNS}}(M_S) \lesssim 39^\circ$$

- From the mass sum rule

$$0.011 \text{ eV} \lesssim m_1 \quad \text{for NO}$$

$$0.028 \text{ eV} \lesssim m_3 \lesssim 0.454 \text{ eV for IO}$$

- From RGEs

$$\theta_{12}^{\text{PMNS}}(M_S) \lesssim 33.5^\circ \text{ for NO}$$

$$\theta_{12}^{\text{PMNS}}(M_S) \lesssim 5.7^\circ \text{ for IO} \Rightarrow \text{☠}$$