Mass and Mixing, Global Analysis

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Rencontres du Vietnam 2017: Neutrinos
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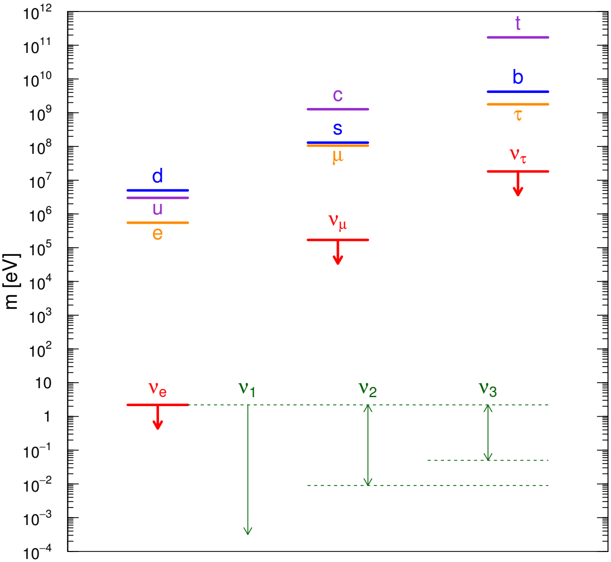
Fermion Mass Spectrum



Neutrino Mixing

Left-handed Flavor Neutrinos produced in Weak Interactions

$$|\nu_e, -\rangle \qquad |\nu_\mu, -\rangle \qquad |\nu_\tau, -\rangle$$

$$\mathcal{H}_{\text{CC}} = \frac{g}{\sqrt{2}} W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right) + \text{H.c.}$$

Fields $\quad \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \Longrightarrow \quad |\nu_\alpha, -\rangle = \sum_k U^*_{\alpha k} |\nu_k, -\rangle \quad$ States

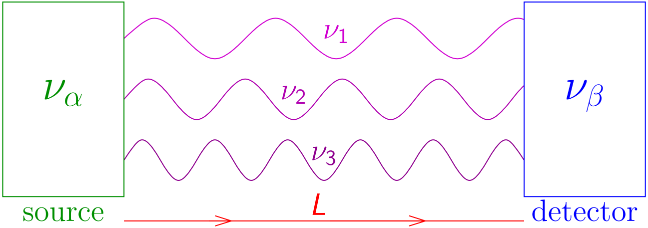
$$|\nu_1, -\rangle \qquad |\nu_2, -\rangle \qquad |\nu_3, -\rangle$$

Left-handed Massive Neutrinos propagate from Source to Detector

3×3 Unitary Mixing Matrix: $\quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

Neutrino Oscillations

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^* \, |\nu_1\rangle + U_{\alpha 2}^* \, |\nu_2\rangle + U_{\alpha 3}^* \, |\nu_3\rangle$$



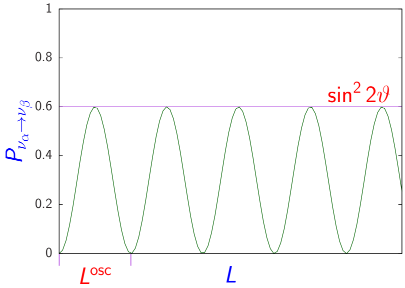
$$|\nu(t>0)\rangle = U_{\alpha 1}^* \, e^{-iE_1 t} \, |\nu_1\rangle + U_{\alpha 2}^* \, e^{-iE_2 t} \, |\nu_2\rangle + U_{\alpha 3}^* \, e^{-iE_3 t} \, |\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

the oscillation probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

2ν-mixing: $P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ $\Longrightarrow$ $L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$



Tiny neutrino masses lead to observable macroscopic oscillation distances!

$$\frac{L}{E} \lesssim \begin{cases} 10 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{short-baseline experiments} & \Delta m^2 \gtrsim 10^{-1}\,\text{eV}^2 \\ 10^3 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{long-baseline experiments} & \Delta m^2 \gtrsim 10^{-3}\,\text{eV}^2 \\ 10^4 \, \frac{\text{km}}{\text{GeV}} & \text{atmospheric neutrino experiments} & \Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2 \\ 10^{11} \, \frac{\text{m}}{\text{MeV}} & \text{solar neutrino experiments} & \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2 \end{cases}$$

Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

Three-Neutrino Mixing Paradigm

Standard Parameterization of Mixing Matrix (as CKM)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $\quad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS

- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent $\Delta m^2_{kj} \equiv m^2_k - m^2_j$: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

(SNO, Borexino; Super-Kamiokande; GALLEX/GNO, SAGE; Homestake, Kamiokande)

VLBL Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$\to$

$$\Delta m^2_{\text{S}} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$$

$$\sin^2\vartheta_{\text{S}} = \sin^2\vartheta_{12} \simeq 0.30$$

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Atmospheric $\nu_\mu \to \nu_\tau$ — (Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2)

LBL Accelerator ν_μ disappearance — (K2K, MINOS, T2K, NOνA)

LBL Accelerator $\nu_\mu \to \nu_\tau$ — (OPERA)

$\rightarrow$

$$\Delta m^2_{\text{A}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{\text{A}} = \sin^2\vartheta_{23} \simeq 0.50$$

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

LBL Accelerator
$\nu_\mu \to \nu_e$ (T2K, MINOS, NOνA)

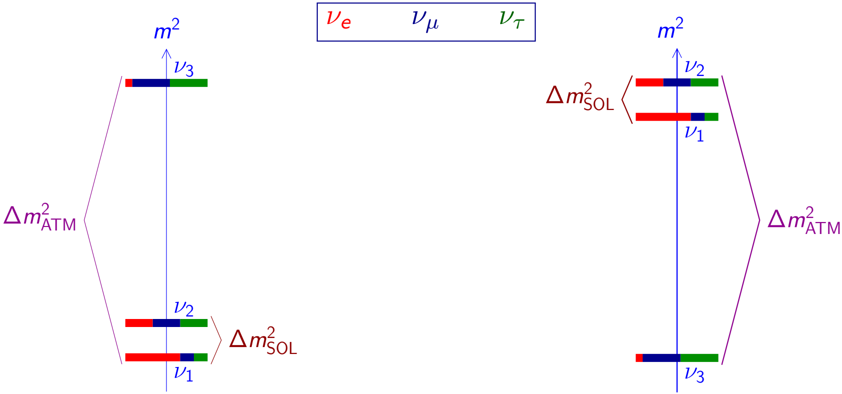
LBL Reactor
$\bar{\nu}_e$ disappearance (Daya Bay, RENO, Double Chooz)

$\rightarrow$

$$\Delta m^2_{\text{A}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{13} \simeq 0.022$$

Mass Ordering



Normal Ordering

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Ordering

$\Delta m^2_{32} < \Delta m^2_{31} < 0$

absolute scale is not determined by neutrino oscillation data

CP Transformation

Right-handed antineutrinos are described by CP-conjugated states

$$\text{Particle} \overset{\text{C}}{\rightleftharpoons} \text{Antiparticle}$$

$$\text{Left-Handed Helicity} \overset{\text{P}}{\rightleftharpoons} \text{Right-Handed Helicity}$$

$$|\nu_\alpha, -\rangle = \sum_k U_{\alpha k}^* |\nu_k, -\rangle \quad \overset{\text{CP}}{\rightleftharpoons} \quad |\bar{\nu}_\alpha, +\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k, +\rangle$$

In oscillation probabilities: Neutrino $U \overset{\text{CP}}{\rightleftharpoons} U^*$ Antineutrino

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re}\left[U_{\alpha k}^* \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^*\right] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right) \quad \leftarrow \quad \text{CP Even}$$

$$+ 2 \sum_{k>j} \text{Im}\left[U_{\alpha k}^* \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^*\right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right) \quad \leftarrow \quad \text{CP Odd}$$

Survival probabilities: $\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ CPT

CP Asymmetries

$$A^{\mathrm{CP}}_{\alpha\beta} = P_{\nu_\alpha\to\nu_\beta} - P_{\bar{\nu}_\alpha\to\bar{\nu}_\beta}$$
$$= 16\, J_{\alpha\beta}\, \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$J_{\alpha\beta} = \mathrm{Im}\left[U^*_{\alpha 1} U_{\beta 1} U_{\alpha 2} U^*_{\beta 2}\right] = \pm J_{\mathrm{CP}} \qquad \text{Jarlskog Invariant}$$

$$J_{\mathrm{CP}} = \mathrm{Im}\left[U^*_{\mu 1}\, U_{e1}\, U_{\mu 2}\, U^*_{e2}\right] = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$

$$J_{\mathrm{CP}} \neq 0 \quad \Longleftrightarrow \quad \vartheta_{12}, \vartheta_{23}, \vartheta_{13} \neq 0, \pi/2 \quad \text{and} \quad \delta_{13} \neq 0, \pi$$

Necessary conditions for observation of CP violation:

- Sensitivity to all mixing angles, including small ϑ_{13}.
- Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}.

LBL $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$

$$\Delta = \frac{\Delta m^2_{31} L}{4E} \qquad A = \frac{2EV}{\Delta m^2_{31}} \qquad V = \sqrt{2} G_F N_e$$

$$\sin\theta_{13} \ll 1 \qquad \Delta m^2_{21}/\Delta m^2_{31} \ll 1$$

$$P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq \sin^2 2\vartheta_{13} \underset{\text{octant}}{\sin^2\vartheta_{23}} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} + \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \underset{\text{CPV}}{\delta_{13}}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} + \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^2 \sin^2 2\vartheta_{12} \cos^2\vartheta_{23} \frac{\sin^2(A\Delta)}{A^2}$$

NO: $\Delta m^2_{31} > 0$ IO: $\Delta m^2_{31} < 0$

for antineutrinos: $\delta_{13} \to -\delta_{13}$ (CPV) and $A \to -A$ (Fake CPV!)

[see: Mezzetto, Schwetz, JPG 37 (2010) 103001]

Global Fits

NuFIT 3.0 (2016)

[arXiv:1611.01514]

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Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity

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[arXiv:1703.04471]

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Global constraints on absolute neutrino masses and their ordering

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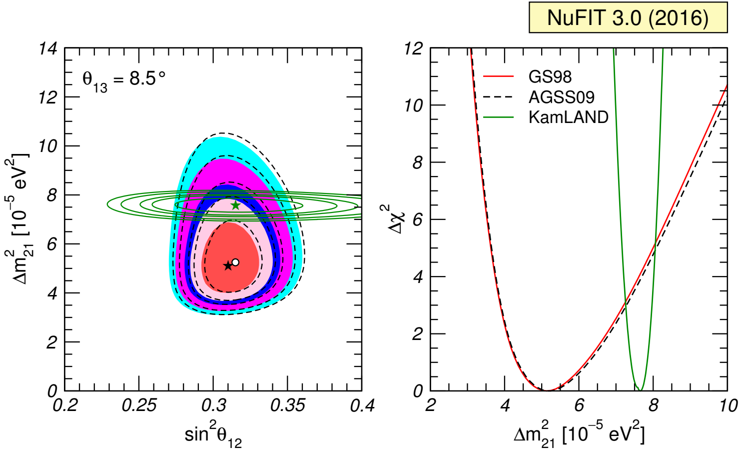
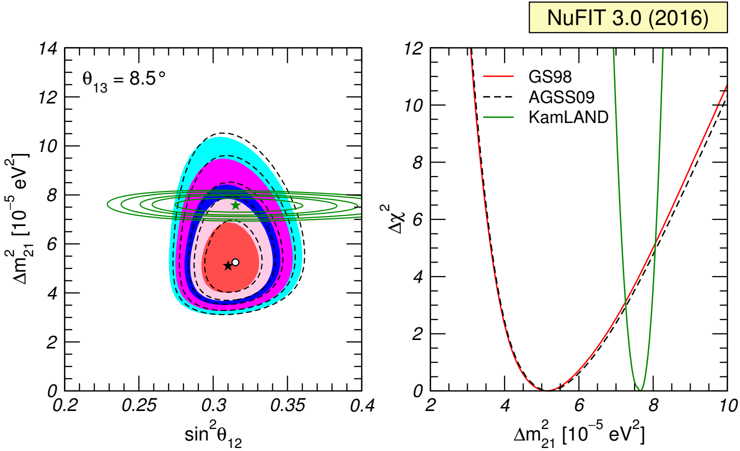
Solar Neutrinos



- Metallicity (C, N , O, Ne, Mg, Si, S, Ar, Fe):
  high (GS98) low (AGSS09)
- A new Generation of Standard Solar Models:
  Vinyoles et al., arXiv:1611.09867

Solar Neutrinos



Solar – KamLAND Tension

- No SK+SNO low-energy spectrum up-turn expected for $(\Delta m^2_{21})_{KL}$
- Larger SK day-night asymmetry than expected for $(\Delta m^2_{21})_{KL}$

Solar Neutrino Spectrum

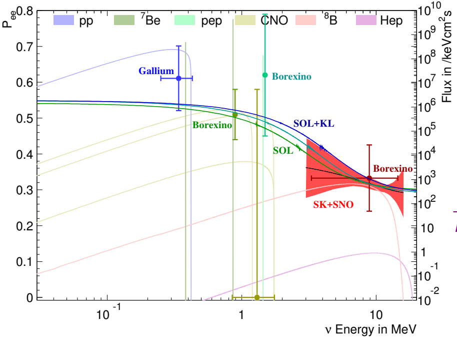
$$\overline{P}_{ee}^{\text{SOL}} = \sum_{k=1}^{3} |U_{ek}|^2 |U_{ek}^0|^2 = \left(\cos^2\vartheta_{12}\cos^2\vartheta_{12}^0 + \sin^2\vartheta_{12}\sin^2\vartheta_{12}^0\right)\cos^4\vartheta_{13} + \sin^4\vartheta_{13}$$

$$= \left(\frac{1}{2} + \frac{1}{2}\cos 2\vartheta_{12}^0 \, \cos 2\vartheta_{12}\right)\cos^4\vartheta_{13} + \sin^4\vartheta_{13}$$

Averaged Vacuum Oscillations

$\theta_{12}^0 \simeq \theta_{12}$

$$\overline{P}_{ee}^{\text{SOL}} \simeq \left(1 - \frac{1}{2}\sin^2\vartheta_{12}\right) \times \left(1 - \sin^2\vartheta_{13}\right)$$



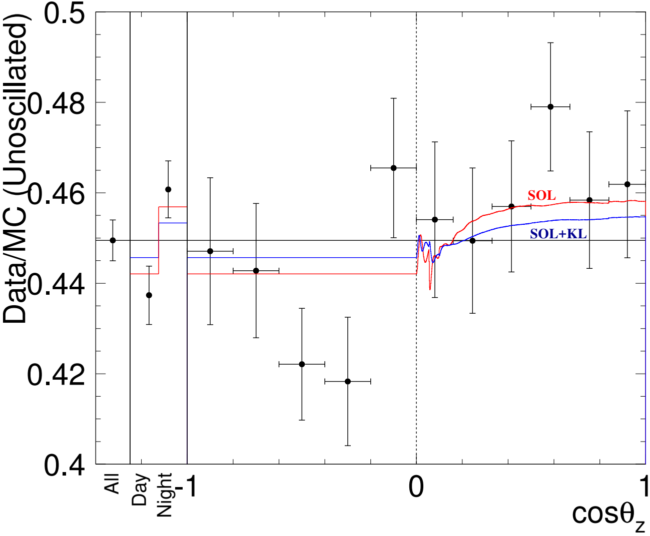
[SK, arXiv:1606.07538]

Adiabatic MSW Transitions

$\theta_{12}^0 \simeq \pi/2$

$$\overline{P}_{ee}^{\text{SOL}} \simeq \sin^2\vartheta_{12} \times \left(1 - \sin^2\vartheta_{13}\right)$$

SK Day-Night Asymmetry

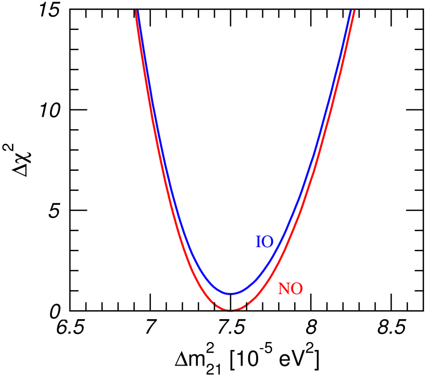


[SK, arXiv:1606.07538]

Δm^2_{21}

NuFIT 3.0 (2016)

[arXiv:1611.01514]



$$\Delta m^2_{21} = 7.50^{+0.19}_{-0.17} \times 10^{-5}\,\mathrm{eV}^2$$

$\sim$ 2.5% precision

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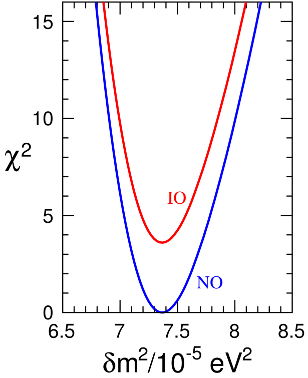
[arXiv:1703.04471]



$$\Delta m^2_{21} = 7.37^{+0.17}_{-0.16} \times 10^{-5}\,\mathrm{eV}^2$$

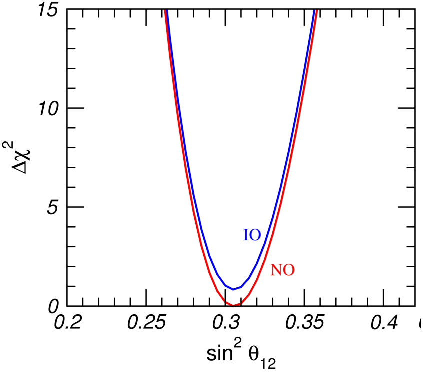
$\sim$ 2.3% precision

$\sim$1.7% difference

$\sin^2\vartheta_{12}$

NuFIT 3.0 (2016)

[arXiv:1611.01514]



$\sin^2\vartheta_{12} = 0.306 \pm 0.012$

$\sim$ 3.9% precision

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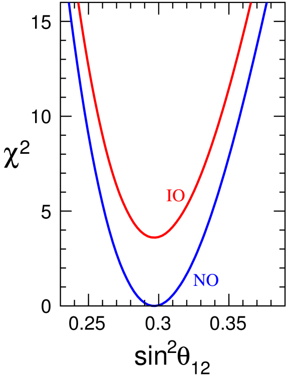
[arXiv:1703.04471]



$\sin^2\vartheta_{12} = 0.297^{+0.017}_{-0.016}$

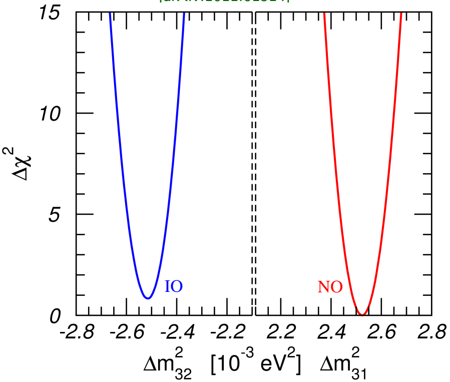
$\sim$ 5.7% precision

$\sim$3.0% difference

Δm^2_{31} & Δm^2_{32}

NuFIT 3.0 (2016)

[arXiv:1611.01514]



$$\frac{|\Delta m^2_{31}|}{10^{-3}\,\text{eV}^2} = \begin{cases} 2.524^{+0.039}_{-0.040} \ \text{(NO)} \\ 2.439^{+0.041}_{-0.038} \ \text{(IO)} \end{cases}$$

$\sim$ 1.7% precision

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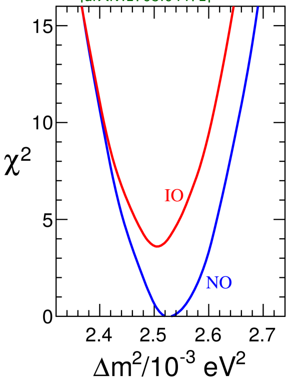
[arXiv:1703.04471]



$$\frac{|\Delta m^2_{31}|}{10^{-3}\,\text{eV}^2} = \begin{cases} 2.562^{+0.042}_{-0.030} \ \text{(NO)} \\ 2.468^{+0.034}_{-0.32} \ \text{(IO)} \end{cases}$$

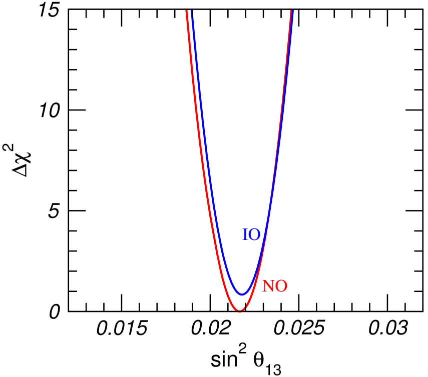
$\sim$ 1.6% precision

$\sim$1.5% difference

$\sin^2\vartheta_{13}$

NuFIT 3.0 (2016)

[arXiv:1611.01514]



$$\sin^2\vartheta_{13} = \begin{cases} 0.02166 \pm 0.00075 \text{ (NO)} \\ 0.02179 \pm 0.00076 \text{ (IO)} \end{cases}$$

$\sim$ 3.5% precision

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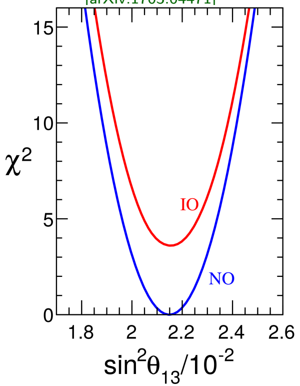
[arXiv:1703.04471]



$$\sin^2\vartheta_{13} = \begin{cases} 0.0215 \pm 0.0007 \quad \text{(NO)} \\ 0.0216^{+0.0008}_{-0.0009} \quad \text{(IO)} \end{cases}$$

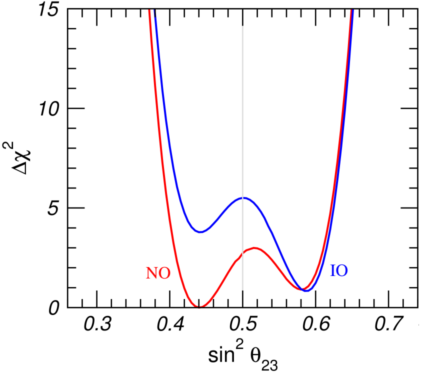
$\sim$ 3.3% precision

$\sim$0.8% difference

$\sin^2\vartheta_{23}$

NuFIT 3.0 (2016)

[arXiv:1611.01514]



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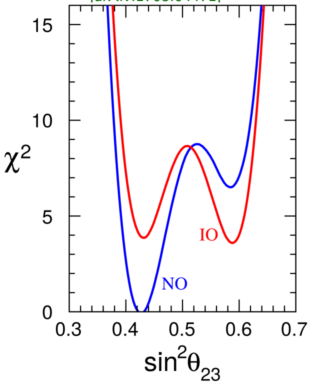


$$\sin^2\vartheta_{23} = \begin{cases} 0.441^{+0.027}_{-0.021} \ \text{(NO)} \\ 0.587^{+0.020}_{-0.024} \ \text{(IO)} \end{cases}$$

$$\sin^2\vartheta_{23} = \begin{cases} 0.425^{+0.021}_{-0.015} \ \text{(NO)} \\ 0.589^{+0.016}_{-0.022} \oplus [0.417, 0.448] \quad \text{(IO)} \end{cases}$$

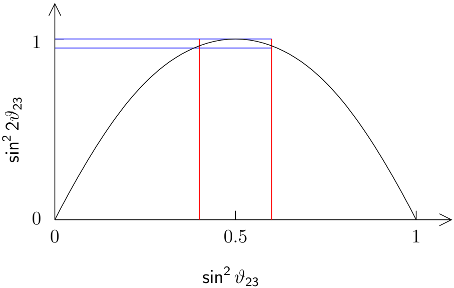
$\sim$ 9% precision

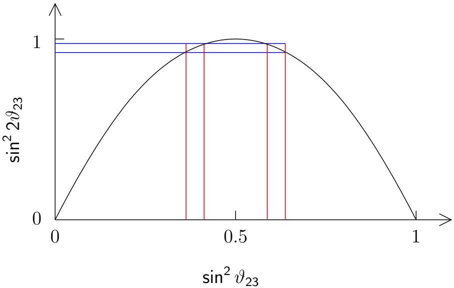
$\sim$ 9% precision

Common NO/IO octant flip

$$P^{\text{LBL}}_{\nu_\mu \to \nu_\mu} \simeq 1 - \sin^2 2\vartheta_{23} \, \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

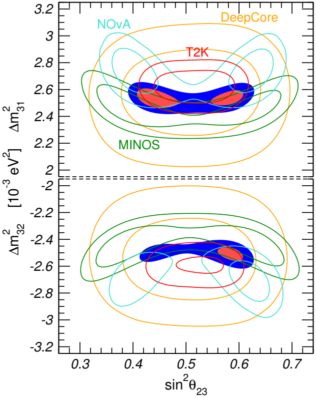
$$\sin^2 2\vartheta_{23} = 4 \sin^2\vartheta_{23} \left(1 - \sin^2\vartheta_{23}\right)$$





NuFIT 3.0 (2016)

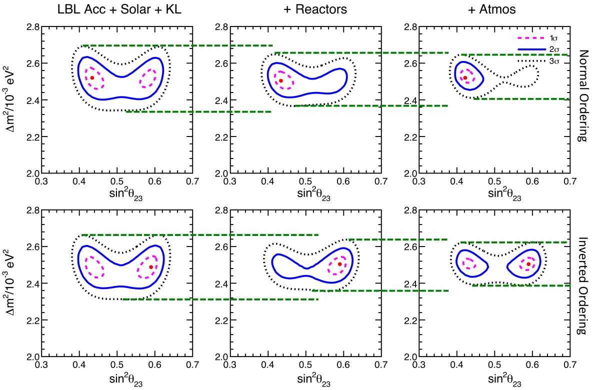
[arXiv:1611.01514]



No SK atmospheric data

Bari 2017

[Lisi @ EPS-HEP 2017]

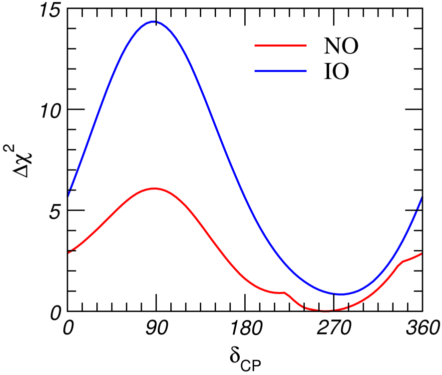


Analyzable subset of SK atmospheric data

δ_{13}

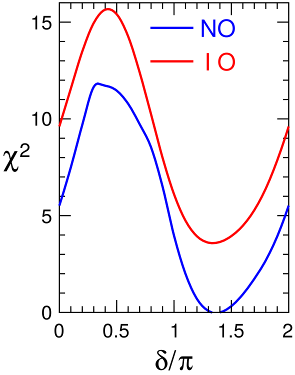
NuFIT 3.0 (2016)

[arXiv:1611.01514]



Bari 2017

[arXiv:1703.04471]

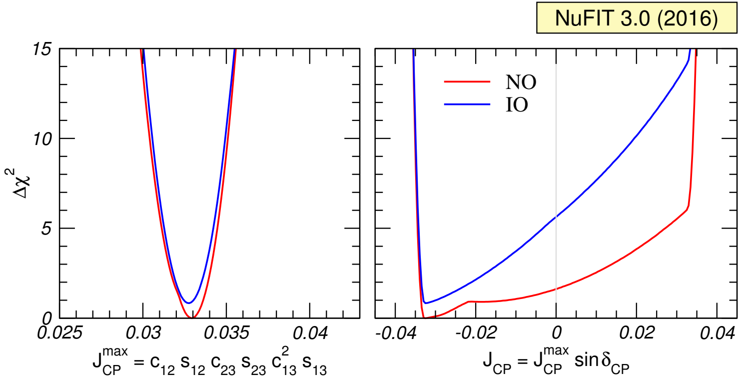


$$\frac{\delta_{13}}{\pi} = \begin{cases} 1.45^{+0.28}_{-0.33} \text{ (NO)} \\ 1.54^{+0.22}_{-0.26} \text{ (IO)} \end{cases}$$

$$\frac{\delta_{13}}{\pi} = \begin{cases} 1.38^{+0.23}_{-0.20} \text{ (NO)} \\ 1.31^{+0.31}_{-0.19} \text{ (IO)} \end{cases}$$

still unknown $\quad \Delta\chi^2_{CPV} = 1.7$

$\sim 20\%$ precision $\quad \Delta\chi^2_{CPV} \simeq 4$

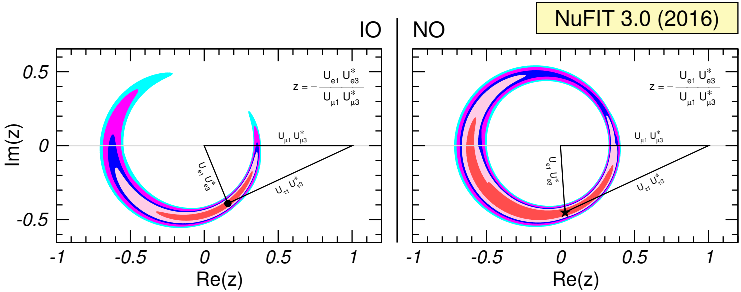


$$J_{CP}^{max} = 0.0329 \pm 0.0007 \, (^{+0.0021}_{-0.0024})$$

$$J_{CP}^{bf} = -0.033$$

About 10^3 larger than $\quad J_{CP}^{quarks} = (3.04^{+0.21}_{-0.20}) \times 10^{-5}$

Unitarity Triangle

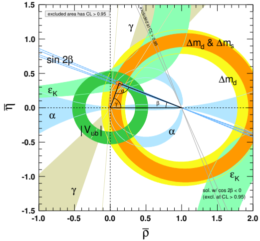


$$z = -\frac{U_{e1}\,U_{e3}^{*}\,U_{\mu 1}^{*}\,U_{\mu 3}}{|U_{\mu 1}|^2|U_{\mu 3}|^2} \implies \text{Im}(z) = \frac{J_{CP}}{|U_{\mu 1}|^2|U_{\mu 3}|^2}$$

Regions defined with respect to the global minimum (NO)

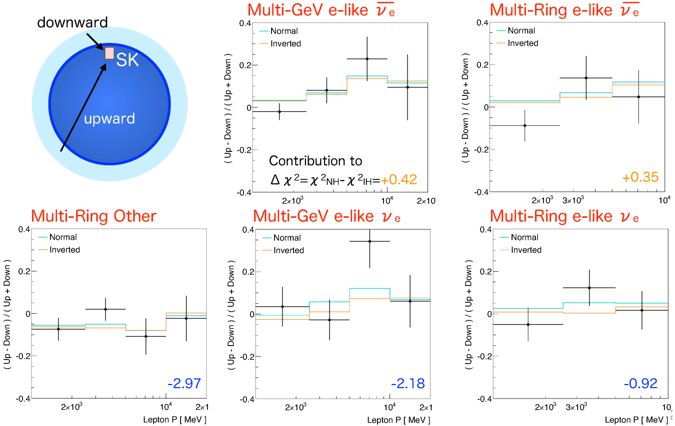
Quark sector:

[PDG 2016]



Mass Ordering

- NuFIT 3.0 (2016): $\Delta\chi^2_{\text{IO-NO}} \simeq 1$ Without SK atmospheric data
- Bari 2017:
  - $\Delta\chi^2_{\text{IO-NO}} = 3.6$ $(\sim 2\sigma)$ With analyzable subset of SK atmospheric data
  - $\Delta\chi^2_{\text{IO-NO}} = 1.1$ Without SK atmospheric data
- SK atmospheric preference for NO due to excess of e-like events

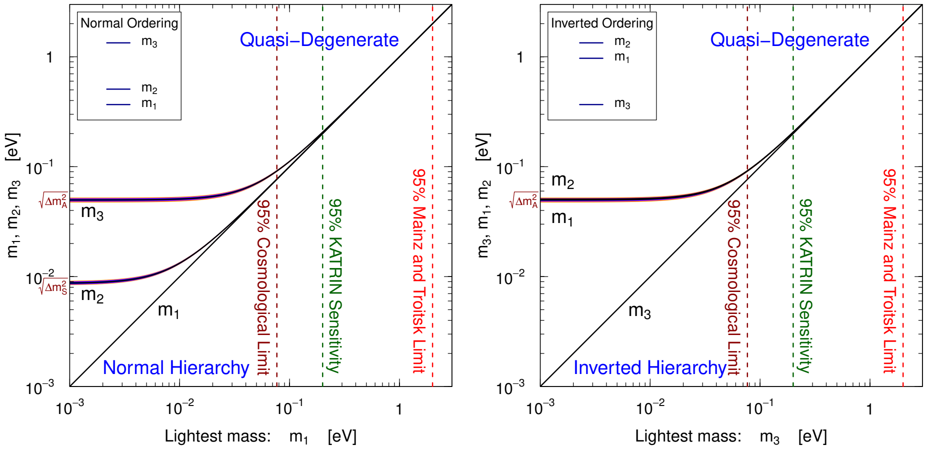


Super-Kamiokande

$\Delta\chi^2_{\text{IO-NO}} = 5.2$

[Koshio @ NOW2016]

Neutrino Masses



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_S^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_A^2$$

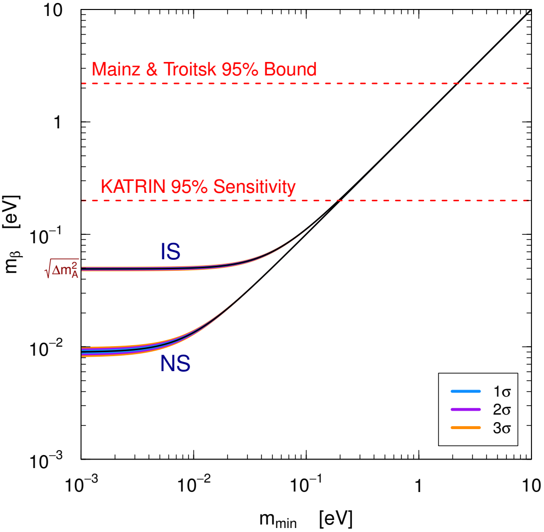
$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_A^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_A^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2}\,\text{eV}$

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

Beta Decay

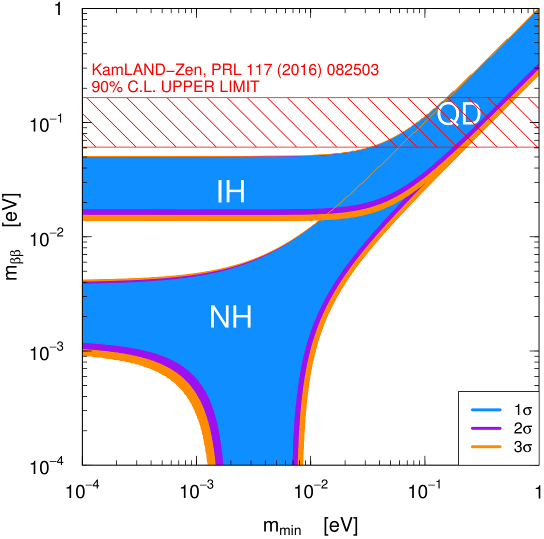
$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



- Quasi-Degenerate:
  $$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$
- Inverted Hierarchy:
  $$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_A^2 \simeq \Delta m_A^2$$
- Normal Hierarchy:
  $$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2$$
  $$\simeq 2 \times 10^{-5} + 6 \times 10^{-5}\,\text{eV}^2$$
- If $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV}$
  $\Downarrow$
  Normal Spectrum

Neutrinoless Double-Beta Decay

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\text{A}} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

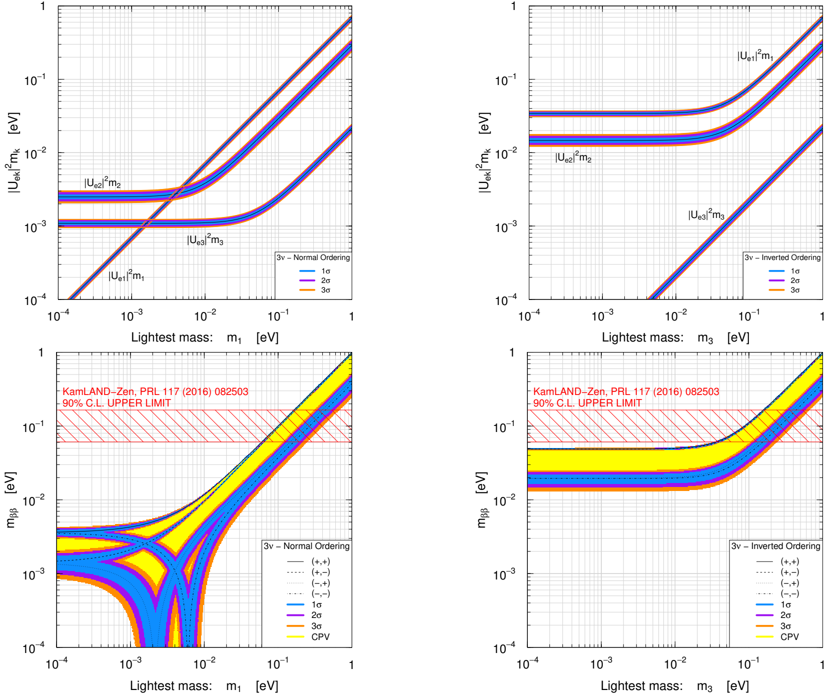
$$|m_{\beta\beta}| \simeq |s^2_{12} \sqrt{\Delta m^2_{\text{S}}} + e^{i\alpha} s^2_{13} \sqrt{\Delta m^2_{\text{A}}}|$$

$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3}\,\text{eV}$$

- If $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV}$

$$\Downarrow$$

Normal Spectrum


|Ue2|²m2
|Ue3|²m3
|Ue1|²m1
3ν – Normal Ordering
1σ
2σ
3σ
Lightest mass: m1 [eV]
|Uek|²mk [eV]
|Ue1|²m1
|Ue2|²m2
|Ue3|²m3
3ν – Inverted Ordering
Lightest mass: m3 [eV]
KamLAND–Zen, PRL 117 (2016) 082503
90% C.L. UPPER LIMIT
3ν – Normal Ordering
(+,+)
(+,−)
(−,+)
(−,−)
CPV
mββ [eV]
3ν – Inverted Ordering

Conclusions

- Robust 3ν-Mixing Paradigm
- Precise determination of the mixing parameters
  - $\Delta m^2_{21} \simeq 7.4 \pm 0.2 \times 10^{-5}\,\mathrm{eV}^2 \quad (\sim 3\%)$
  - $\sin^2\vartheta_{12} \simeq 0.30 \pm 0.02 \quad (\sim 6\%)$
  - $\dfrac{|\Delta m^2_{31}|}{10^{-3}\,\mathrm{eV}^2} \simeq \begin{cases} 2.54 \pm 0.04 & (\mathrm{NO}) \quad (\sim 2\%) \\ 2.45 \pm 0.04 & (\mathrm{IO}) \quad (\sim 2\%) \end{cases}$
  - $\sin^2\vartheta_{13} \simeq 0.0216 \pm 0.0007 \quad (\sim 3\%)$
- Open Problems:
  - $\vartheta_{23} \lesseqgtr 45^\circ$? [T2K, NOνA, ...]
  - CP violation ? $\delta_{13} \approx 3\pi/2$? [T2K, NOνA, DUNE, HyperK]
  - Mass Ordering ? [JUNO, RENO-50, PINGU, ORCA, INO]
  - Absolute Mass Scale ? [β Decay, Neutrinoless Double-β Decay, Cosmology]
  - Dirac or Majorana ? [Neutrinoless Double-β Decay]
  - Physics Beyond Three-Neutrino Mixing ?
  - Theory: Why lepton mixing $\neq$ quark mixing ? Is there any connection ?
    Why $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$?