

Statical properties of the CMB B-mode polarisation signal

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Santos et al., 2017, JCAP, 01, 043 [arXiv:1612.03564]

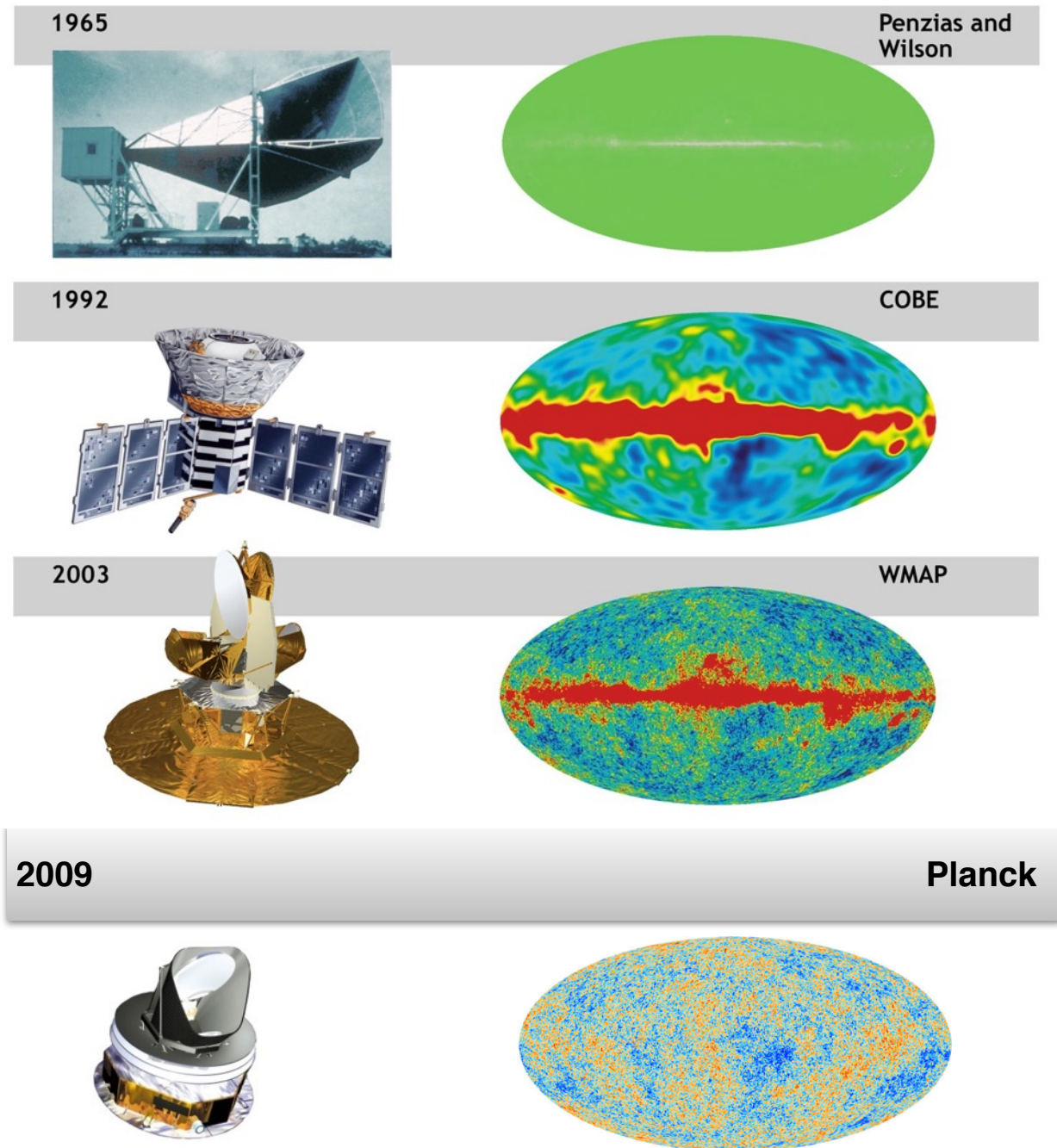
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Outline

- Introduction
 - The cosmic microwave background, CMB
 - The polarised signal
 - The B-map: different contributions
- Motivations
- Statistical method
- The E-to-B leakage contribution

CMB: A brief introduction

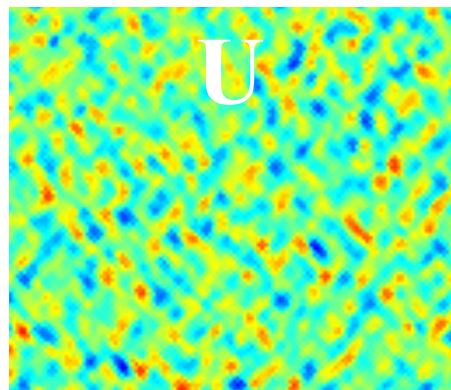
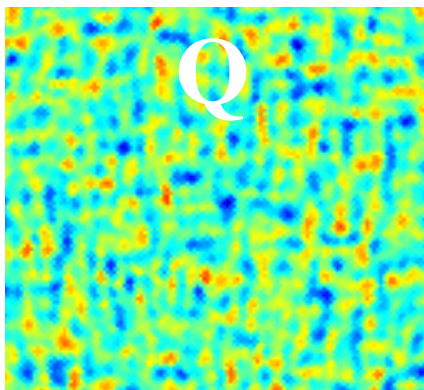
- Discovered in 1965 by Penzias and Wilson
- Temperature anisotropies measured by COBE in 1992
- Improved measurements by WMAP and Planck



The polarisation signal



- By rotating the local coordinate system, Q is rotated into U and vice-versa.

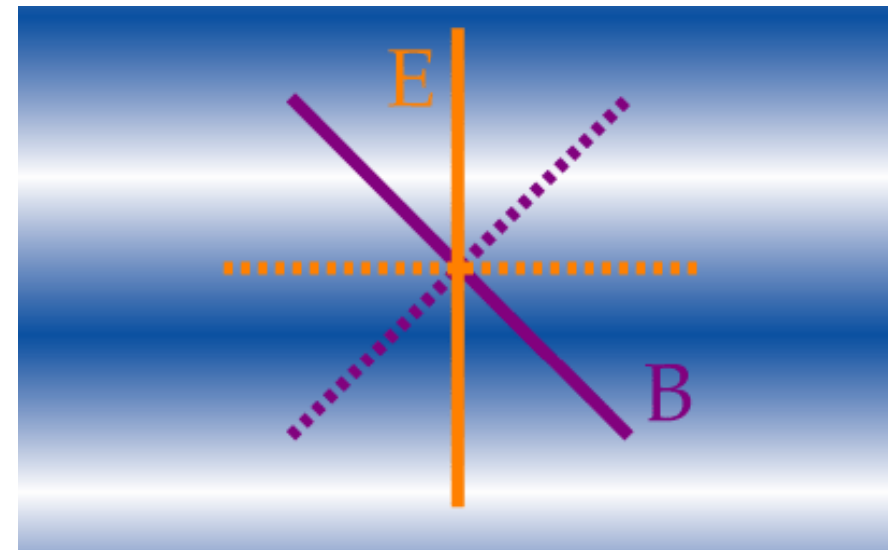


The linearly polarised CMB is completely described by a spin 2 and spin -2 field

$$P_{\pm 2}(\vec{n}) = Q(\vec{n}) \pm iU(\vec{n})$$

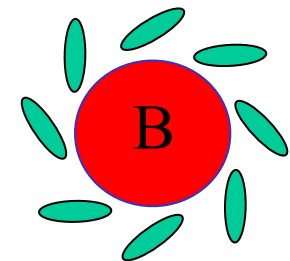
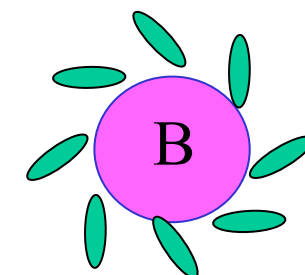
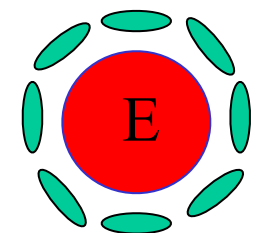
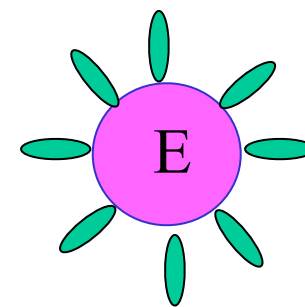
E/B decomposition

- The description North-south and East-West (more formally, the stokes parameters) depends on the arbitrary choices of coordinates
- We then describe the polarisation by its orientation relative to itself: E-mode and B-mode



Cold Spot

Hot Spot



Spherical harmonics expansion

$$P_{\pm}(\vec{n}) = \sum_{lm} a_{\pm 2, lm} Y_{lm}(\vec{n})$$

$$E_{lm} \equiv -\frac{1}{2} [a_{2, lm} + a_{-2, lm}]$$

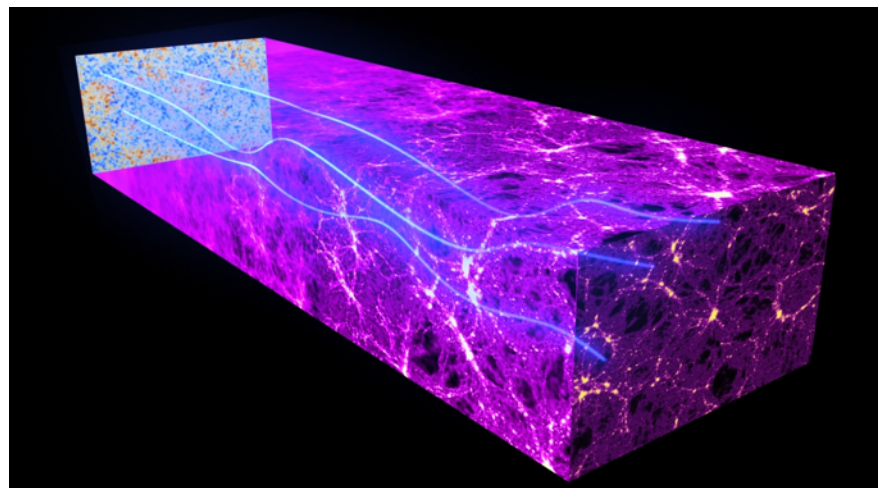
$$B_{lm} \equiv -\frac{1}{2i} [a_{2, lm} - a_{-2, lm}]$$

Our target

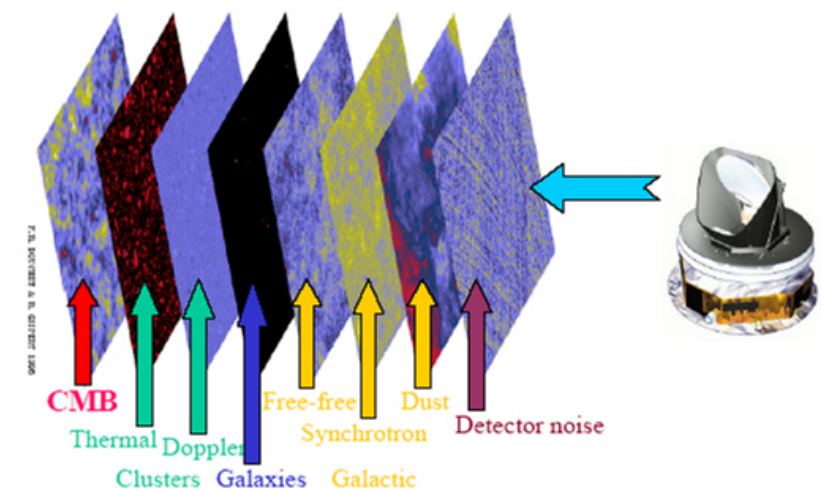


But, what can generate B-mode?

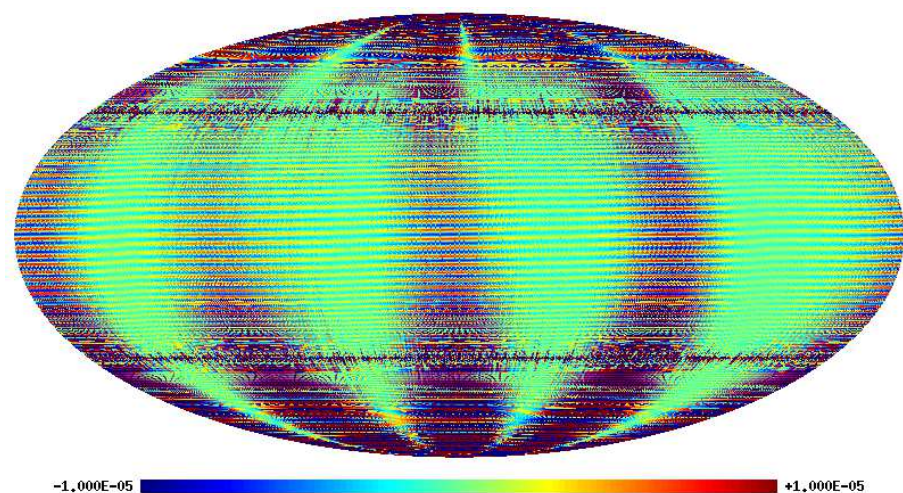
CMB lensing



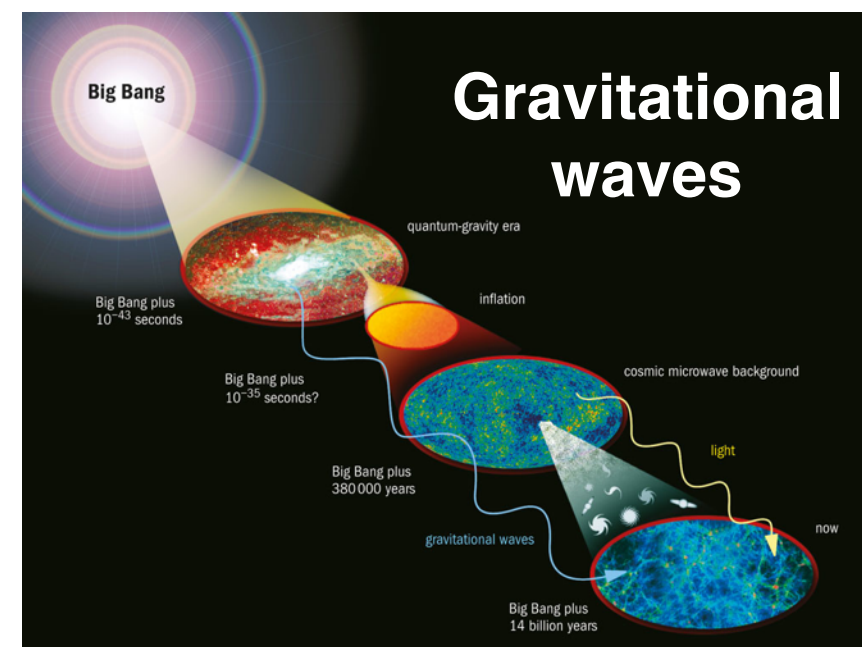
Foreground separation



E-to-B leakage

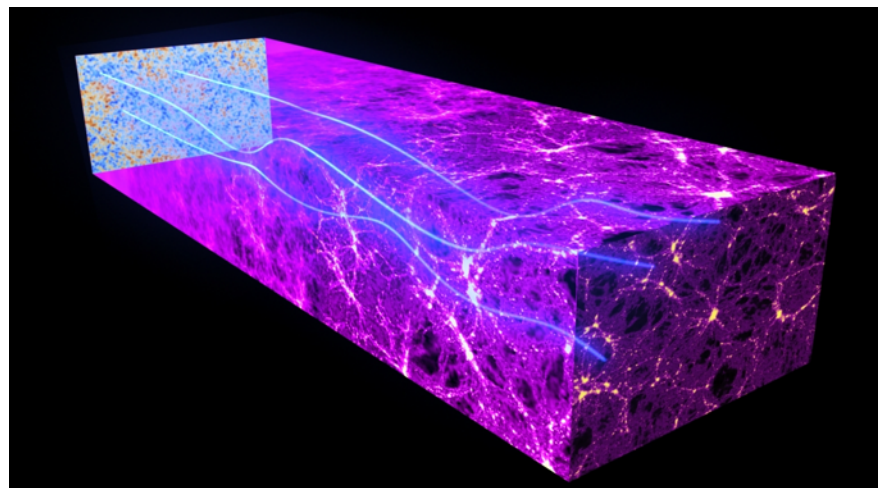


Gravitational waves

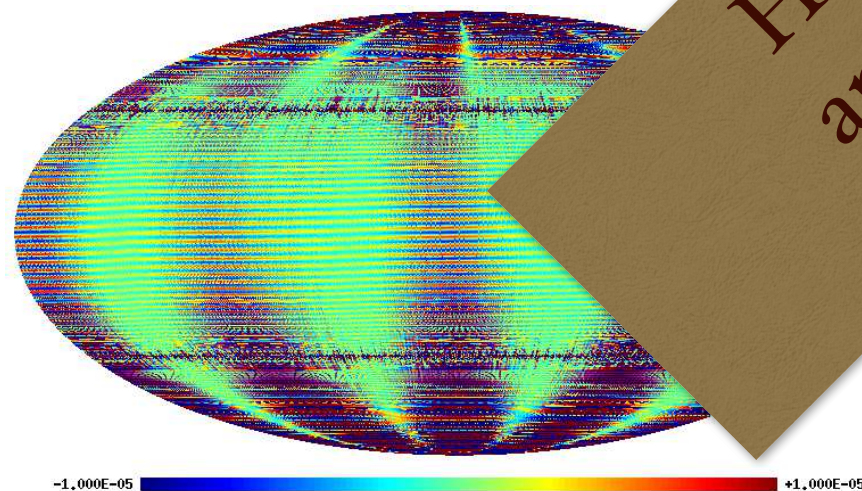


But, what can generate B-mode?

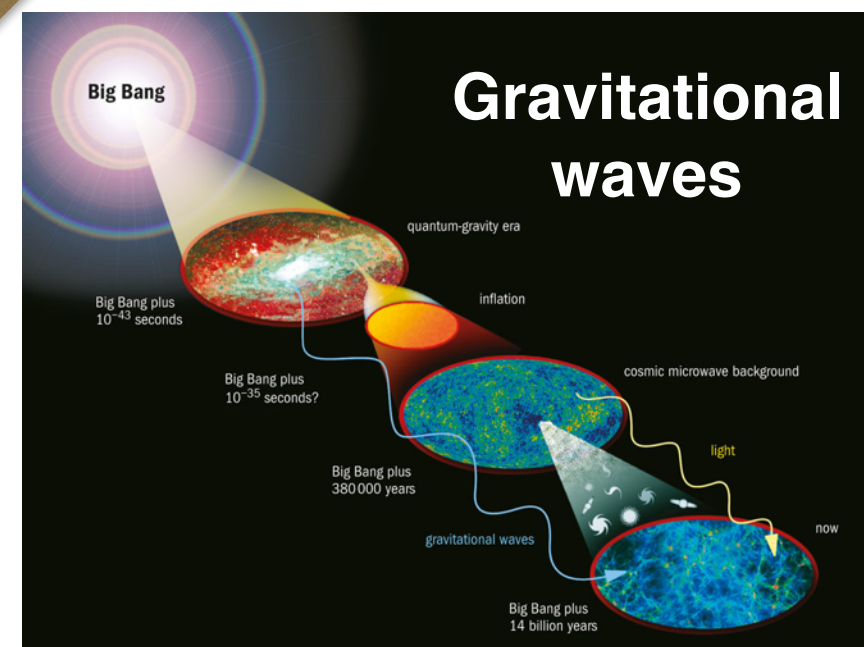
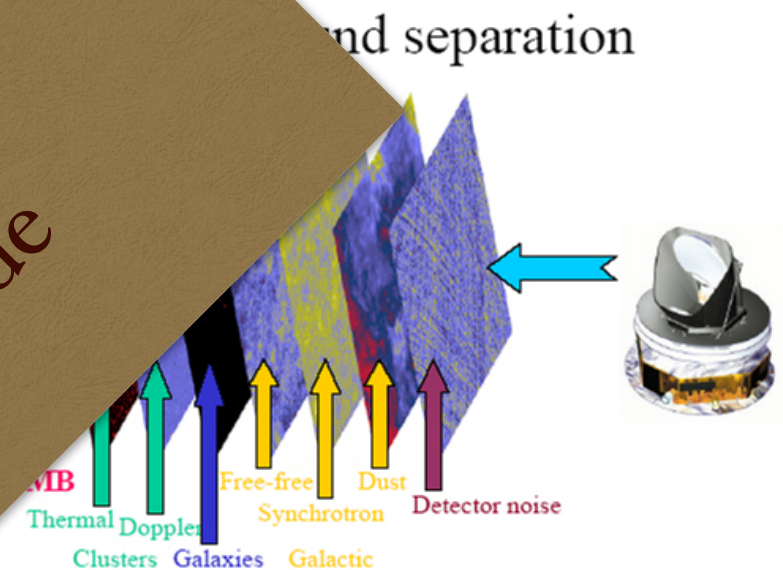
CMB lensing



E-to-B leakage



How can we distinguish among different B-mode signals?



Motivations

- In the linear theory, the CMB fluctuation satisfy the Gaussian distribution
- For nearly Gaussian random fields (CMB T-map and E-map), the power spectrum can completely describe the statical properties of the field
- The CMB B-mode polarisation field is a non-gaussian random field due to the CMB lensing effect
 - Santos et al., 2016, JCAP, 07, 029 [arXiv:1510.07779]
- To extract all the information from a non-Gaussian field we need extra statical tools besides the power spectrum, as for example, the Minkowski functionals

- Using mainly the Minkowski functionals we aim to look for imprints of different contributions to the B-mode map
- Today, I will present the results for the E-to-B leakage when considering a partial sky analysis
 - We include the effect of primordial gravitational waves (GW) and extra sky cuts

The Minkowski functionals

- The morphological properties of the excursion sets can be quantified in terms of the MF
- There are 3 MF for the two dimensional CMB excursion sets.

$$v_0(v) = \int_{Q_v} \frac{da}{4\pi}$$

The total fractional area of the map above a certain threshold

$$v_1(v) = \int_{\partial Q_v} \frac{dl}{16\pi}$$

The boundary length of the excursion set per unit area

$$v_2(\nu) = \int_{\partial Q_\nu} \frac{k dl}{8\pi^2}$$

The Euler Characteristic per unit area (the difference between the numbers of hot spots and cold spots)

Quantifying the result:

$$\chi^2 = \sum_{aa'} \left[\bar{v}_a^{ideal} - \langle v_a^{real} \rangle \right] C_{aa'}^{-1} \left[\bar{v}_{a'}^{ideal} - \langle v_{a'}^{real} \rangle \right]$$

where

$$C_{aa'} \equiv \frac{1}{499} \sum_{k=1}^{500} \left[\left(v_a^{k,real} - \bar{v}_a^{real} \right) \left(v_{a'}^{k,real} - \bar{v}_{a'}^{real} \right) \right]$$

A partial sky survey

- In the ideal case (full sky map), we directly derive E and B from Q and U
- However, even for satellite surveys we will not get a full-sky map. Why?
- We must mask out the unavoidable Galactic foreground!

E/B decomposition in partial sky

$$\mathcal{B}(\hat{n}) = -\frac{1}{2i} [\bar{\partial}\bar{\partial}P_+(\hat{n}) - \partial\partial P_-(\hat{n})]$$

$$\partial f \equiv -\sin^s \theta \left(\frac{\partial}{\partial \theta} + \frac{i}{\sin \theta} \frac{\partial}{\partial \phi} \right) (f \sin^{-s} \theta),$$

$$\bar{\partial} f \equiv -\sin^{-s} \theta \left(\frac{\partial}{\partial \theta} - \frac{i}{\sin \theta} \frac{\partial}{\partial \phi} \right) (f \sin^s \theta).$$

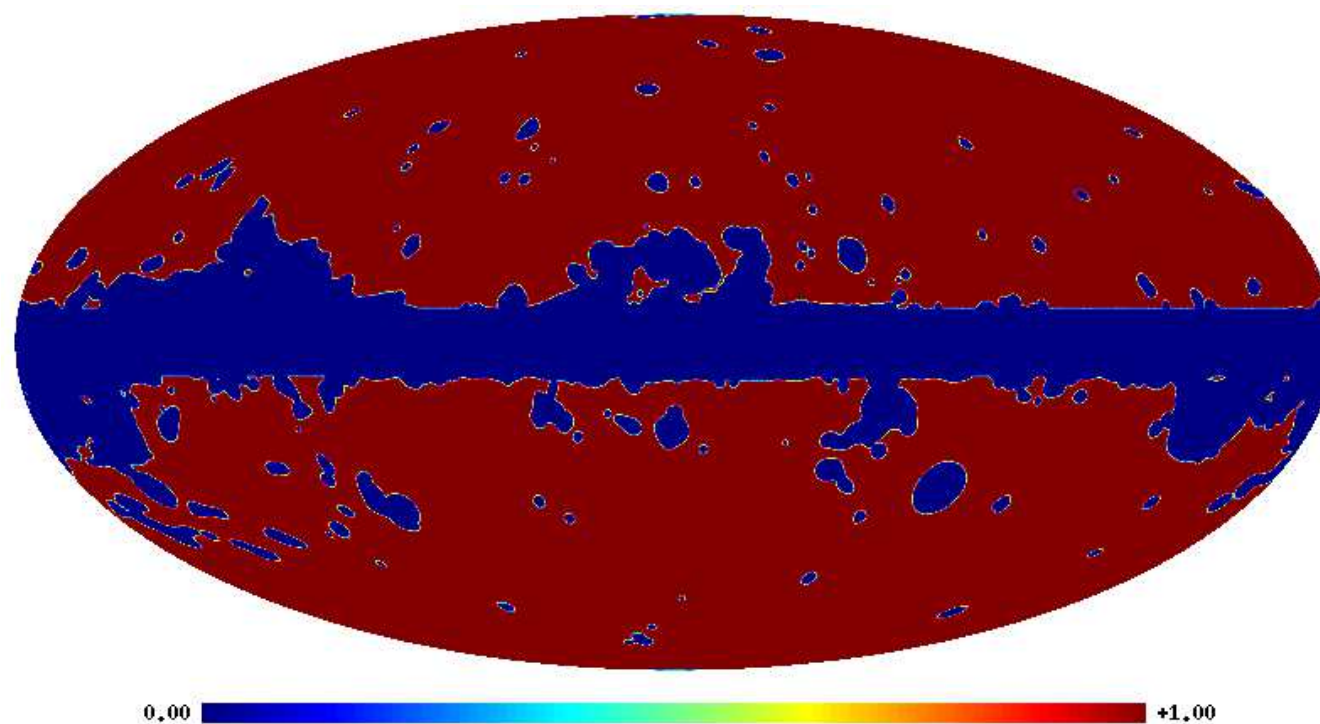
$$\mathcal{B}(\hat{n}) \equiv \sum_{\ell, m} \mathcal{B}_{\ell m} Y_{\ell m}(\hat{n}) \longrightarrow \mathcal{B}_{\ell m} = \int \mathcal{B}(\hat{n}) Y_{\ell m}^*(\hat{n}) d\hat{n}, \quad = N_{\ell, 2} B_{\ell m}$$

$$N_{\ell, s} = \sqrt{(\ell + s)! / (\ell - s)!}$$

$$\mathcal{B}_{\ell m}^{\text{pure}} \equiv -\frac{1}{2i} \int d\hat{n} \left\{ P_+(\hat{n}) [\bar{\partial}\bar{\partial} (W(\hat{n}) Y_{\ell m}(\hat{n}))]^* - P_-(\hat{n}) [\partial\partial (W(\hat{n}) Y_{\ell m}(\hat{n}))]^* \right\}$$

Sky apodization (Planck polarisation mask)

Yi-Fan Wang, Kai Wang , Wen Zhao, arXiv:1511.01220



$$W_i = \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\delta_i - \frac{\delta_c}{2}}{\sqrt{2}\sigma} \right) \quad \delta_i < \delta_c \quad W_i = 1 \quad \delta_i > \delta_c$$

E/B decomposition numerical method

- SZ approach [Smith, (2006); Smith and Zaldarriaga (2007)]: the most efficient method for estimating the CMB B-mode power spectrum in partial sky [Fertè et al. (2013)]
- Step 1: To compute the spin-0, spin-1 and spin-2 rendition of the window function

$$W_0 = W$$

$$W_1 = \partial W$$

$$W_2 = \partial \partial W$$

$$\partial W = -\frac{\partial W}{\partial \theta} - \frac{i}{\sin \theta} \frac{\partial W}{\partial \phi}$$

$$\partial \partial W = -\cot \theta \frac{\partial W}{\partial \theta} + \frac{\partial^2 W}{\partial \theta^2} - \frac{1}{\sin^2 \theta} \frac{\partial^2 W}{\partial \phi^2} - \frac{2i \cot \theta}{\sin \theta} \frac{\partial W}{\partial \phi} + \frac{2i}{\sin \theta} \frac{\partial^2 W}{\partial \theta \partial \phi}$$

- Step 2: Construct 3 apodized maps

$$P_{\pm 2} = W_0 P_{\pm 2} \quad P_{\pm 1} = W_{\mp 1} P_{\pm 2} \quad P_{\pm 0} = W_{\mp 2} P_{\pm 2}$$

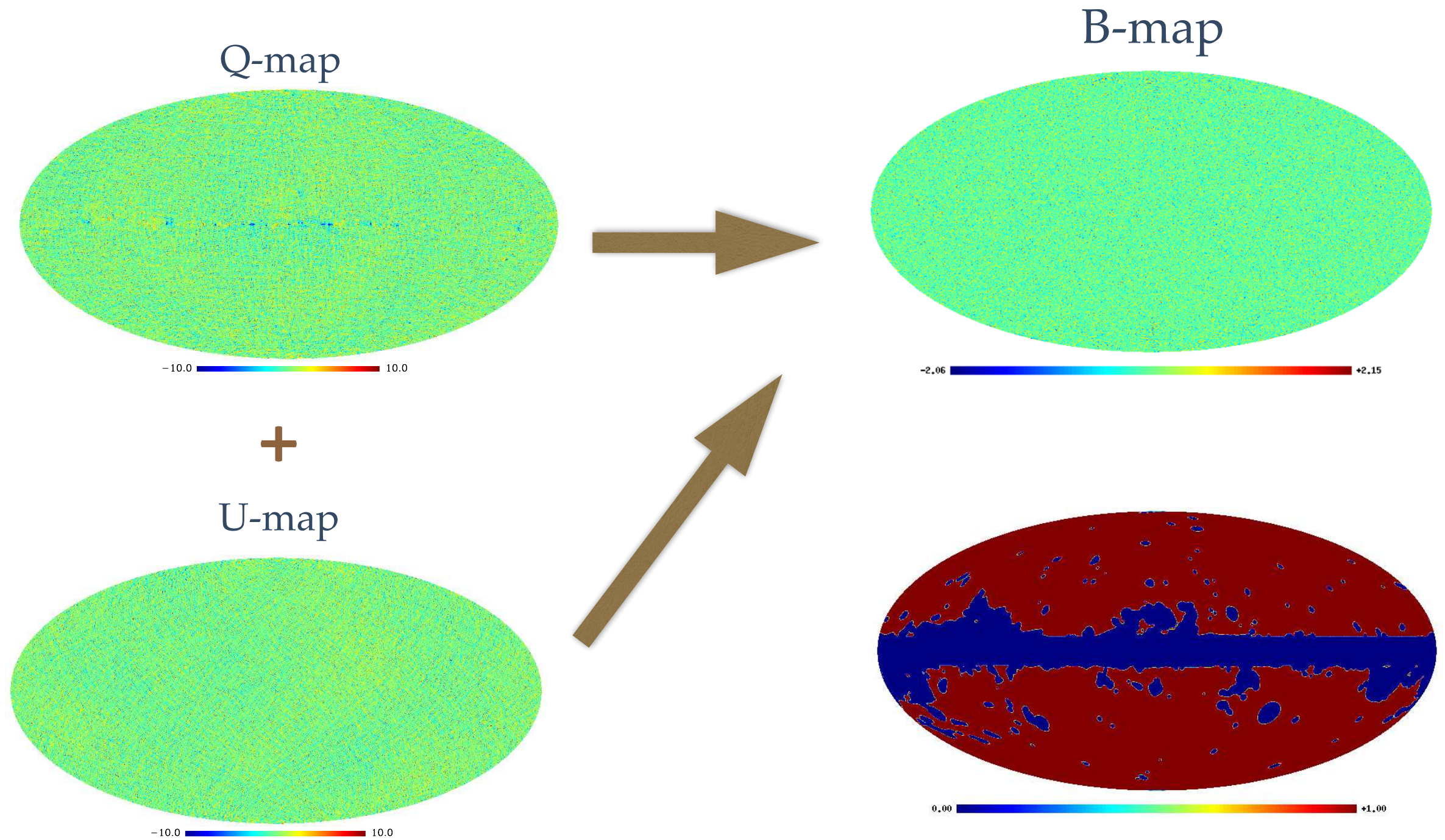
- Step 3: Generating the new B_{lm} and finally the B-map

$$\tilde{B}_{lm} = \left(B_{0,lm} + 2N_{l,1} B_{1,lm} + N_{l,2} B_{2,lm} \right)$$

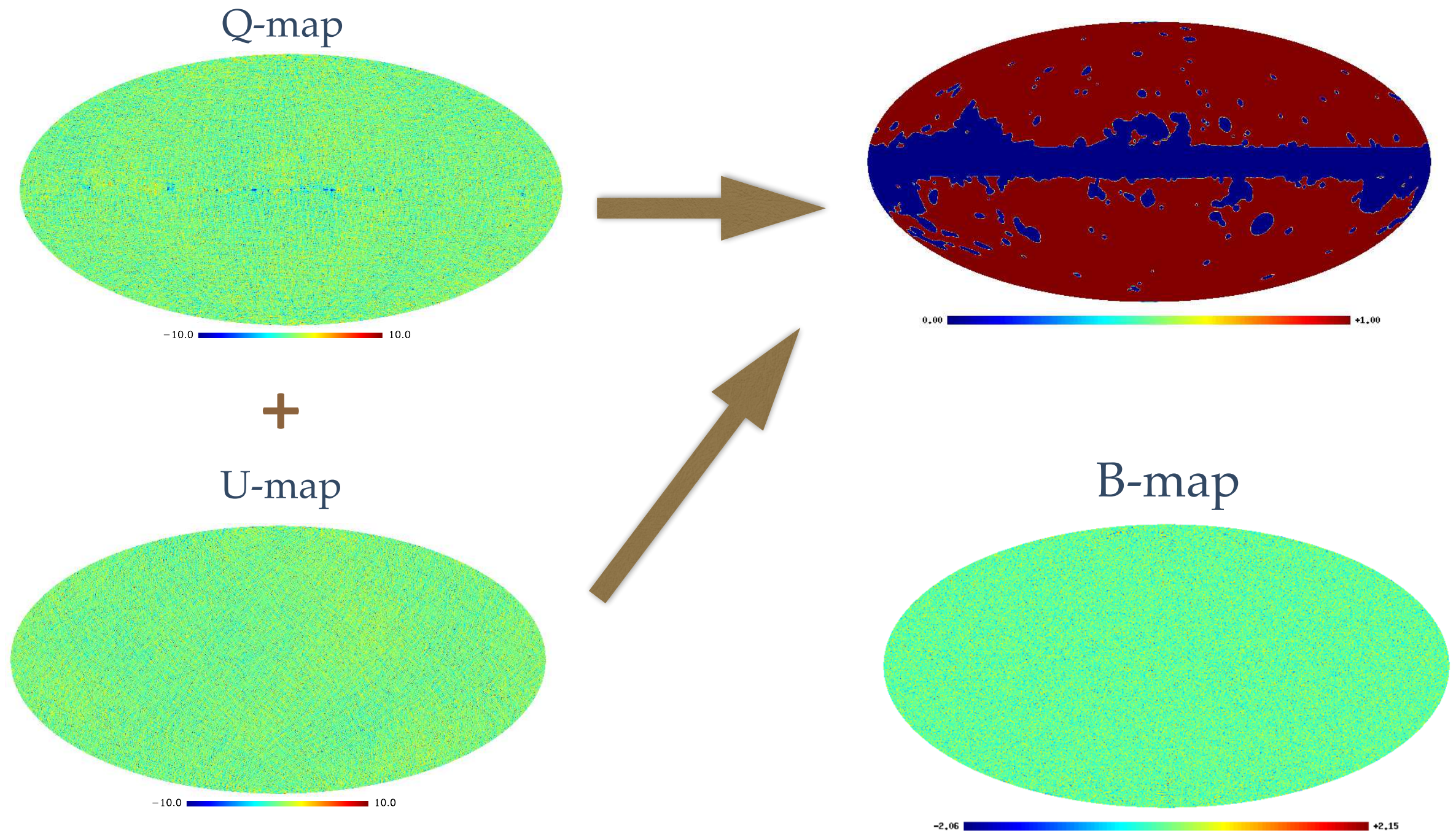
Simulations

- We used CAMB to generate the lensed CMB power spectra
 - Planck best fit parameters for the LCDM model
- These power spectra are the input of lenspix to produce Q and U maps
 - 500 simulations with nside=1024 (i.e. 1.25829×10^7 pixels)

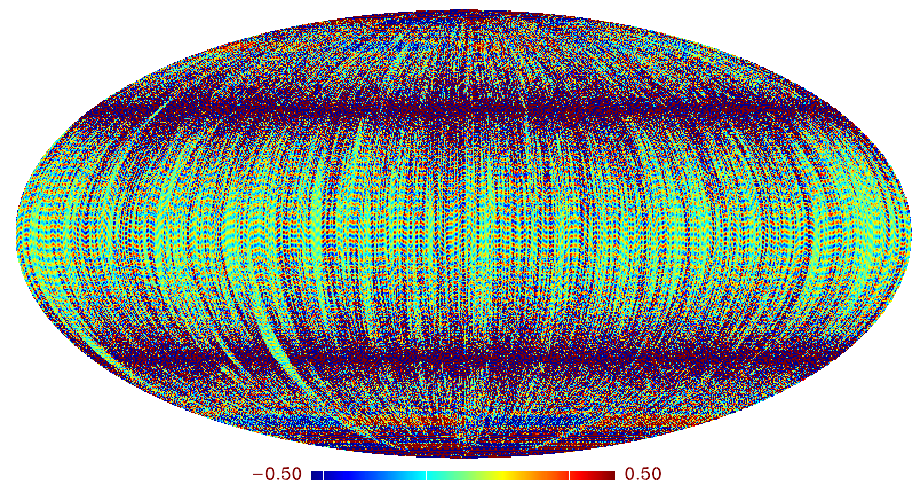
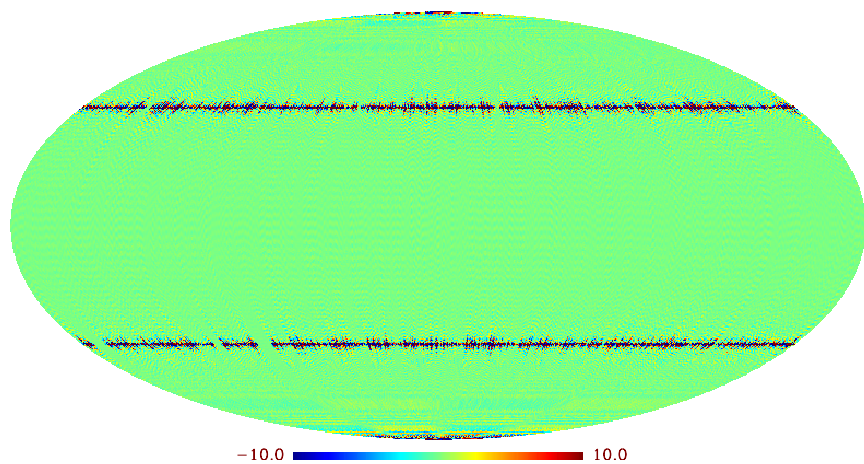
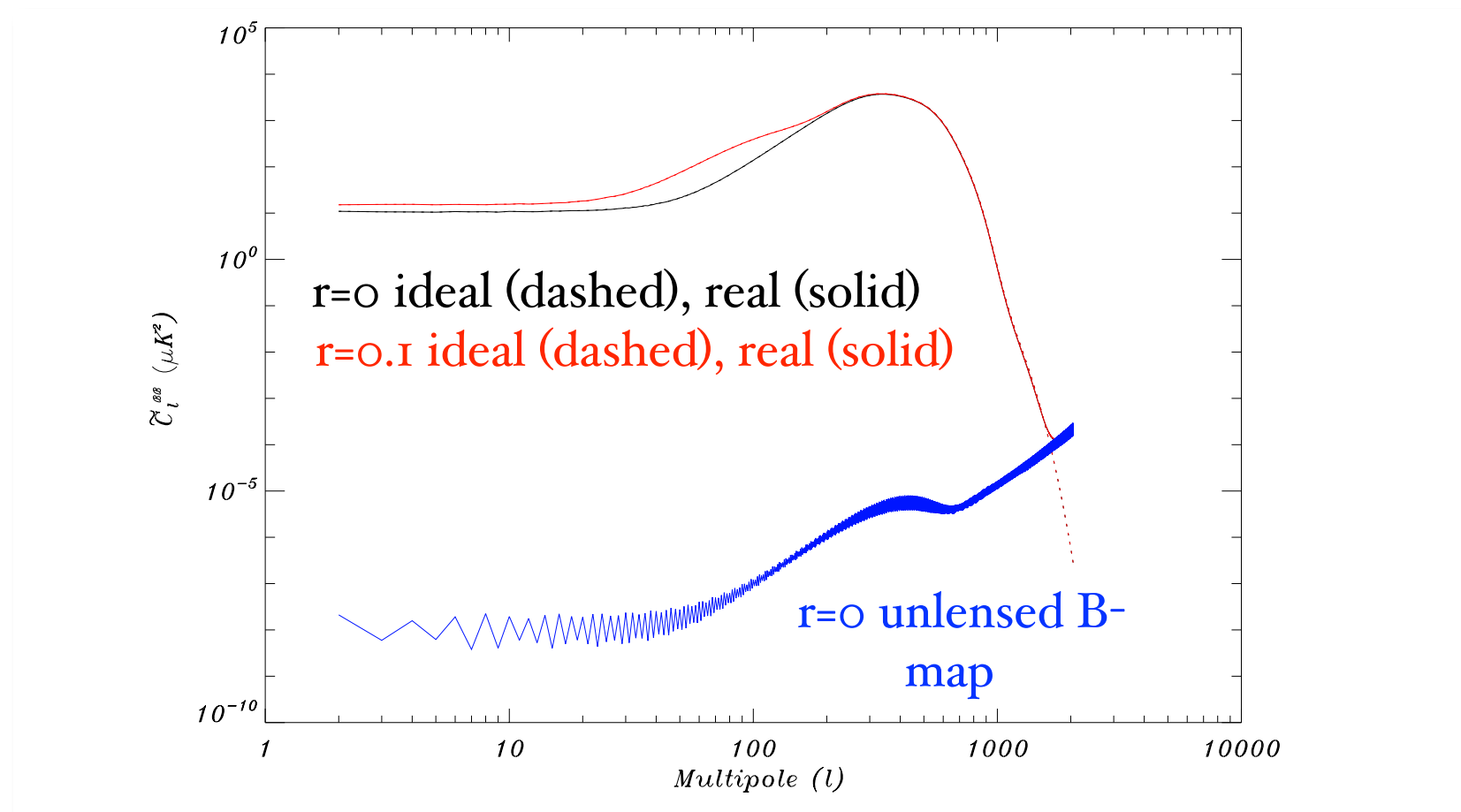
Ideal case



Real case



Real vs ideal



Testing different soothing scales

- In order to remove the multipoles dominated by noise, the calculation of the MF requires smoothing the maps to be analysed
- Even though different smoothing scales of the same CMB map have a high correlation, they must be taken into account in order to extract all its available information

$$\theta_s = 10', 20', 30', 40', 50', 60'$$

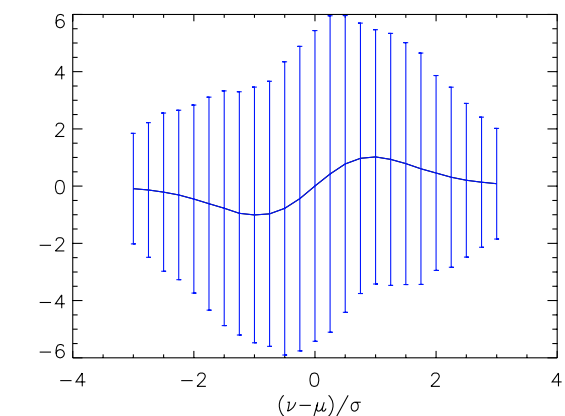
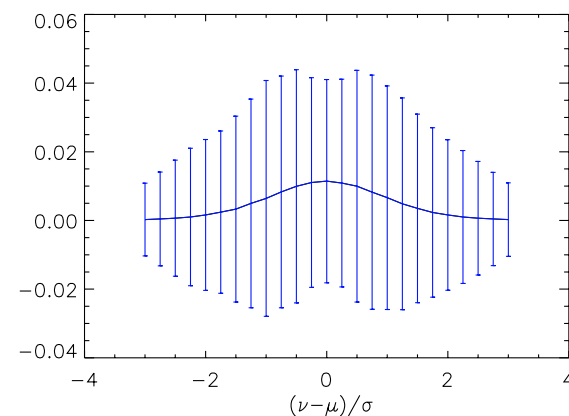
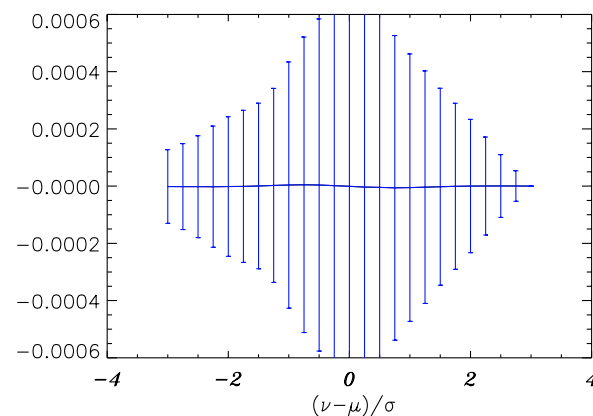
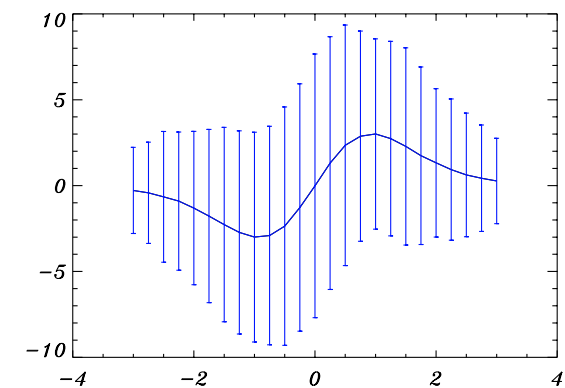
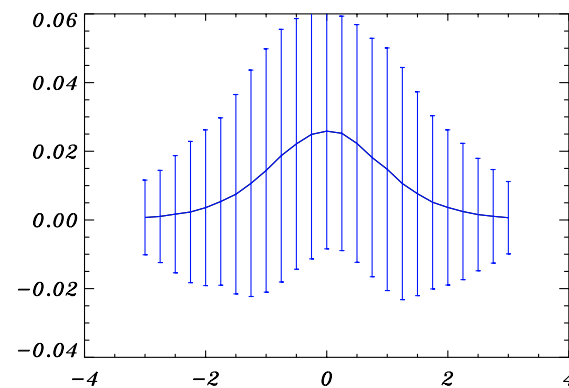
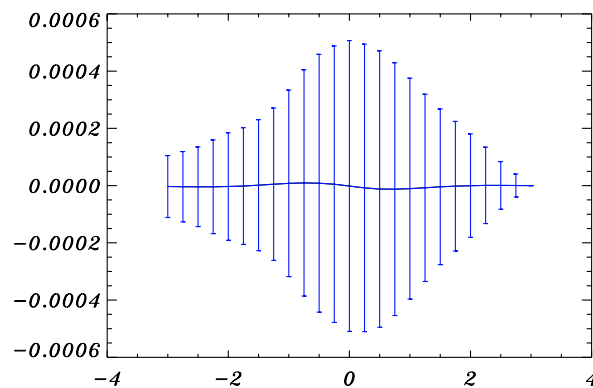
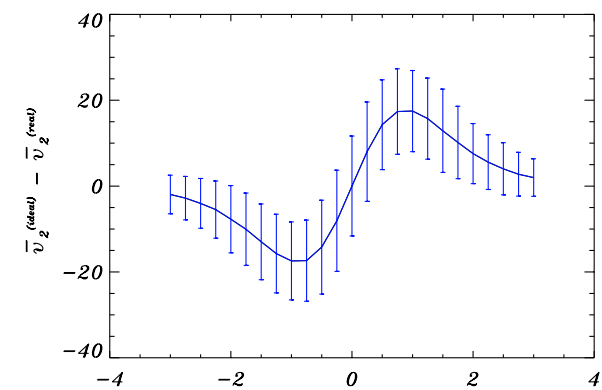
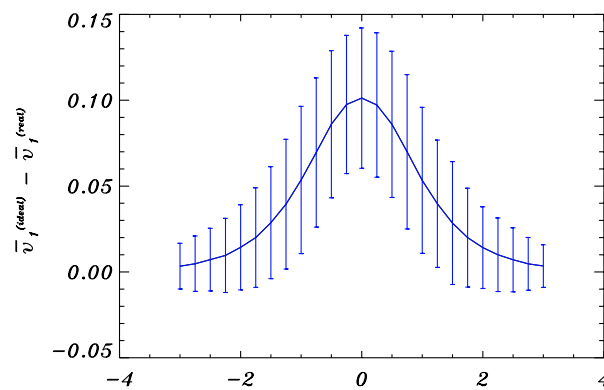
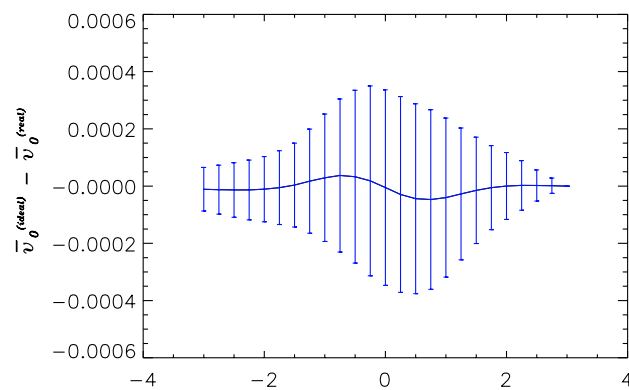
r=0

MF: ideal case-real case

$$\theta_s = 10'$$

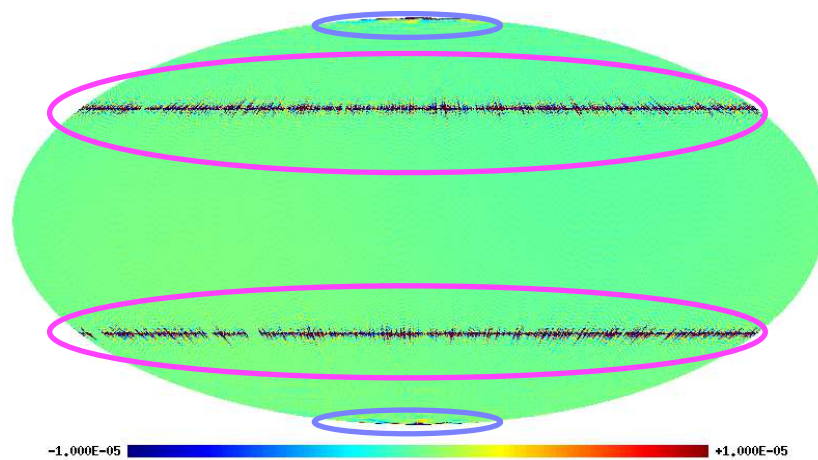
$$\theta_s = 40'$$

$$\theta_s = 60'$$

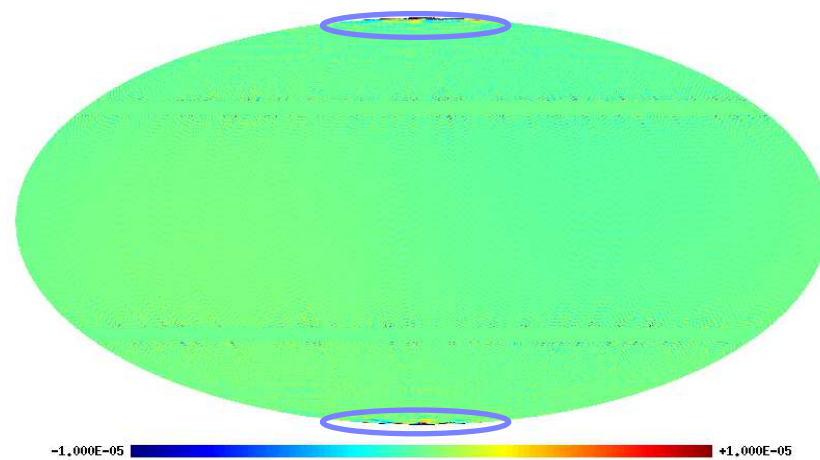


Adding sky cuts and avoiding the leakage signal: χ^2 statistics

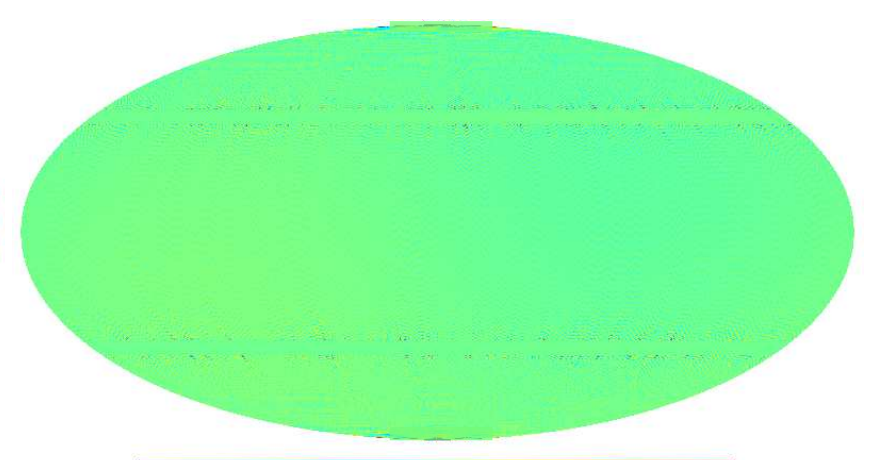
Sky cut 1



Sky cut 2



Sky cut 3



$r=0$	10'	20'	30'	40'	50'	60'
cut 1	18.93	9.53	4.27	1.62	0.71	0.34
cut 2	17.27	8.55	3.80	1.42	0.63	0.30
cut 3	15.78	7.97	3.56	1.28	0.57	0.27

Comparing different tensor-to-scalar ratios

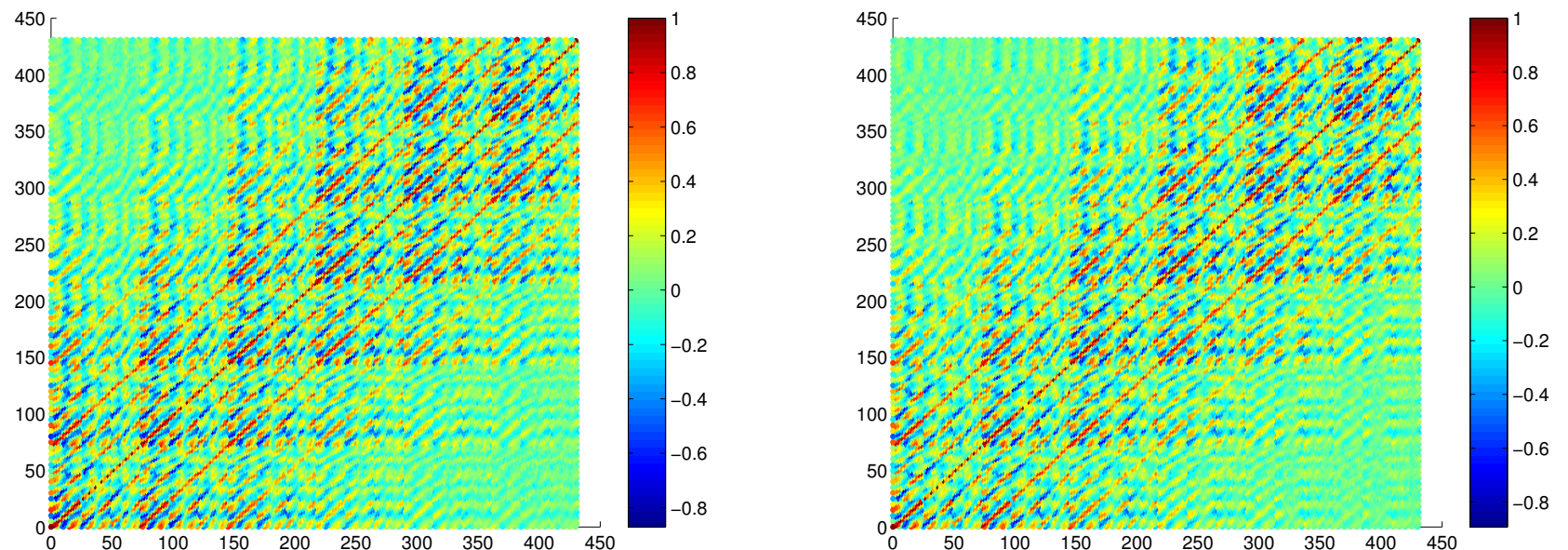
	10'	20'	30'	40'	50'	60'
$\chi^2_{(r=0)}$	18.93	9.53	4.27	1.62	0.71	0.34
$\chi^2_{(r=0.1)}$	21.18	11.00	4.41	1.63	0.62	0.29

	r=0	r=0.1
Total χ^2	220.23	268.74

The correlation coefficient of the MF

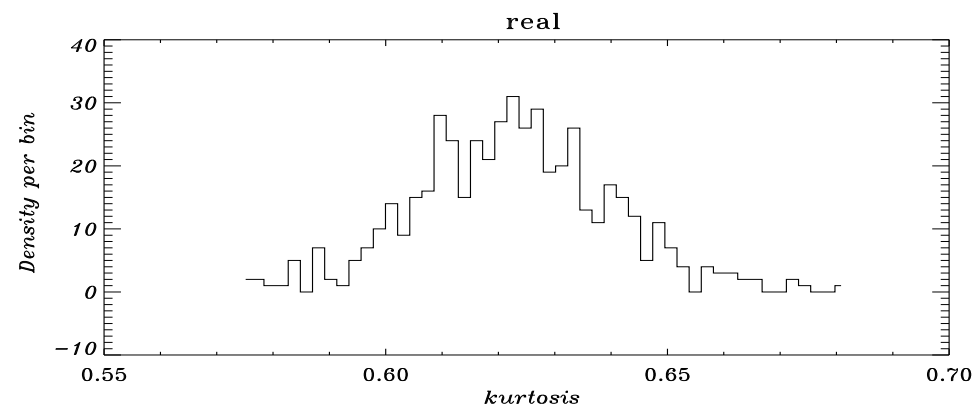
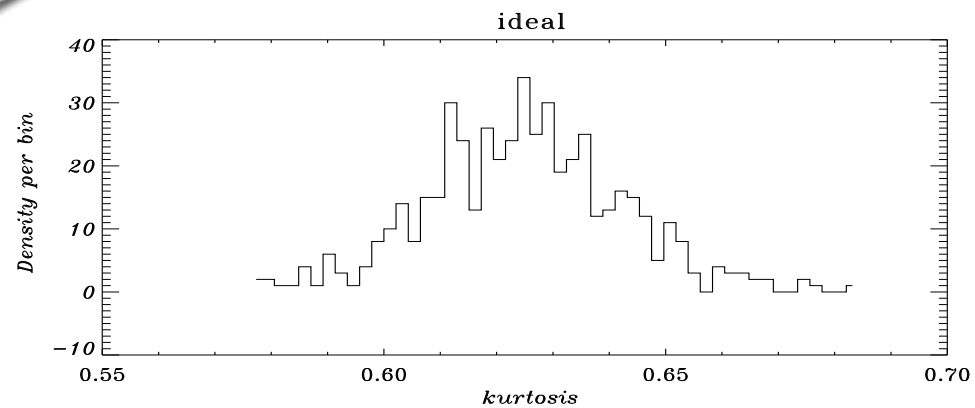
- The large values of the total χ^2 for both $r=0$ and $r=0.1$ show that the MF for different smoothing scales are very correlated
- The E-to-B leakage is not a stochastic noise, and it is always relevant in the same sky regions
- The total χ^2 depends on the correlation coefficients instead of a direct sum of the χ^2 values for each smoothing scale

$$\rho_{aa'} = C_{aa'} / \sqrt{C_{aa} C_{a'a'}}$$

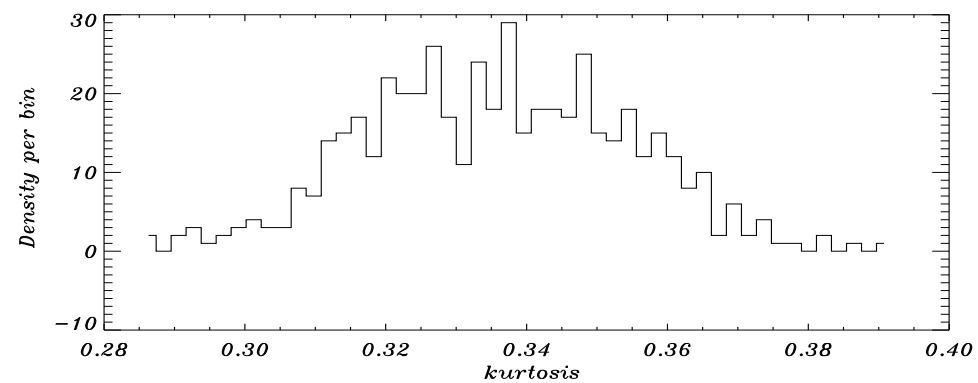
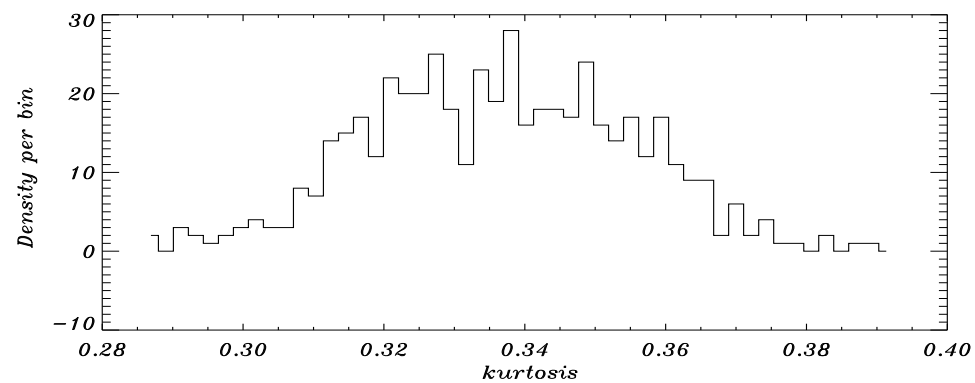


Kurtosis: Ideal case - real case

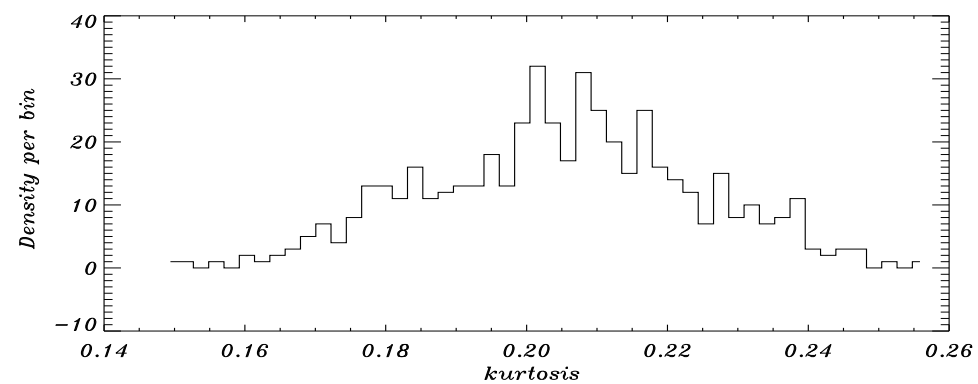
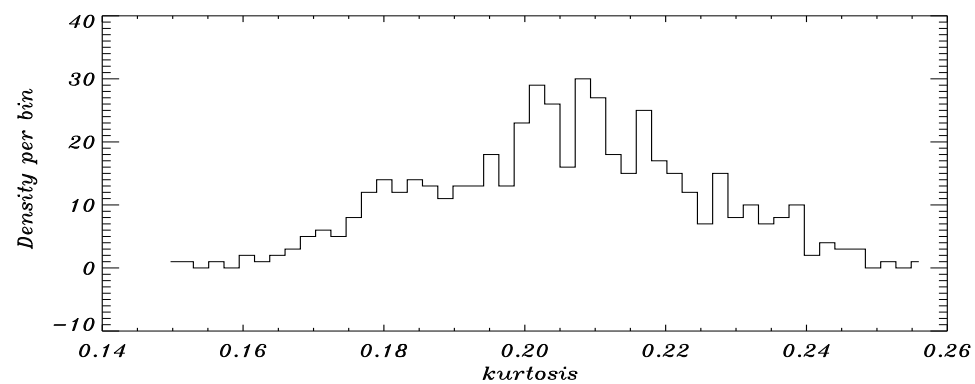
$r=0$



$$\theta_s = 10'$$



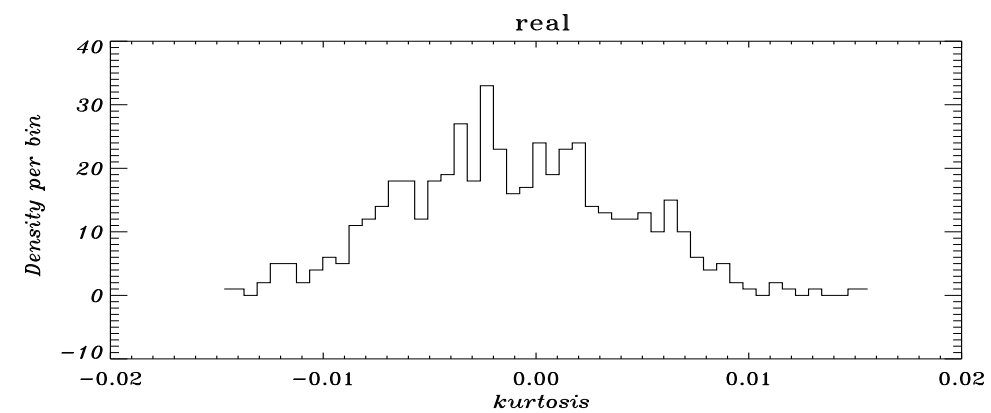
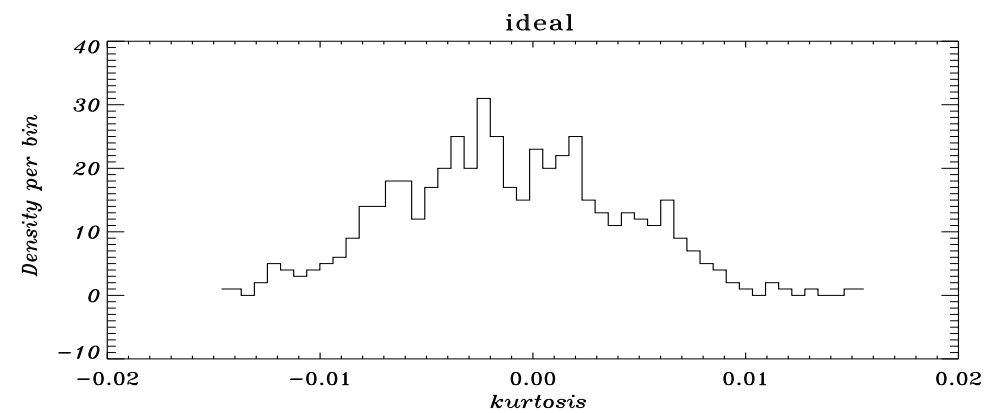
$$\theta_s = 40'$$



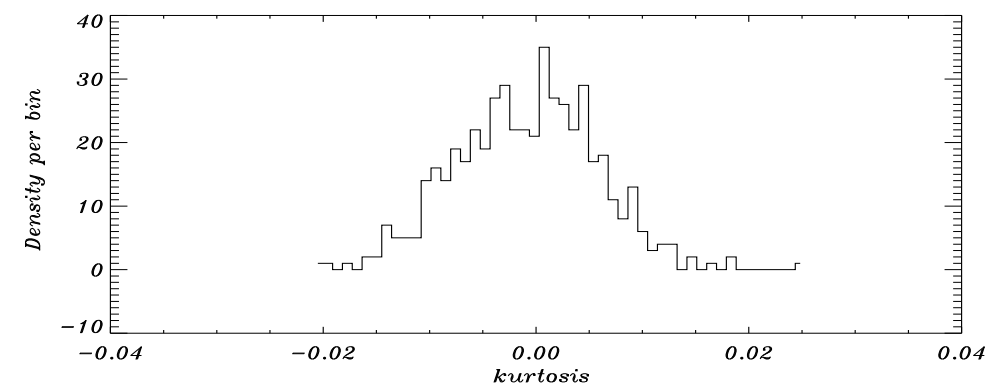
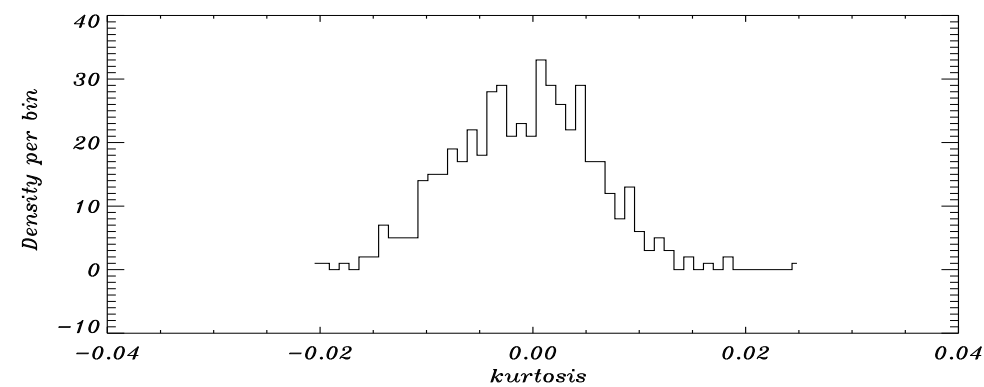
$$\theta_s = 60'$$

Skewness: ideal case - real case

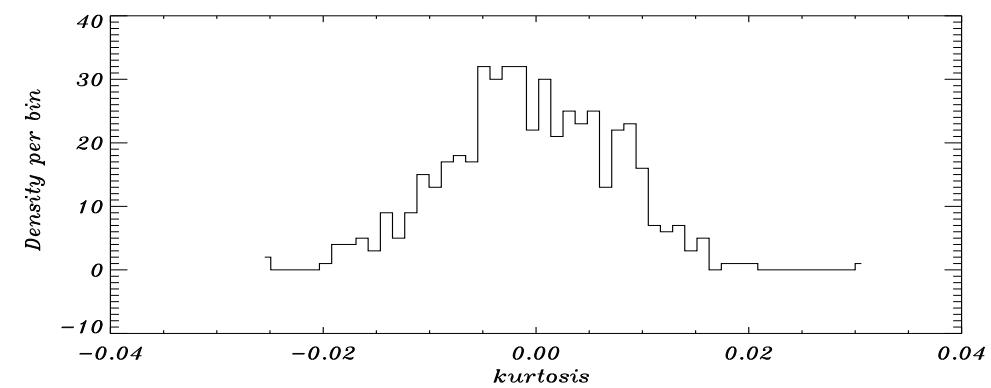
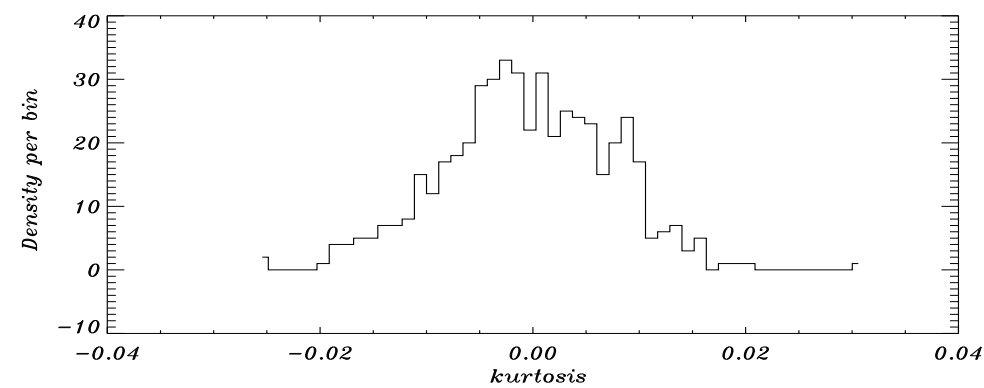
$r=0$



$\theta_s = 10'$



$\theta_s = 40'$



$\theta_s = 60'$

Conclusions

- The leakage is dominated by high multipoles: its significance becomes smaller by increasing the smoothing scale
- The imprint of the leakage is still noticeable in the MF analysis even when the contamination bands and the poles are excluded
- These regions do not play an important roll in the overall leakage contribution

- Even though the leakage seems not relevant for individual smoothing scales, it is definitely relevant when they are all combined
- The E-to-B leakage should be carefully considered when analysing partial sky CMB B-mode data in order to avoid misinterpretation of the data
- We did not find any imprint of the E-to-B leakage in the simulated B-maps when considering both skewness and kurtosis statistics for both $r=0$ and $r=0.1$