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Dark energy in vector-tensor theories

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Collaboration with
A. De Felice, L. Heisenberg, S. Mukohyama, S. Nakamura, S. Tsujikawa, Y. Zhang.

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1. Introduction

- **The late-time cosmic acceleration**

The discovery of late-time cosmic acceleration is reported in 1998.
The source for this unknown phenomenon is named dark energy (DE).

➡ **DE may originate from modification of gravity at large distances.**

- **DE models based on modified gravity**

1. **Scalar-tensor theories**
   Scalar fields coupled to gravity. e.g.) $\underbrace{\text{Brans-Dicke gravity, f(R) gravity, galileon gravity,}}_{\text{Horndeski theories}}$ …

2. **Vector-tensor theories**
   Vector fields coupled to gravity. e.g.) Generalized Proca (GP) theories,
   Beyond-generalized Proca (BGP) theories,
   Naruko-san's talk ➡ Extended vector-tensor theories.

3. **Massive gravity**

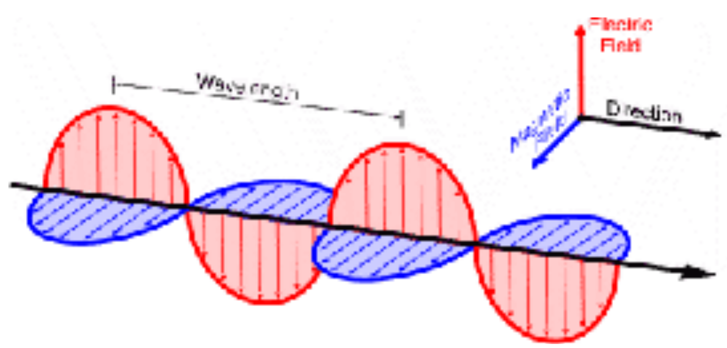
1. Introduction

◆ **Generalized Proca (GP) theories**

Maxwell field (massless)

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \,,$$
$$(F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu)$$

- U(1) gauge invariant
- DOF: 2 transverse polarizations



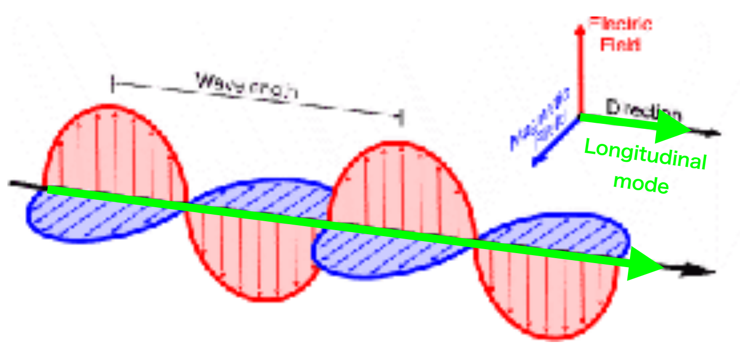
- One cannot introduce galileon-like interactions keeping U(1) symmetry

Deffayet, Gumrukcuoglu, Mukohyama and Wang (2014)

Proca field (massive)

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 A^\mu A_\mu \,,$$

- mass term breaks U(1) gauge invariance
- DOF: 2 transverse polarizations and + 1 longitudinal mode



- What happens if one generalizes Proca theories by keeping propagating DOF?

1. Introduction

- **Generalized Proca (GP) theories** L. Heisenberg (2014)

These terms are introduced in order to keep EOMs up to second-order.

They corresponds to intrinsic vector modes.

$$
\begin{aligned}
\mathcal{L}_2 =& G_2(X)\,, \\
\mathcal{L}_3 =& G_3(X)\nabla_\mu A^\mu\,, \\
\mathcal{L}_4 =& \underline{G_4(X)R} + G_{4,X}(X)\left[(\nabla_\mu A^\mu)^2 + \underline{c_2}\nabla_\mu A_\nu \nabla^\mu A^\nu - (1+\underline{c_2})\nabla_\mu A_\nu \nabla^\nu A^\mu\right]\,, \\
\mathcal{L}_5 =& \underline{G_5(X)G_{\mu\nu}\nabla^\mu A^\nu} - \frac{1}{6}G_{5,X}(X)\big[(\nabla_\mu A^\mu)^3 - \underline{3d_2}\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma \\
& - 3(1-\underline{d_2})\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + (2-\underline{3d_2})\nabla_\mu A_\nu \nabla^\rho A^\mu \nabla^\nu A_\rho + \underline{3d_2}\nabla_\mu A_\nu \nabla^\rho A^\mu \nabla_\rho A_\nu\big]\,, \\
\mathcal{L}_6 =& \underline{\underline{G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta} + \frac{1}{2}G_{6,X}(X)\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu}\,.
\end{aligned}
$$

$$
X = -\frac{1}{2}A_\mu A^\mu\,, \quad F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \quad Y = A^\mu A^\nu F_\mu{}^\alpha F_{\nu\alpha}\,, \quad L^{\mu\nu\alpha\beta} = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}R_{\rho\sigma\gamma\delta}\,, \quad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}
$$

$(\epsilon^{\mu\nu\rho\sigma}$: Levi-Civita tensor$)$

- DOF: 2 vector + 1scalar (+ 2 tensor)
- In the scalar-limit $(A_\mu \to \nabla_\mu \phi)$, these theories reduces to shift-symmetric Horndeski theories.

2. Cosmology in GP theories

- **BG dynamics**

action: $S = \int d^4x\sqrt{-g}\,(\mathcal{L}_{\rm GP} + \mathcal{L}_M)$

metric: $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$ $\Rightarrow$ vector filed: $A^\mu = (\phi(t), 0, 0, 0)$

modified Einstein equation

$H \equiv \dot{a}/a$

(0,0): $G_2 - G_{2,X}\phi^2 - 3G_{3,X}H\phi^3 + 6G_4H^2 - 6(2G_{4,X} + G_{4,XX}\phi^2)H^2\phi^2 + 5G_{5,X}H^3\phi^3 + G_{5,XX}H^3\phi^5 = \rho_M\,,$

(1,1): $G_2 - \dot{\phi}\phi^2G_{3,X} + 2G_4\,(3H^2 + 2\dot{H}) - 2G_{4,X}\phi\,(3H^2\phi + 2H\dot{\phi} + 2\dot{H}\phi) - 4G_{4,XX}H\dot{\phi}\phi^3 + G_{5,X}H\phi^2(2\dot{H}\phi + 2H^2\phi + 3H\dot{\phi}) + G_{5,XX}H^2\dot{\phi}\phi^4 = -P_M\,.$

Intrinsic vector modes do not appear at the BG level.

field equation

$$\phi\left(G_{2,X} + 3G_{3,X}H\phi + 6G_{4,X}H^2 + 6G_{4,XX}H^2\phi^2 - 3G_{5,X}H^3\phi - G_{5,XX}H^3\phi^3\right) = 0\,.$$

In the branch $\phi \neq 0$, this is the algebraic equation of ϕ and H. Thus, there exists de sitter solutions characterized by $\boxed{\phi = \text{constant and } H = \text{constant.}}$

2. Cosmology in GP theories

◆ **BG dynamics (extended vector Galileon)**

Let us consider the following generalization of the vector Galileon:

$$\boxed{G_2(X) = b_2 X^{p_2}\,, \quad G_3(X) = b_3 X^{p_3}\,, \quad G_4(X) = \frac{M_{\rm pl}^2}{2} + b_4 X^{p_4}\,, \quad G_5(X) = b_5 X^{p_5}\,.}$$

where $p_3 = (p + 2p_2 - 1)/2\,, \quad p_4 = p + p_2\,, \quad p_5 = (3p + 2p_2 - 1)/2\,.$ This model reduces to the vector Galileon when $p_2 = p = 1$.

The field equation gives

$$\boxed{1 + 3\beta_3 + 6(2p + 2p_2 - 1)\beta_4 - (3p + 2p_2)\beta_5 = 0\,.}$$

$$\beta_i \equiv \frac{p_i b_i}{2^{p_i - p_2} p_2 b_2} (\phi^p H)^{i-2}\,, \qquad \boxed{\phi^p \propto H^{-1}}$$

2. Cosmology in GP theories

◆ **BG dynamics (extended vector Galileon)**

DE equation of state

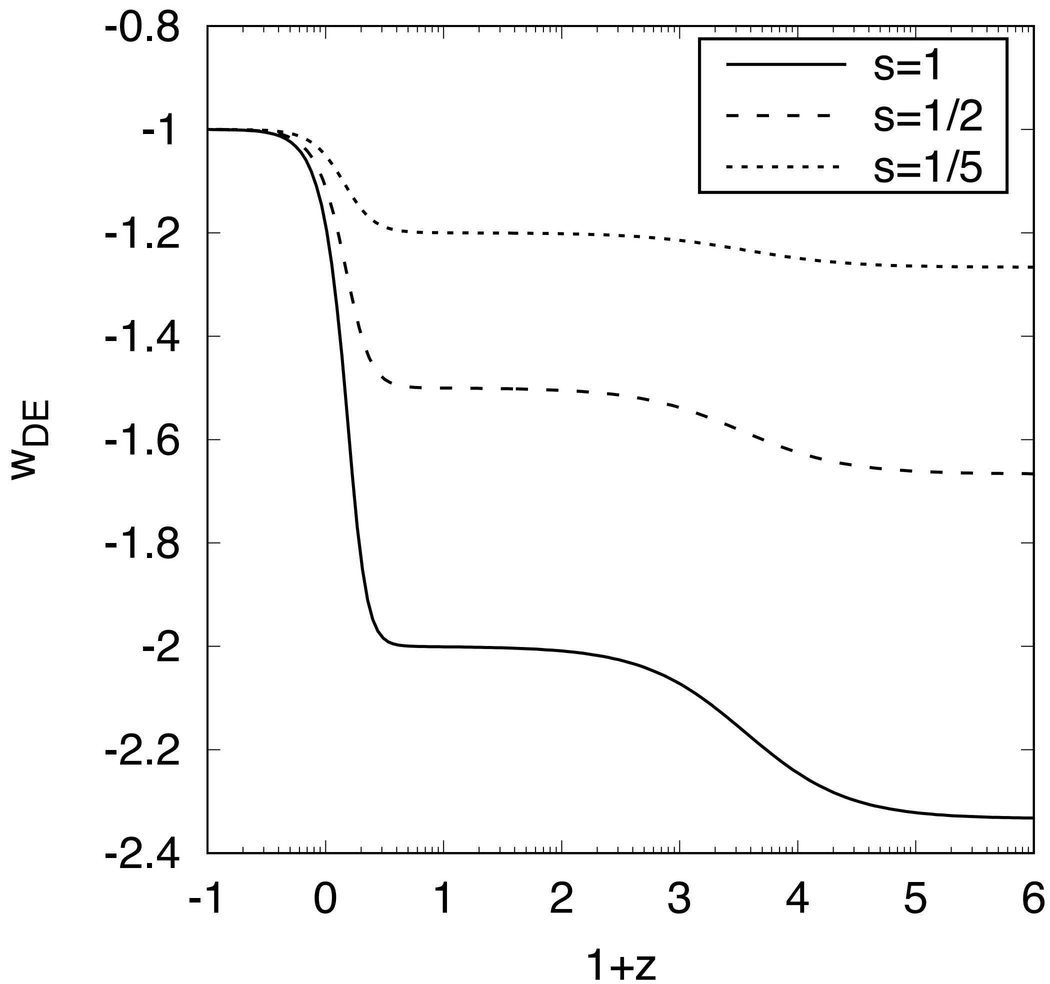
$$\boxed{w_{\rm DE} = -\frac{3(1+s)+s\,\Omega_r}{3(1+s\,\Omega_{\rm DE})}\,.}$$

$$s \equiv p_2/p$$

The combined analysis of SNIa, CMB and BAO gives the bound on s as

$$\boxed{0 \le s \le 0.36 \ (95\% \ \text{C.L.})}$$

De Felice, Tsujikawa (2012)



For the vector Galileon model $(s=1)$, we have $w_{\rm DE}=-2$ during the matter $(\Omega_r=\Omega_{\rm DE}=0)$.
For smaller s , this value approaches -1.

2. Cosmology in GP theories

- **cosmological perturbations**

In order to study stability conditions and observational signatures for the matter distribution, we need to study the cosmological perturbations. Perturbations of metric (in flat gauge), the vector field can be written as

metric: $ds^2 = -(1+2\alpha)\,dt^2 + 2\,(\partial_i\chi + V_i)\,dt dx^i + a^2(t)\,(\delta_{ij}+h_{ij})\,dx^i dx^j$

vector field: $A^\mu = \left(A^0, A^i\right), \quad A^0 = \phi(t) + \delta\phi\,, \quad A^i = \frac{1}{a^2}\delta^{ij}\,(\partial_j\chi_V + E_j)\ .$

scalar perturbation: $\alpha\,,\ \chi\,,\ \delta\phi\,,\ \chi_V$

vector perturbation: $V_i\,,\ E_j \quad (\partial^i V i = 0 = \partial^j E_j)$

tensor perturbation: $h_{ij} \quad \left(\partial^i h_{ij} = h^i_i = 0\right)$

2. Cosmology in GP theories

- **stability conditions**

tensor perturbation: $$S_T^{(2)} = \sum_{\lambda=+,\times} \int dt\, d^3x\, a^3 \frac{q_T}{8} \left[\dot{h}_\lambda^2 - \frac{c_T^2}{a^2} (\partial h_\lambda)^2 \right] ,$$

vector perturbation: $$S_V^{(2)} \simeq \sum_{i=1}^{2} \int dt\, d^3x\, \frac{a q_V}{2} \left(\dot{Z}_i^2 + \frac{k^2}{a^2} c_V^2 Z_i^2 \right) ,$$

The no ghost $(q_T > 0\,,\ q_V > 0)$ and no-gradient instability $(c_T^2 > 0\,,\ c_V^2 > 0)$ conditions are satisfied for $\boxed{|\beta_4| \ll 1 \ \ |\beta_5| \ll 1}$,

scalar perturbation: $$L_S^{(2)} = a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} + \frac{k^2}{a^2} \vec{\mathcal{X}}^t \boldsymbol{G} \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{B} \dot{\vec{\mathcal{X}}} \right) ,$$

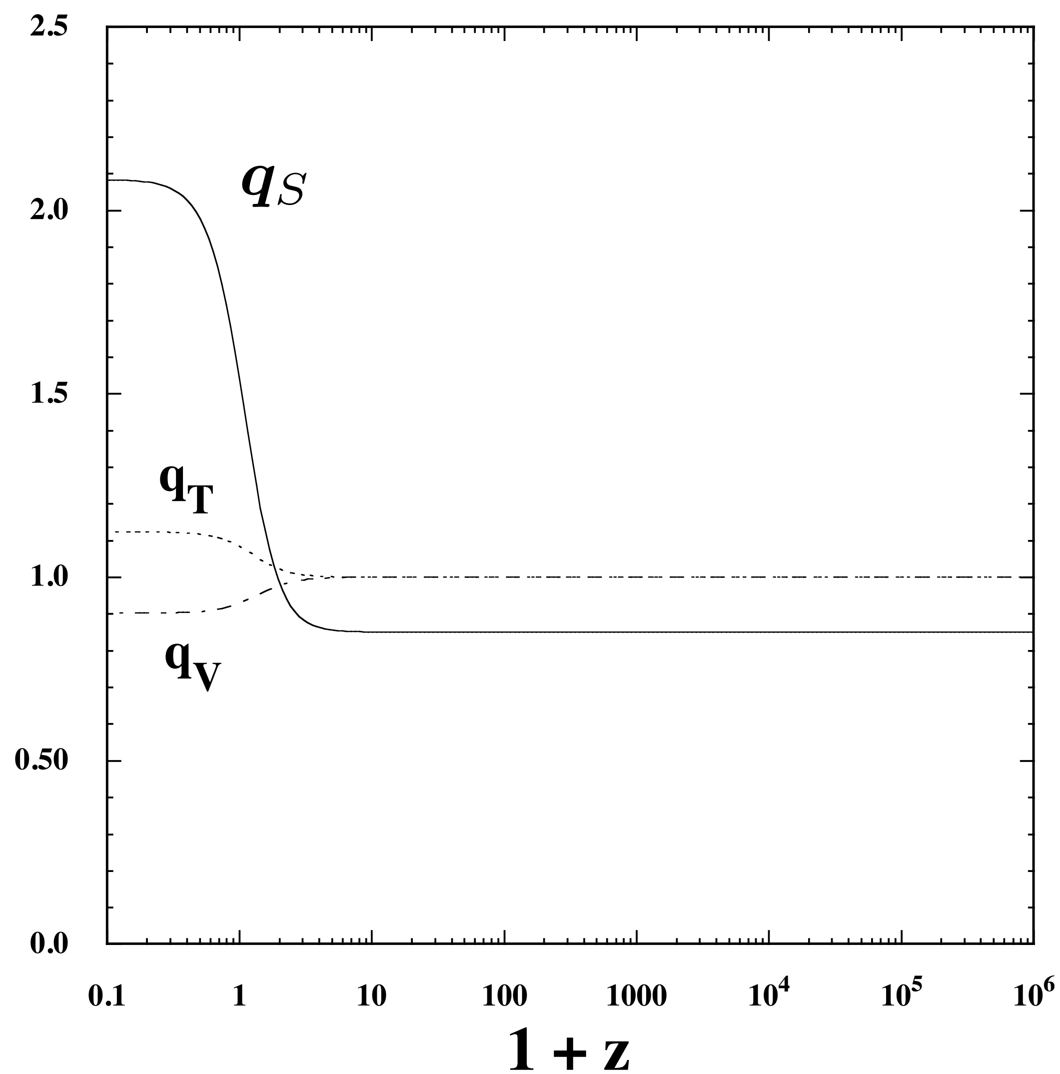
$$\vec{\mathcal{X}}^t = (\psi, \delta\rho_M)\ ,\ \ \psi \equiv \chi_V + \phi(t)\chi$$

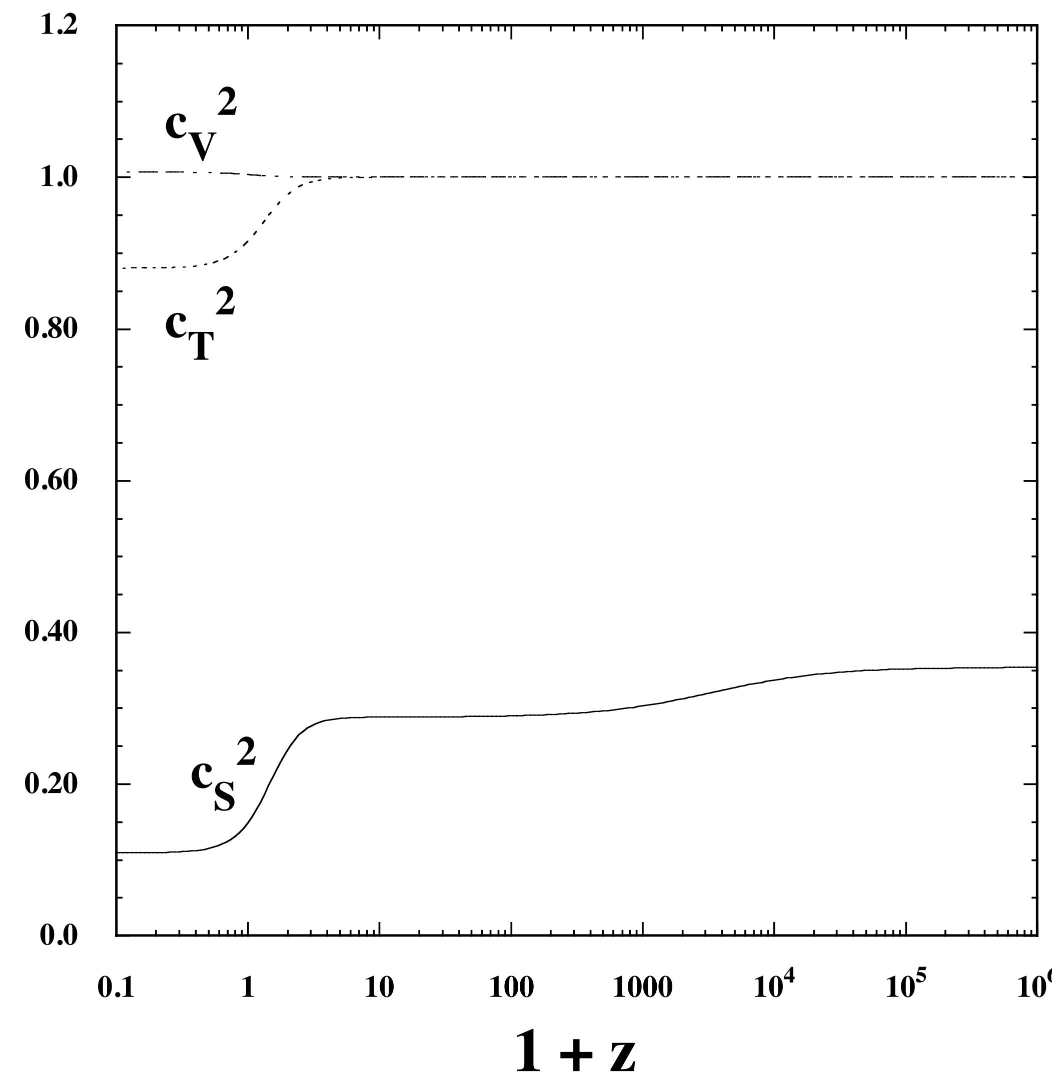
- no-ghost condition: Two eigenvalues of the kinetic matrix should be positive. One of them is $\rho_M + P_M > 0$, the other eigenvalue q_S is positive if $\boxed{b_2 < 0.}$

- no-gradient instability condition: $|\beta_4| \ll 1 \ \ |\beta_5| \ll 1\,,$

2. Cosmology in GP theories

- **An example of healthy models**





$$p_2 = 1\,, \quad p = 5\,, \quad \beta_4 = 0.01\,, \quad \beta_5 = 0\,, \quad c_2 = -1\,.$$

2. Cosmology in GP theories

- **Effective gravitational coupling**

In order to see observational signatures for the matter distribution, we study the matter perturbation under the quasi-static approximation on the sub-horizon-scale. By combining scalar perturbation equations, we obtain

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho_M \delta \simeq 0 \qquad (\delta \simeq \delta\rho_M/\rho_M)$$

Here $G_{\rm eff}$ is the effective gravitational coupling described by

$$G_{\rm eff} = \frac{\xi_2 + \xi_3}{\xi_1}$$

$$\xi_1 = 4\pi\phi^2 (w_2 + 2Hq_T)^2 \,,$$ positive

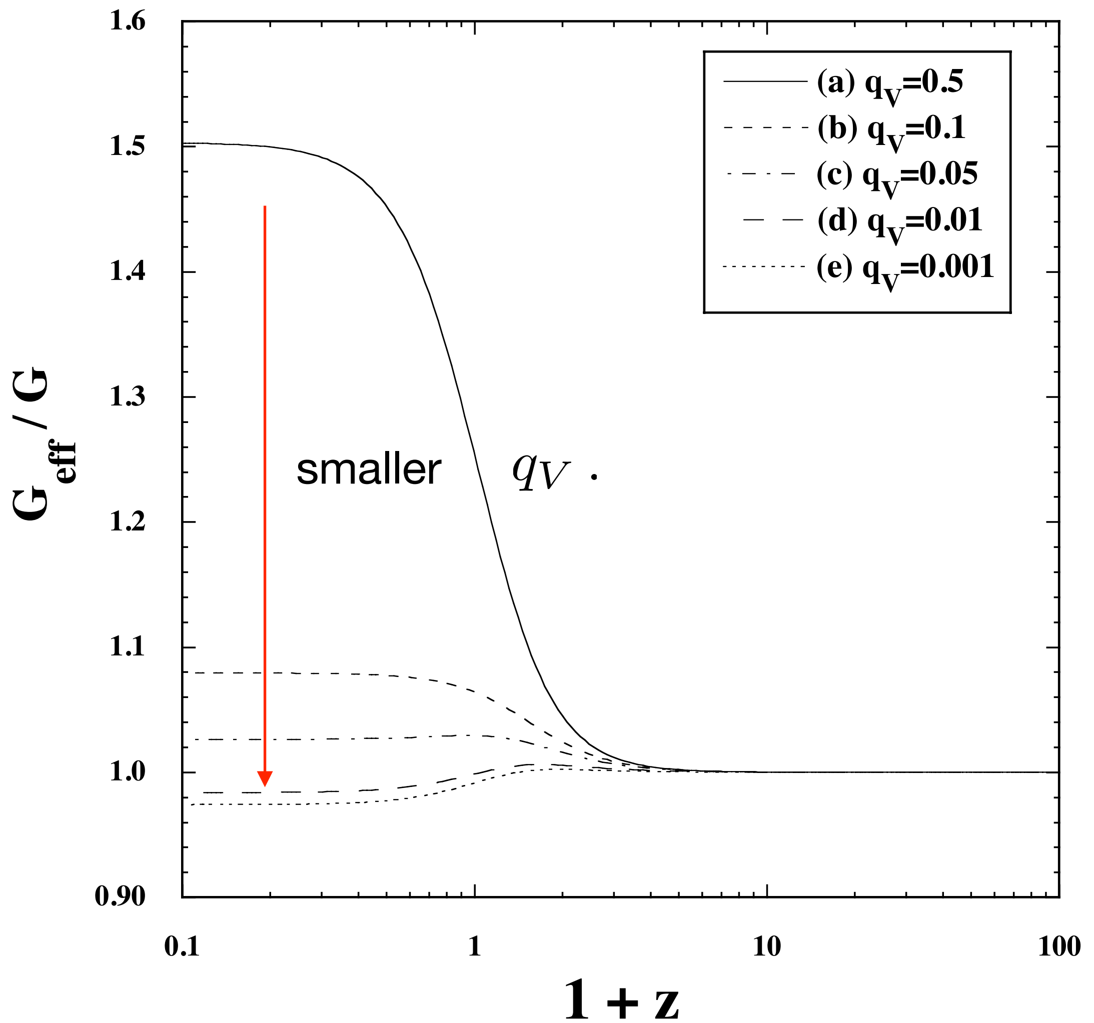
$$\xi_2 = [H(w_2 + 2Hq_T) - \dot{w}_1 + 2\dot{w}_2 + \rho_M]\phi^2 - \frac{w_2^2}{q_V} \,,$$ positive/negative

$$\xi_3 = \frac{1}{8H^2\phi^2 q_S^3 q_T c_S^2}\Big[2\phi^2 \{q_S[w_2\dot{w}_1 - (w_2 - 2Hq_T)\dot{w}_2] + \rho_M w_2[3w_2(w_2 + 2Hq_T) - q_S]\} + \frac{q_S}{q_V} w_2 \{w_2(w_2 - 2Hq_T) - w_6\phi(w_2 + 2Hq_T)\}\Big]^2 \,,$$ positive

When $q_V \ll 1$, $G_{\rm eff}$ tends to decrease.

2. Cosmology in GP theories

◆ **Effective gravitational coupling**



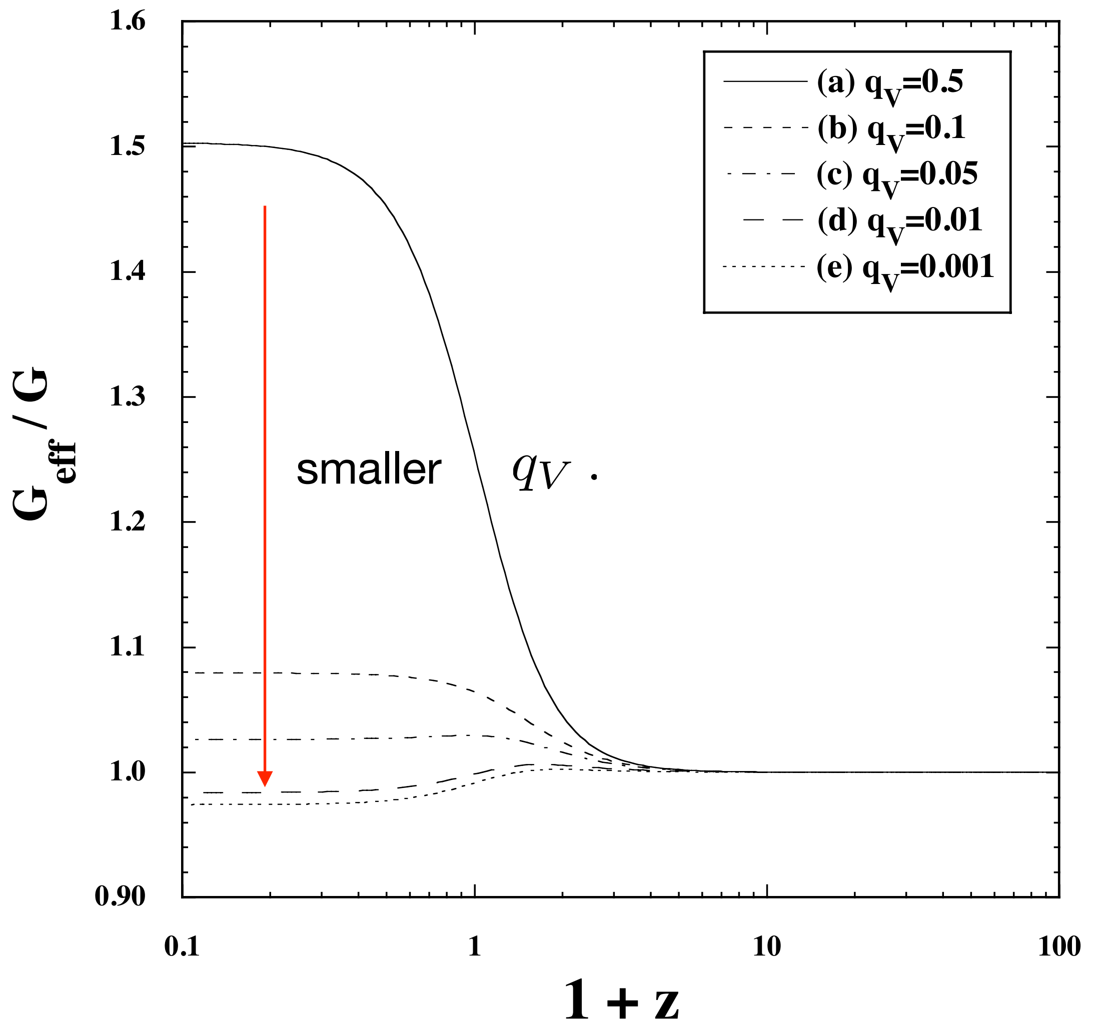
$p_2 = 1/2,\ p = 5/2,\ \beta_4 = 10^{-4},\ \beta_5 = 0.052,\ c_2 = d_2 = 0.$

$$q_V = G_{2,F} + 2G_{2,Y}\phi^2 - 4g_5 H\phi + 2G_6 H^2 + 2G_{6,X} H^2\phi^2 \,,$$

For $G_6 = b_6 X^{p_6}$, the last term is constant when $p_6 = p$. By tuning the constant b_6, one can easily decrease q_V .

2. Cosmology in GP theories

- **Effective gravitational coupling**



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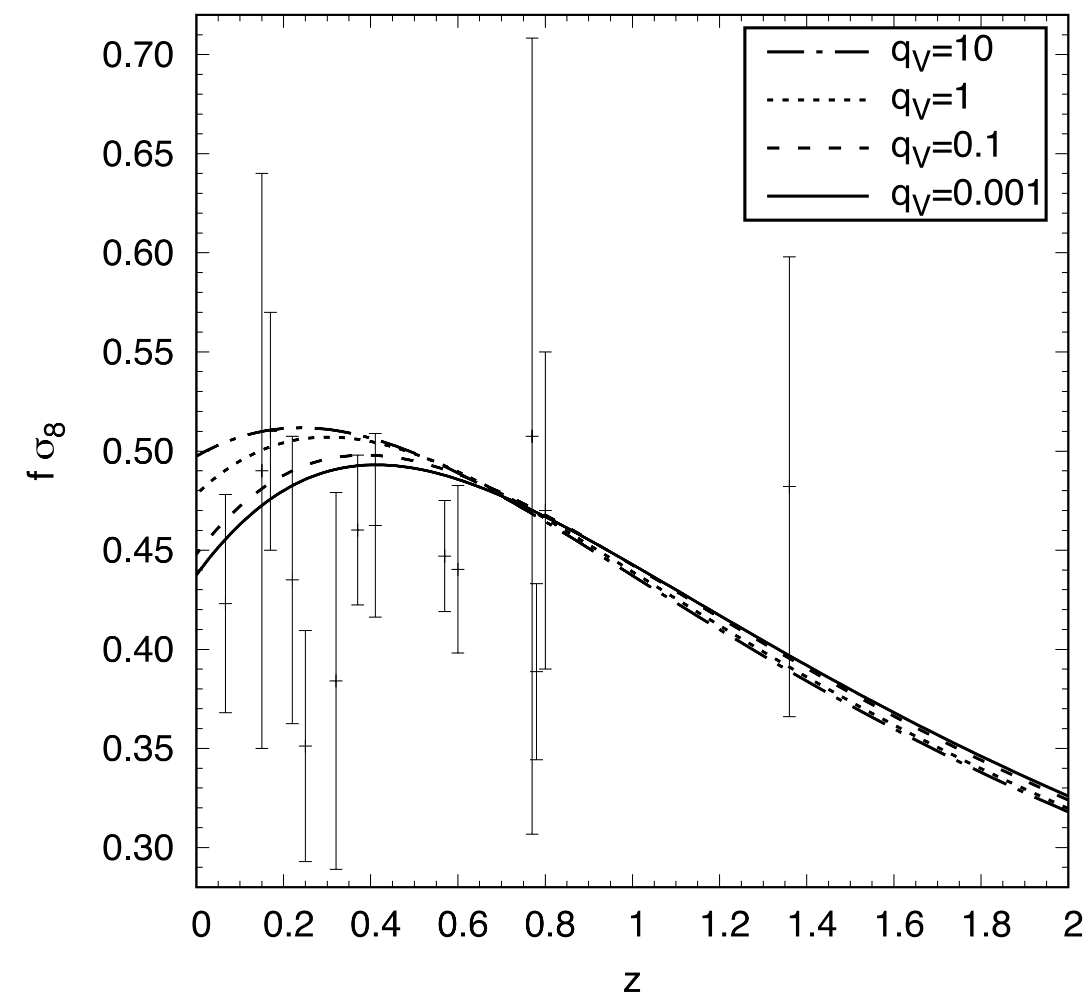


$f \equiv \dot{\delta}/(H\delta)$: the growth rate of density perturbation

σ_8 : the amplitude of density perturbation

We normalized σ_8 by using Planck best-fit value.

3. Cosmology in BGP theories

- **Beyond-generalized Proca (BGP) theories** L. Heisenberg, RK and S. Tsujikawa (2016)

The Lagrangians of GP theories are constructed by using the Levi-Civita tensor and the derivative of the vector field as follows:

$$\mathcal{L}_{i+2} = g_{i+2}(X)\, \hat{\delta}^{\beta_1 \cdots \beta_i \gamma_{i+1} \cdots \gamma_4}_{\alpha_1 \cdots \alpha_i \gamma_{i+1} \cdots \gamma_4} \nabla_{\beta_1} A^{\alpha_1} \cdots \nabla_{\beta_i} A^{\alpha_i} \,,$$

$$\left(\hat{\delta}^{\beta_1 \beta_2 \gamma_3 \gamma_4}_{\alpha_1 \alpha_2 \gamma_3 \gamma_4} \equiv \epsilon_{\alpha_1 \alpha_2 \gamma_3 \gamma_4} \epsilon^{\beta_1 \beta_2 \gamma_3 \gamma_4} \right)$$

As long as this structure is being kept, higher order derivatives do not appear in EOMs. On the other hand, the following Lagrangians $\mathcal{L}_4^{\rm N}, \mathcal{L}_5^{\rm N}$ break this feature:

$$\mathcal{L}_4^{\rm N} = f_4(X) \hat{\delta}^{\beta_1 \beta_2 \beta_3 \gamma_4}_{\alpha_1 \alpha_2 \alpha_3 \gamma_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3} \,,$$

$$\mathcal{L}_5^{\rm N} = f_5(X) \hat{\delta}^{\beta_1 \beta_2 \beta_3 \beta_4}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3} \nabla^{\alpha_4} A_{\beta_4} \,,$$

In the scalar limit, these terms reduce to a part of GLPV theories (beyond Horndeski). Those terms give rise to higher order spatial derivatives but no higher order time derivatives.

Deffayet *et al.* (2015)

3. Cosmology in BGP theories

- **Lagrangians describing BGP theories** L. Heisenberg, RK and S. Tsujikawa (2016)

$$
\begin{aligned}
\mathcal{L}_2 &= G_2(X, F, Y)\,, \\
\mathcal{L}_3 &= G_3(X)\nabla_\mu A^\mu\,, \\
\mathcal{L}_4 &= G_4(X)R + G_{4,X}(X)\left[(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho\right]\,, \\
\mathcal{L}_5 &= G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}(X)[(\nabla_\mu A^\mu)^3 - 3\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma] \\
&\quad - g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^\beta{}_\mu \nabla_\alpha A_\beta\,, \\
\mathcal{L}_6 &= G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2}G_{6,X}(X)\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu\,,
\end{aligned}
$$

with

$$
\begin{aligned}
\mathcal{L}_4^{\rm N} &= f_4(X)\hat{\delta}^{\beta_1\beta_2\beta_3\gamma_4}_{\alpha_1\alpha_2\alpha_3\gamma_4} A^{\alpha_1}A_{\beta_1}\nabla^{\alpha_2}A_{\beta_2}\nabla^{\alpha_3}A_{\beta_3}\,, \\
\mathcal{L}_5^{\rm N} &= f_5(X)\hat{\delta}^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} A^{\alpha_1}A_{\beta_1}\nabla^{\alpha_2}A_{\beta_2}\nabla^{\alpha_3}A_{\beta_3}\nabla^{\alpha_4}A_{\beta_4}\,, \\
\tilde{\mathcal{L}}_5^{\rm N} &= \tilde{f}_5(X)\hat{\delta}^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} A^{\alpha_1}A_{\beta_1}\nabla^{\alpha_2}A^{\alpha_3}\nabla_{\beta_2}A_{\beta_3}\nabla^{\alpha_4}A_{\beta_4}\,, \\
\mathcal{L}_6^{\rm N} &= \tilde{f}_6(X)\hat{\delta}^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} \nabla_{\beta_1}A_{\beta_2}\nabla^{\alpha_1}A^{\alpha_2}\nabla_{\beta_3}A^{\alpha_3}\nabla_{\beta_4}A^{\alpha_4}\,,
\end{aligned}
$$

3. Cosmology in BGP theories

- **Concrete models**

GP theories

$$G_4 = \frac{M_{\rm pl}^2}{2} + b_4 X^{p_4}\,, \quad G_5 = b_5 X^{p_5}\,, \quad f_4 = f_5 = \tilde{f}_5 = \tilde{f}_6 = 0\,.$$

BGP theories

$$G_4 = \frac{M_{\rm pl}^2}{2}\,, \quad G_5 = 0\,, \quad f_4 = \frac{1}{4}(2p_4 - 1) b_4 X^{p_4 - 2}\,, \quad f_5 = -\frac{1}{12} p_5 b_5 X^{p_5 - 2}\,,$$
$$\tilde{f}_5 = c_5 X^{q_5}\,, \quad \tilde{f}_6 = c_6 X^{q_6}\,.$$

with $G_2 = b_2 X^{p_2} + F\,, \quad G_3 = b_3 X^{p_3}\,, \quad g_5 = G_6 = 0\,.$

In these two models, the BG dynamics is completely the same.
Differences between two models appear at the level of perturbations.

3. Cosmology in BGP theories

- **stability conditions in BGP theories**

tensor and vector perturbations:

The no-ghost and no-gradient instability conditions are satisfied for

$$\boxed{|\beta_4| \ll 1 \quad |\beta_5| \ll 1\,, \quad |\tilde{f}_6| H^2 \phi^2 \ll 1\,, |\tilde{f}_5| H \phi^3 \ll 1}$$

scalar perturbations:

- The no-ghost condition in BGP is the same as that in GP.
- The no gradient instability condition is

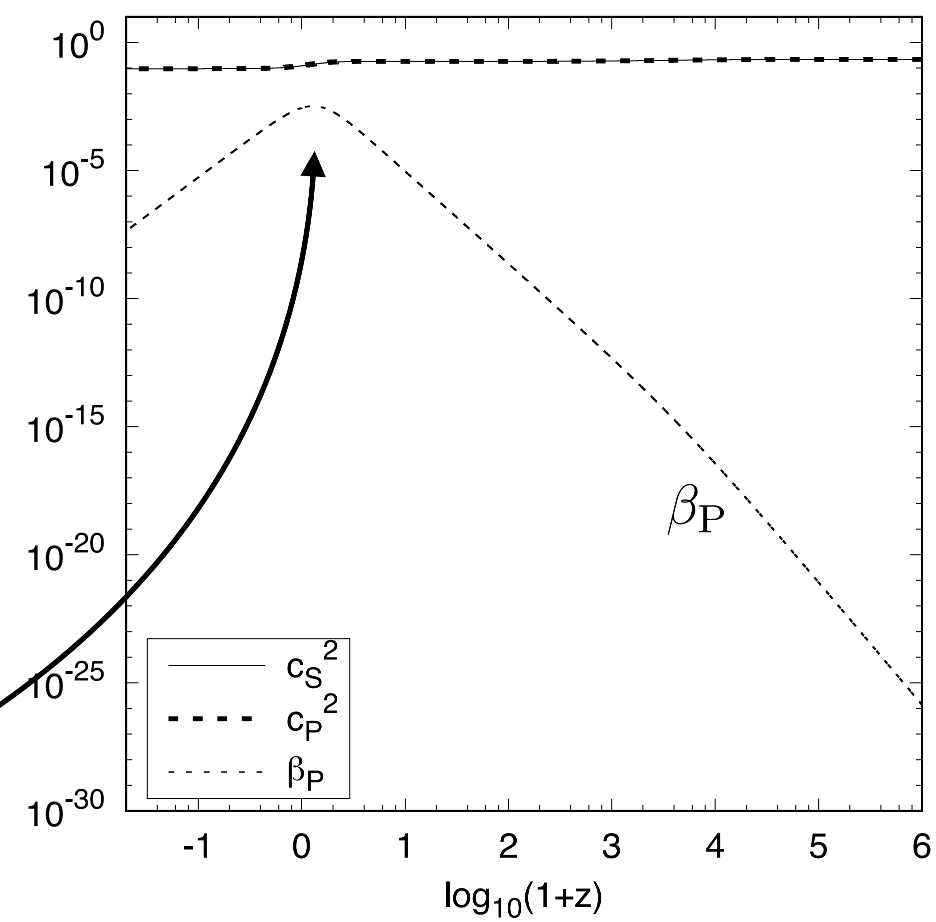
$$c_S^2 = c_{\rm P}^2 - \beta_P$$

c_P^2 : scalar propagation speed squared in GP theories

β_P : parameter characterizing the deviation from GP theories

$$\boxed{\beta_{\rm P} \propto \Omega_{\rm DE} \left(\Omega_r + \Omega_m\right) \left(f_4 + 3H\phi f_5\right)}$$

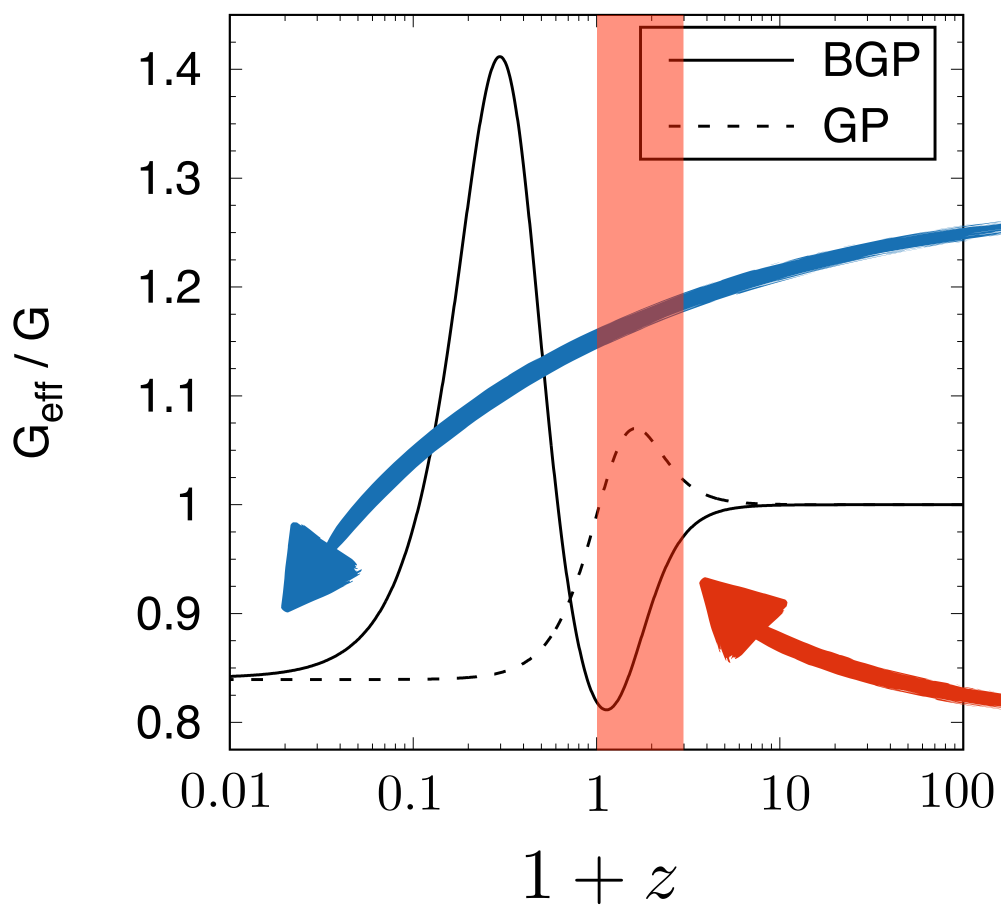
The deviation from GP theories affects only around the present epoch.



3. Cosmology in BGP theories

- **matter perturbation**

  - **effective gravitational coupling**



$\beta_4 = 5.00 \times 10^{-2}, \beta_5 = 6.78 \times 10^{-2},$
$p_2 = 1, p = 5.$ (health both in GP and BGP)

In this parameter choices, $G_{\rm eff}$ become smaller than that in GR at the de Sitter point both in GP and BGP.

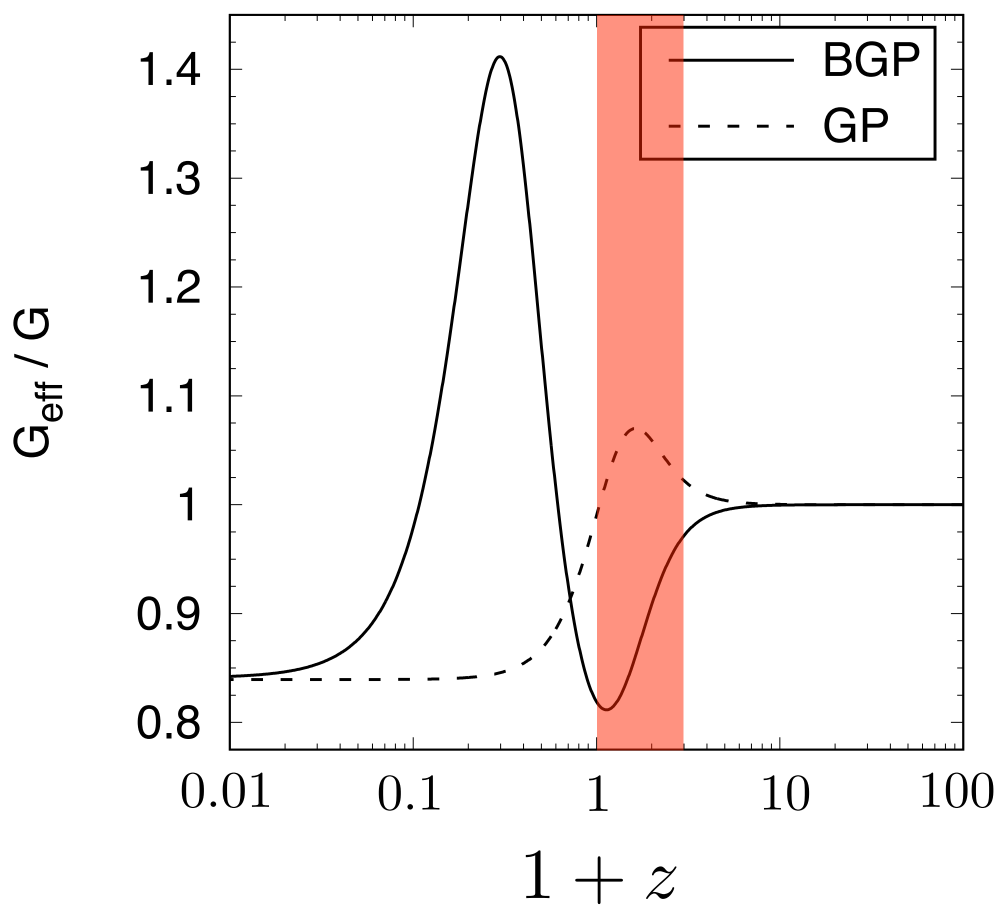
However, it shows a temporal growth around today in GP, while it temporally decreases in BGP.

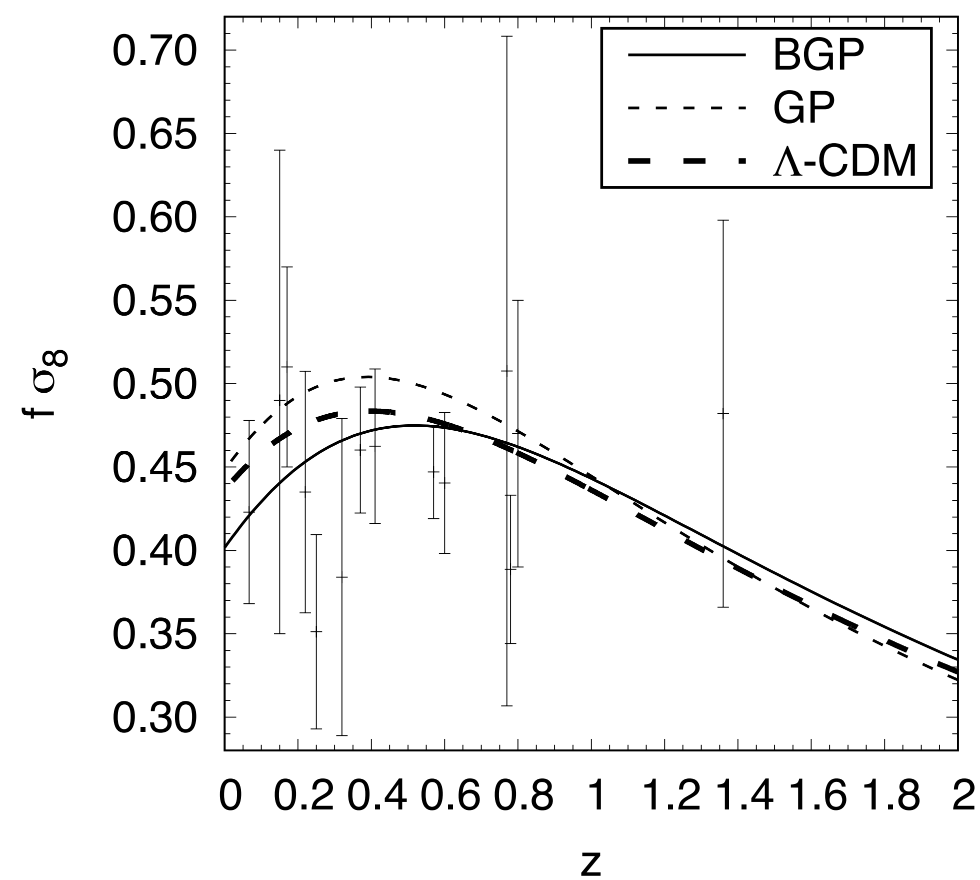
In BGP theories, the lower growth rate can be realized compared to GP theories and Λ-CDM.

3. Cosmology in BGP theories

◆ **matter perturbation**

- **effective gravitational coupling**





$\beta_4 = 5.00 \times 10^{-2}, \beta_5 = 6.78 \times 10^{-2},$
$p_2 = 1, p = 5.$ (health both in GP and BGP)

In BGP theories, the lower growth rate can be realized compared to GP theories and $\Lambda\text{-}CDM$.

4. Conclusions

- Generalized Proca (GP) and Beyond-generalized Proca (BGP) theories give rise to interesting cosmological solutions with a stable late-time de Sitter attractor.
- We derived 6 no-ghost and no-gradient instability conditions associated with tensor, vector, scalar perturbations.
- We constructed a class of models in which all the stability conditions are satisfied during the whole cosmic expansion history.
- We also derived the effective gravitational coupling that can be used to put observational constraints on the models.
- In BGP theories, the effective gravitational coupling can be even smaller than that in GP theories. By virtue of this behavior, it can be compatible with the recent RSD data even by using Planck best-fit value of $\sigma_8(z=0)=0.82.$
- It will be of interest to put observational constraints on the viable parameter spaces for our proposed models.