A simple three-shape description of the matter bispectrum of large-scale structure

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based on

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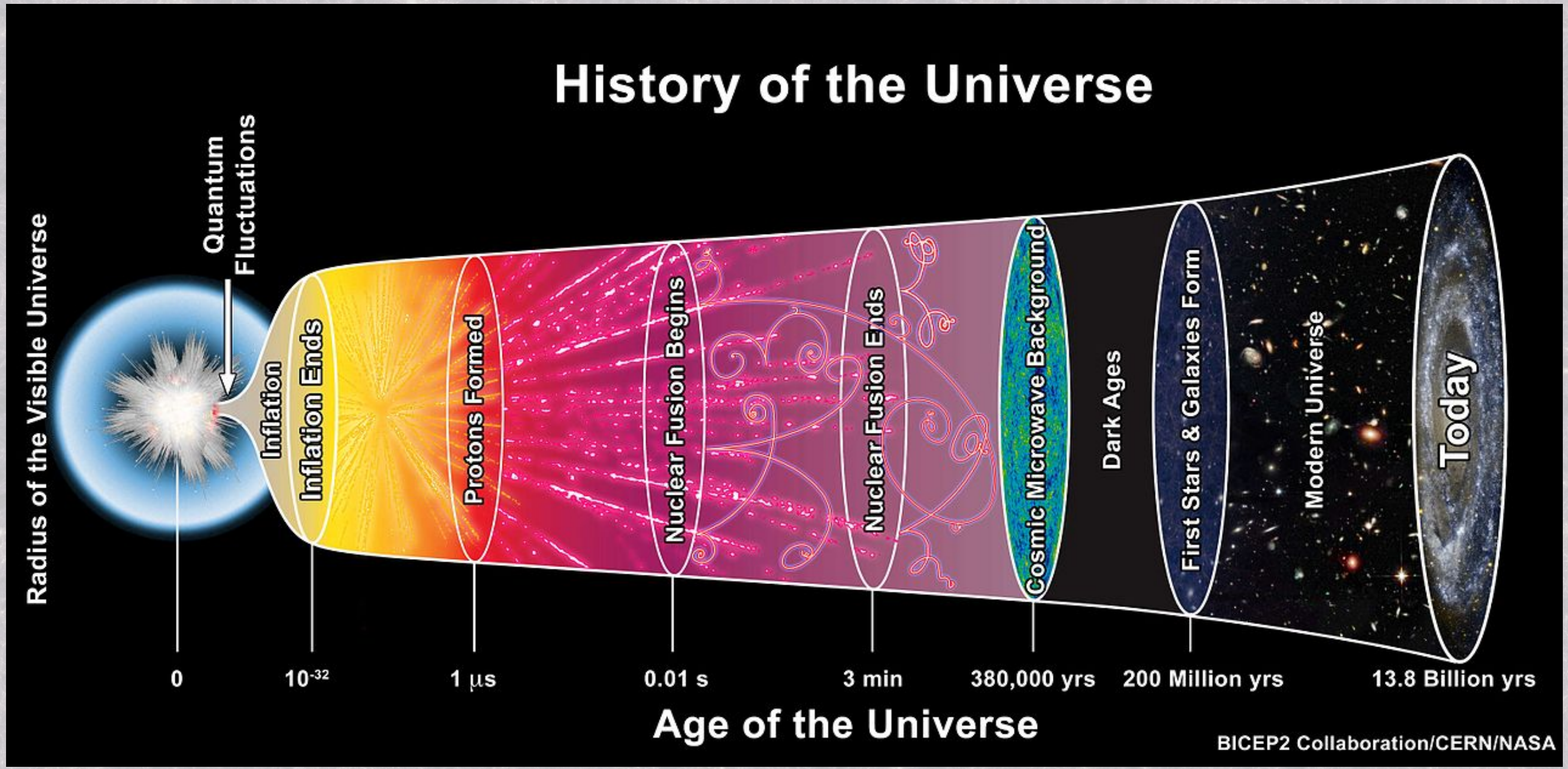
Outline

- Introduction
- Perturbation theories
- Halo models
- N-body simulations
- Building a phenomenological model
- Perturbation theories results
- Halo models results
- Non-Gaussianity: shapes of simulations and three-shape model for local and equilateral types
- Extension - galaxy bispectrum
- Conclusions and future extensions

Cosmological Probes

- Cosmic Microwave Background (CMB)
  - Surface of last scattering
  - $z \sim 1100$
- Large Scale Structure
  - Data from multiple redshifts
  - Low redshift

Complementary information



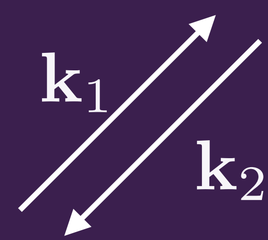
Probes of Large Scale Structure

- Galaxy surveys
  - SDSS, BOSS, DES, Euclid
- Weak lensing surveys
  - DES, KiDS, LSST, Euclid, WFIRST
- Lyman α forest
- 21 cm
- Radio surveys
  - ASKAP, SKA

Correlation functions

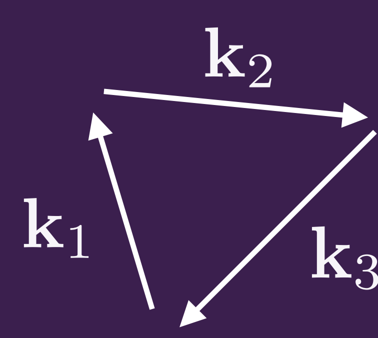
Power spectrum

$$\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2)P(k_1)$$



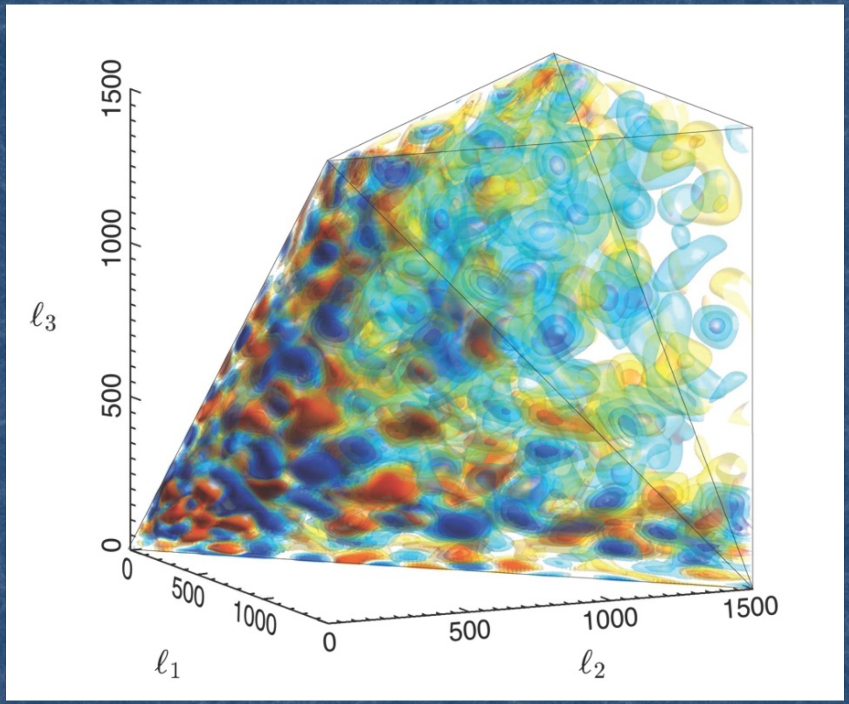
Bispectrum

$$\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

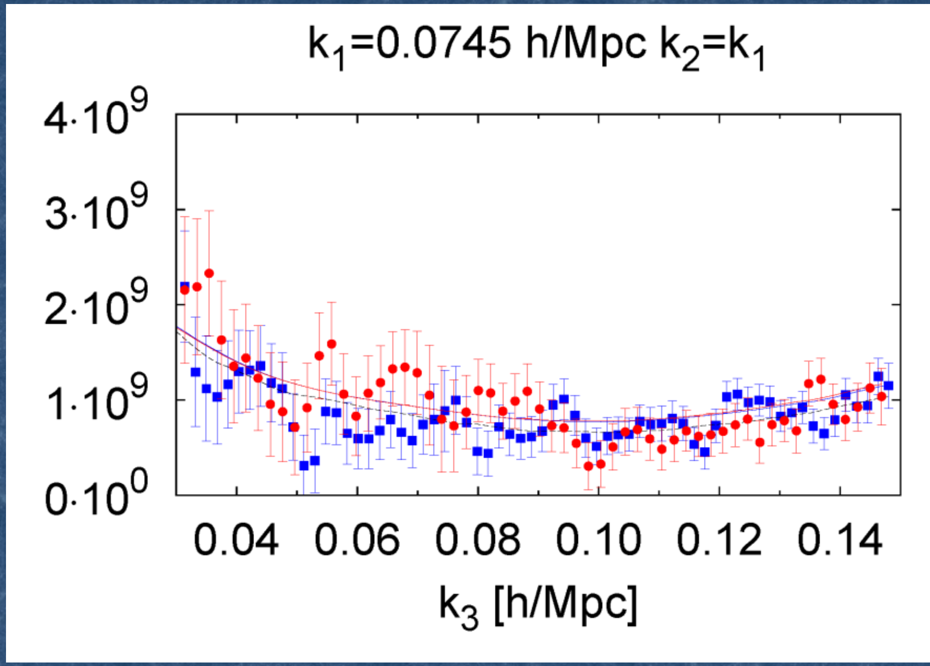


Measurements of bispectra from cosmological data

CMB, from Planck 2015 temperature data



LSS, from BOSS data (Gil-Marin et al 2014) - blue (North Galactic Cap), red (South Galactic Cap)



Shape and amplitude correlators

Scalar Product between 2 bispectra

$$\langle B_i B_j \rangle \equiv \frac{V}{\pi} \int \frac{k_1 k_2 k_3 B_i(k_1,k_2,k_3) B_j(k_1,k_2,k_3)}{P(k_1)P(k_2)P(k_3)}$$

Cumulative scalar product
$k_1,\ k_2,\ k_3 \leq k_{max}$

Sliced scalar product
$k_1 + k_2 + k_3 \in [K-\Delta K, K+\Delta K]$

Shape correlator

$$S(B_i, B_j) = \frac{\langle B_i, B_j \rangle}{\sqrt{\langle B_i, B_i \rangle \langle B_j, B_j \rangle}}$$

analogous to a cosine

Amplitude correlator

$$A(B_i, B_j) = \sqrt{\frac{\langle B_i, B_i \rangle}{\langle B_j, B_j \rangle}}$$

analogous to a ratio

Shape decomposition of the bispectrum

Ansatz

Decompose bispectrum as

$$B(k_1, k_2, k_3) = \sum_{i=1}^{3} f_i(K) S_i(k_1, k_2, k_3)$$

$$(K \equiv k_1 + k_2 + k_3)$$

Shape functions

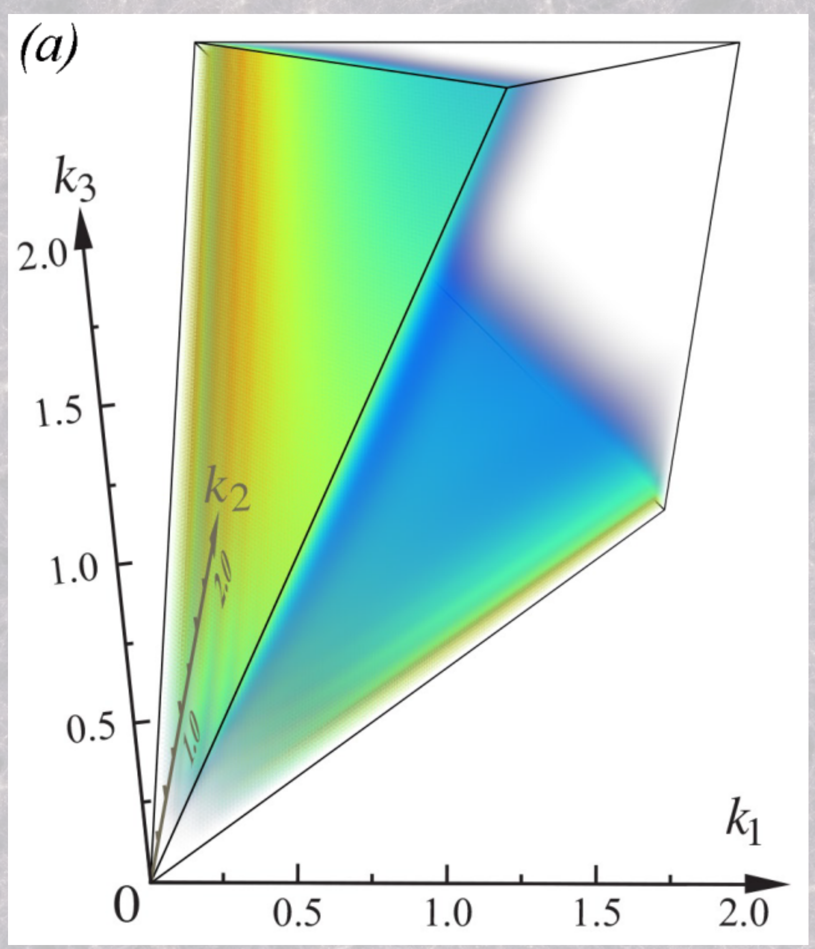
can replace by HALOFIT power spectrum

1. **Tree-level**: $S^{\text{tree}}(k_1, k_2, k_3) = 2P_{lin}(k_1)P_{lin}(k_2)F_2^{(s)}(k_1, k_2) + 2\,\text{perms}$
2. **Squeezed**: $S^{\text{squeezed}}(k_1, k_2, k_3) = \frac{1}{3}\left[P_{lin}(k_1)P_{lin}(k_2) + 2\,\text{perms}\right]$
3. **Constant**: $S^{\text{constant}}(k_1, k_2, k_3) = 1(\text{Mpc}/h)^6$

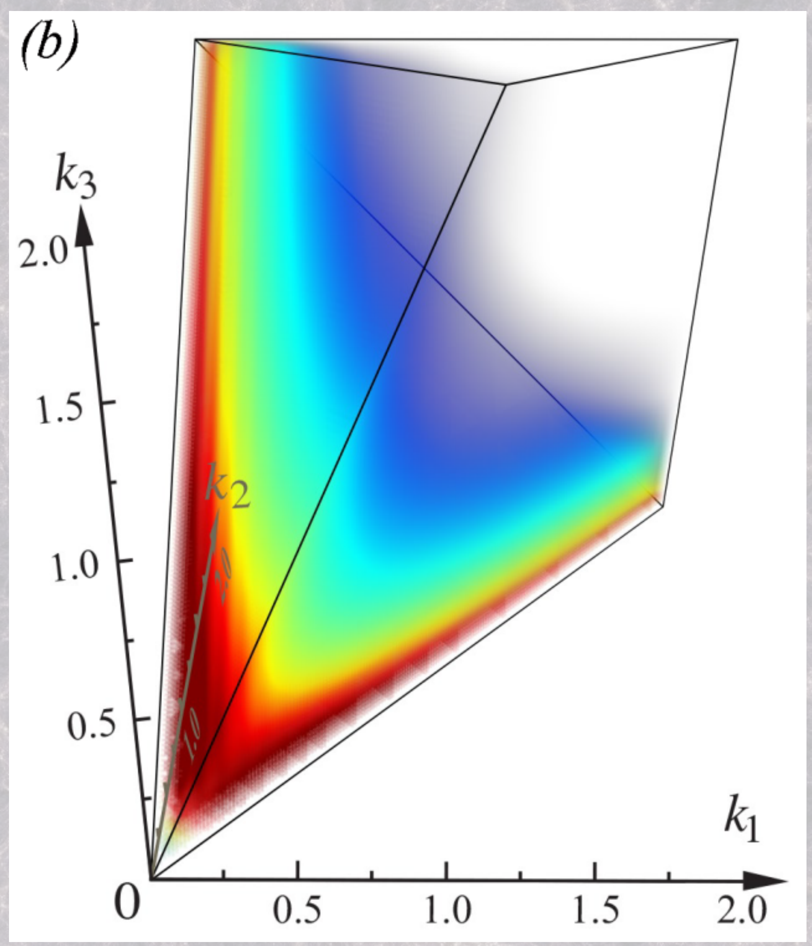
Signal-to-noise weighted bispectra

$$B^{SN}(k_1,k_2,k_3) \equiv \sqrt{\frac{k_1k_2k_3}{P(k_1)P(k_2)P(k_3)}}B(k_1,k_2,k_3)$$

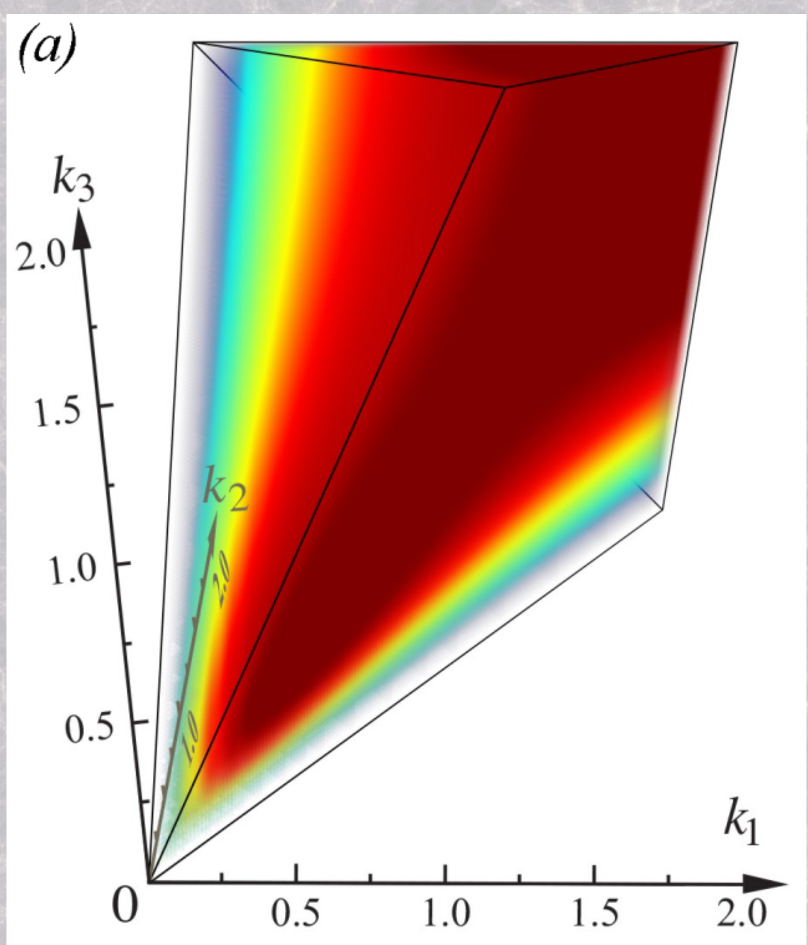
Tree-level

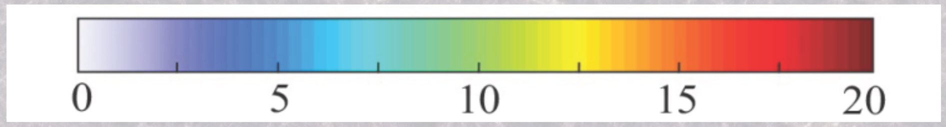


Squeezed

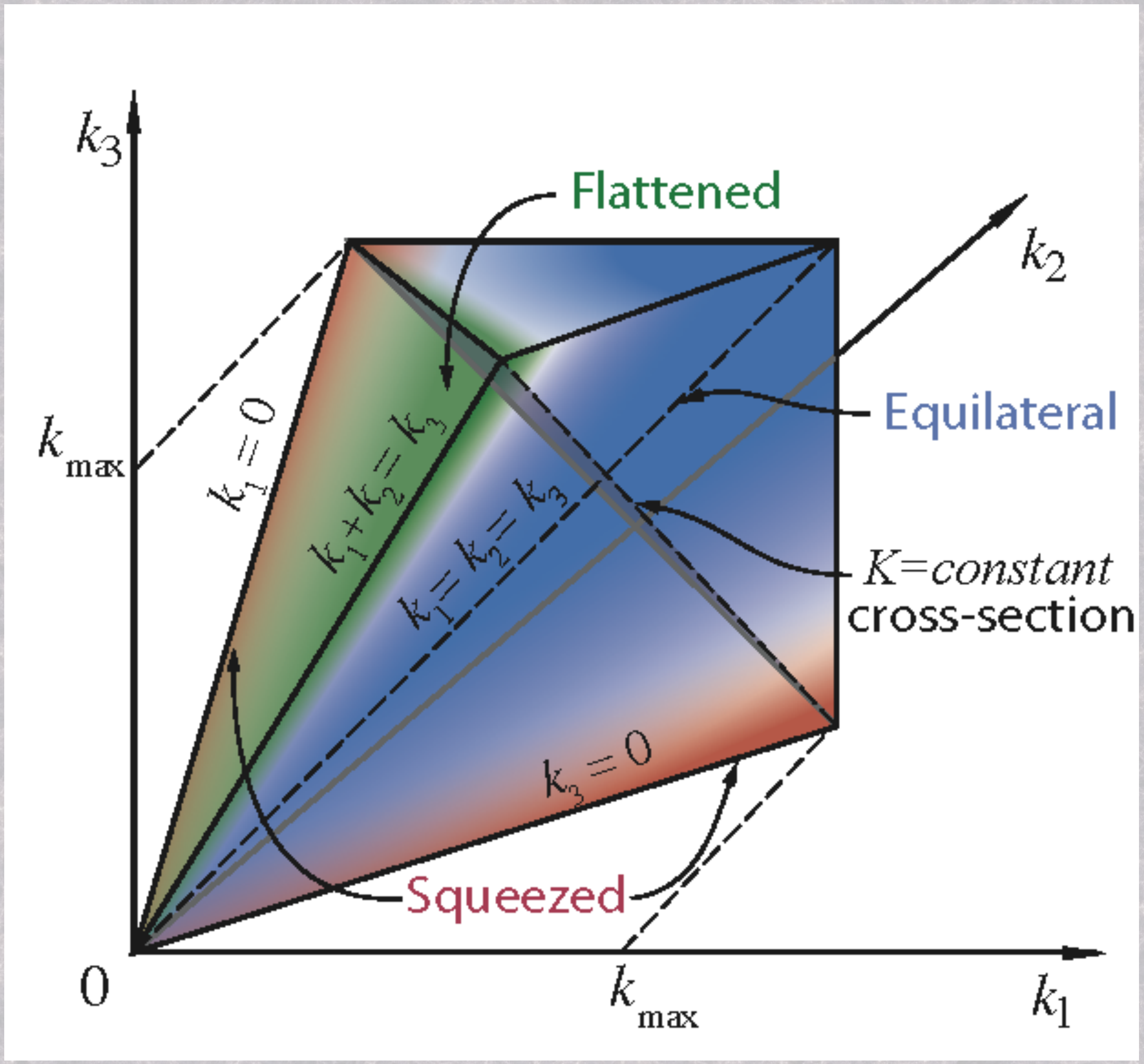


Constant





Where to look for the three shapes



Theoretical models of LSS

Perturbation theories

- Deviations from linear scales
- Expected to work in the mildly non-linear regime, as long as perturbations are small
- Not valid into the nonlinear regime
- Various approaches considered to improve accuracy and/or range of validity
- Mainly 1-loop results

- Eulerian Standard PT (SPT).
- Effective Field Theory of LSS (EFT).
- Renormalised PT (MPTbreeze formalism).
- Resummed Lagrangian PT (RLTP).

Halo models

- Phenomenological models describing matter clustering on all scales
- Based on the spherical collapse model

- Standard halo model
- Improved halo model

Eulerian Standard PT

- Fluid approximation of matter in the universe
- Based on loop expansion

➢ External lines represent external wavevectors k_1, k_2, k_3

➢ Internal lines represent P_{lin} with arguments the relevant wavevector

➢ Sum of wavevectors is 0 at each vertex

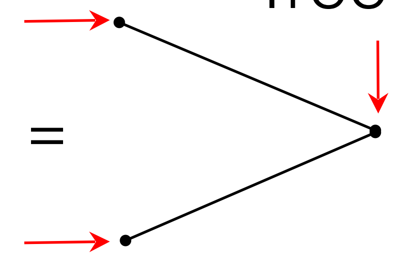
➢ Internal lines are integrated over

➢ Vertices represent kernels

➢ The number of internal lines from each vertex • gives the order of the kernel $F_n^{(s)}$

Tree level

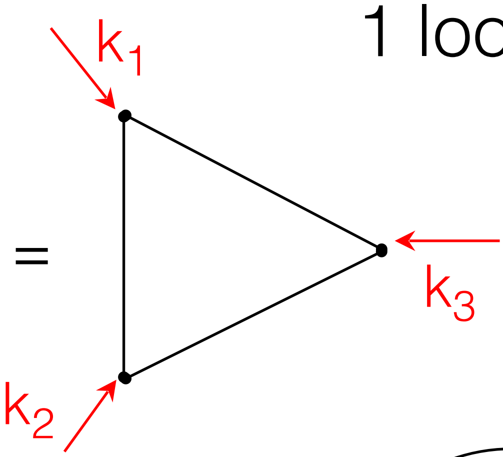
B_{211} =



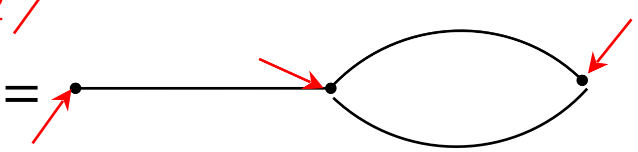
+ 2 perms.

1 loop

B_{222} =

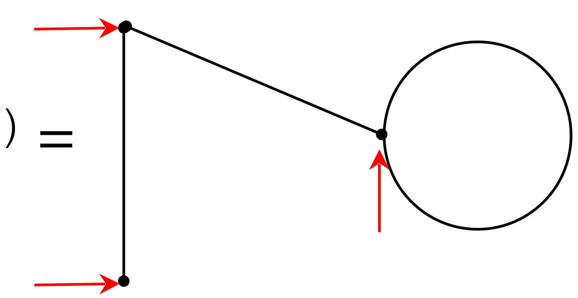


$B_{321}^{(I)}$ =



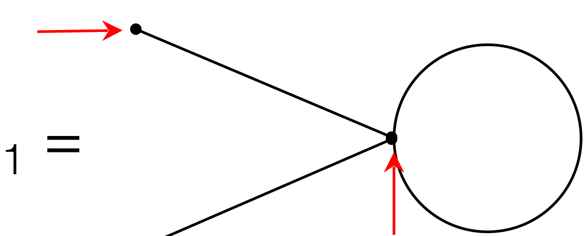
+ 5 perms.

$B_{321}^{(II)}$ =



+ 5 perms.

B_{411} =



+ 2 perms.

Other perturbation theories

Effective Field Theory [Baldauf et al 2015, Angulo et al 2015]

- Solves the problem of the excess of power in SPT in the mildly nonlinear regime
- Considers the effect of short wavelength modes on the long ones
- This is achieved by adding counterterms
- Counterterms from the bispectrum have coefficients fitted from the power spectrum

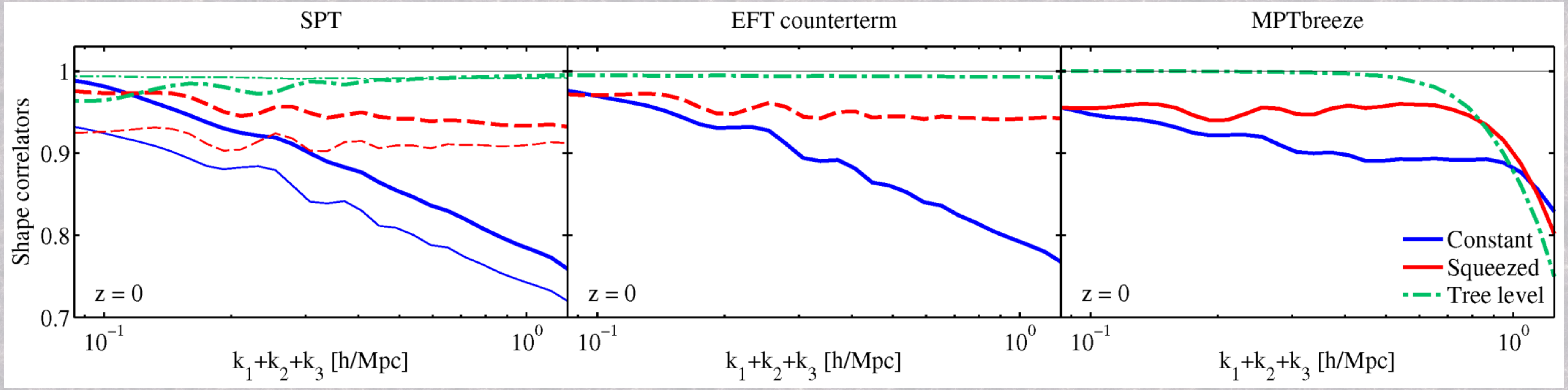
Renormalised PT [Bernardeau et al 2008]

- Infinite expansion of SPT is resummed to obtain a convergent expansion
- Further simplification: **MPTbreeze** [Crocce et al 2012]
  - Formalism is significantly simplified
  - Bispectrum can be expressed in terms of some of the SPT terms and is exponentially supressed
  - Possible to compute at 2 loops

Resummed Lagrangian PT [Matsubara et al 2008, Rampf et al 2012]

- Uses Lagrangian coordinates for initial displacements
- Bispectrum can be expressed in terms of the SPT bispectrum, but is exponentially supressed

Shapes of PT models



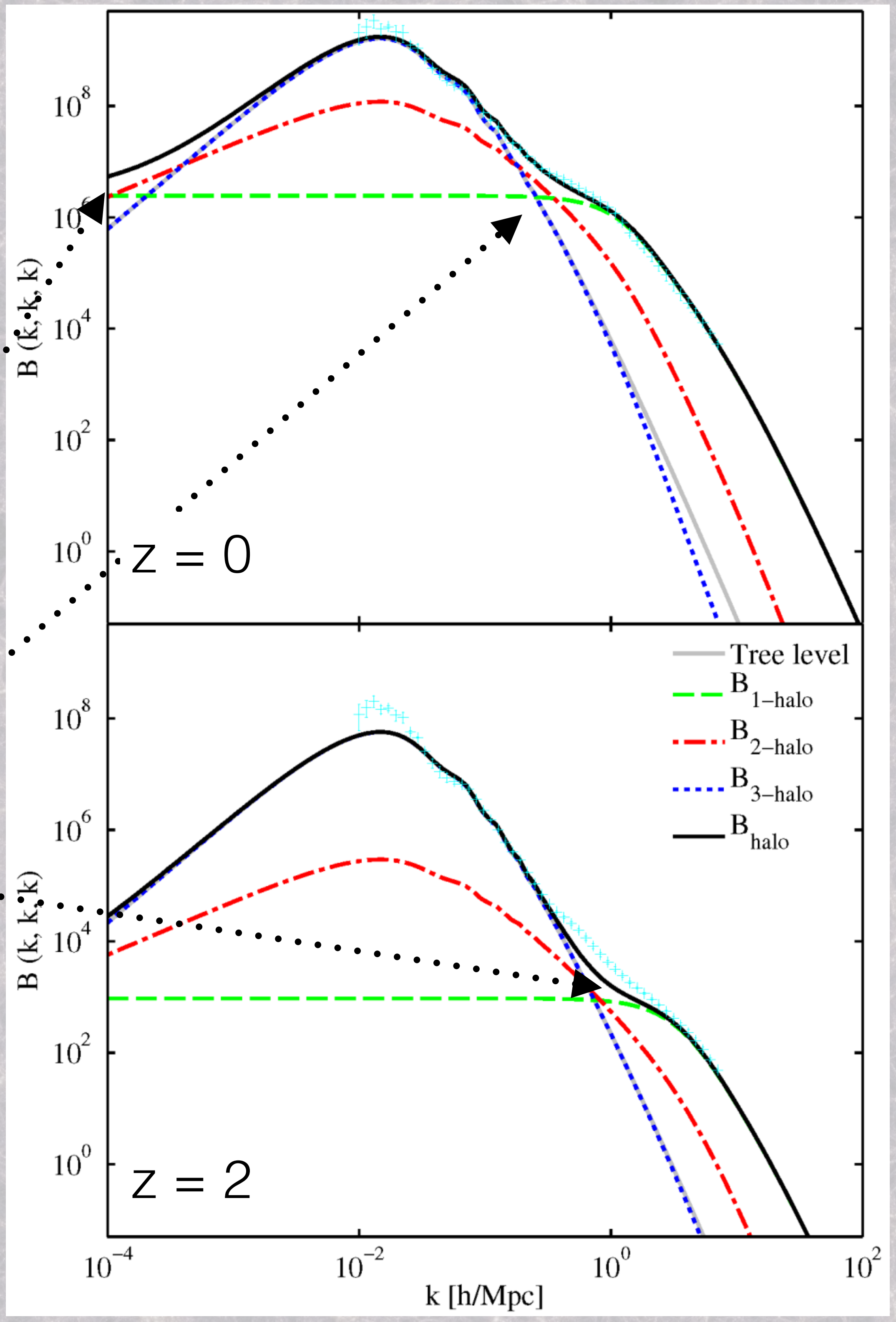
Sliced shape correlator between PT models and the 3 shapes considered

All 1 loop PT models have a flattened shape

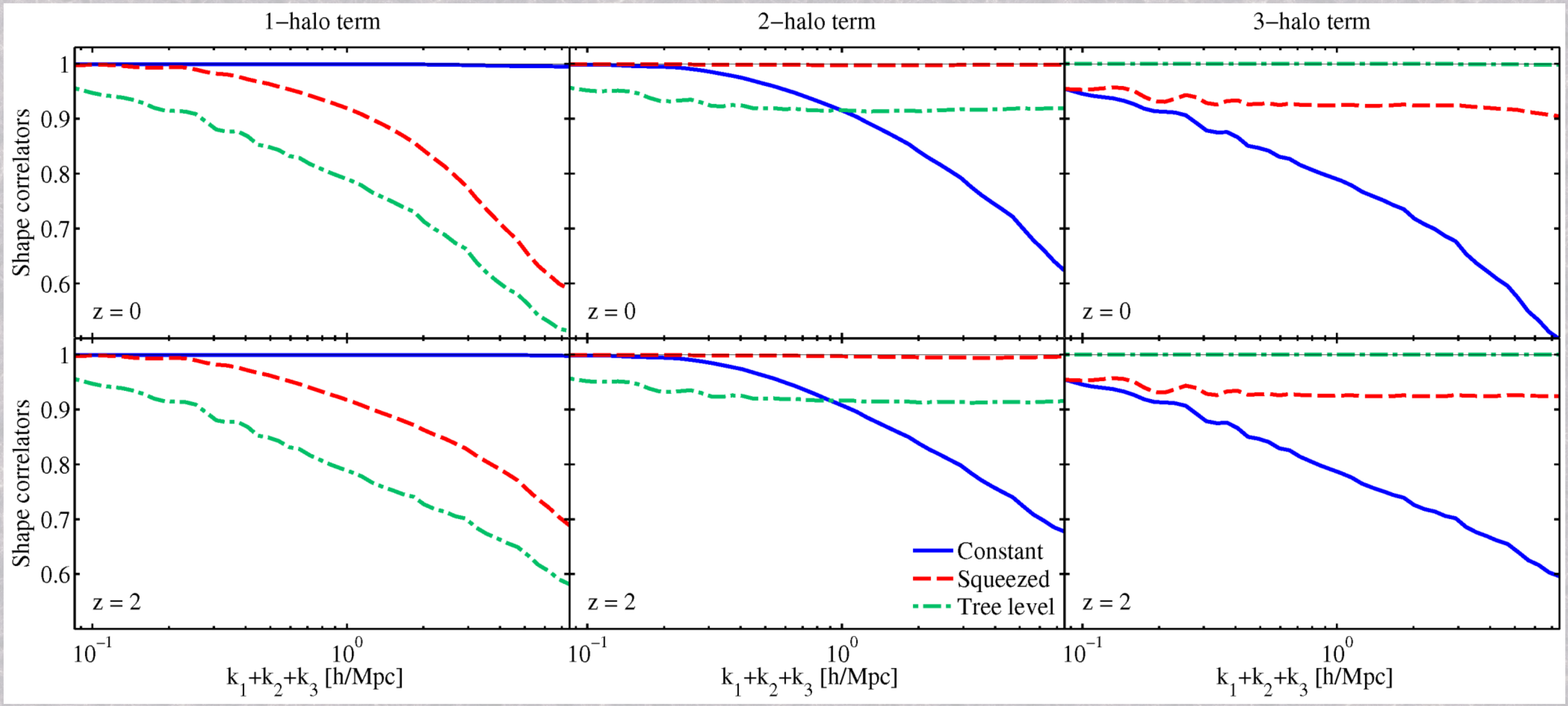
Halo models

- Profile – Navarro-Frenk-White (NFW): $u(k|m,z)$ – spatial mass distribution of halos.
- Mass function – Tinker mass function: $n(m,z)$ – number density of halos.
- Clustering of dark matter halo centres – deterministic bias.

- Physically, on very large scales should have only 3-halo term contributions; however, 1- and 2-halo terms don't decay to 0 in this regime → **excess** on large scales.
- In transition regime between 1- and 3-halo terms, there is a **deficit** of power at $z > 0$.
- First problem solved in halo-PT model (Valageas et al 2011) – make sure that positions corresponding to triplet (k_1,k_2,k_3) contribute to only one term of halo model and 3-halo contribution substituted with PT.



Shapes of the halo model



Constant shape

Squeezed shape

Tree-level shape

Fit to the halo model

Use ansatz & previous slide

$$B_{1h}(k_1,k_2,k_3) = f_{1h}(K)S^{\text{constant}}(k_1,k_2,k_3)$$
$$B_{2h}(k_1,k_2,k_3) = f_{2h}(K)S^{\text{squeezed}}(k_1,k_2,k_3)$$
$$B_{3h}(k_1,k_2,k_3) = f_{3h}(K)S^{\text{treeNL}}(k_1,k_2,k_3)$$

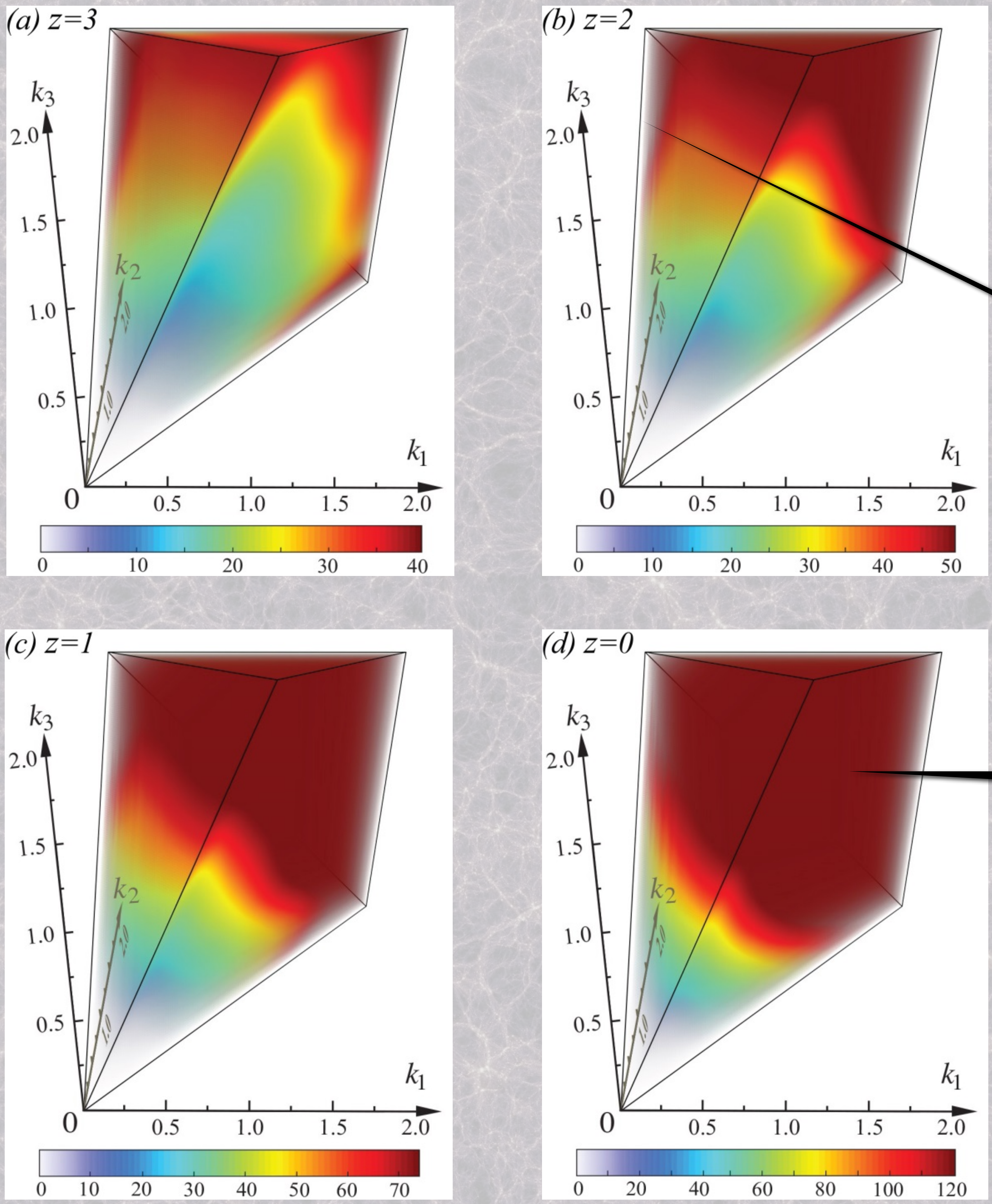
Halo model
(equilateral configuration)

$$f_{1h}(K) = \frac{A}{(1+bK^2)^2}$$
$$f_{2h}(K) = \frac{C}{(1+DK^{-1})^3}$$
$$f_{3h}(K) = 1 \quad \text{(if using } S^{\text{tree}}\text{)}$$

Numerical simulations

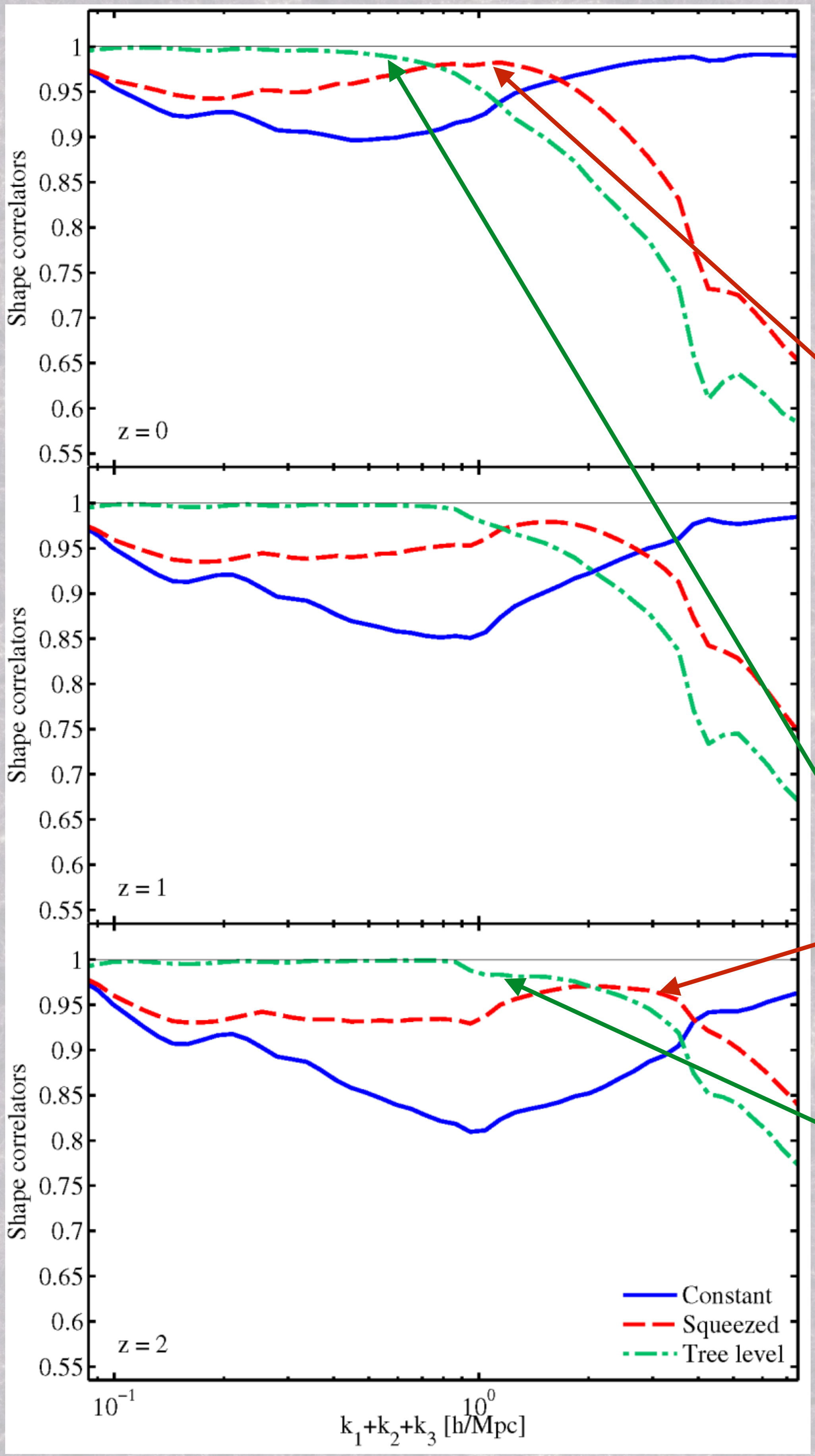
- Use modal method (Fergusson et al 2009, Schmittfull et al 2013), that compresses information given by the data
- Only around 50 modes are required to accurately recover full bispectrum
- Simulations
  - 3 simulations covering in total $z \leq 7.8$ h/Mpc
  - 3 realisations each
  - Starting redshift: 49
  - 512^3 points in simulation box

Shapes of N-body simulations



Squeezed shape (z = 2,3)

Constant shape (all z)



- Large scales: flattened shape
- Intermediate scales: squeezed shape
- Small scales: constant shape

Shapes correspond to the shapes of the 3 components of the halo model

Significant squeezed component at all redshifts

Flat shape dominating even after the tree-level has decayed

Three-shape model

Use decomposition **ansatz** (beginning of talk)

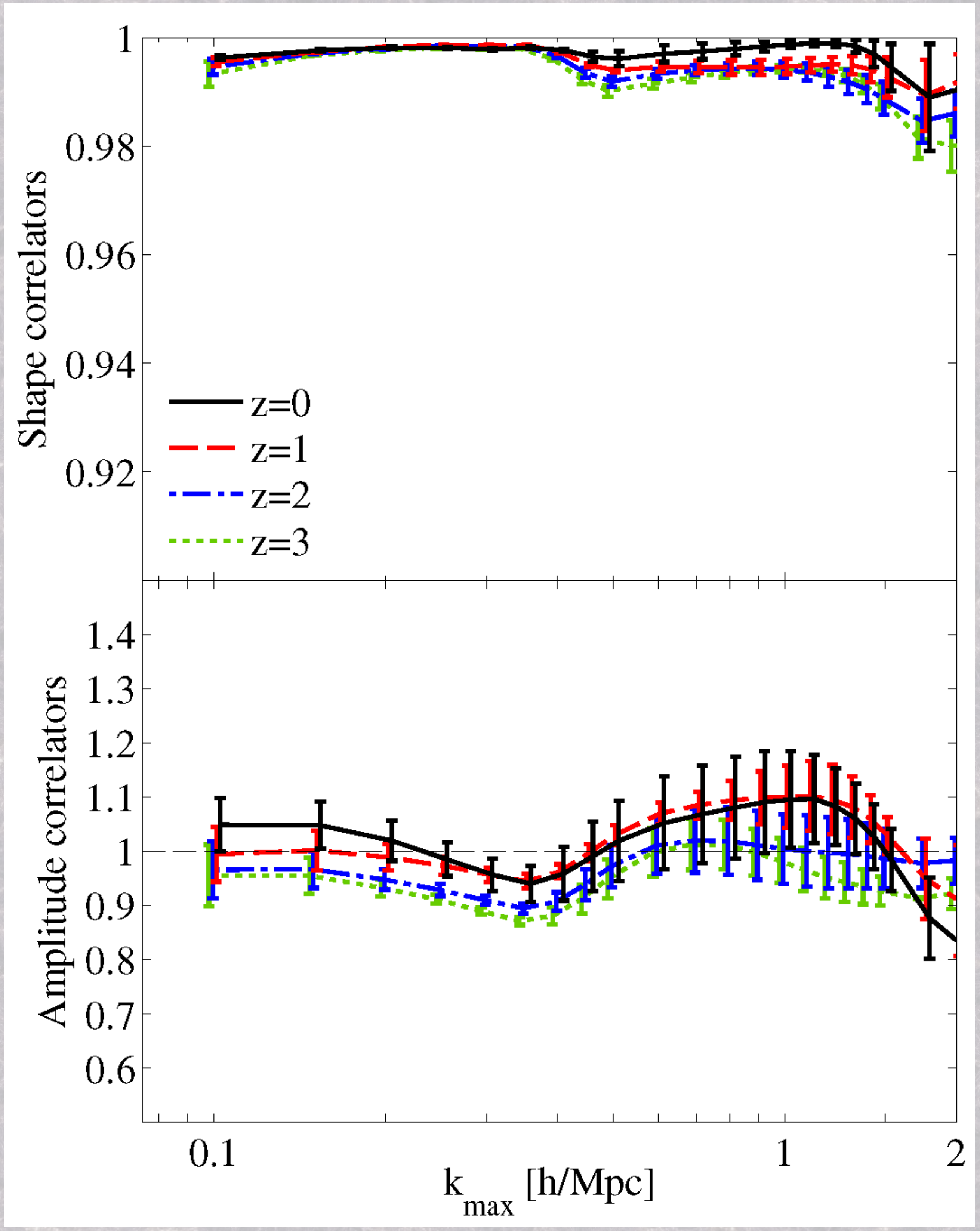
$$B(k_1, k_2, k_3) = \sum_{i=1}^{3} f_i(K) S_i(k_1, k_2, k_3)$$

Choosing functions f_i:

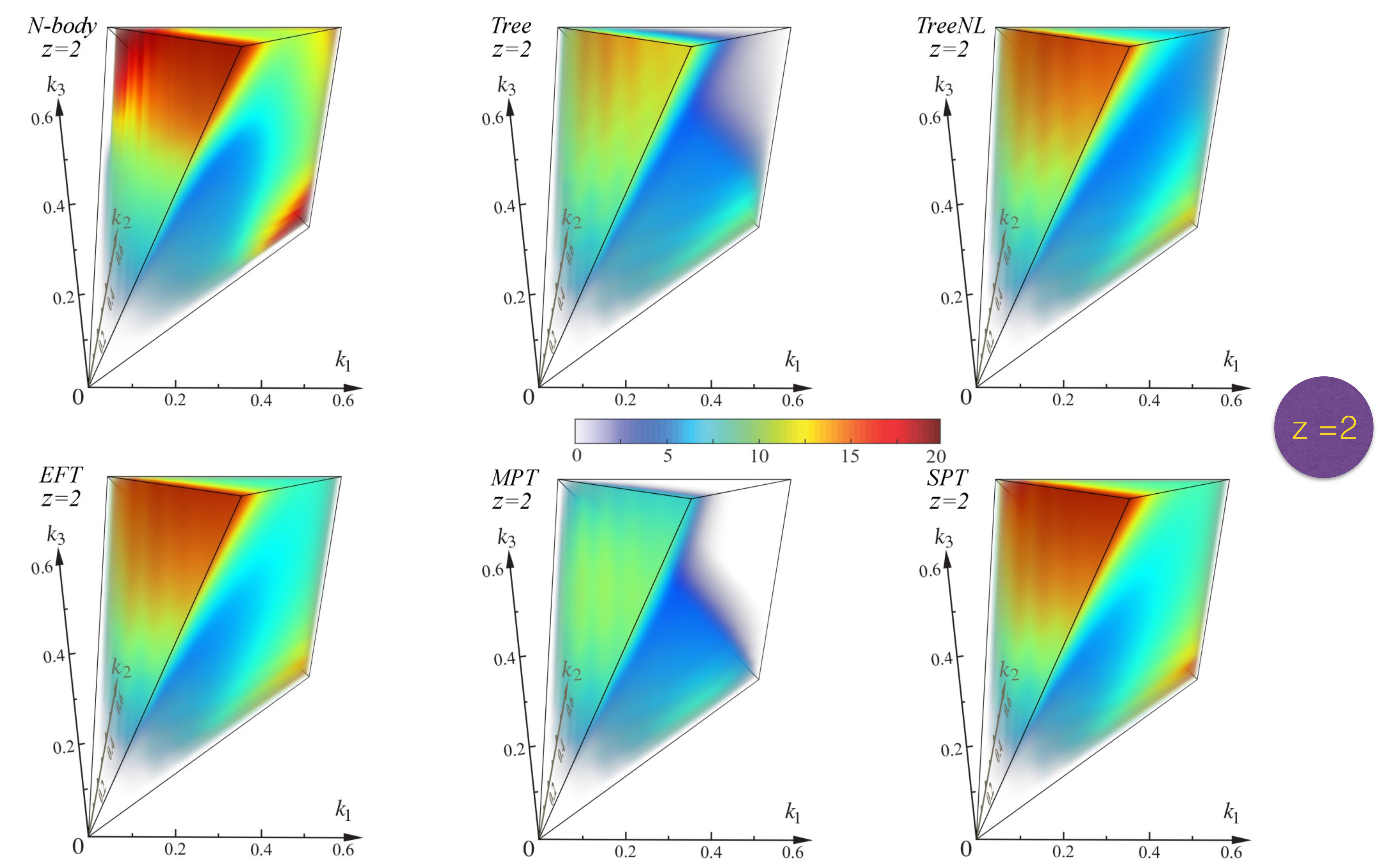
- Constant shape (f_1)
  In the strongly nonlinear regime the one-halo term provides a good fit, hence use the fit to the 1-halo term
- Squeezed shape (f_2)
  Keep functional form of the 2-halo term fitting function, but refit the two coefficients
- Tree-level shape (f_3)
  Significant flat shape after tree-level has decayed, hence use non-linear tree-level with cut-off

$$f_{3h}(K) = \exp\left(-K/E\right)$$

Best-fit model



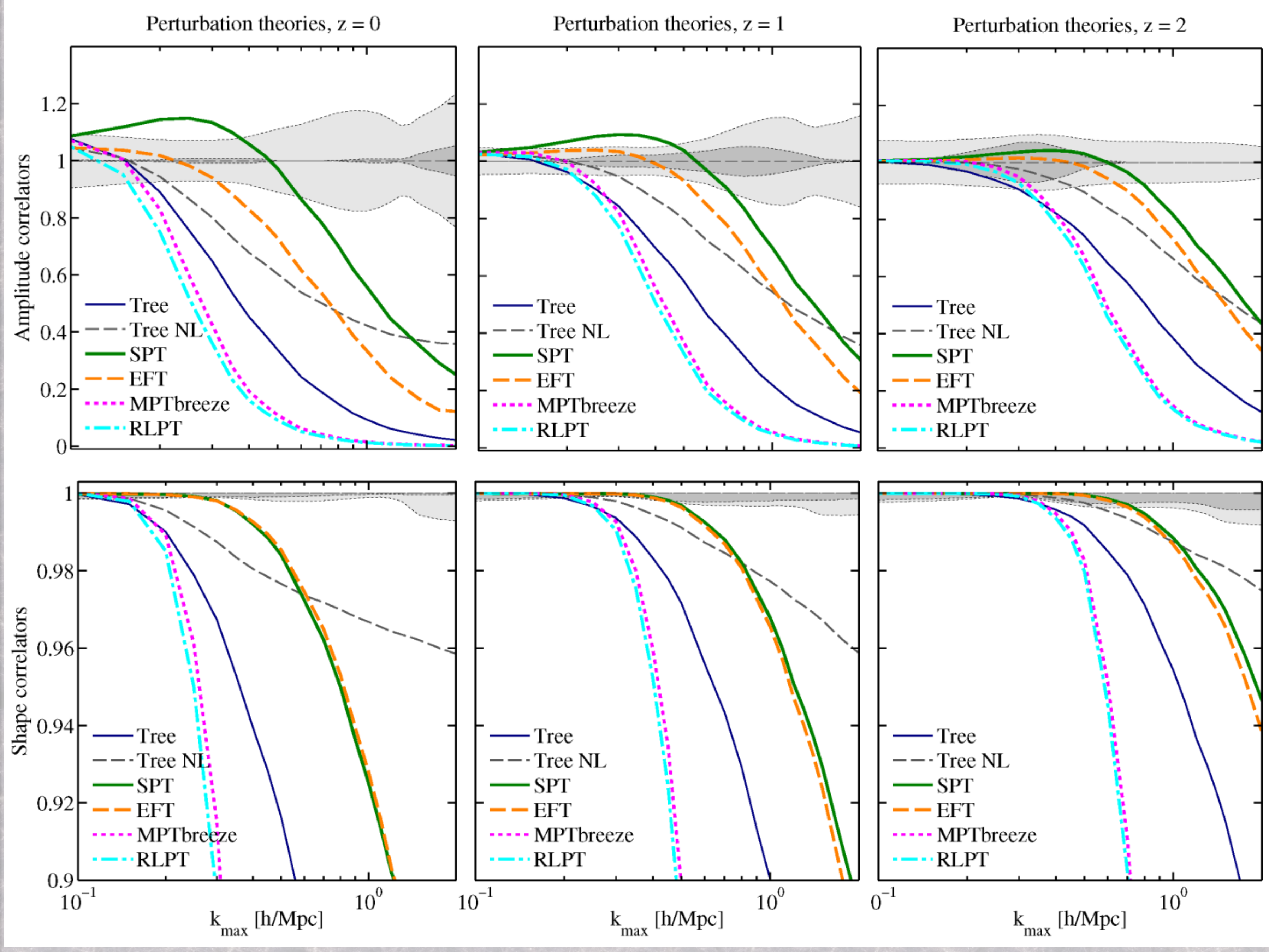
Perturbation theories 3D bispectra



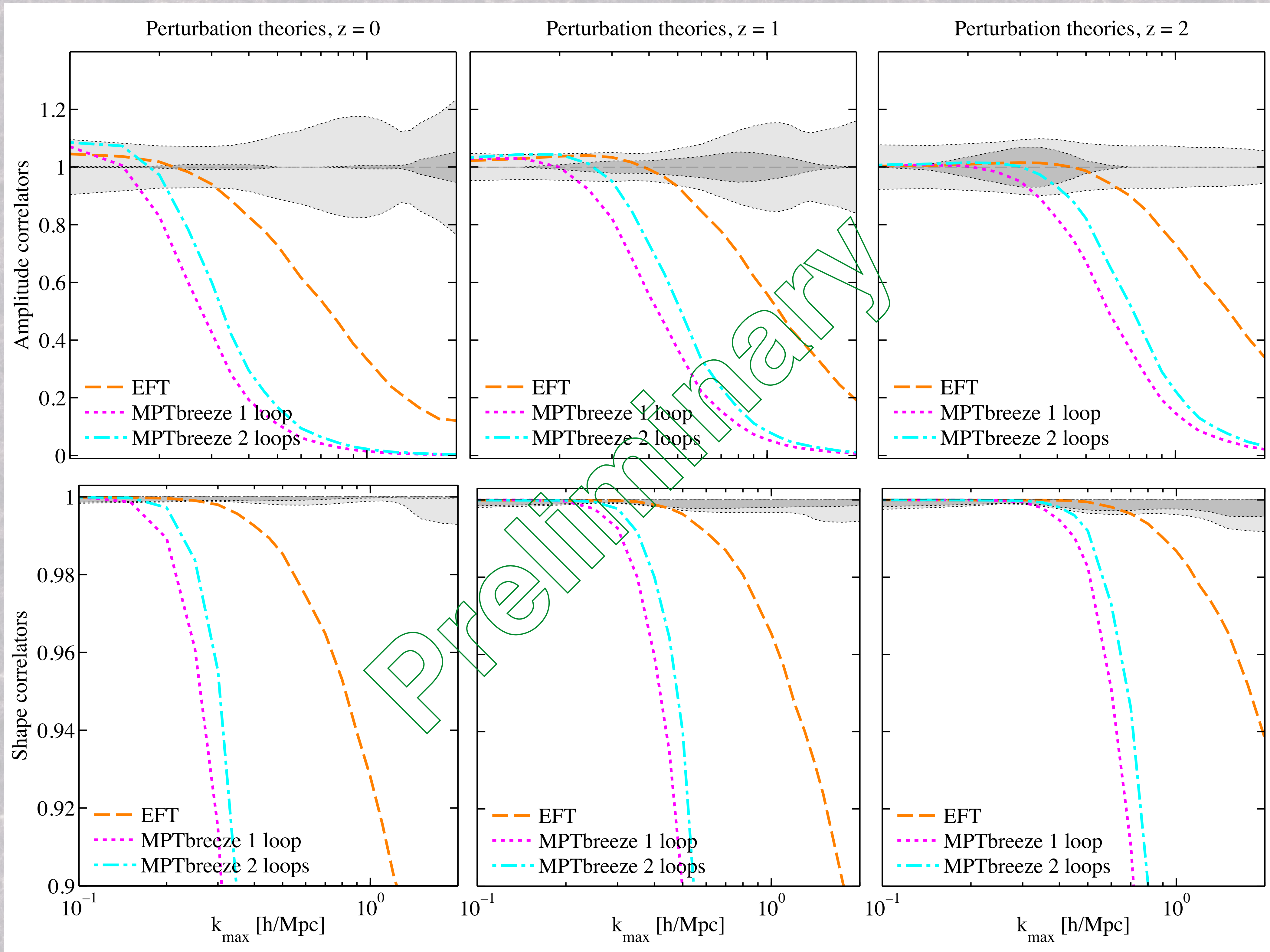
z =2

3D comparison between models and simulations for perturbation theories

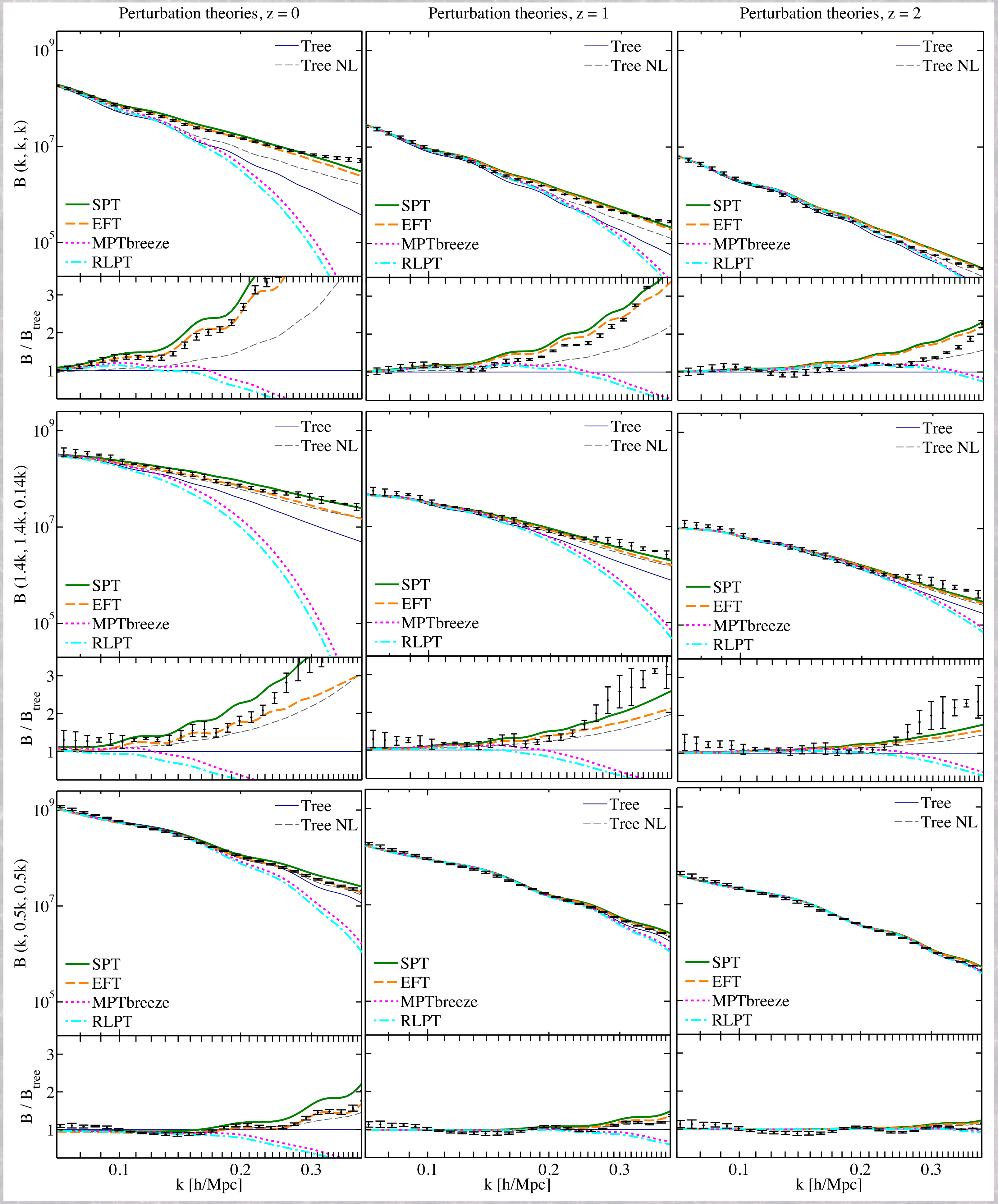
- Comparison is wrt three-shape model
- Dark grey: fitting errors
- Light grey: fitting errors + simulation error bars
- There is excess of power on large scales at $z = 0$ (too much power from 1, 2-halo terms) in the 3-shape model
- Exponential suppression of the MPTbreeze and RLPT, otherwise accurate
- Well-known excess of power of SPT on mildly nonlinear scales
- Problem solved in EFT with suitable counterterm
- NL tree level does not decay to 0 (exponential suppression is required in the 3-shape model)
- Shape correlator is not a good way to compare models
- Amplitude is significantly better



MPTbreeze 2-loop results



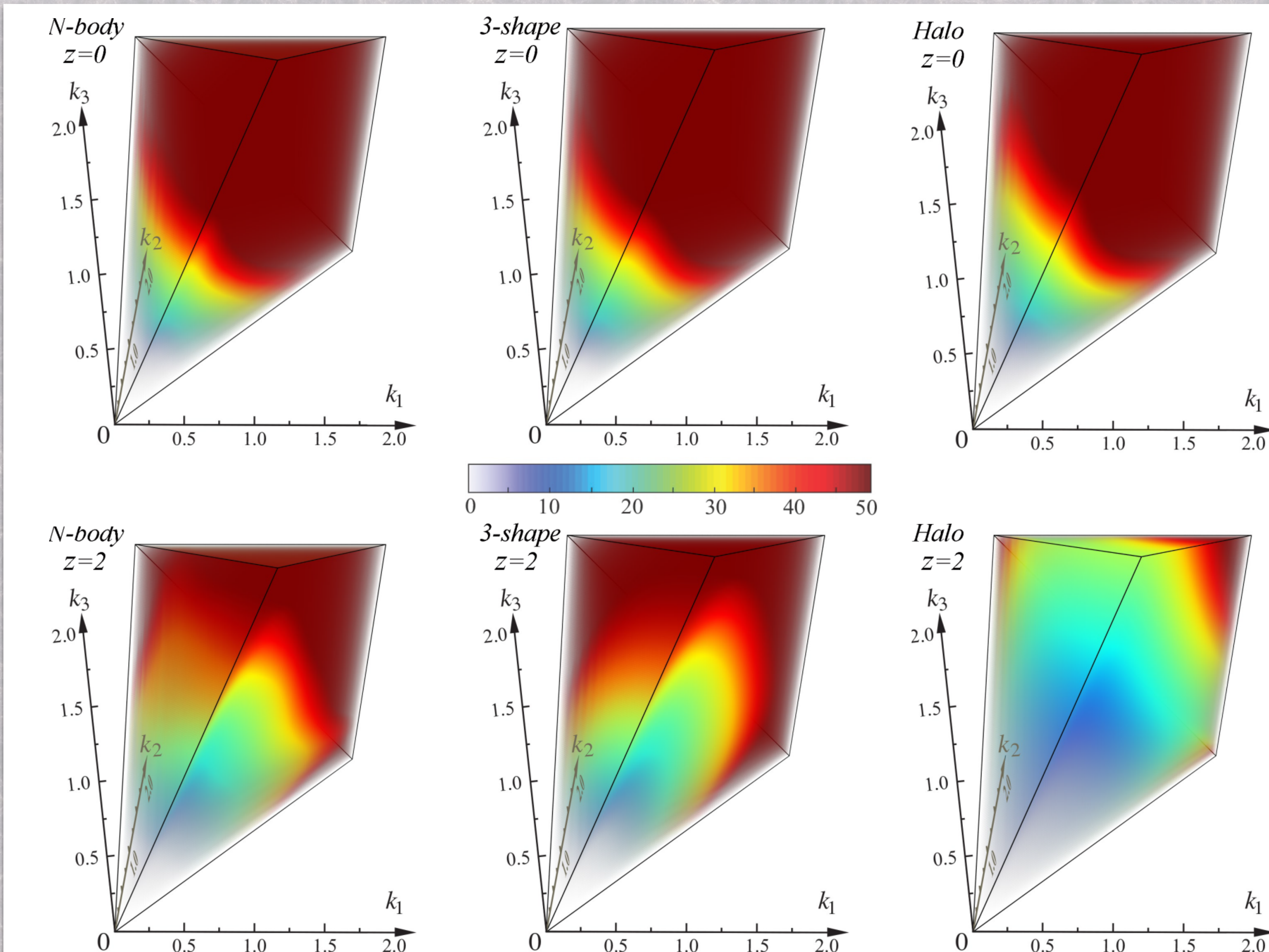
- 2-loop bispectrum computation
- Significant improvement over 1-loop (Δk~0.1 *h*/Mpc)
- 1-loop EFT still getting further
- 2-loop shape is still flat (as tree-level)



Slice comparison

- Slices show what models work in each configuration
- EFT is the most accurate on mildly nonlinear scales for all redshifts and configurations, even though it is the most accurate at $z = 0$
- SPT is relatively close to EFT, but there is always in excess of power
- As in the 3D correlators, RLPT and MPTbreeze predict very similar results
- NL tree-level shows a relative accuracy with respect to linear tree level

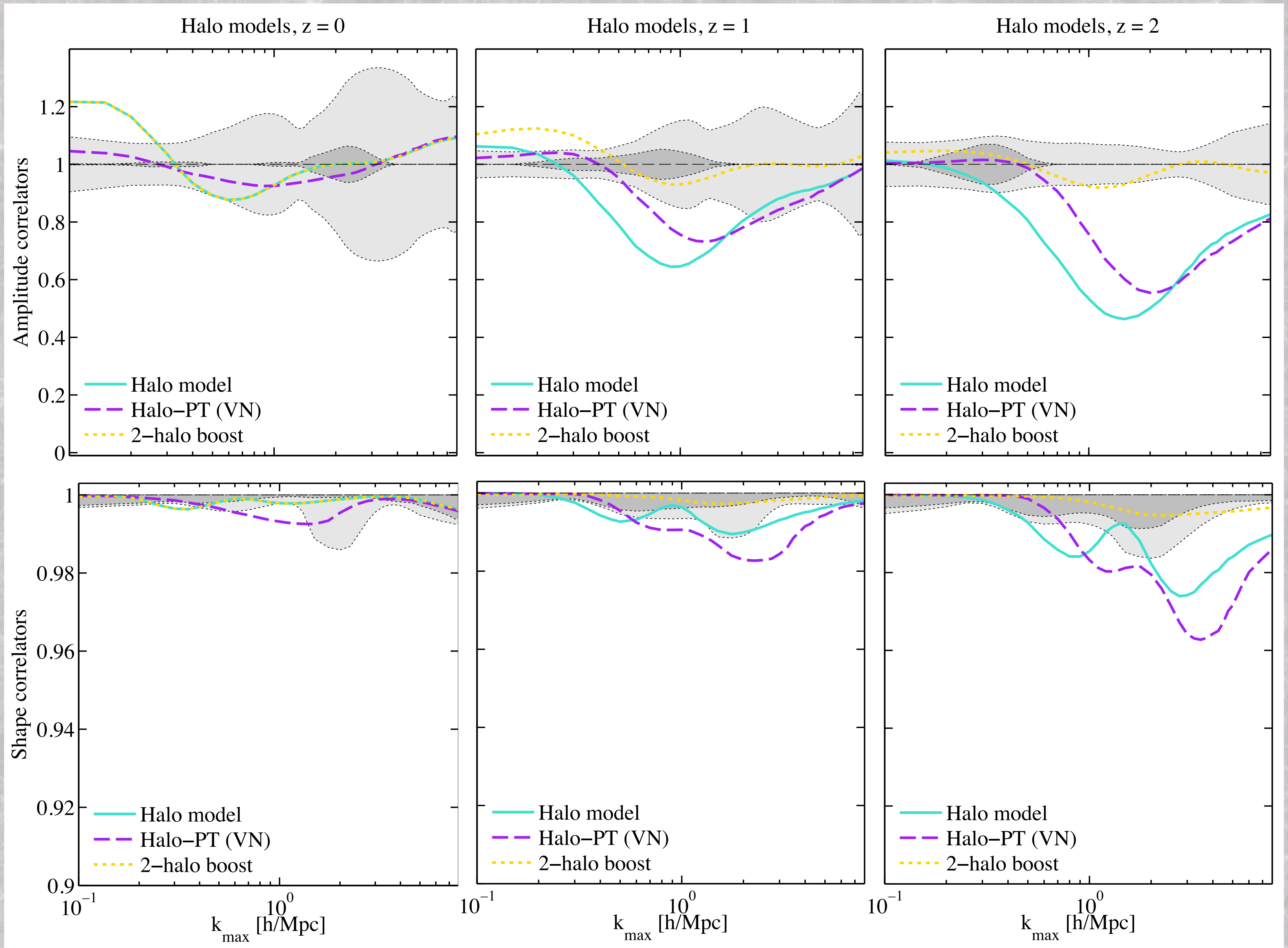
Halo models 3D bispectra



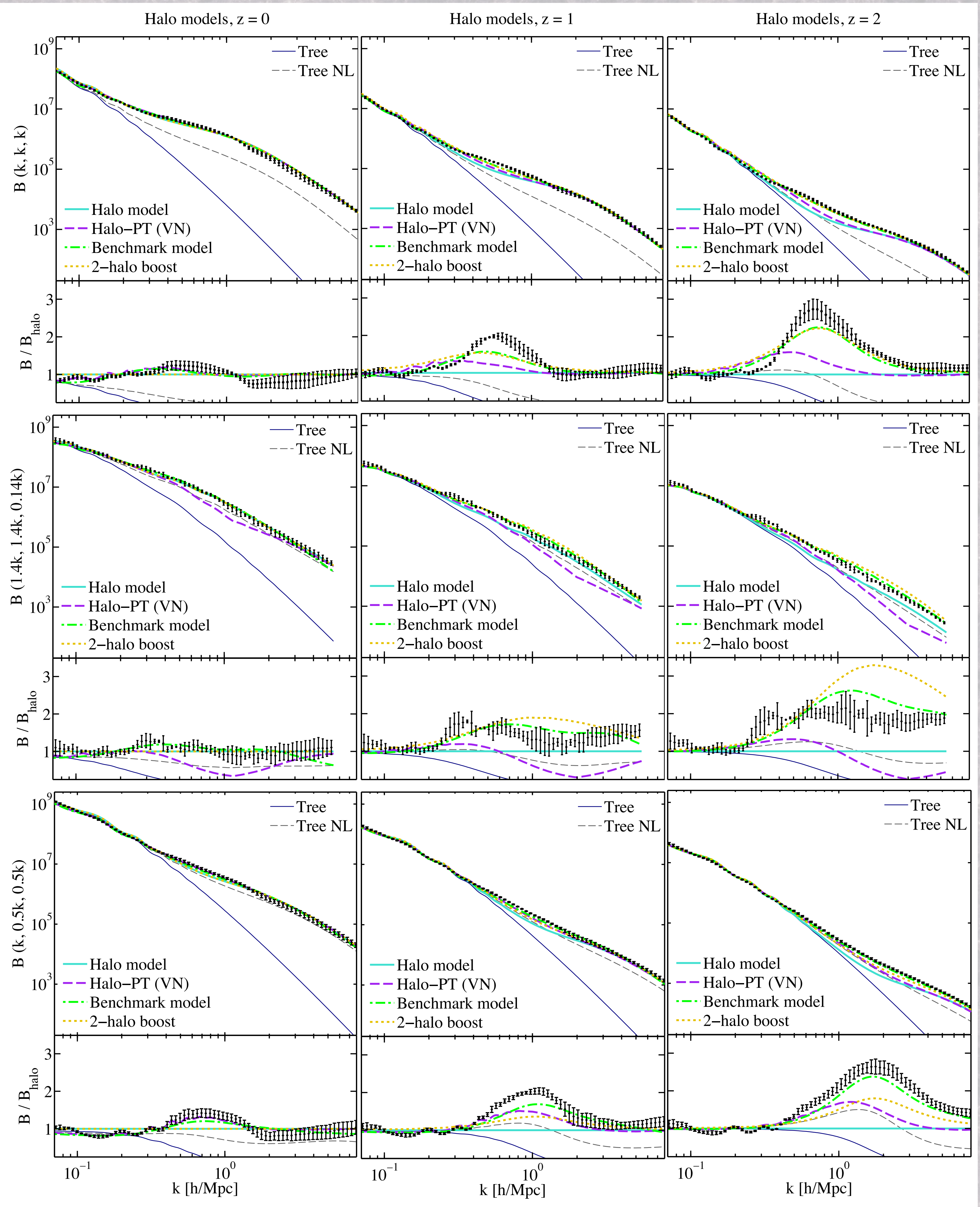
😀
✔ Good agreement halo-simulations at z=0

😩
✘Bad agreement halo-simulations at z=2

3D comparison between models and simulations for halo models



- Again comparison is wrt three-shape model
- The small dark grey areas show that the 3-shape model is a good fit to the data
- Significant excess on large scales in the halo model, especially at $z = 0$
- Halo-PT model OK at $z = 0$
- At $z > 0$, there is a significant deficit in the halo model on intermediate scales, but prediction is accurate again at high k; this issue is only partially solve in the halo-PT model
- The 2-halo boost model gives very similar prediction to the 3-shape model, except the excess as $k \to 0$



Slices comparison

- Slices show that the three-shape model provides a good fit for all configurations for all the three redshifts considered in the nonlinear regime.
- Significant deficit in the halo and halo-PT models at $z > 0$, which is increasing as the redshift is increased.
- At $z = 2$, the 2-halo boost provides a good fit in the equilateral configuration, but an excess of power in the squeezed limit and a deficit in the flattened limit; same behaviour at $z = 1$, but less evident.

Primordial non-Gaussianity

Basic theory

Matter density contrast related to primordial potential via $\delta(\mathbf{k}, z) = M(k, z)\Phi(\mathbf{k})$

Linear power spectrum $P_{lin}(k, z) = M(k, z)^2 P_\Phi(k)$

Non-Gaussian bispectrum $B_0 = B^{NG}_{\rm tree} = M(k_1, z)M(k_2, z)M(k_3, z)B_\Phi(k_1, k_2, k_3)$

Consider 2 shapes for primordial bispectrum

$$B^{\rm local}_\Phi(k_1, k_2, k_3) = 2f^{\rm local}_{\rm NL}\left[P_\Phi(k_1)P_\Phi(k_2) + 2 \text{ perms}\right]$$

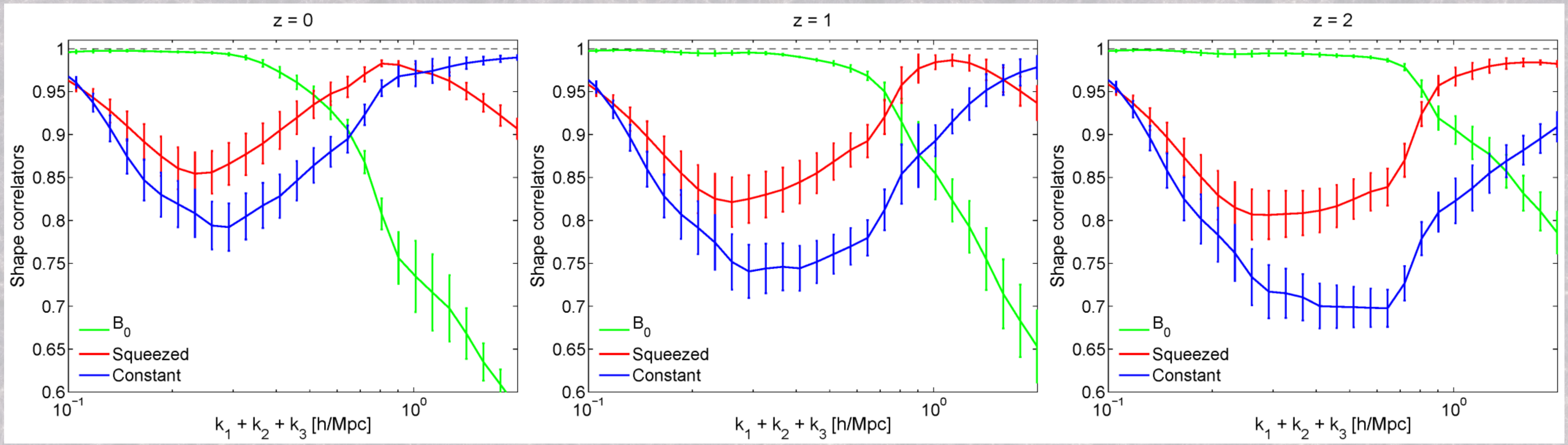
$$\begin{aligned} B^{\rm eq}_\Phi(k_1, k_2, k_3) = 6f^{\rm eq}_{\rm NL}\,\{&-[P_\Phi(k_1)P_\Phi(k_2) + 2 \text{ perms}] \\ &- 2[P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)]^{2/3} \\ &+ [P^{1/3}_\Phi(k_1)P^{2/3}_\Phi(k_2)P_\Phi(k_3) + 5 \text{ perms}]\ \} \end{aligned}$$

Simulations: Schmittfull et al 2013, $f^{\rm local}_{\rm NL} = 10$ and $f^{\rm eq}_{\rm NL} = 100$.

PNG shapes (local-type NG)

Correlators between $\Delta B_{NG} \equiv B(f_{\rm NL} = 10) - B(f_{\rm NL} = 0)$ and the 3 shapes

- **Constant** shape ✓ (same as before)
- **Squeezed** shape ✓ (same as before)
- **Tree-level** shape ⇒ Non-Gaussian tree-level shape



NG extension of 3-shape model

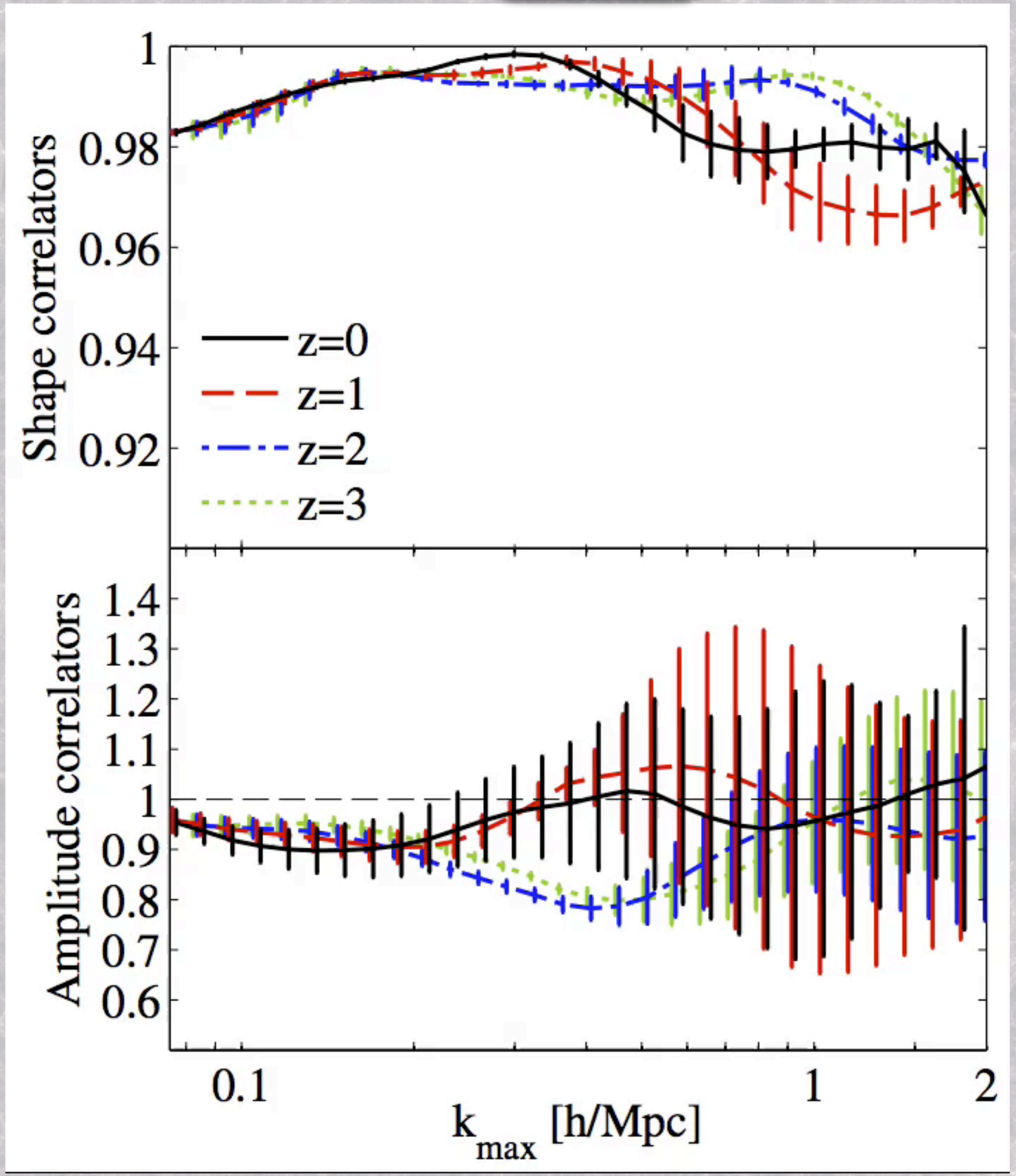
$$\Delta B_{NG}(k_1,k_2,k_3) = B^{NG,NL}_{\text{tree}} + c_{2h} f_{2h}(K) S^{\text{squeezed}} + c_{1h} f_{1h}(K) S^{\text{constant}}$$

fit c_{1h}, c_{2h}

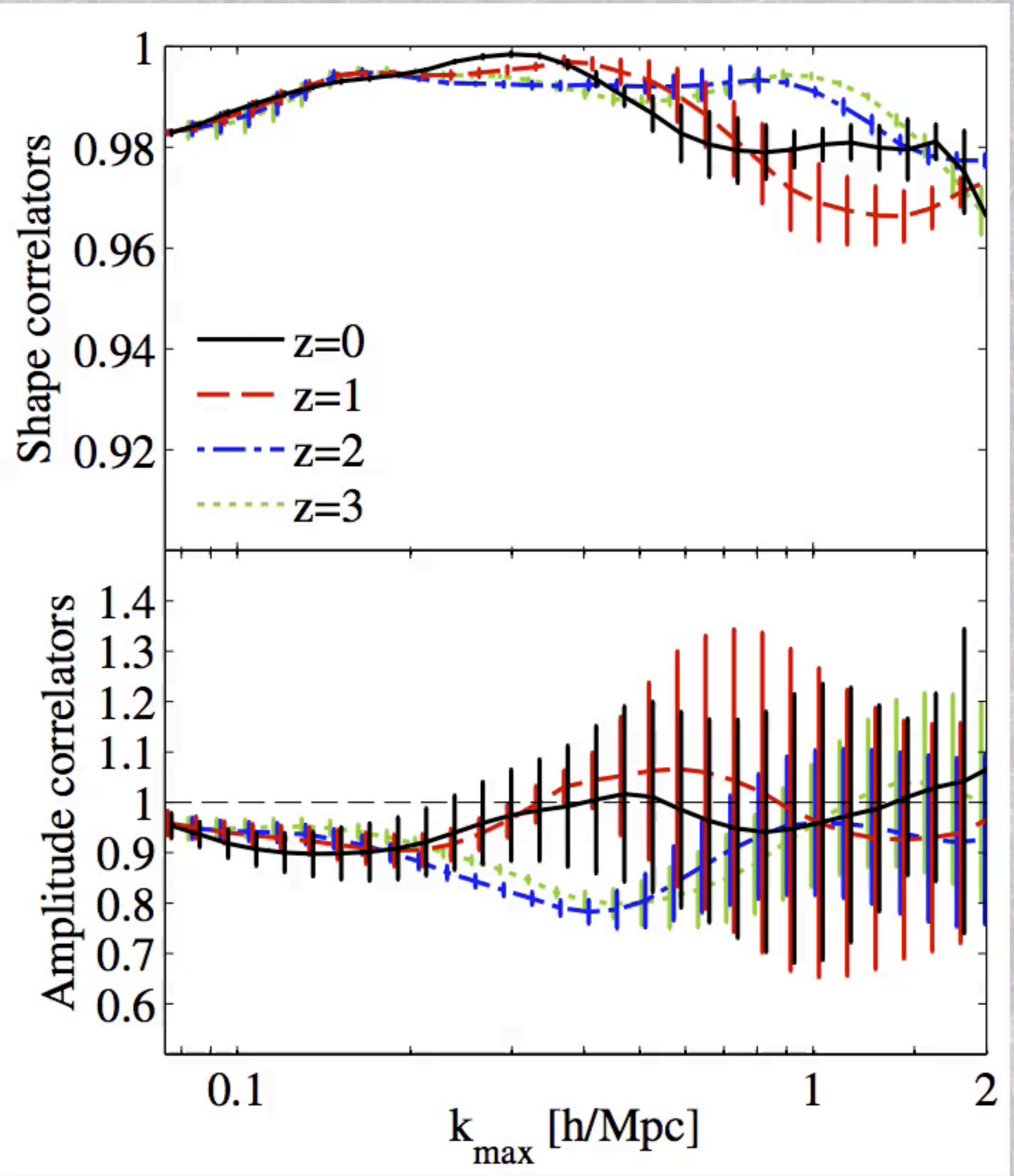
replaces Gaussian tree-level bispectrum depends on type of NG considered

same functions as for Gaussian simulations

Local NG, f_{NL}=10



Equilateral PNG



Equilateral NG, f_{NL}=100

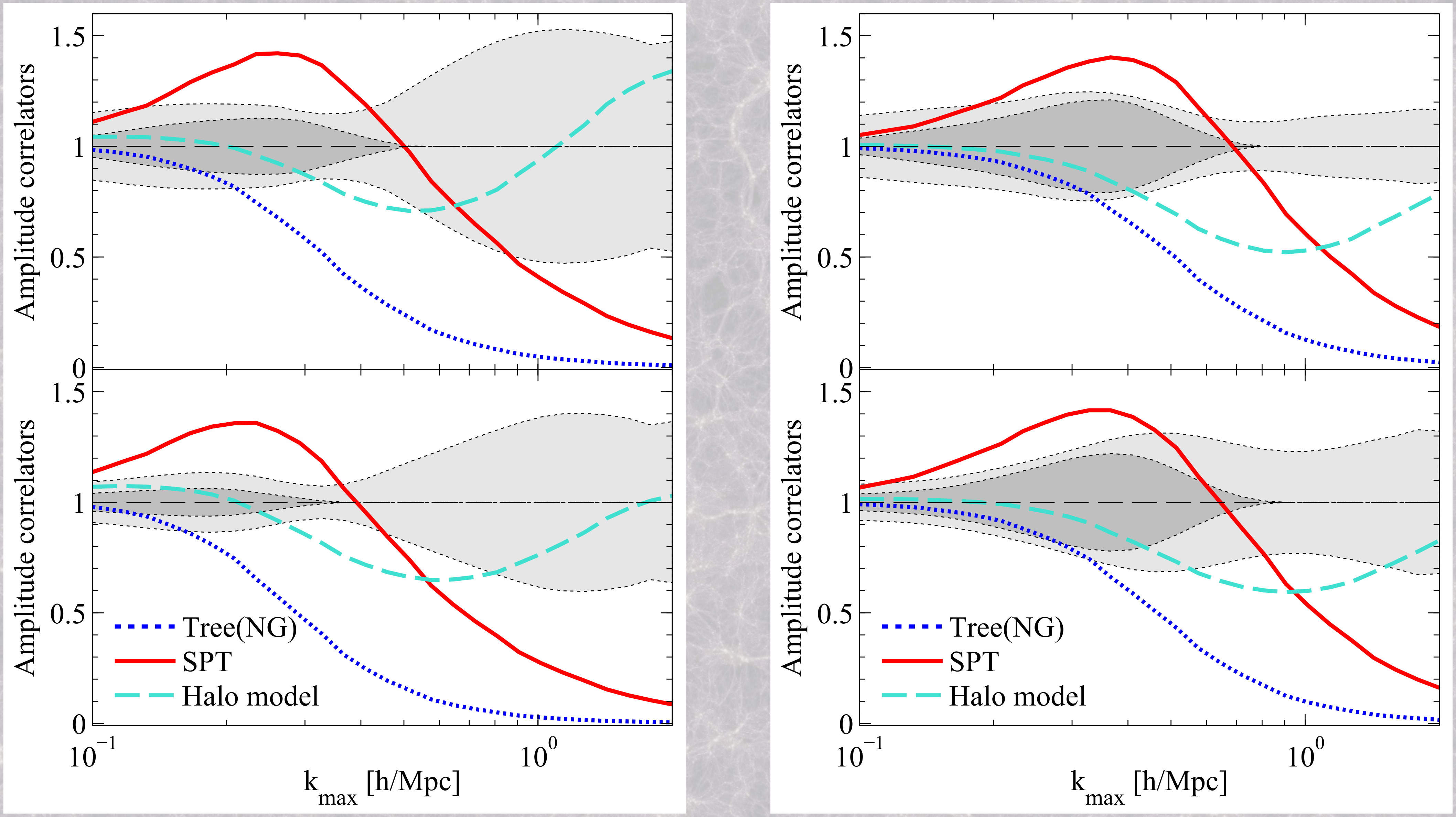
✦Simple extension to 3-shape model
✦Agreement at 20-25% level

Comparison to SPT and halo model

$z=1$ | $z=2$

Local $f_{NL}=10$

Equilateral $f_{NL}=10$



Extension - Galaxy Bispectrum

(arXiv:1707.XXXXX, with D. Karagiannis, M. Liguori, A. Racannelli, L. Verde)

Modelling of galaxy bispectrum

- Bias expansion (up to second order):

$$\delta_h^{E,(G)}(\mathbf{x},\tau) = b_1^E(\tau)\delta(\mathbf{x},\tau) + \epsilon^E(\mathbf{x},\tau) + \frac{b_2^E(\tau)}{2}\delta^2(\mathbf{x},\tau) + b_{s^2}^E(\tau)s^2(\mathbf{x},\tau) + \epsilon_\delta^E(\mathbf{x},\tau)\delta(\mathbf{x},\tau)$$

- Galaxy bispectrum expansion

$$\begin{aligned} B_{ggg}^{(G)}(k_1,k_2,k_3,z) &= b_1^3 B_G(k_1,k_2,k_3,z) + B_\epsilon + 2b_1 B_{\epsilon\epsilon_\delta} \\ &+2b_1^2\left(\frac{b_2}{2} + b_{s^2}S_2(\mathbf{k}_1,\mathbf{k}_2)\right)P_m^L(k_1,z)P_m^L(k_2,z) + 2\text{ perm} \\ &+b_1^2\int_{\mathbf{q}}\left(\frac{b_2}{2} + b_{s^2}S_2(\mathbf{q},\mathbf{k}_3-\mathbf{q})\right)T_\delta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{k}_3-\mathbf{q}) + 2\text{ perm}\end{aligned}$$

- Theoretical uncertainties for DM bispectrum (Baldauf et al 2016)
  use fitting formula for one-loop MPTbreeze method

Forecasts for radio surveys

Fisher matrix f_{NL} constraints for ASKAP/EMU and SKA

	ASKAP/ EMU	SKA	Planck
Local	0.092	0.083	5.0
Equilateral	4.53	6.57	43
Orthogonal	1.14	1.6	21

Preliminary

Future directions

- Direct application – the bispectrum of weak gravitational lensing.
- Full study of the shapes of primordial non-Gaussianity for local, equilateral and orthogonal.
- Better calibration of the model by running more accurate and higher-resolution numerical simulations.
- Testing of parameter dependencies and creating HALOFIT-like methodology for the bispectrum.
- Three-shape model for galaxy bispectrum

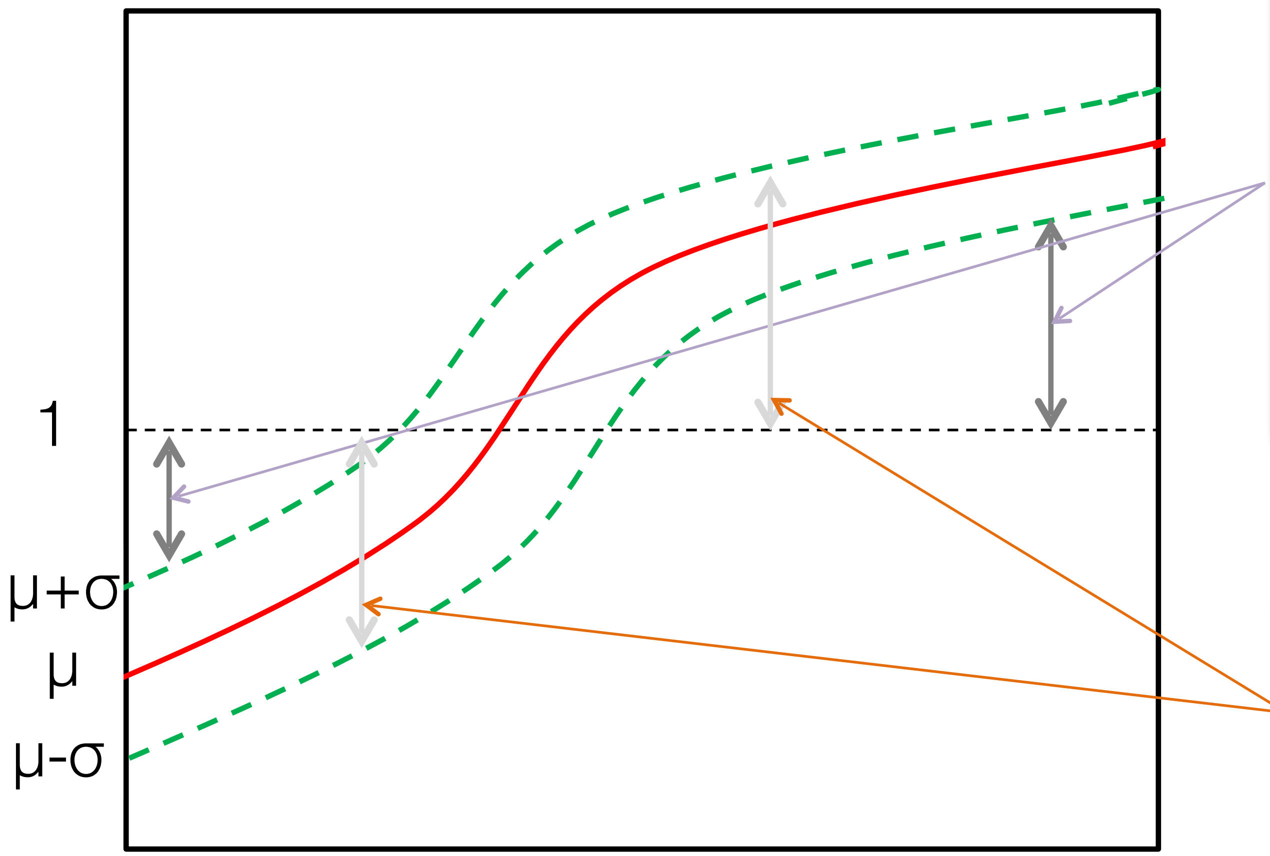
Conclusions

- Study on the different theoretical models to see on which scales each of them is accurate.
- PT: EFT extends the furthest on intermediate scales; however it has free parameters that require fitting to numerical data.
- Halos: identification of deficiencies in the halo model bispectrum at early times, due to inaccuracies in the two-halo term; the assumption that all matter is in the form of virialised halos is increasingly invalid at high redshift.
- Shape analysis of the perturbative models and also of the components of the halo model.
- Building of a **simple three-shape model**, partly inspired by the 1-, 2- and 3-halo terms of the halo model bispectrum, that gives a remarkable fit to the *N*-body bispectrum, at both early and late times into nonlinear scales.
- First step towards building a phenomenological model describing primordial non-Gaussianity.

Thank you for your attention!

Cám ơn vì sự quan tâm của bạn!

Grey areas



Dark grey area: area between 1 and closed dashed green curve

$$\sigma_{\text{dark}} = \begin{cases} 0 & \text{if } 1 \in [\mu - \sigma, \mu + \sigma] \\ \mu - \sigma - 1 & \text{if } \mu - \sigma > 1 \\ 1 - \mu - \sigma & \text{if } \mu + \sigma < 1 \end{cases}$$

Light grey area: add error bar from realisations to difference between mean and 1

$$\sigma_{\text{light}} = |\mu - 1| + \sigma$$

Total correlator

$$\mathcal{T}\left(B_i, B_j\right) \equiv 1 - \sqrt{\frac{\langle B_j - B_i, B_j - B_i \rangle}{\langle B_j, B_j \rangle}} = 1 - \sqrt{1 - 2\mathcal{S}\left(B_i, B_j\right)\mathcal{A}\left(B_i, B_j\right) + \mathcal{A}^2\left(B_i, B_j\right)}$$

- Total correlator quantifies differences in any triangle configuration unlike shape or amplitude (it decreases whenever $B_i \neq B_j$)
- Represents a better measure of the goodness of fit
- Can be related to the traditional χ^2
- Used for fitting the 3-shape model with the simulations