Conversion of Bound Muons: Lepton Flavour Violation from Doubly Charged Scalars

Tanja Geib

with Alexander Merle: *Phys. Rev. D93 (2016) 055039* → technical details

with Stephen King, Alexander Merle, Jose Miguel No, Luca Panizzi: *Phys. Rev. D93 (2016) 073007* → complementarity

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NuFact 2016, August 25, 2016

Introduction: Why Loop Level Neutrino Masses??

Why are radiative models so interesting?

- naturally small neutrino mass
- possibility to introduce new particles (e. g. DM candidate, further Higgs doublets or doubly charged scalars)
- neutrinos massive $\rightarrow$ LFV/LNV couplings exist $\rightarrow$ rich phenomenology:
  - $\mu \rightarrow e\,\gamma$ (LFV)
  - $\mu \rightarrow 3\,e$ (LFV)
  - neutrinoless double beta decay (LNV)

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Testability: complementary between neutrino and LHC physics

- low energy precision physics: indirect detection of e. g. charged scalars due to contribution to LFV/LNV processes
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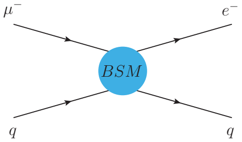
μ–e Conversion

within **muonic atom**:
bound μ^- captured by nucleus
and re-emitted as e^-



→ **coherent** μ–e conversion: same initial and final state of the nucleus

past: SINDRUM II for ^{48}Ti (1993), ^{208}Pb (1995), ^{197}Au (2006)

future: DeeMee for ^{28}Si, COMET and Mu2e (taking data $\sim$ 2018) for ^{27}Al, PRISM/PRIME for ^{48}Ti

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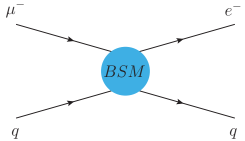

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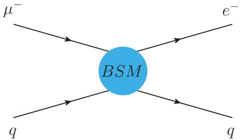

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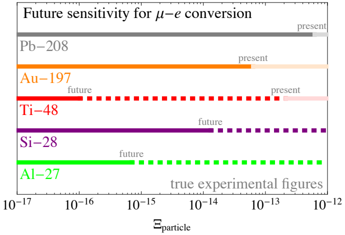

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Effective theory of a doubly charged scalar singlet

based on King, Merle, Panizzi JHEP 1411 (2014) 124

Minimal extension of SM:

- only **one** extra particle: S^{++}
  $\rightarrow$ lightest of possible new particles (UV completion e.g. Cocktail model)
  $\rightarrow$ reduction of input parameters
- tree-level coupling to SM (to charged right-handed leptons)
  $\rightarrow$ **LNV and LFV!**
- effective Dim-7 operator (necessary to generate neutrino mass)

$$\mathcal{L} = \mathcal{L}_{\rm SM} - V(H,S)$$

$$+ (D_\mu S)^\dagger (D^\mu S) + f_{ab}\, \overline{(\ell_{Ra})^c} \ell_{Rb}\, S^{++} + \text{h.c.} - \frac{g^2 v^4 \xi}{4\Lambda^3} S^{++} W^-_\mu W^{-\mu} + \text{h.c.}$$

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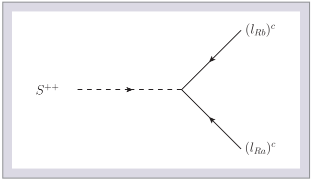

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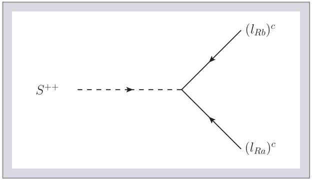
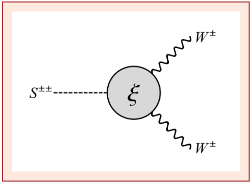
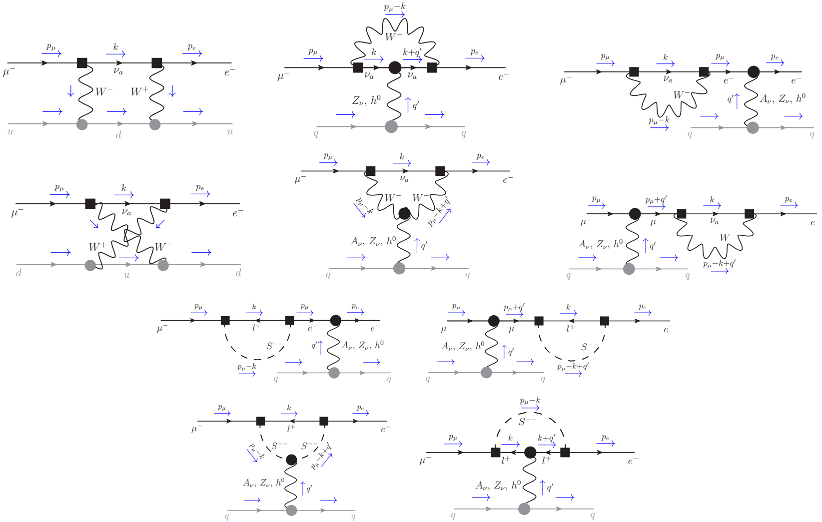




μ–e Conversion: Universally Valid for Models Involving Doubly Charged Singlet Scalars based on TG, Merle Phys.Rev. D93 (2016) 055039

μ–e conversion realised at **one-loop** level



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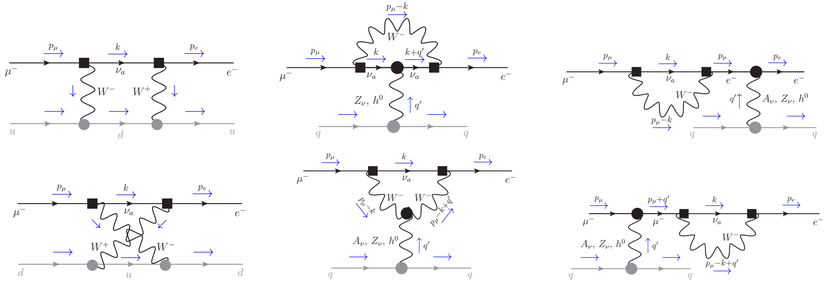
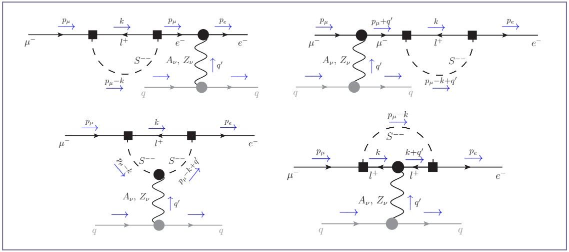




relevant diagrams

Energy Scales of the Process

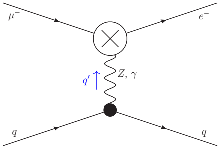
What we know about the process:

- muon **bound** in **1s state** with binding energy
  $\epsilon_B \simeq \frac{m_\mu}{m_e} \cdot 13.6\ \mathrm{eV} \cdot Z \ll m_\mu \xrightarrow{Z\leq 100}$ **non-relativistic**
- consider **coherent** process $\rightarrow$ initial and final nucleus in **ground state**
  $+$ in good approximation: both nuclei at rest

$$\Rightarrow E_e = \underbrace{m_\mu - \epsilon_B}_{\substack{E_\mu \\ \sim \mathcal{O}(100\ \mathrm{MeV})}} + \underbrace{E_i - E_f}_{\sim \mathcal{O}(\mathrm{MeV})} \sim \mathcal{O}(100\ \mathrm{MeV})$$

$\Rightarrow e^-$ is **relativistic** particle under influence of Coulomb potential:
$E_e \simeq E_\mu \simeq m_\mu$ and $m_e \simeq 0$

- for 4-momentum transfer $q' = p_e - p_\mu$
  In this set-up $\Rightarrow$ $\boxed{q'^2 \simeq -m_\mu^2}$



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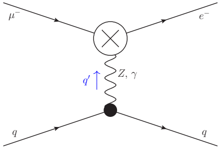

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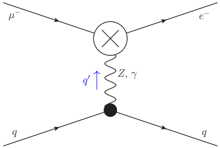

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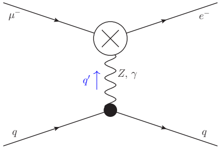
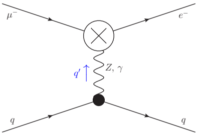


Different Contributions to μ-e Conversion

- estimate nuclear radius: $R = \overbrace{r_0}^{\sim\mathcal{O}(10^{-15}\ \mathrm{m})} A^{1/3} \sim \mathcal{O}(10^{-15}\ \mathrm{m})$
- reduced Bohr radius: $\underbrace{a_0}_{\mathcal{O}(10^{-10}\ \mathrm{m})} \frac{m_e}{m_\mu} \sim \mathcal{O}(10^{-13}\ \mathrm{m})$
- estimate interaction range: $r_\gamma \to \infty$ and $r_Z \leq 10^{-18}$ m
  $\Rightarrow$ for Z-exchange: μ^- has to be **within nucleus**! Probability?!



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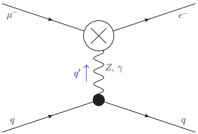

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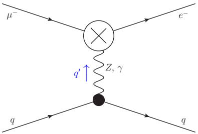

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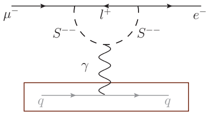

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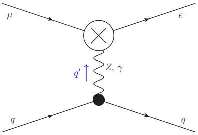

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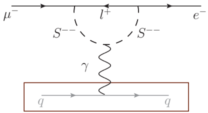

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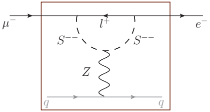

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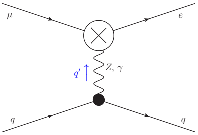

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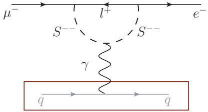

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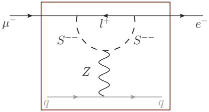

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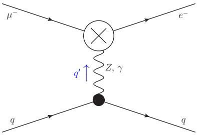

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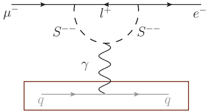

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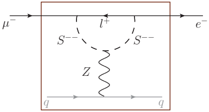

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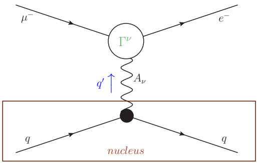

$$\mathcal{M} \propto \int \mathrm{d}^3 r\, \overline{\psi^e_{jlm}}(p_e, r)\, \Gamma^\nu\, \psi^\mu_{j_\mu l_\mu m_\mu}(p_\mu, r)\, \underbrace{\langle N | \overline{q}\, \gamma_\nu\, q | N \rangle}_{Ze\rho^{(P)}(r)\,\delta_{\nu 0}}$$

$\rightarrow$ **wave functions** for μ^- and e^- obtained by solving modified Dirac equation (+ Coulomb potential)

$\rightarrow$ Most **general** (Lorentz-) invariant **expression** for Γ^ν:

$$\Gamma^\nu = \left(\gamma^\nu - \frac{\not{q}'\, q'^\nu}{q'^2}\right) F_1(q'^2) + \frac{i\,\sigma^{\nu\rho}\, q'_\rho}{m_\mu} F_2(q'^2) + \left(\gamma^\nu - \frac{\not{q}'\, q'^\nu}{q'^2}\right)\gamma_5\, G_1(q'^2) + \frac{i\,\sigma^{\nu\rho}\, q'_\rho}{m_\mu}\gamma_5\, G_2(q'^2)$$

with $q' = p_e - p_\mu$.

In non-relativistic limit:

$\Rightarrow$ ψ_{jlm} and $Ze\rho^{(P)}(r)$ factorise from Γ^0 on matrix element level

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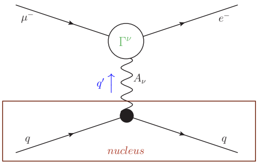

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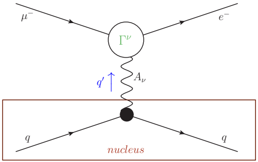

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→ Most **general** (Lorentz-) invariant **expression** for Γ^ν:

$$\Gamma^\nu = \left(\gamma^\nu - \frac{\not{q}' q'^\nu}{q'^2}\right) F_1(q'^2) + \frac{i\,\sigma^{\nu\rho}\,q'_\rho}{m_\mu} F_2(q'^2) + \left(\gamma^\nu - \frac{\not{q}' q'^\nu}{q'^2}\right) \gamma_5 \, G_1(q'^2) + \frac{i\,\sigma^{\nu\rho}\,q'_\rho}{m_\mu} \gamma_5 \, G_2(q'^2)$$

with $q' = p_e - p_\mu$.

In non-relativistic limit:
⇒ ψ_{jlm} and $Ze\rho^{(P)}(r)$ factorise from Γ^0 on matrix element level

Photonic Contribution

Write **branching ratio** as product of nuclear and particle physics parts

$$\mathrm{BR}(\mu^- N \to e^- N) = \frac{8\alpha^5 m_\mu Z_{\mathrm{eff}}^4 Z F_p^2}{\Gamma_{\mathrm{capt}}} \, \Xi^2$$

see Kuno, Okada Rev. Mod. Phys. 73 (2001) 151-202

→ **factorisation** works perfectly for **photonic** (long-range) contributions
→ Ξ has to be modified for **non-photonic** contributions to be a function of the nuclear characteristics (A,Z)

Particle physics information absorbed into

$$\Xi^2 = \left| -F_1(-m_\mu^2) + F_2(-m_\mu^2) \right|^2 + \left| G_1(-m_\mu^2) + G_2(-m_\mu^2) \right|^2$$

⇒ determine **form factors** from amputated diagrams with off-shell photon with help of Mathematica package *Package*-X (Patel, Comput. Phys. Commun. 197 (2015) 276)

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In good approximation (up to a **few per cent**), we use

$$F_1(q'^2) = G_1(q'^2) = -f_{ea}^* f_{a\mu} \left[\frac{2m_a^2 + m_\mu^2 \log\left(\frac{m_a}{M_S}\right)}{12\pi^2 M_S^2} + \frac{\sqrt{m_\mu^2 + 4m_a^2}(m_\mu^2 - 2m_a^2)}{12\pi^2 m_\mu M_S^2} \text{Arctanh}\left(\frac{m_\mu}{\sqrt{m_\mu^2 + 4m_a^2}}\right) \right]$$

$$F_2(q'^2) = -G_2(q'^2) = f_{ea}^* f_{a\mu} \frac{m_\mu^2}{24\pi^2 M_S^2}$$

with $q'^2 = -m_\mu^2$ for the particle physics factor:

$$\Xi^2_{\text{photonic}} = \frac{1}{288\,\pi^4\, m_\mu^2\, M_S^4} \left| \sum_{a=e,\,\mu,\,\tau} f_{ea}^* f_{a\mu} \left(4m_a^2\, m_\mu - m_\mu^3 + 2\left(-2m_a^2 + m_\mu^2\right)\sqrt{4m_a^2 + m_\mu^2}\; \text{Arctanh}\left[\frac{m_\mu}{\sqrt{4m_a^2 + m_\mu^2}}\right] + m_\mu^3 \ln\left[\frac{m_a^2}{M_S^2}\right] \right) \right|^2$$

→ while F_2 is independent of m_a, $|F_1|$ decreases with increasing m_a

→ hierarchy: $|F_2| < |F_1|$ **but** for $M_S \sim 10$ GeV of order 10 %

→ compare to $\mu \to e\gamma$: $F_1(q'^2 = 0) = G_1(q'^2 = 0) = 0$ and $F_2(q'^2 = 0) = -G_2(q'^2 = 0) = F_2(q'^2 = -m_\mu^2) \Rightarrow \mu^- - e^-$ conversion enhanced by F_1 contribution

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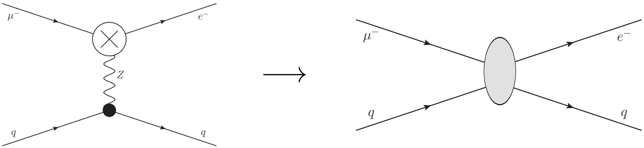
Non-Photonic Contribution

Short-range $\leftrightarrow$ takes place inside the nucleus:
EFT treatment $\Rightarrow$ **Integrating out** the Z-boson:



$\rightarrow$ four-point vertices
$\rightarrow$ consider operators up to **dimension six**
$\rightarrow$ for the coherent μ–e conversion, the only vertex realised in this model is described by

$$\mathcal{L}_{\text{short-range}} = -\frac{G_F}{\sqrt{2}} \underbrace{\frac{2(1 + k_q \sin^2\theta_W)\cos\theta_W}{g} A_R(q'^2)}_{g_{RV(q)}} \overline{e_R}\,\gamma_\nu\,\mu_R\,\overline{q}\,\gamma^\nu\,q$$

in terms of the chiral form factor $A_R(q'^2)$

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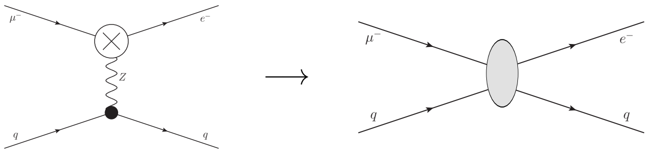

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→ **no perfect factorisation** anymore: Ξ modified to be function of **nuclear characteristics**

→ instead of lines we do have bands with finite widths for Ξ

⇒ determine form factors from amputated diagrams with off-shell Z-Boson

Combining photonic and non-photonic contributions:

$$\Xi_{\text{particle}} \to \Xi_{\text{combined}}(Z, N) = \Xi_{\text{photonic}} + \Xi_{\text{non-photonic}}(Z, N)$$

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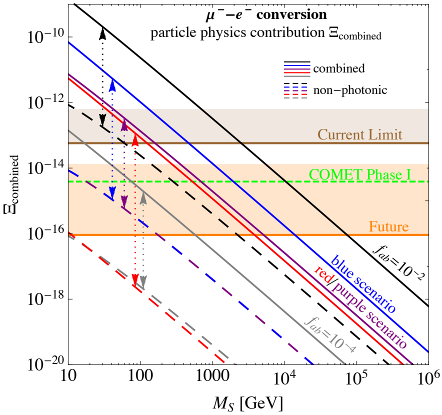
see TG, Merle Phys.Rev. D93 (2016) 055039



Benchmark Points:

f_{ab} such that LFV/LNV bounds fulfilled + suitable neutrino mass matrix reproduced

- 'red': $f_{ee} \simeq 0$ and $f_{e\tau} \simeq 0$
- 'purple': $f_{ee} \simeq 0$ and $f_{e\mu} \simeq \frac{f^*_{\mu\tau}}{f^*_{\mu\mu}} f_{e\tau}$
- 'blue': $f_{e\mu} \simeq \frac{f^*_{\mu\tau}}{f^*_{\mu\mu}} f_{e\tau}$

$\Downarrow$

choose **representative 'average' set** for each scenario to display M_S dependence

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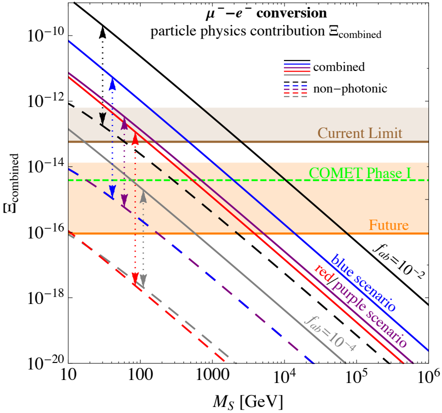


→ widths of the bands so small that appear as lines

→ non-photonic (DASHED) contributions **negligibly small**

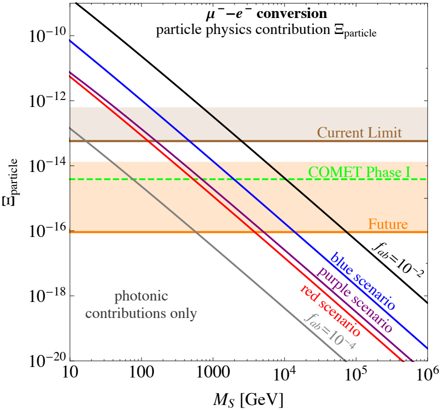
⇓

→ approximate process by its purely photonic (SOLID) contribution

→ **factorisation**: dependence on isotope only in width of limit

Results: Photonic Contribution vs $\mu \to e\gamma$

see TG, Merle Phys.Rev. D93 (2016) 055039 and King, Merle, Panizzi JHEP 1411 (2014) 124



For $\mu^- \to e^- \gamma$:
strongest bound for red, weakest for blue points

$$A \propto \left| f^*_{ee} f_{e\mu} + f^*_{e\mu} f_{\mu\mu} + f^*_{e\tau} f_{\tau\mu} \right| \cdot C$$

$\to$ some amount of cancellation

For $\mu^- - e^-$ **conversion**:
!! other way around !!

$$A \propto \left| C_e\, f^*_{ee} f_{e\mu} + C_\mu\, f^*_{e\mu} f_{\mu\mu} + C_\tau\, f^*_{e\tau} f_{\tau\mu} \right|$$

$\to$ flavour-dependent coefficients: prevent similar cancellations

$\to$ **shape of amplitude leads to drastical change** (not mainly off-shell contributions)

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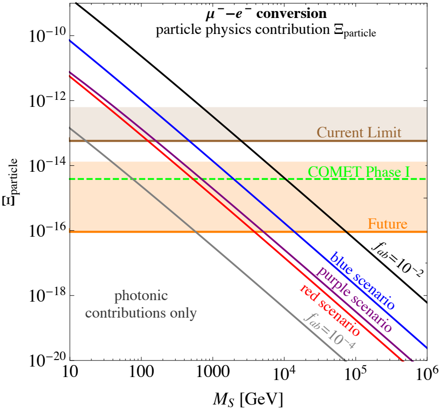

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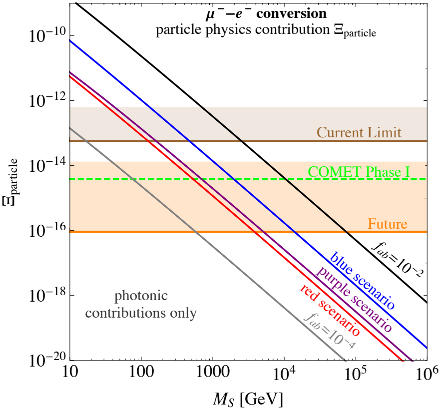

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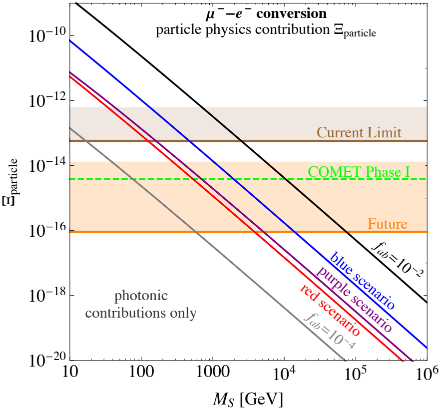

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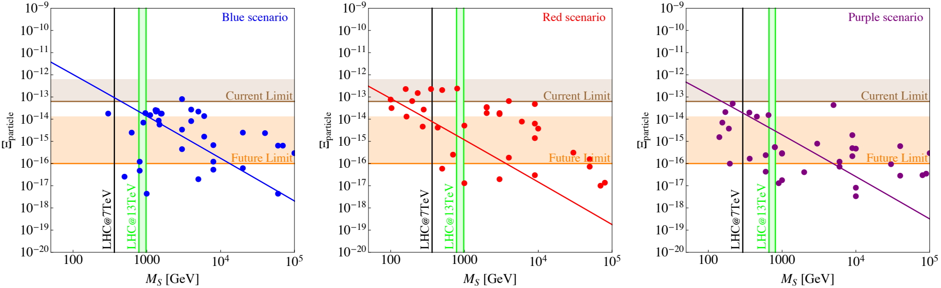
Results: Complementarity

see TG, King, Merle, No, Panizzi Phys.Rev. D93 (2016) 073007



From **'average scenarios'** (depicted by lines), we can estimate the **lower limits on M_S** resulting from μ-e conversion:

	current limit [GeV]	future sensitivity [GeV]	COMET I (Al-27) [GeV]
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purple curve	$M_S > 42.5 - 152.3$	$M_S > 360.7 - 4885.2$	$M_S > 694.5$
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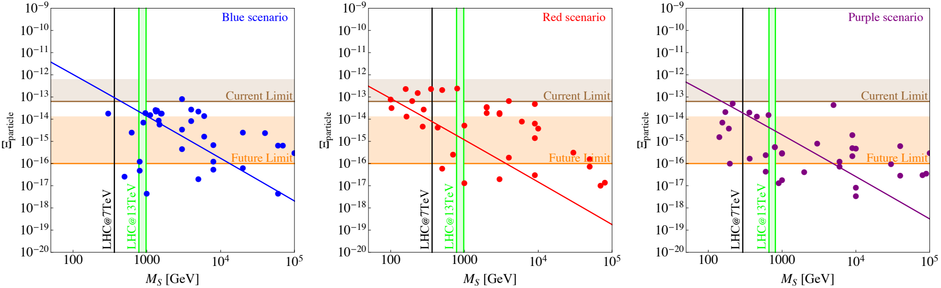

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- **complementarity**: rich phenomenology of loop models $\rightarrow$ high- and low-energy processes $\rightarrow$ μ^-–e^- conversion important part of study

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  $\rightarrow$ analytic expression for Ξ_{particle}

- **COMET**: expecting to take first data in **2018**

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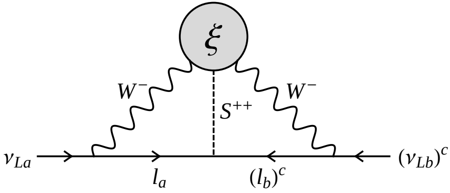
Thank you for your attention!!

Any questions?

Backup Slides

Generating the Neutrino Mass

The mass is generated at **two-loop level** via the diagram



which leads to the **neutrino mass**

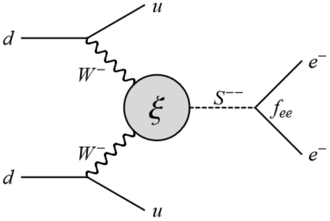
$$\mathcal{M}_{\nu,ab}^{\text{2-loop}} = \frac{2\,\xi\, m_a\, m_b\, M_S^2\, g_{ab}(1+\delta_{ab})}{\Lambda^3}\;\mathcal{I}\left[M_W,\, M_S,\, \mu\right]$$

$\longrightarrow$ Majorana mass term
$\longrightarrow$ further LNV processes

Testing the Model

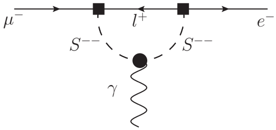
based on King, Merle, Panizzi arXiv:1406.4137

Selection of interesting processes: **low energy physics**



- neutrinoless double beta decay:

$$\frac{\xi\, f_{ee}}{M_S^2\, \Lambda^3} < \frac{4.0 \cdot 10^{-3}}{\mathrm{TeV}^5}$$



- $\mu^- \to e^- \gamma$:

$$\left| f_{ee}^*\, f_{e\mu} + f_{e\mu}^*\, f_{\mu\mu} + f_{e\tau}^*\, f_{\mu\tau} \right| < 3.2 \cdot 10^{-4}\, M_S^2 [\mathrm{TeV}]$$

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benchmark points:

f_{ab} such that bounds fulfilled + suitable light neutrino mass matrix reproduced

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$\downarrow$

complementary check with **high energy experiments**:
compute cross sections for e.g.

- $S^{\pm\pm} \to W^{\pm\pm}$
- $S^{\pm\pm} \to l_a^{\pm\pm}\, l_b^{\pm\pm}$
- ...

$\to$ some of the benchmark points already excluded by LHC data (7 TeV run)

Testing the Model

based on King, Merle, Panizzi arXiv:1406.4137

benchmark points:

f_{ab} such that bounds fulfilled + suitable light neutrino mass matrix reproduced

- 'red': $f_{ee} \simeq 0$ and $f_{e\tau} \simeq 0$
- 'purple': $f_{ee} \simeq 0$ and $f_{e\mu} \simeq \frac{f^*_{\mu\tau}}{f^*_{\mu\mu}} f_{e\tau}$
- 'blue': $f_{e\mu} \simeq \frac{f^*_{\mu\tau}}{f^*_{\mu\mu}} f_{e\tau}$

$$\downarrow$$

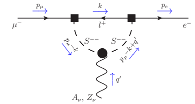
complementary check with **high energy experiments**:
compute cross sections for e.g.

- $S^{\pm\pm} \to W^{\pm\pm}$
- $S^{\pm\pm} \to l_a^{\pm\pm}\, l_b^{\pm\pm}$
- ...

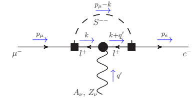
$\to$ some of the benchmark points already excluded by LHC data (7 TeV run)

Photonic Contribution: Cross Check via UV Divergences

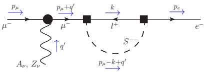
In form of $\boxed{i\mathcal{M} = e\, f_{ea}^*\, f_{a\mu}\, A_\nu(q')\, \overline{u}_e(p_e)\, \mathcal{I}^\nu\, u_\mu(p_\mu)}$:



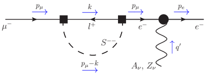
$$-4Q_S \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L \not k (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L (\not k + \not q' + m_a)\, \gamma^\nu\, (\not k + m_a) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2 \varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L\, \not k\, (\not p_e + m_\mu)\, \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not p_e (\not p_e + m_\mu) \gamma^\nu$$

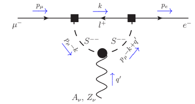


$$4Q_{e^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\gamma^\nu\, \not p_\mu\, P_L\, \not k}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2 \varepsilon} \frac{Q_{e^-}}{m_\mu^2} \gamma^\nu \not p_\mu P_L \not p_\mu$$

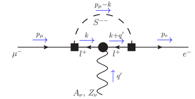
$$\boxed{\Rightarrow \Sigma \mathcal{I}^\nu = \frac{i}{(4\pi)^2 \varepsilon} [(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-}) P_L \gamma^\nu] = 0} \quad \checkmark$$

Photonic Contribution: Cross Check via UV Divergences

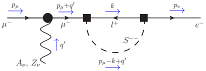
In form of $\boxed{i\mathcal{M} = e\, f_{ea}^{*}\, f_{a\mu}\, A_\nu(q')\, \overline{u}_e(p_e)\, \mathcal{I}^\nu\, u_\mu(p_\mu)}$:



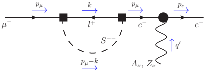
$$-4Q_S \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L \not k (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L (\not k + \not q' + m_a)\, \gamma^\nu\, (\not k + m_a) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2 \varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L \not k (\not p_e + m_\mu)\, \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not p_e (\not p_e + m_\mu) \gamma^\nu$$

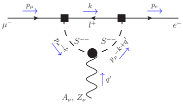


$$4Q_{e^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\gamma^\nu\, \not p_\mu\, P_L\, \not k}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2 \varepsilon} \frac{Q_{e^-}}{m_\mu^2} \gamma^\nu \not p_\mu P_L \not p_\mu$$

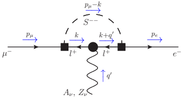
$$\boxed{\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2 \varepsilon} [(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-}) P_L \gamma^\nu] = 0} \quad \surd$$

Photonic Contribution: Cross Check via UV Divergences

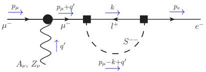
In form of $\boxed{i\mathcal{M} = e\, f_{ea}^{*}\, f_{a\mu}\, A_\nu(q')\, \overline{u}_e(p_e)\, \mathcal{I}^\nu\, u_\mu(p_\mu)}$:



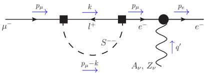
$$-4Q_S \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L \not{k} (2p_\mu - 2k + q')^\nu}{[k^2-m_a^2][(p_\mu-k+q')^2-M_S^2][(p_\mu-k)^2-M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L(\not{k}+\not{q}'+m_a)\,\gamma^\nu\,(\not{k}+m_a)P_R}{[k^2-m_a^2][(p_\mu-k)^2-M_S^2][(k+q')^2-m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2\varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L\, \not{k}\, (\not{p}_e+m_\mu)\,\gamma^\nu}{[p_e^2-m_\mu^2][(p_e-k)^2-M_S^2][k^2-m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not{p}_e(\not{p}_e + m_\mu)\gamma^\nu$$

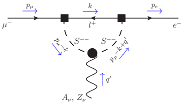


$$4Q_{e^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\gamma^\nu\, \not{p}_\mu\, P_L\, \not{k}}{[p_\mu^2][(p_\mu-k)^2-M_S^2][k^2-m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2\varepsilon} \frac{Q_{e^-}}{m_\mu^2} \gamma^\nu \not{p}_\mu P_L \not{p}_\mu$$

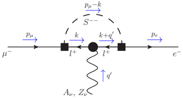
$$\boxed{\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2\varepsilon}[(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-})P_L\gamma^\nu] = 0} \quad \surd$$

Photonic Contribution: Cross Check via UV Divergences

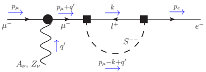
In form of $i\mathcal{M} = e\, f_{ea}^{*}\, f_{a\mu}\, A_\nu(q')\, \overline{u}_e(p_e)\, \mathcal{I}^\nu\, u_\mu(p_\mu)$:



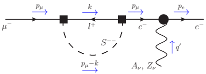
$$-4Q_S \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L \not{k}(2p_\mu - 2k + q')^\nu}{[k^2-m_a^2][(p_\mu-k+q')^2-M_S^2][(p_\mu-k)^2-M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L(\not{k}+\not{q}'+m_a)\,\gamma^\nu\,(\not{k}+m_a)P_R}{[k^2-m_a^2][(p_\mu-k)^2-M_S^2][(k+q')^2-m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2\varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L\,\not{k}\,(\not{p}_e+m_\mu)\,\gamma^\nu}{[p_e^2-m_\mu^2][(p_e-k)^2-M_S^2][k^2-m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not{p}_e(\not{p}_e + m_\mu)\gamma^\nu$$



$$4Q_{e^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\gamma^\nu\, \not{p}_\mu\, P_L\, \not{k}}{[p_\mu^2][(p_\mu-k)^2-M_S^2][k^2-m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2\varepsilon} \frac{Q_{e^-}}{m_\mu^2} \gamma^\nu \not{p}_\mu P_L \not{p}_\mu$$

$$\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2\varepsilon}[(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-})P_L\gamma^\nu] = 0$$ √

Photonic Contribution: Cross Check via UV Divergences

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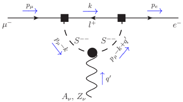
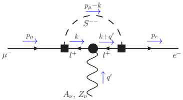
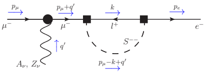


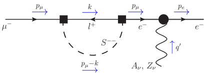
$$-4Q_S \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L \not{k} (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L(\not{k} + \not{q}' + m_a)\,\gamma^\nu\,(\not{k} + m_a) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2\varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L\, \not{k}\, (\not{p}_e + m_\mu)\, \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not{p}_e (\not{p}_e + m_\mu) \gamma^\nu$$

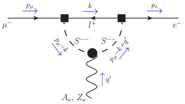


$$4Q_{e^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\gamma^\nu\, \not{p}_\mu\, P_L\, \not{k}}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2\varepsilon} \frac{Q_{e^-}}{m_\mu^2} \gamma^\nu \not{p}_\mu P_L \not{p}_\mu$$

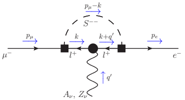
$$\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2\varepsilon}[(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-}) P_L \gamma^\nu] = 0 \quad \checkmark$$

Photonic Contribution: Cross Check via UV Divergences

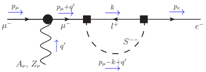
In form of $i\mathcal{M} = e\, f^*_{ea}\, f_{a\mu}\, A_\nu(q')\, \overline{u}_e(p_e)\, \mathcal{I}^\nu\, u_\mu(p_\mu)$:



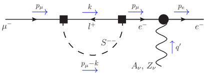
$$-4Q_S \int \frac{d^dk}{(2\pi)^d} \frac{P_L \not{k} (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{d^dk}{(2\pi)^d} \frac{P_L (\not{k} + \not{q}' + m_a)\, \gamma^\nu\, (\not{k} + m_a) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2\varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{d^dk}{(2\pi)^d} \frac{P_L\, \not{k}\, (\not{p}_e + m_\mu)\, \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not{p}_e (\not{p}_e + m_\mu) \gamma^\nu$$

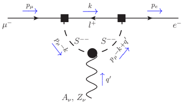


$$4Q_{e^-} \int \frac{d^dk}{(2\pi)^d} \frac{\gamma^\nu\, \not{p}_\mu\, P_L\, \not{k}}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2\varepsilon} \frac{Q_{e^-}}{m_\mu^2} \gamma^\nu \not{p}_\mu P_L \not{p}_\mu$$

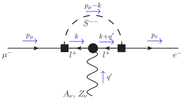
$$\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2\varepsilon}[(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-}) P_L \gamma^\nu] = 0$$ ✓

Photonic Contribution: Cross Check via UV Divergences

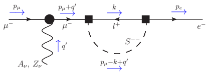
In form of $i\mathcal{M} = e\, f^*_{ea}\, f_{a\mu}\, A_\nu(q')\, \overline{u}_e(p_e)\, \mathcal{I}^\nu\, u_\mu(p_\mu)$:



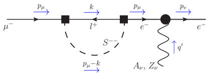
$$-4Q_S \int \frac{d^dk}{(2\pi)^d} \frac{P_L \not{k}(2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{d^dk}{(2\pi)^d} \frac{P_L(\not{k} + \not{q}' + m_a)\, \gamma^\nu\, (\not{k} + m_a)P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2\varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{d^dk}{(2\pi)^d} \frac{P_L\, \not{k}\, (\not{p}_e + m_\mu)\, \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not{p}_e(\not{p}_e + m_\mu)\gamma^\nu$$



$$4Q_{e^-} \int \frac{d^dk}{(2\pi)^d} \frac{\gamma^\nu\, \not{p}_\mu\, P_L\, \not{k}}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2\varepsilon} \frac{Q_{e^-}}{m_\mu^2} \gamma^\nu \not{p}_\mu P_L \not{p}_\mu$$

$$\Rightarrow \Sigma\, \mathcal{I}^\nu = \frac{i}{(4\pi)^2\varepsilon}[(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-})P_L\, \gamma^\nu] = 0 \quad \checkmark$$

Photonic Contribution: Results I

Determine **form factors** with help of Mathematica package *Package*-X (Patel, arXiv:1503.01469):

$$\mathbf{F}_1(-m_\mu^2) = \mathbf{G}_1(-m_\mu^2) =$$

$$= -\frac{1}{128\,\pi^2 m_\mu^4} \sum_{a=e,\,\mu,\,\tau} f_{ea}^* f_{a\mu} \Big[2\, m_\mu^2 \Big(-5m_a^2 + 6m_\mu^2 + 5M_S^2 \Big) - 2\, S_a\, m_\mu^2 \Big(m_a^2 + 3m_\mu^2 - M_S^2 \Big)$$
$$\ln\left[\frac{2m_a^2}{2m_a^2 + m_\mu^2(1+S_a)}\right] + 4\, S_S\, m_\mu^2 \Big(m_a^2 + m_\mu^2 - M_S^2 \Big) \ln\left[\frac{2M_S^2}{2M_S^2 + m_\mu^2(1+S_S)}\right] + \Big(3m_a^2 \Big(2m_a^2 - m_\mu^2$$
$$-4M_S^2 \Big) + 5m_\mu^4 - 7m_\mu^2 M_S^2 + 6M_S^4 \Big) \ln\left[\frac{m_a^2}{M_S^2}\right] + 2\, T_a \Big(-6m_a^2 + m_\mu^2 + 6M_S^2 \Big) \ln\left[\frac{2m_a M_S}{m_a^2 - m_\mu^2 + M_S^2 - T_a}\right]$$
$$+2\, m_\mu^2 \Big[\Big(m_a^4 + 8m_a^2 m_\mu^2 + M_S^4 - 2M_S^2 \Big(m_a^2 + 2m_\mu^2 \Big) \Big) C_0 \Big[0,\, -m_\mu^2,\, m_\mu^2;\, m_a,\, M_S,\, m_a \Big]$$
$$+2 \Big(m_a^4 - 2M_S^2 \Big(m_a^2 - 2m_\mu^2 \Big) + M_S^4 \Big) C_0 \Big[0,\, -m_\mu^2,\, m_\mu^2;\, M_S,\, m_a,\, M_S \Big] \Big] \Big]$$

$$\xrightarrow{M_S \gg m_a} -f_{ea}^* f_{a\mu} \left[\frac{2m_a^2 + m_\mu^2 \log\left(\frac{m_a}{M_S}\right)}{12\pi^2 M_S^2} + \frac{\sqrt{m_\mu^2 + 4m_a^2}(m_\mu^2 - 2m_a^2)}{12\pi^2 m_\mu M_S^2} \text{ Arctanh}\left(\frac{m_\mu}{\sqrt{m_\mu^2 + 4m_a^2}}\right) \right] + \mathcal{O}(M_S^{-4})$$

Note: $\mathcal{O}(M_S^{-4})$ gives corrections of up to a **few per cent**

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$$-4M_S^2\Big)+5m_\mu^4-7m_\mu^2\,M_S^2+6M_S^4\Big)\ln\left[\frac{m_a^2}{M_S^2}\right] + 2\,T_a\Big(-6m_a^2+m_\mu^2+6M_S^2\Big)\ln\left[\frac{2m_a\,M_S}{m_a^2-m_\mu^2+M_S^2-T_a}\right]$$
$$+2\,m_\mu^2\Big[\Big(m_a^4+8m_a^2\,m_\mu^2+M_S^4-2M_S^2\Big(m_a^2+2m_\mu^2\Big)\Big)\,C_0\Big[0,\,-m_\mu^2,\,m_\mu^2;\,m_a,\,M_S,\,m_a\Big]$$
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$$\left.\left. +2\left(-3m_a^4 + 2m_a^2 \Big(3M_S^2 + 2m_\mu^2 \Big) - 3M_S^4 \right) C_0\left[0,\,-m_\mu^2,\,m_\mu^2;\,M_S,\,m_a,\,M_S\right] \right]\right]$$

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$$\ln\left[\frac{2m_a\,M_S}{m_a^2 - m_\mu^2 + M_S^2 - T_a}\right] + 2\,m_\mu^2 \Bigg[\bigg(-3m_a^4 - 3M_S^4 + 2M_S^2 \Big(3m_a^2 + 2m_\mu^2 \Big) \bigg) C_0\Big[0,\, -m_\mu^2,\, m_\mu^2;\, m_a,\, M_S,\, m_a\Big]$$
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'Average Scenario' Couplings

	red	purple	blue
f_{ee}	10^{-16}	10^{-15}	10^{-1}
$f_{e\mu}$	10^{-2}	10^{-3}	10^{-4}
$f_{e\tau}$	10^{-19}	10^{-2}	10^{-2}
$f_{\mu\mu}$	10^{-4}	10^{-3}	10^{-3}
$f_{\mu\tau}$	10^{-5}	10^{-4}	10^{-4}
$f_{ee}\, f_{e\mu}$	10^{-18}	10^{-18}	$\mathbf{10^{-5}}$
$f_{e\mu}\, f_{\mu\mu}$	$\mathbf{10^{-6}}$	$\mathbf{10^{-6}}$	10^{-7}
$f_{e\tau}\, f_{\mu\tau}$	10^{-24}	$\mathbf{10^{-6}}$	10^{-6}

Table: First part: 'average scenario' couplings for the benchmark points as extracted from Tab. 7 in *King, Merle, Panizzi: arXiv:1406.4137*. Second part: combination of couplings that enter the μ–e conversion amplitude. The bold values indicate the dominant photonic contribution.

Non-Photonic Bands

- The amplitude that enters the non-photonic Ξ takes the form

$$\mathcal{A} \propto \left| f^*_{ee}\, f_{e\mu}\, D(m_e) + f^*_{e\mu}\, f_{\mu\mu}\, D(m_\mu) + f^*_{e\tau}\, f_{\tau\mu}\, D(m_\tau) \right| .$$

- The function $D(m_a)$ strongly varies with m_a.
  → **dominant term** stems from the tau propagating within the loop, i.e. $\boldsymbol{D(m_\tau)}$
  → exeeds the muon and electron contribution by three to four orders of magnitude
- blue/purple scenario: neither $f^*_{ee} f_{e\mu}$ nor $f^*_{e\mu} f_{\mu\mu}$ bypasses this difference
  + **identic** $\boldsymbol{f^*_{e\tau} f_{\tau\mu}}$ in both scenarios
  → indistinguishable curves
- red/grey scenario:
  dominant contributions: $f^*_{e\mu}\, f_{\mu\mu}\, D(m_\mu) \sim f^*_{e\tau}\, f_{\tau\mu}\, D(m_\tau)$
  → same order of magnitude, i.e. **comparable values** of non-photonic contribution