# *NME in the deep inelastic* $l^{\pm}/\nu(\bar{\nu}) - A$ *scattering*

**Huma Haider**

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### Deep Inelastic scattering(DIS)

General process for the deep inelastic scattering is

$$l(k) + N(p) \longrightarrow l(k') + X(p'), \quad l = e^{\pm}, \mu^{\pm}, \nu_l, \bar{\nu}_l, \ N = n, p$$

### Kinematics(Nucleon in the rest frame)

$$Q^2 = -q^2 = -(k-k')^2 = 4EE' \sin^2\frac{\theta}{2}$$
$$M^2 = p^2$$
$$\nu = p.q = M(E - E')$$
$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2p.q} = \frac{Q^2}{2MEy}$$
$$y = \frac{p.q}{p.k} = 1 - \frac{E'}{E}$$
$$W^2 = M^2 + 2p.q - Q^2$$

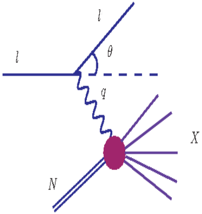



*Figure:* Deep Inelastic Scattering

### *MINERνA*

MINERνA is using neutrino/antineutrino beam to study Nuclear medium Effect(NME) using several nuclear targets in the energy region of $1 - 20$GeV. Recently they have presented the results for cross section in the DIS region.

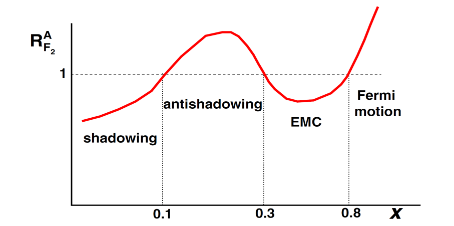
### *JLab*

JLab has used high intensity electron beam in the energy region of 6 GeV and 12 GeV and performed scattering cross section measurement.

MINOS, MicroBooNE, NOvA, DUNE...

It is important to understand nucleon dynamics and reduce the cross section uncertainty(∼20-25%) which is contributing to the systematic errors.

## NME is broadly divided into four parts



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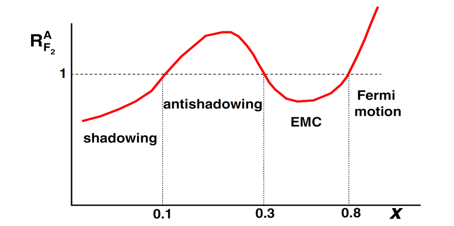
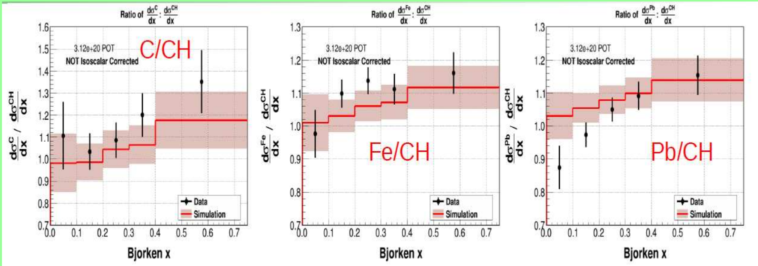



## MINER$\nu$A: PRD93 071101(2016)



## *Phenomenological Efforts*

| Phenomenological group | data types used |
|---|---|
| EKS98 | $l$+$A$ DIS, p+$A$ DY |
| HKM | $l$+$A$ DIS |
| HKN04 | $l$+$A$ DIS, p+$A$ DY |
| nDS | $l$+$A$ DIS, $p$+$A$ DY |
| EKPS | $l$+$A$ DIS, p+$A$ DY |
| HKN07 | $l$+$A$ DIS, p+$A$ DY |
| EPS08 | $l$+$A$ DIS, p+$A$ DY, $h^{\pm}, \pi^0, \pi^{\pm}$ in d+Au |
| EPS09 | $l$+$A$ DIS, p+$A$ DY, $\pi^0$ in d+Au |
| nCTEQ | $l$+$A$ DIS, p+$A$ DY |
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| DSSZ | $l$+$A$ and $\nu$+$A$ DIS, p+$A$ DY, $\pi^0, \pi^{\pm}$ in d+Au |

Paukkunen and Salgado:JHEP2010: "find no apparent disagreement with the nuclear effects in neutrino DIS and those in charged lepton DIS."

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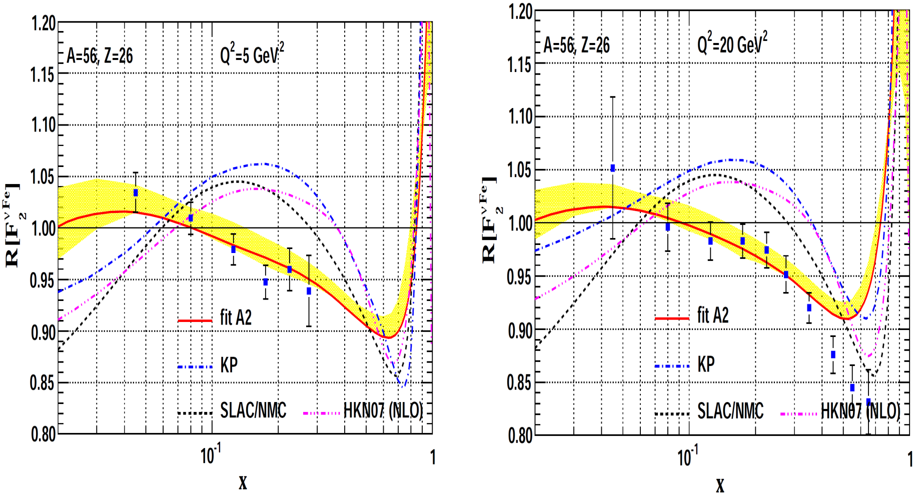



J G Morfin J. of Physics: Conf. Ser. 408 (2013) 012054;
Kovarik et al. Phys.Rev.Lett. 106 (2011) 122301

### *Theoretical Efforts*

Very few efforts have been made:

### *Kulagin and Petti*

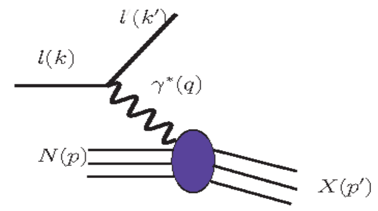
Nucl.Phys A 765(2006)126 and Phys. Rev. D 76(2007)094023

### *Our group at Aligarh*

Nucl.Phys A 857(2011)29
Phys. Rev. C 84(2011)054610
Phys. Rev. C 85(2012)055201
Phys. Rev. C 87(2013)035502
Nucl.Phys A 940(2015)138
Nucl.Phys A 943(2015)58
Nucl.Phys A 955(2016)58
arXiv:1606.04645

## $l^{\pm} - N$ *scattering*

$$l^{\pm}(k) + N(p) \rightarrow l^{\pm}(k') + X(p'),$$




$l^{\pm}$-N DCX:

$$\frac{d^2\sigma^N}{d\Omega' dE'} = \frac{\alpha^2}{q^4}\frac{|\mathbf{k}'|}{|\mathbf{k}|} L^{\alpha\beta}\, W^N_{\alpha\beta}$$

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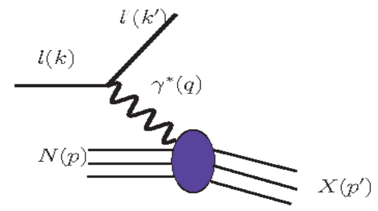


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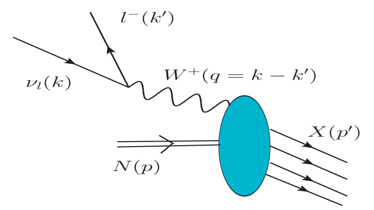
$$\nu_l(\bar\nu_l)(k) + N(p) \to l^\pm(k') + X(p'),$$




$\nu$-N DCX:

$$\frac{d^2\sigma^N}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2 - m_W^2}\right)^2 L^{\alpha\beta} W^N_{\alpha\beta}$$

## $l^{\pm} - N$ *scattering*

**Leptonic Tensor**

$$L^{\alpha\beta} = 2(k^{\alpha}k'^{\beta} + k^{\beta}k'^{\alpha} - k.k' g^{\alpha\beta})$$

**Hadronic tensor**

$$\begin{aligned} W_{\alpha\beta}^{N} &= \left(\frac{q_{\alpha}q_{\beta}}{q^2} - g_{\alpha\beta}\right) W_{1N} + \frac{1}{M^2} \\ &\times \left(p_{\alpha} - \frac{p.q}{q^2} q_{\alpha}\right) \left(p_{\beta} - \frac{p.q}{q^2} q_{\beta}\right) W_{2N} \end{aligned}$$

$$MW_{1N}(\nu, Q^2) = F_1^N(x, Q^2)$$

$$\nu W_{2N}(\nu, Q^2) = F_2^N(x, Q^2)$$

$$\begin{aligned} F_2^{ep}(x) &= x\left[\frac{4}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(d(x) + \bar{d}(x))\right. \\ &\left. + \frac{1}{9}(s(x) + \bar{s}(x)) + \frac{4}{9}(c(x) + \bar{c}(x))\right] \end{aligned}$$

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$$L^{\alpha\beta} = k^\alpha k'^\beta + k^\beta k'^\alpha - k.k' g^{\alpha\beta} \pm i\epsilon^{\alpha\beta\rho\sigma} k_\rho k'_\sigma$$

**Hadronic tensor**

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$$\begin{aligned} MW_{1N}(\nu, Q^2) &= F_1^N(x, Q^2) \\ \nu W_{2N}(\nu, Q^2) &= F_2^N(x, Q^2) \\ \nu W_{3N}(\nu, Q^2) &= F_3^N(x, Q^2) \\ F_2^{\nu p} &= 2x[d(x) + s(x) + \bar u(x) + \bar c(x)], \\ xF_3^{\bar\nu p} &= 2x[u(x) + c(x) - \bar d(x) - \bar s(x)] \end{aligned}$$

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### *QCD evolution*

The QCD evolution of DIS structure functions are taken from the works of Vermaseren and van Neerven et al.NPB724(2005)3 van Neerven and Vogt NPB568(2000)263.

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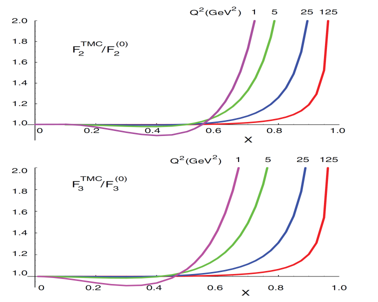
*The TMC correction has been taken from the works of Schienbein et al. JPG 35 (2008) 053101.*



## $l^{\pm} - N$ *scattering*

The differential cross section:

$$\frac{d^2\sigma^l}{dxdy} = \frac{8M_N E_l \pi\alpha^2}{Q^4}\left\{xy^2 F_{1N}(x,Q^2) + \left(1 - y - \frac{xyM_N}{2E_l}\right) F_{2N}(x,Q^2)\right\}.$$

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## *$\nu(\bar{\nu}) - N$ scattering*

The differential cross section:

$$\begin{aligned}\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx\ dy} &= \frac{G_F^2 M E_\nu}{\pi(1+Q^2/M_W^2)^2}\left(\left[y^2 x + \frac{m_l^2 y}{2E_\nu M}\right] F_{1N}(x,Q^2)\right.\\ &+ \left[(1 - \frac{m_l^2}{4E_\nu^2}) - (1 + \frac{Mx}{2E_\nu})y\right] F_{2N}(x,Q^2)\\ &\left.\pm \left[xy(1 - \frac{y}{2}) - \frac{m_l^2 y}{4E_\nu M}\right] F_{3N}(x,Q^2)\right)\end{aligned}$$

# *If we look inside the nucleus*

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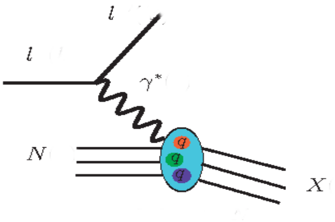

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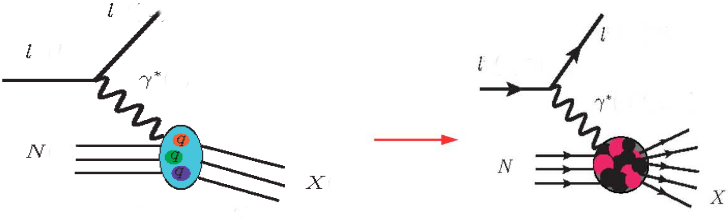

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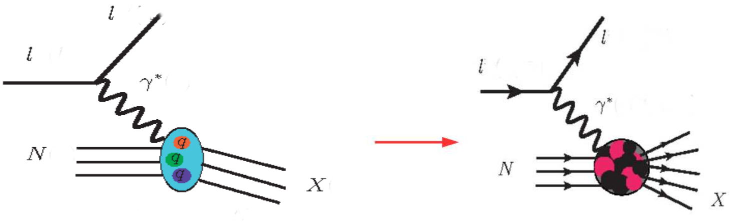


**We have considered the following NME:**

1. Fermi motion
2. Pauli blocking
3. Nucleon correlations
4. Meson cloud contributions
5. Shadowing and antishadowing

*The cross section for an element of volume $dV$ in the nucleus is related to the probability per unit time ($\Gamma$) of the lepton interacting with the nucleons:*

$$d\sigma = \Gamma dt dS = \Gamma \frac{dt}{dl} dS dl = \Gamma \frac{1}{v} dV = \Gamma \frac{E_l}{\mid \mathbf{k} \mid} dV = \Gamma \frac{E_l}{\mid \mathbf{k} \mid} d^3 r,$$

d$l$ is the length of the interaction, $v(= \frac{dl}{dt})$ is the velocity of the incoming lepton and we have used $\mathbf{k} = \mathbf{v} E_l$.

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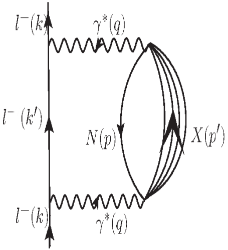
$\Gamma$ is also related to imaginary part of lepton self energy:

$$-\frac{\Gamma}{2} = \frac{m_l}{E_l(\mathbf{k})} Im\Sigma$$

$$d\sigma = \frac{-2m}{E_l(\mathbf{k})} Im\Sigma(k) \frac{E_l(\mathbf{k})}{|\mathbf{k}|} d^3r,$$



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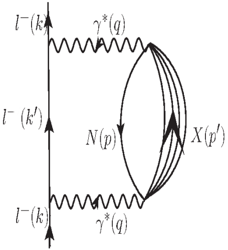


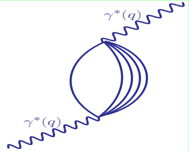

### *Lepton self energy $\Sigma(k)$:*

$$-i\Sigma(k) = \int \frac{d^4q}{(2\pi)^4} \bar{u}_l(\mathbf{k})\, ie\gamma^{\mu}\, i\frac{\not{k}' + m}{k'^2 - m^2 + i\epsilon} ie\gamma^{\nu} u_l(\mathbf{k}) \frac{-ig_{\mu\rho}}{q^2} (-i)\, \Pi^{\rho\sigma}(q) \frac{-ig_{\sigma\nu}}{q^2}$$

### *Imaginary part of lepton self energy:*

$$Im\Sigma(k) = e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_l} \theta(q^0)\, Im(\Pi^{\alpha\beta}) \frac{1}{q^4} \frac{1}{2m} L_{\alpha\beta}$$

## photon self-energy $\Pi^{\alpha\beta}(q)$ in the nuclear medium:




$$\Pi^{\alpha\beta}(q) = e^2 \int \frac{d^4p}{(2\pi)^4} G(p) \sum_X \sum_{s_p, s_l} \prod_{i=1}^{N} \int \frac{d^4 p'_i}{(2\pi)^4} \prod_l G_l(p'_l) \prod_j D_j(p'_j)$$

$$< X|J^{\mu}|H >< X|J^{\nu}|H >^* (2\pi)^4 \, \delta^4(q + p - \sum_{i=1}^{N} p'_i)$$

*Relativistic Dirac propagator $G^0(p_0, \mathbf{p})$ for a free nucleon:*

$$G^0(p_0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \left\{ \frac{\sum_r u_r(p)\bar{u}_r(p)}{p^0 - E(\mathbf{p}) + i\epsilon} + \frac{\sum_r v_r(-p)\bar{v}_r(-p)}{p^0 + E(\mathbf{p}) - i\epsilon} \right\}$$

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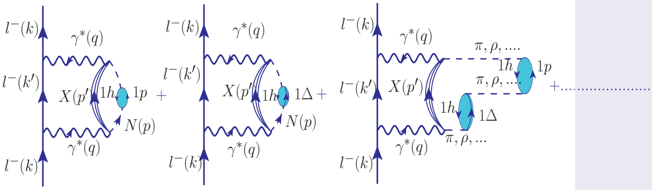

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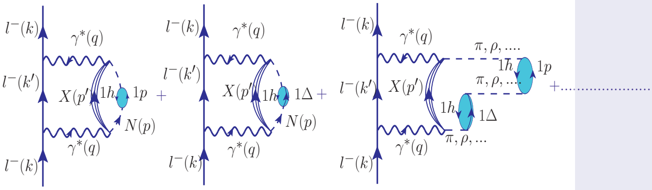


$$G(p_0,\mathbf{p}) = \frac{M}{E(\mathbf{p})}\frac{\sum_r u_r(p)\bar{u}_r(p)}{\left(p^0 - E(\mathbf{p}) + \mathbf{i}\epsilon\right)} + \left(\frac{M}{E(\mathbf{p})}\right)^2 \frac{1}{\left(p^0 - E(\mathbf{p}) + \mathbf{i}\epsilon\right)} \sum \frac{\sum_r u_r(p)\bar{u}_r(p)}{\left(p^0 - E(\mathbf{p}) + \mathbf{i}\epsilon\right)} + ..$$

$$= \frac{M}{E(\mathbf{p})}\frac{\sum_r u_r(p)\bar{u}_r(p)}{\left(p^0 - E(\mathbf{p}) + \mathbf{i}\epsilon \frac{\mathbf{M}}{\mathbf{E(p)}}\sum\right)}$$

## *Relativistic nucleon propagator in the nuclear medium:*

$$G(p^0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \sum_r u_r(\mathbf{p}) \bar{u}_r(\mathbf{p}) \left[ \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, \mathbf{p})}{p^0 - \omega - i\epsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega, \mathbf{p})}{p^0 - \omega + i\epsilon} \right]$$

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for $p^0 \leq \mu$

$$S_h(p^0, \mathbf{p}) = \frac{1}{\pi} \frac{\frac{M}{E(\mathbf{p})} Im\Sigma(p^0, \mathbf{p})}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma(p^0, \mathbf{p}))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma(p^0, \mathbf{p}))^2}$$

for $p^0 > \mu$

$$S_p(p^0, \mathbf{p}) = -\frac{1}{\pi} \frac{\frac{M}{E(\mathbf{p})} Im\Sigma(p^0, \mathbf{p})}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma(p^0, \mathbf{p}))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma(p^0, \mathbf{p}))^2}$$

P.Fernandez de Cordoba and E. Oset, PRC 46, 1697(1992)

Spectral function is normalized to mass number 'A':

$$4\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, \mathbf{p}, \rho(r))\, d\omega = A\,,$$

where $\rho(r)$ is the baryon density for the nucleus.

Kinetic energy $< T >$:

$$< T > = \frac{4}{A}\int d^3r \int \frac{d^3p}{(2\pi)^3}(E(\mathbf{p}) - M)\int_{-\infty}^{\mu} S_h(p^0, \mathbf{p}, \rho(r))\, dp^0\,,$$

$$< E > = \frac{4}{A}\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(p^0, \mathbf{p}, \rho(r))\, p^0 dp^0\,,$$

and the binding energy per nucleon:

$$|E_A| = -\frac{1}{2}(< E - M > + \frac{A-2}{A-1} < T >)$$

Spectral function is normalized to mass number 'A':

$$4\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega, \mathbf{p}, \rho(r))\, d\omega = A\,,$$

where $\rho(r)$ is the baryon density for the nucleus.

Kinetic energy $< T >$:

$$< T > = \frac{4}{A}\int d^3r \int \frac{d^3p}{(2\pi)^3}(E(\mathbf{p}) - M)\int_{-\infty}^{\mu} S_h(p^0, \mathbf{p}, \rho(r))\, dp^0\,,$$

$$< E > = \frac{4}{A}\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(p^0, \mathbf{p}, \rho(r))\, p^0 dp^0\,,$$

and the binding energy per nucleon:

$$|E_A| = -\frac{1}{2}(< E - M > + \frac{A-2}{A-1} < T >)$$

# RECALL

## *Lepton self energy $\Sigma(k)$ in the nuclear medium:*

$$Im\Sigma(k) = e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_l} \theta(q^0)\, Im(\Pi^{\alpha\beta}) \frac{1}{q^4} \frac{1}{2m} L_{\alpha\beta}$$

Scattering cross section: $d\sigma = -\frac{2m_\nu}{|\mathbf{k}|}\, \mathrm{Im}\,\Sigma\, d^3r\,.$

## *Differential scattering cross section for $l^\pm - A$ interaction:*

$$\frac{d^2\sigma}{d\Omega' dE'} = -\frac{\alpha}{(q)^4} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \frac{1}{(2\pi)^2} L_{\alpha\beta} \int d^3r \mathrm{Im}\Pi^{\alpha\beta}(q)\,.$$

$$l^\pm\text{-N DCX: } \frac{d^2\sigma^N}{d\Omega' dE'} = \frac{\alpha^2}{q^4} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L^{\alpha\beta}\, W^N_{\alpha\beta}$$

$$l^\pm\text{-A DCX: } \frac{d^2\sigma^A}{d\Omega' dE'} = \frac{\alpha^2}{q^4} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L^{\alpha\beta}\, W^A_{\alpha\beta}$$

$$W^A_{\alpha\beta} = -\int d^3r \mathrm{Im}\Pi_{\alpha\beta}(q)$$

*Nuclear hadronic tensor:*

It is written as a convolution of nucleonic hadronic tensor with the hole spectral function

$$W^A_{\alpha\beta} = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(\mathbf{p})} S_h(p^0, \mathbf{p}, \rho(r)) W^N_{\alpha\beta}(p, q)$$

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$$W^A_{\mu\nu} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) W^A_1(\nu, Q^2) + \frac{W^A_2(\nu, Q^2)}{M^2_A} \left(p_\mu - \frac{p.q}{q^2} q_\mu\right) \left(p_\nu - \frac{p.q}{q^2} q_\nu\right)$$

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$$W^{N}_{\mu\nu} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) W_{1N}(\nu,Q^2) + \frac{W_{2N}(\nu,Q^2)}{M^2}\left(p_\mu - \frac{p.q}{q^2}q_\mu\right)\left(p_\nu - \frac{p.q}{q^2}q_\nu\right)$$

Taking the xx component

$$W_{xx}^N = \left(\frac{q_x q_x}{q^2} - g_{xx}\right) W_1^N + \frac{1}{M^2}\left(p_x - \frac{p.q}{q^2}\, q_x\right)\left(p_x - \frac{p.q}{q^2}\, q_x\right) W_2^N$$

Taking the xx component

$$W^N_{xx} = \left(\frac{q_x q_x}{q^2} - g_{xx}\right) W^N_1 + \frac{1}{M^2}\left(p_x - \frac{p.q}{q^2}\, q_x\right)\left(p_x - \frac{p.q}{q^2}\, q_x\right) W^N_2$$

Choosing **q** along the z-axis

$$W^N_{xx}(\nu_N, Q^2) = W^N_1(\nu_N, Q^2) + \frac{1}{M^2} p_x^2 W^N_2(\nu_N, Q^2)$$

Similarly taking xx component of nuclear hadronic tensor

$$W_{xx}^{A}(\nu_A, Q^2) = W_1^{A}(\nu_A, Q^2) = \frac{F_1^{A}(x_A)}{AM}$$

Similarly taking xx component of nuclear hadronic tensor

$$W^A_{xx}(\nu_A, Q^2) = W^A_1(\nu_A, Q^2) = \frac{F^A_1(x_A)}{AM}$$

### $F_1(x) = M\ W_1(\nu, Q^2), \quad F_2(x) = \nu\ W_2(\nu, Q^2)$

$$\frac{F^A_1(x_A)}{AM} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \times \left[\frac{F^N_1(x_N)}{M} + \frac{1}{M^2} {p_x}^2 \frac{F^N_2(x_N)}{\nu}\right]$$

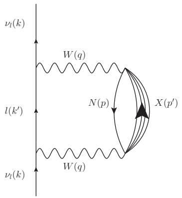
$$F_2(x) = \nu W_2(\nu, Q^2)$$

## $F_2^A(x_A)$ *in nuclear medium*

$$F_2^A(x_A) = 2\sum_{p,n}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h^{p,n}(p^0, \mathbf{p}, \rho_{p,n}(\mathbf{r}))F_2^N(x_N)C$$

$$C = \left[\frac{Q^2}{q_z^2}\left(\frac{p^2 - p_z^2}{2M^2}\right) + \frac{(p.q)^2}{M^2\nu^2}\left(\frac{p_z\ Q^2}{p.qq_z} + 1\right)^2 \frac{q_0 M}{p_0\ q_0 - p_z\ q_z}\right]$$

# *Weak Interaction*




## $\nu$ *self energy* $\Sigma(k)$:

$$\Sigma(k) = (-i)\frac{G_F}{\sqrt{2}}\frac{4}{m_\nu}\int\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2 - m_l^2 + i\epsilon}\left(\frac{m_W}{q^2 - m_W^2}\right)^2 L_{\alpha\beta}\Pi^{\alpha\beta}(q)$$

## $\Pi^{\alpha\beta}(q)$ is the $W$ self-energy in the nuclear medium:

$$-i\Pi^{\alpha\beta}(q) = (-)\int\frac{d^4p}{(2\pi)^4}iG(p)\sum_X\sum_{s_p,s_i}\prod_{i=1}^{n}\int\frac{d^4p'_i}{(2\pi)^4}\prod_l iG_l(p'_l)\prod_j iD_j(p'_j)$$
$$\left(\frac{-G_F m_W^2}{\sqrt{2}}\right)\langle X|J^\alpha|N\rangle\langle X|J^\beta|N\rangle^*(2\pi)^4\delta^4(q+p-\Sigma_{i=1}^n p'_i)$$

## Weak Nuclear Structure Function

$$F_1^A(x_A) = 4AM \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \left[ \frac{F_1^N(x_N)}{M} + \frac{1}{M^2} {p_x}^2 \frac{F_2^N(x_N)}{\nu} \right]$$

$$F_2^A(x_A) = 2 \sum_{p,n} \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h^{p,n}(p^0, \mathbf{p}, \rho_{p,n}(\mathbf{r})) F_2^N(x_N) C$$

$$C = \left[ \frac{Q^2}{q_z^2} \left( \frac{p^2 - p_z^2}{2M^2} \right) + \frac{(p.q)^2}{M^2 \nu^2} \left( \frac{p_z\, Q^2}{p.q q_z} + 1 \right)^2 \frac{q_0\, M}{p_0\; q_0 - p_z\; q_z} \right]$$

$$F_3^A(x_A, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \frac{p^0\gamma - p_z}{(p^0 - p_z\gamma)\gamma} F_3^N(x_N)$$

# $\pi$ and $\rho$ mesons contributions

### "Significant at low and mid $x$"

1. There are virtual mesons associated with each nucleon bound inside the nucleus.
2. These meson clouds get strengthened by the strong attractive nature of nucleon-nucleon interactions.
3. This leads to an increase in the interaction probability of virtual photons with the meson cloud.
4. The effect of meson cloud is more pronounced in heavier nuclear targets and dominate in the intermediate region of x(0.2<x<0.6).

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## For meson cloud contribution

$$2\pi \frac{M}{E(\mathbf{p})} S_h(p_0, \mathbf{p}) W_N^{\alpha\beta}(p,q) \rightarrow 2ImD(p)\theta(p_0) W_\pi^{\alpha\beta}(p,q)$$

Pion propagator in the nuclear medium

$$D(p) = [{p_0}^2 - \mathbf{p}^2 - m_\pi^2 - \Pi_\pi(p_0, \mathbf{p})]^{-1}$$

$$\Pi_\pi = \frac{f^2/m_\pi^2 F^2(p) \mathbf{p}^2 \Pi^*}{1 - f^2/m_\pi^2 V_L' \Pi^*}$$

$\pi NN$ form factor $F(p) = (\Lambda^2 - m_\pi^2)/(\Lambda^2 + \mathbf{p}^2)$

## $F_{1,\pi}^{A}(x_A)$:

$$F_{1,\pi}^{A}(x_\pi) = -6AM\int d^3r\int \frac{d^4p}{(2\pi)^4}\theta(p_0)\ \delta ImD(p)\, 2m_\pi \times \left[\frac{F_{1\pi}(x_\pi)}{m_\pi} + \frac{|\mathbf{p}|^2 - p_z^2}{2(p_0\ q_0 - p_z q_z)}\frac{F_{2\pi}(x_\pi)}{m_\pi}\right]$$

## $F_{2,\pi}^{A}(x_A)$:

$$F_{2,\pi}^{A}(x_\pi) = -6\int d^3r\int \frac{d^4p}{(2\pi)^4}\theta(p_0)\ \delta ImD(p)\, 2m_\pi\ \frac{m_\pi}{p_0 - p_z\ \gamma}C_1 F_{2\pi}(x_\pi)$$

$$C_1 = \frac{Q^2}{q_z^2}\left(\frac{|\mathbf{p}|^2 - p_z^2}{2m_\pi^2}\right) + \frac{(p_0 - p_z\ \gamma)^2}{m_\pi^2}\left(\frac{p_z\ Q^2}{(p_0 - p_z\ \gamma)q_0 q_z} + 1\right)^2$$

## $F_{1,\rho}^{A}(x_A)$:

$$F_{1,\rho}^{A}(x_\rho) = -12AM \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\ \delta ImD_\rho(p)\, 2m_\rho \times \left[\frac{F_{1\rho}(x_\rho)}{m_\rho} + \frac{|\mathbf{p}|^2 - p_z^2}{2(p_0\ q_0 - p_z\ q_z)} \frac{F_{2\rho}(x_\rho)}{m_\rho}\right]$$

## $F_{2,\rho}^{A}(x_A)$:

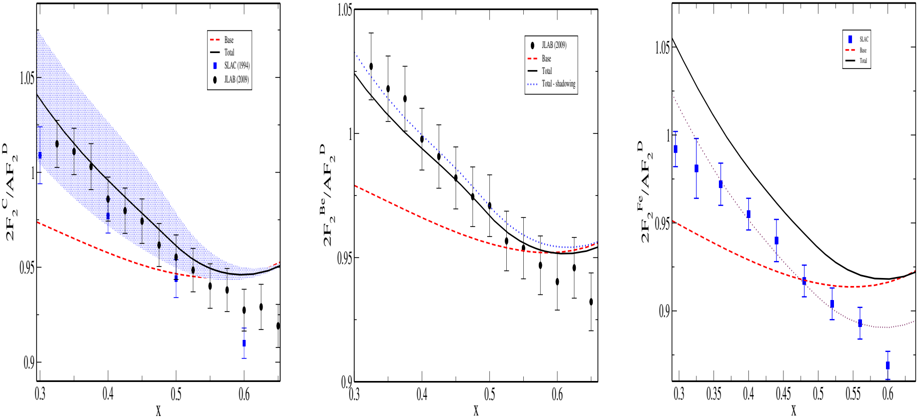
$$F_{2,\rho}^{A}(x_\rho) = -12 \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\ \delta ImD_\rho(p)\, 2m_\rho\ \frac{m_\rho}{p_0 - p_z\ \gamma} C_2 F_{2\rho}(x_\rho)$$

$$C_2 = \frac{Q^2}{q_z^2}\left(\frac{|\mathbf{p}|^2 - p_z^2}{2m_\rho^2}\right) + \frac{(p_0 - p_z\ \gamma)^2}{m_\rho^2}\left(\frac{p_z\ Q^2}{(p_0 - p_z\ \gamma)q_0 q_z} + 1\right)^2$$

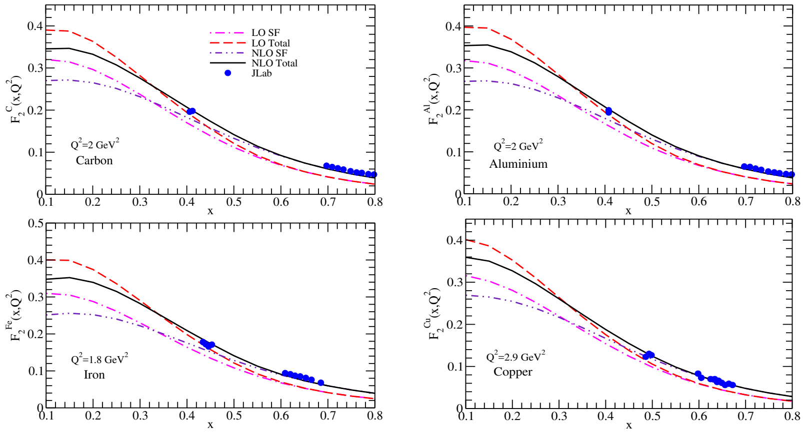
*"Significant at low-$x$ and low-$Q^2$"*

For the shadowing and antishadowing effects, Glauber-Gribov multiple scattering model has been used following the works of Kulagin and Petti. PRD76(2007)094033.

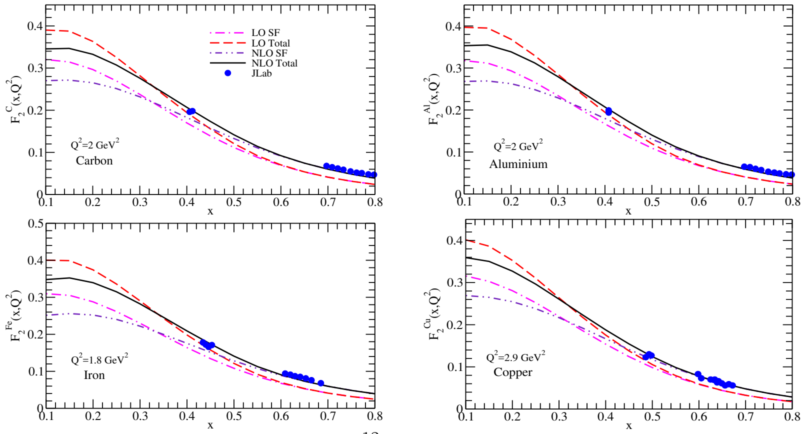
# *Electromagnetic Interactions*



# $F_{2A}^{EM}(x, Q^2)$ $vs$ $x$



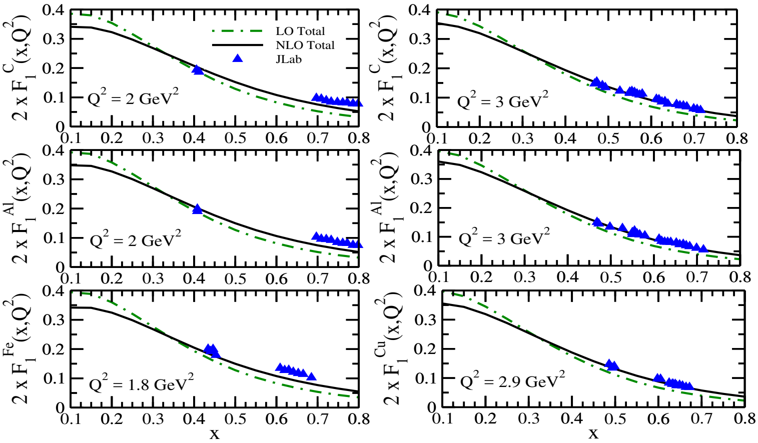
# $F_{2A}^{EM}(x,Q^2)$ *vs* $x$



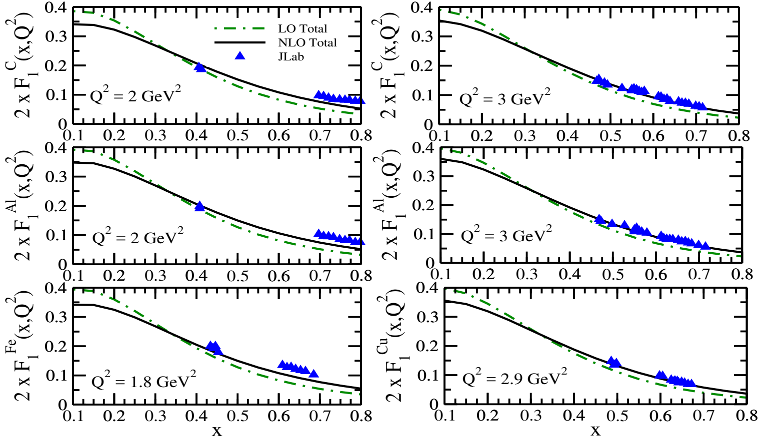

- At LO(SF→Full): $\sim 15\%$ increase at low x in $^{12}C$, increases with x and negligible at high x.
- At NLO: Results at low x get suppressed while at high $x$ results get enhanced compared to LO results.

- NME depends on 'A'

# $2xF_{1A}(x, Q^2)$ vs $x$

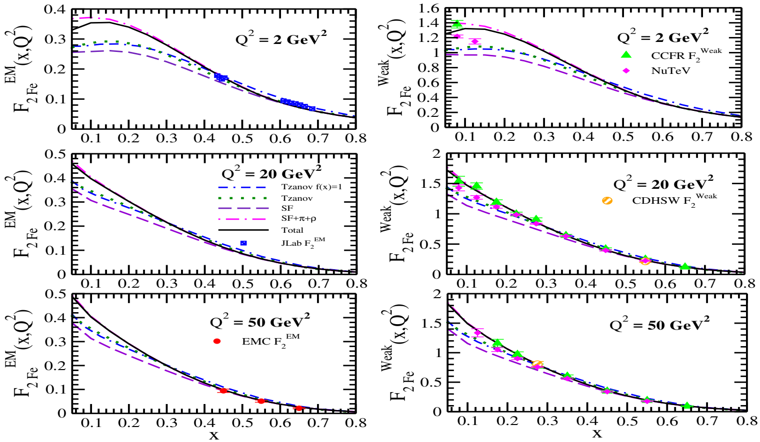


# $2xF_{1A}(x, Q^2)$ $vs$ $x$

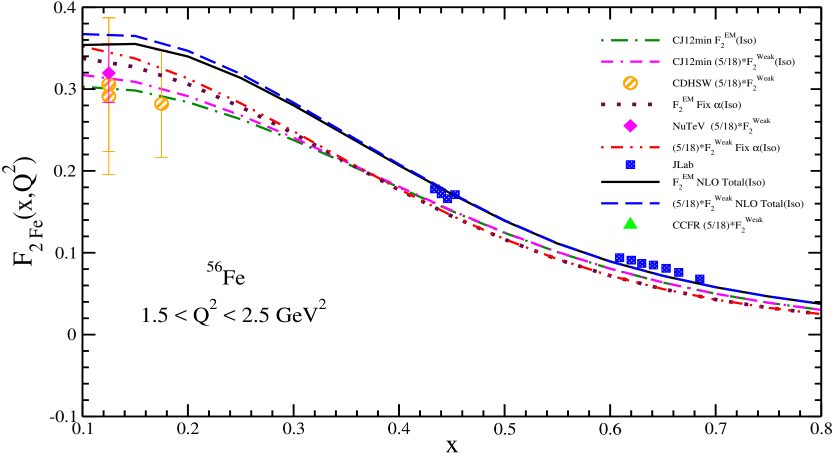


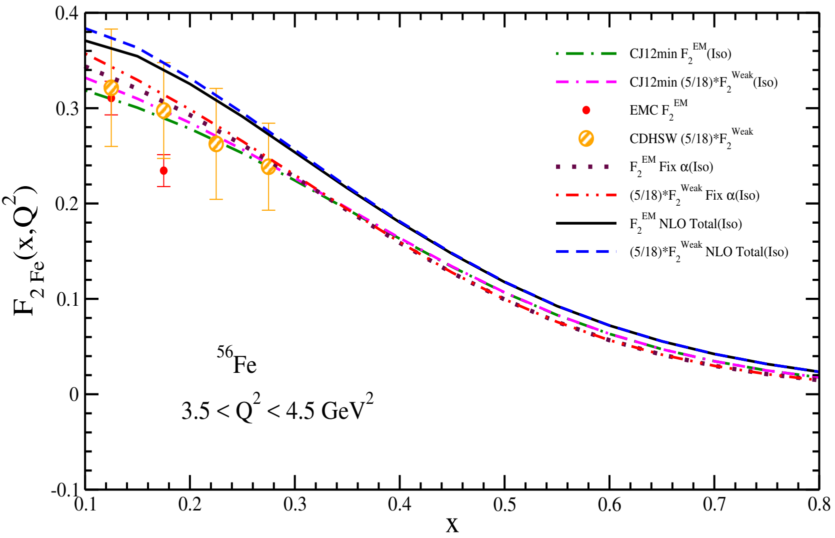

- Qualitatively similar in nature to that found in $F_{2A}^{EM}(x, Q^2)$.
- Quantitatively some variation, specially in low x region.

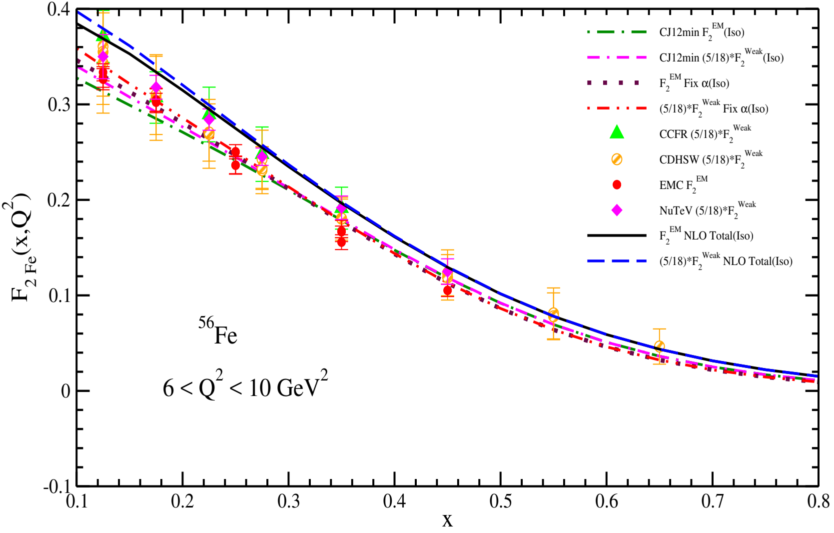
# $F_{2A}^{EM}(x,Q^2)$ vs $x$ and $F_{2A}^{Weak}(x,Q^2)$ vs $x$

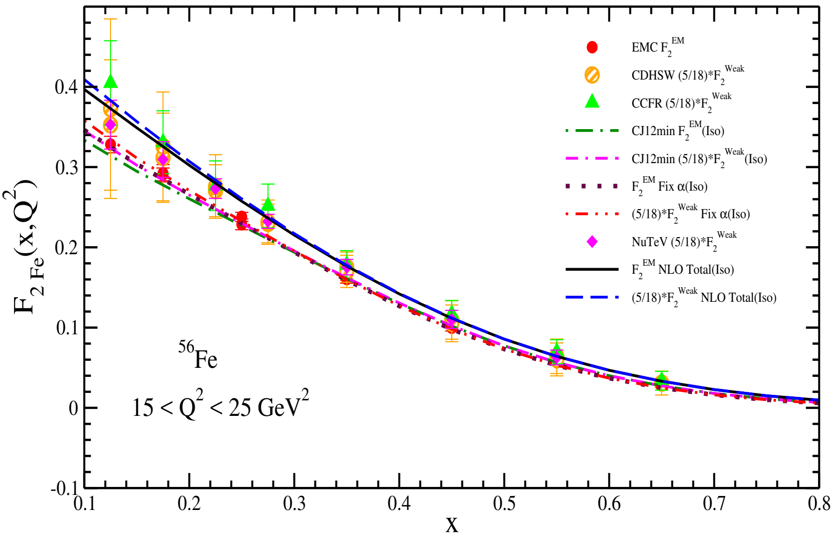


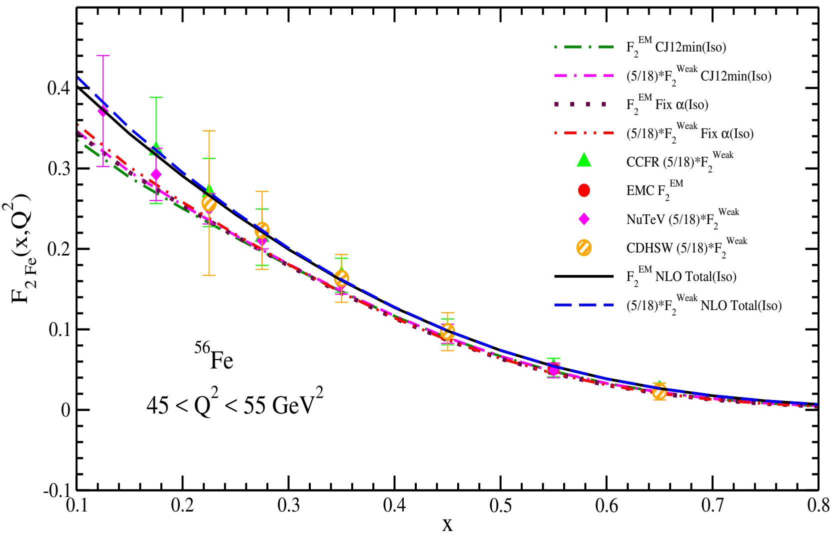

- Free→SF: Reduction of ∼ 8% at $x = 0.1$; ∼ 18% at $x = 0.4$; ∼ 3% at $x = 0.7$.
- SF→ $\pi\&\rho$: Increase in results at low and mid values of $x$ i.e ∼ 30% at $x$=0.1; 15% at $x$=0.4.
- shadowing effects reduces results at low x i.e ∼ 10% at $x$=0.05 and ∼ 5% at $x$=0.1.


CJ12min F2 EM(Iso)
CJ12min (5/18)*F2 Weak(Iso)
CDHSW (5/18)*F2 Weak
F2 EM Fix α(Iso)
NuTeV (5/18)*F2 Weak
(5/18)*F2 Weak Fix α(Iso)
JLab
F2 EM NLO Total(Iso)
(5/18)*F2 Weak NLO Total(Iso)
CCFR (5/18)*F2 Weak
56Fe
1.5 < Q2 < 2.5 GeV2
F2 Fe(x,Q2)
x
0.4
0.3
0.2
0.1
0
-0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

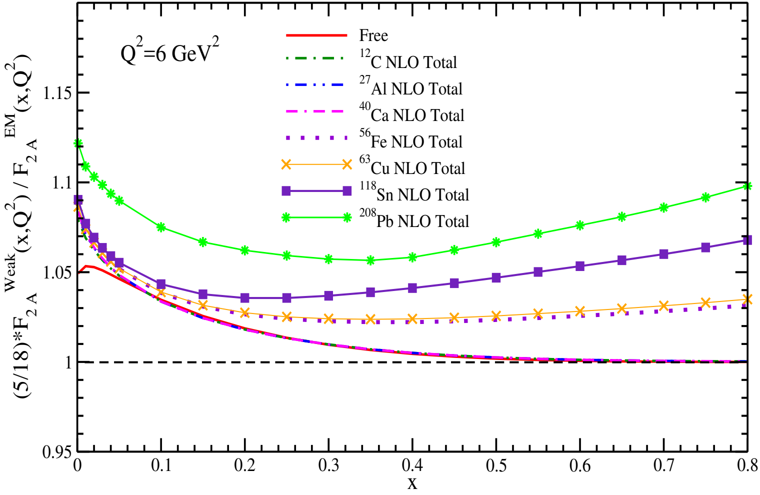

CJ12min F2 EM(Iso)
CJ12min (5/18)*F2 Weak(Iso)
EMC F2 EM
CDHSW (5/18)*F2 Weak
F2 EM Fix α(Iso)
(5/18)*F2 Weak Fix α(Iso)
F2 EM NLO Total(Iso)
(5/18)*F2 Weak NLO Total(Iso)
56Fe
3.5 < Q2 < 4.5 GeV2
F2 Fe(x,Q2)
x
0.4
0.3
0.2
0.1
0
-0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

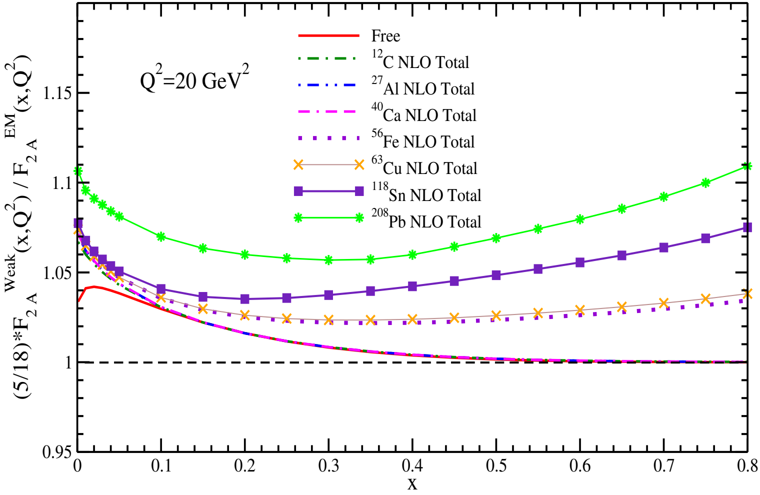

CJ12min F$_2^{EM}$(Iso)
CJ12min (5/18)*F$_2^{Weak}$(Iso)
F$_2^{EM}$ Fix α(Iso)
(5/18)*F$_2^{Weak}$ Fix α(Iso)
CCFR (5/18)*F$_2^{Weak}$
CDHSW (5/18)*F$_2^{Weak}$
EMC F$_2^{EM}$
NuTeV (5/18)*F$_2^{Weak}$
F$_2^{EM}$ NLO Total(Iso)
(5/18)*F$_2^{Weak}$ NLO Total(Iso)
$^{56}$Fe
$6 < Q^2 < 10$ GeV$^2$
F$_{2\,Fe}$(x,Q$^2$)
x
0.4
0.3
0.2
0.1
0
-0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

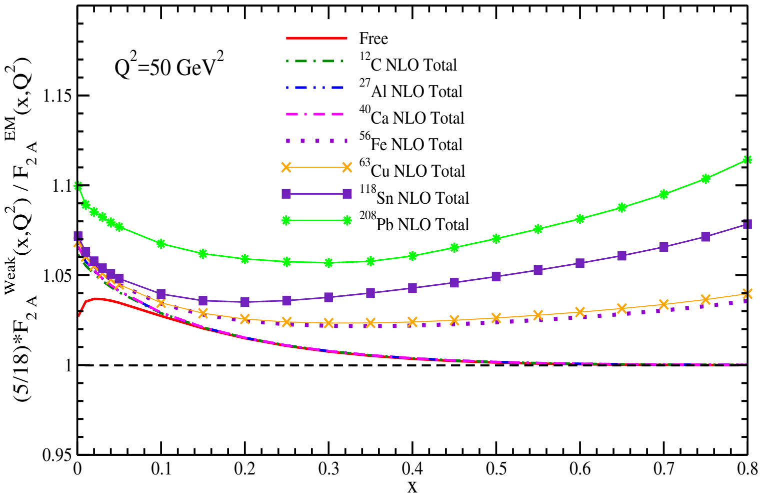

EMC F2 EM
CDHSW (5/18)*F2 Weak
CCFR (5/18)*F2 Weak
CJ12min F2 EM(Iso)
CJ12min (5/18)*F2 Weak(Iso)
F2 EM Fix α(Iso)
(5/18)*F2 Weak Fix α(Iso)
NuTeV (5/18)*F2 Weak
F2 EM NLO Total(Iso)
(5/18)*F2 Weak NLO Total(Iso)
56Fe
15 < Q2 < 25 GeV2
F2 Fe(x,Q2)
x


$F_2^{EM}$ CJ12min(Iso)
$(5/18)*F_2^{Weak}$ CJ12min(Iso)
$F_2^{EM}$ Fix α(Iso)
$(5/18)*F_2^{Weak}$ Fix α(Iso)
CCFR $(5/18)*F_2^{Weak}$
EMC $F_2^{EM}$
NuTeV $(5/18)*F_2^{Weak}$
CDHSW $(5/18)*F_2^{Weak}$
$F_2^{EM}$ NLO Total(Iso)
$(5/18)*F_2^{Weak}$ NLO Total(Iso)
$^{56}$Fe
$45 < Q^2 < 55$ GeV$^2$
$F_{2\,Fe}(x,Q^2)$
x
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
-0.1
0
0.1
0.2
0.3
0.4

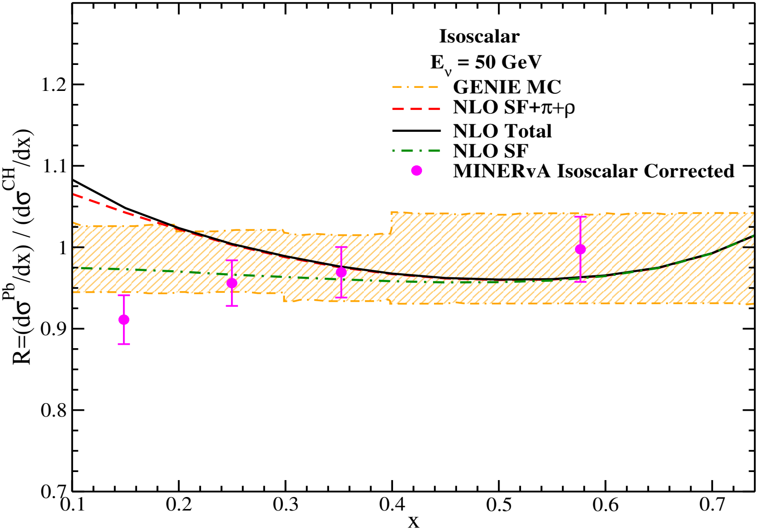
# $\frac{\frac{5}{18}F_{2A}^{Weak}(x,Q^2)}{F_{2A}^{EM}(x,Q^2)}$ vs $x$



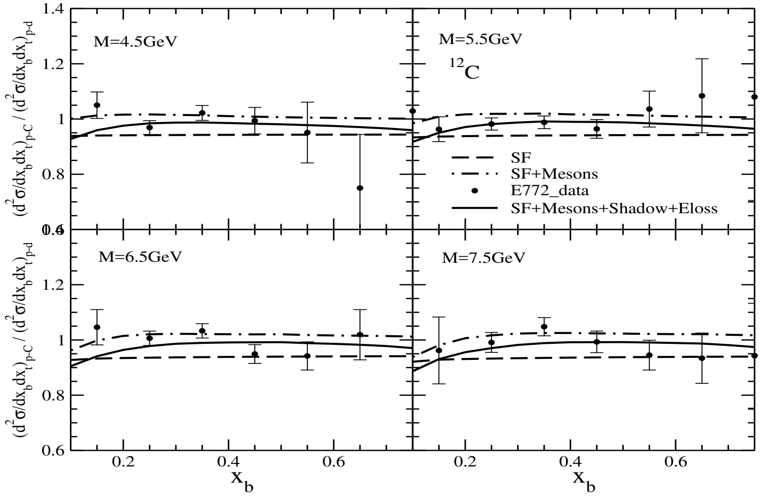

Free
12C NLO Total
27Al NLO Total
40Ca NLO Total
56Fe NLO Total
63Cu NLO Total
118Sn NLO Total
208Pb NLO Total
Q2=20 GeV2
(5/18)*F2 A Weak(x,Q2) / F2 A EM(x,Q2)
x
0.95
1
1.05
1.1
1.15
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8


Q^2=50 GeV^2
Free
12C NLO Total
27Al NLO Total
40Ca NLO Total
56Fe NLO Total
63Cu NLO Total
118Sn NLO Total
208Pb NLO Total
(5/18)*F2 A Weak(x,Q^2) / F2 A EM(x,Q^2)
x
0.95
1
1.05
1.1
1.15
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

# *MINERνA: PRD93 071101(2016)*



# *Drell-Yan:1606.04645 [nucl-th]*



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4. *The experiments at JLab and MINERvA using several nuclear targets in the region of low as well as high $x$ and $Q^2$, should be able to determine more precisely $F_2^{EM}(x,Q^2)$ and $F_2^{Weak}(x,Q^2)$ structure functions.*


thank
you . . .