Interrelationship

among

ν physics, $(g-2)_l$, EDMs and LFV

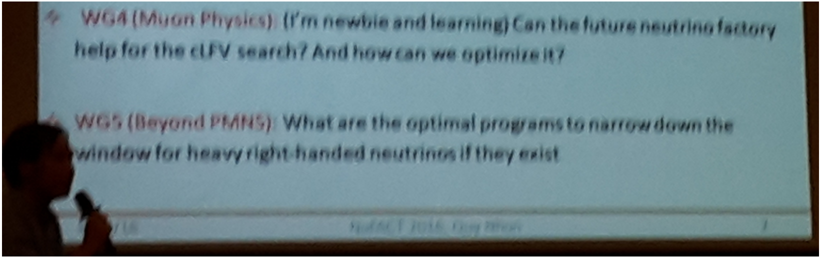
Giovanni Marco Pruna

Paul Scherrer Institut
Villigen, CH

NuFact, Quy Nhơn, 25 August 2016

Outline

- Interrelationship among searches
- Interrelationship among theories
- Interrelationship among energy scales
- Interrelationship among different scientific communities



Interrelationship and observables

Interrelationship: the way in which two or more things are connected and affect one another.

ν physics: phenomena connected with neutrinos.

$(g-2)$ and EDM of leptons: observables.

cLFV: a clear signal of new physics.

Research associated to these points is actually testing the same overall picture.

The Standard Model: $SU(3) \times SU(2) \times U(1)$, and 3 flavours.

Standard Model and its problems

The Standard Model does not take into account the following observations:

- neutrino oscillations;
- dark matter observation;
- baryogenesis;
- gravity.

It does not provide a convincing explanation for:

- hierarchy problem;
- flavour puzzle;
- QCD theta term;
- gauge couplings unification.

Experiment: testing the Standard Model

At the high energy frontier:

- searches for new particles;
- tests for non-standard properties at high energy.

At the low energy frontier:

- neutrino physics;
- $g - 2$, EDM;
- charged LFV;
- Kaon, B-meson, D-meson physics;
- ...

Theory: Bottom-up “versus” top-down

Top-down approach:

- a system is broken down to gain insight into its compositional sub-systems;
- an overview of the system is formulated, specifying but not detailing any first-level subsystems;
- each subsystem is then refined in yet greater detail, sometimes in many additional subsystem levels, until the entire specification is reduced to base elements.

Bottom-up approach:

- systems are put together to give rise to grander systems, thus making the original systems sub-systems of the emergent system;
- the individual base elements of the system are first specified in great detail;
- these elements are then linked together to form larger subsystems, which then in turn are linked, sometimes in many levels, until a complete top-level system is formed.

Tackling the baryogenesis problem

Testable Baryogenesis in Seesaw Models

P. Hernández,[a] **M. Kekic,**[a] **J. López-Pavón,**[b] **J. Racker,**[a] **J. Salvado.**[a]

[a]*Instituto de Física Corpuscular, Universidad de Valencia and CSIC, Edificio Institutos Investigación, Catedrático José Beltrán 2, 46980 Spain*

[b]*INFN, Sezione di Genova, via Dodecaneso 33, 16146 Genova, Italy*

ABSTRACT: We revisit the production of baryon asymmetries in the minimal type I seesaw model with heavy Majorana singlets in the GeV range. In particular we include for the first time "washout" effects from scattering processes with gauge bosons and higgs decays and inverse decays, besides the dominant top scatterings. We show that in the minimal model with two singlets, and for an inverted light neutrino ordering, future measurements from SHiP and neutrinoless double beta decay could in principle provide sufficient information to predict the matter-antimatter asymmetry in the universe up to a sign. We also show that SHiP measurements could provide very valuable information on the PMNS CP phases.

Dressing the model

Spoiling the original idea:

A $U(1)_{B-L}$ extension of the SM

$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

New states:

- A scalar (χ, SM-singlet)
  $V = \cdots + \lambda_1 (H^\dagger H)^2 + \lambda_2 \mid \chi \mid^4 + \lambda_3 H^\dagger H \mid \chi \mid^2$
- 3 RH neutrinos: $\nu_R \xrightarrow{\text{see-saw}} \nu_h$
  $\mathscr{L}_Y = \cdots - y^\nu \overline{l_L} \nu_R \widetilde{H} - y^M \overline{(\nu_R)^c} \nu_R \chi + \text{H.c.}$
- A new gauge boson (Z')

ψ	$SU(3)_C$	$SU(2)_L$	Y	$B-L$
q_L	3	2	$\frac{1}{6}$	$\frac{1}{3}$
u_R	3	1	$\frac{2}{3}$	$\frac{1}{3}$
d_R	3	1	$-\frac{1}{3}$	$\frac{1}{3}$
l_L	1	2	$-\frac{1}{2}$	-1
e_R	1	1	-1	-1
ν_R	1	1	0	-1

ψ	$SU(3)_C$	$SU(2)_L$	Y	$B-L$
H	1	2	$\frac{1}{2}$	0
χ	1	1	0	2

See-saw mechanism

Neutrinos combine, and the mass matrix is:

$$\mathscr{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix},$$

where

$$m_D = \frac{(y^\nu)^*}{\sqrt{2}} v, \qquad M = \sqrt{2} y^M x.$$

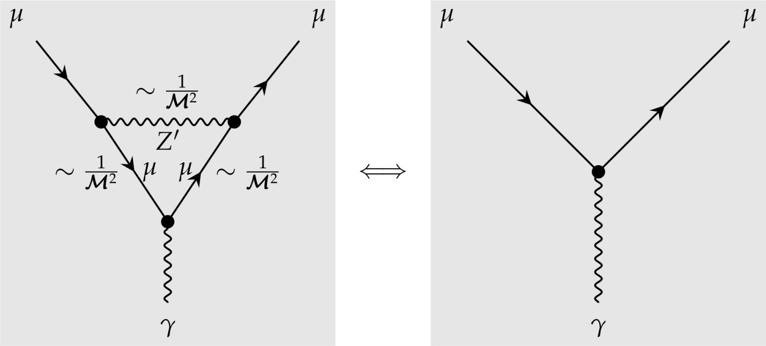
The diagonalisation of the mass matrix realises the "see-saw" mechanism.

3×3 mass matrices of the Majorana neutrinos are given by:

$$\begin{aligned} M_l &\simeq m_D M^{-1} m_D^T = \frac{1}{2\sqrt{2}} y^\nu (y^M)^{-1} (y^\nu)^T \frac{v^2}{x}, \\ M_h &\simeq M = \sqrt{2} y^M x. \end{aligned}$$

"See-saw" effect: the greater is M, the smaller is M_l.

Many synergies



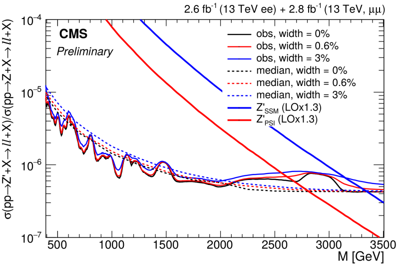
We can now explain:

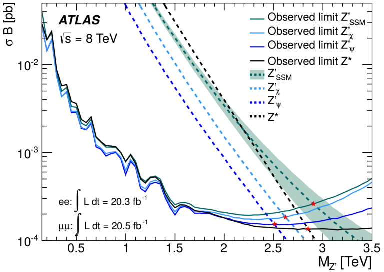
- neutrino masses, maybe excess in $g-2$ (or maybe not);
- baryogenesis;
- the conservation of $B-L$.

Direct searches on Z'

Limits on the Z'_{B-L} mass are $\sim$ with respect to the Z'_{SSM} case.

From CMS&ATLAS we have:





The model is surely under experimental investigation.

It provides many testable features: experiments suggest that such model (if existing) prefers to be considerably decoupled.

Lepton Flavour Violation: a conceptual challenge

The Dim-4 SM provides an accidental flavour symmetry:

- it holds in QCD and EM interactions;
- in the quark sector, it's broken by EW interactions.

The lepton sector strictly conserves the flavour.

At the same time, we have remarkable phenomenological evidences of FV in the neutrino sector, but…

…No evidence of the following phenomenological realisations:

- $l_h^\pm \to \gamma + l_i^\pm$ where $h, i = e, \mu, \tau$,
- $l_h^\pm \to l_i^\pm l_j^\pm l_k^\mp$ where $h, i, j, k = e, \mu, \tau$,
- $Z \to l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau$,
- $H \to l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau$.

Experimental "observations"

MUONIC AND TAUONIC LFV TRANSITIONS - A SELECTION

- BR($\mu \to 3e$)$< 1.0 \times 10^{-12}$ at the 90% C.L.
  SINDRUM Collaboration, Nucl. Phys. B **299** (1988) 1;
- BR($\mu \to \gamma + e$)$< 4.2 \times 10^{-13}$ at the 90% C.L.
  MEG Collaboration, Eur. Phys. J. C **76** (2016) 434;
- BR($Z \to e + \mu$)$< 7.5 \times 10^{-7}$ at the 95% C.L.
  ATLAS Collaboration, Phys. Rev. D **90** (2014) 072010;

- BR($\tau \to 3e$)$< 2.1 \times 10^{-8}$ at the 90% C.L.
  BELL Collaboration, Phys. Lett. B **687** (2010) 139-143;
- BR($\tau \to \gamma + \mu$)$< 4.4 \times 10^{-8}$ at the 90% C.L.
  BaBar Collaboration, Phys. Rev. Lett. **104** (2010) 021802;
- BR($Z \to \tau + \mu$)$< 1.2 \times 10^{-5}$ at the 95% C.L.
  DELPHI Collaboration, Z. Phys. C **73** (1997) 243-251;
- BR($H \to \tau + \mu$)$< 1.8 \times 10^{-2}$ at the 90% C.L.
  ATLAS/CMS Collaboration, arXiv:1508.03372/arXiv:1502.07400.

Extending the interactions of the SM

Assumptions: SM is merely an effective theory, valid up to some scale Λ. It can be extended to a field theory that satisfies the following requirements:

- its gauge group should contain $SU(3)_C \times SU(2)_L \times U(1)_Y$;
- all the SM degrees of freedom must be incorporated;
- at low energies (i.e. when $\Lambda \to \infty$), it should reduce to SM.

Assuming that such reduction proceeds via decoupling of New Physics (NP), the Appelquist-Carazzone theorem allows us to write such theory in the form:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

Dimension-5 operator

Only one dimension 5 operator is allowed by gauge symmetry:

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m(l_p^k)^T C l_r^n \; \equiv \; (\widetilde{\varphi}^\dagger l_p)^T C(\widetilde{\varphi}^\dagger l_r).$$

After the EW symmetry breaking, it can generate neutrino masses and mixing.

Its contribution to LFV has been studied since the late 70s:

- in the context of higher dimensional effective realisations;
  S. T. Petcov, Sov. J. Nucl. Phys. **25** (1977) 340 [Yad. Fiz. **25** (1977) 641]
- in connection with the "see-saw" mechanism.
  P. Minkowski, Phys. Lett. B **67**, 421 (1977)

Plenary talk from Patrick Huber.

Dimension-6 operators

2-leptons

$$\boxed{Q_{eW}} = (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu};$$
$$\boxed{Q_{eB}} = (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}.$$
$$\boxed{Q^{(1)}_{\varphi l}} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$$
$$\boxed{Q^{(3)}_{\varphi l}} = (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$$
$$\boxed{Q_{\varphi e}} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$$
$$\boxed{Q_{e\varphi}} = (\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$$

4-leptons

$$\boxed{Q_{ll}} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$
$$\boxed{Q_{ee}} = (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$$
$$\boxed{Q_{le}} = (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$$

4-fermions

$$Q^{(1)}_{lq} = (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$$
$$Q^{(3)}_{lq} = (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$$
$$Q_{eu} = (\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$$
$$Q_{ed} = (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$$
$$Q_{lu} = (\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$$
$$Q_{ld} = (\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$$
$$Q_{qe} = (\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$$
$$Q_{ledq} = (\bar{l}^j_p e_r)(\bar{d}_s q^j_t)$$
$$\boxed{Q^{(1)}_{lequ}} = (\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{q}^k_s u_t)$$
$$\boxed{Q^{(3)}_{lequ}} = (\bar{l}^j_p \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t)$$

They all provide LF-violation

Dim-6 operators: $l_2 \to l_1\gamma$ at the tree level

Only one dim-6 term can produce $l_2 \to l_1\gamma$ at the tree level:

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010** (2010) 085

Working in the physical basis, we consider:

$$
\begin{aligned}
Q_{eB} &\to Q_{e\gamma} c_W - Q_{eZ} s_W , \\
Q_{eW} &\to -Q_{e\gamma} s_W - Q_{eZ} c_W ,
\end{aligned}
$$

where $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$ are the sine and cosine of the weak mixing angle. The term

$$
\mathcal{L}_{e\gamma} \equiv \frac{C_{e\gamma}}{\Lambda^2} Q_{e\gamma} + \text{h.c.} = \frac{C_{e\gamma}^{pr}}{\Lambda^2} (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi F_{\mu\nu} + \text{h.c.},
$$

where $F_{\mu\nu}$ is the electromagnetic field-strength tensor, is then the only term in the D-6 Lagrangian that induces a $l_2 \to l_1\gamma$ transition at tree level.

Dim-6 operators: $H \to l_i l_j$ at the tree level

Only one dim-6 term provides $H \to l_i l_j$ at the tree level:

$$Q_{e\varphi} = (\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi),$$

that sums to the SM Yukawa sector:

$$\begin{aligned}
\mathcal{L}_{\mathrm{D4}} + \mathcal{L}_{e\varphi} = & \frac{v}{\sqrt{2}} \left(-y_{pr} + \frac{v^2}{2\Lambda^2} C_{e\varphi}^{pr} \right) \bar{e}_p e_r \\
& + \frac{1}{\sqrt{2}} \left(-y_{pr} + \frac{v^2}{2\Lambda^2} C_{e\varphi}^{pr} \right) \bar{e}_p e_r h + \boxed{\frac{v^2}{\sqrt{2}\Lambda^2} C_{e\varphi}^{pr}} \bar{e}_p e_r h \\
& + \frac{i}{\sqrt{2}} \left(-y_{pr} + \frac{v^2}{2\Lambda^2} C_{e\varphi}^{pr} \right) \bar{e}_p e_r \widehat{Z} \\
& + i \left(-y_{pr} + \frac{v^2}{2\Lambda^2} C_{e\varphi}^{pr} \right) \bar{e}_p \nu_r \widehat{W}^+ + [\ldots] \,.
\end{aligned}$$

Other operators that are relevant at the tree level

Other LFV processes such as $Z \to l_i l_j$ or $l_j \to 3 l_i$ are phenomenologically present at the tree-level if the following operators appear in the Lagrangian:

$\boxed{\text{2-leptons}}$

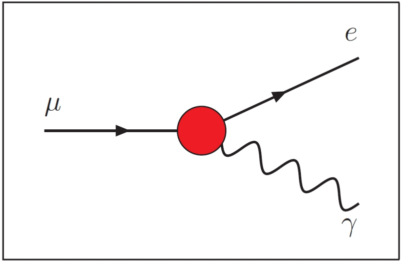
$$
\begin{aligned}
Q_{eW} &= (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu} \\
Q_{eB} &= (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu} \\
Q^{(1)}_{\varphi l} &= (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r) \\
Q^{(3)}_{\varphi l} &= (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r) \\
Q_{\varphi e} &= (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)
\end{aligned}
$$

$\boxed{\text{4-leptons}}$

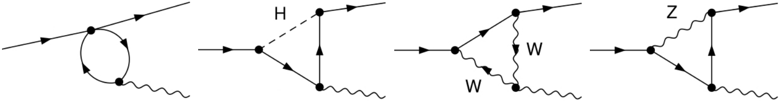
$$
\begin{aligned}
Q_{ll} &= (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t) \\
Q_{ee} &= (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t) \\
Q_{le} &= (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)
\end{aligned}
$$

Dim-6 operators: $\mu(\tau) \to e(\mu/e)\gamma$ at one loop

For those who have good sight, even a point-like interaction. . .



. . . looks like a wild place to explore!



Interaction and branching ratio

Dim-6 operators contribute to the coefficients C_{TL} and C_{TR} of the photon-mediated FV interaction:

$$V^{\mu} = \frac{1}{\Lambda^2} i\sigma^{\mu\nu} \left(C_{TL}\, \omega_L + C_{TR}\, \omega_R \right) \left(p_{\gamma} \right)_{\nu} .$$

Being the partial width of the process $\mu \to e\gamma$

$$\Gamma_{\mu\to e\gamma} = \frac{1}{16\pi m_{\mu}} \left| \mathcal{M} \right|^2 , \quad \text{with} \quad \left| \mathcal{M} \right|^2 = \frac{4 \left(\left| C_{TL} \right|^2 + \left| C_{TR} \right|^2 \right) m_{\mu}^4}{\Lambda^4},$$

then the branching ratio is

$$\mathrm{BR}(\mu \to e\gamma) = \frac{\Gamma_{\mu\to e\gamma}}{\Gamma_{\mu}} = \frac{m_{\mu}^3}{4\pi \Lambda^4 \Gamma_{\mu}} \left(\left| C_{TL} \right|^2 + \left| C_{TR} \right|^2 \right) .$$

By calculating the dim-6 contributions to C_{TL} and C_{TR} one obtain the connection between effective coefficients and BR.

Dim-6 effective contributions to C_{TL} and C_{TR}

Operator	C_{TL} or $C_{TR}(l_2 \longleftrightarrow l_1)$
$Q_{e\gamma}$	$-C_{e\gamma}\dfrac{\sqrt{2}m_W s_W}{e}$
Q_{eZ}	$-C_{eZ}\dfrac{e m_Z}{16\sqrt{2}\pi^2}\left(3-6c_W^2+4c_W^2\log\left[\dfrac{m_W^2}{m_Z^2}\right]+(12c_W^2-6)\log\left[\dfrac{m_Z^2}{\lambda^2}\right]\right)$
$Q_{\varphi l}^{(1)}$	$-C_{\varphi l}^{(1)}\dfrac{e m_1\left(1+s_W^2\right)}{24\pi^2}$
$Q_{\varphi l}^{(3)}$	$C_{\varphi l}^{(3)}\dfrac{e m_1\left(3-2s_W^2\right)}{48\pi^2}$
$Q_{\varphi e}$	$C_{\varphi e}\dfrac{e m_2\left(3-2s_W^2\right)}{48\pi^2}$
$Q_{e\varphi}$	$C_{e\varphi}\dfrac{m_W s_W}{48\sqrt{2}m_H^2\pi^2}\left(4m_1^2+4m_2^2+3m_1^2\log\left[\dfrac{m_1^2}{m_H^2}\right]+3m_2^2\log\left[\dfrac{m_2^2}{m_H^2}\right]\right)$
$Q_{lequ}^{(3)}$	$-\dfrac{e}{2\pi^2}\sum_u m_u\left(C_{lequ}^{(3)}\right)^{21uu}\log\left[\dfrac{m_u^2}{\lambda^2}\right]$

Operator	C_{TL}	C_{TR}
Q_{le}	$\dfrac{e}{16\pi^2}\left(m_e C_{le}^{2ee1}+m_\mu C_{le}^{2\mu\mu 1}+m_\tau C_{le}^{2\tau\tau 1}\right)$	$\dfrac{e}{16\pi^2}(m_e C_{le}^{1ee2}+m_\mu C_{le}^{1\mu\mu 2}+m_\tau C_{le}^{1\tau\tau 2})$

$\mu \to 3e$ and $Z \to e\mu$ branching ratios

For the three-body decay $l_1^\pm \to l_2^\pm l_2^\mp l_2^\pm$, one has

$$\Gamma(l_1^+ \to l_2^+ l_2^- l_2^+) = \Bigg(40 e^2 v^2 \left(\left|C_{e\gamma}^{12}\right|^2 + \left|C_{e\gamma}^{21}\right|^2 \right) \left(8 \ln\left[\frac{m_1}{m_2}\right] - 11 \right)$$

$$+ \frac{2m_1^4}{m_Z^2} \left(\left(5 - 20 s_W^2 + 36 s_W^4\right) \left|C_{eZ}^{12}\right|^2 + 4\left(1 - 4 s_W^2 + 9 s_W^4\right) \left|C_{eZ}^{21}\right|^2 \right)$$

$$+ \frac{15 m_2^2 m_1^2 v^2 \left(\left|C_{e\varphi}^{12}\right|^2 + \left|C_{e\varphi}^{21}\right|^2 \right)}{8 m_H^4} + 10 m_1^2 \left(1 - 4 s_W^2 + 12 s_W^4\right) \left|C_{\varphi e}^{12}\right|^2$$

$$+ 20 m_1^2 \left(1 - 4 s_W^2 + 6 s_W^4\right) \left(\left|C_{\varphi l(1)}^{12}\right|^2 + \left|C_{\varphi l(3)}^{12}\right|^2 \right) +$$

$$+ 10 m_1^2 \left(\left|C_{le}^{1112}\right|^2 + \left|C_{le}^{1211}\right|^2 \right) + 80 m_1^2 \left(\left|C_{ee}^{1112}\right|^2 + \left|C_{ll}^{1112}\right|^2 \right) \Bigg) \frac{m_1^3}{30 (8\pi)^3 \Lambda^4}.$$

Flavour-violating Z decays can be parametrised as follows:

$$\Gamma(Z \to l_1^\pm l_2^\mp) = \frac{m_Z^3 v^2}{12 \pi \Lambda^4} \left(\left|C_{eZ}^{12}\right|^2 + \left|C_{eZ}^{21}\right|^2 + \left|C_{\varphi e}^{12}\right|^2 + \left|C_{\varphi l(1)}^{12}\right|^2 + \left|C_{\varphi l(3)}^{12}\right|^2 \right).$$

No correlation: limits from some muonic transition

Coefficient	MEG ($\mu \to e\gamma$) $BR \leq 5.7 \cdot 10^{-13}$	ATLAS ($Z \to e\mu$) $BR \leq 7.5 \cdot 10^{-7}$	SINDRUM ($\mu \to 3e$) $BR \leq 1.0 \cdot 10^{-12}$
$C_{eZ}^{\mu e}(m_Z)$	$1.4 \cdot 10^{-13} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.8 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(1)}$	$2.5 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.5 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(3)}$	$2.4 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.5 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi e}$	$2.4 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.6 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{e\varphi}^{\mu e}$	$2.7 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$		$6.1 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{le}^{eee\mu}$	$4.2 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$		$2.2 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{le}^{e\mu\mu\mu}$	$2.0 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{le}^{e\tau\tau\mu}$	$1.2 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{ee}^{eee\mu}$			$7.7 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{ll}^{eee\mu}$			$7.7 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}$

No correlation: limits from some tauonic transition

Coefficient	BaBar ($\tau \to \mu\gamma$) $BR \leq 4.4 \cdot 10^{-8}$	LEP ($Z \to \tau\mu$) $BR \leq 1.2 \cdot 10^{-5}$	BELL ($\tau \to 3\mu$) $BR \leq 2.1 \cdot 10^{-8}$	ATLAS&CMS ($H \to \tau\mu$) $BR \leq 1.85 \cdot 10^{-2}$
$C_{eZ}^{\tau\mu}(m_Z)$	$1.5 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$6.1 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{\varphi l}^{(1)}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$9.0 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{\varphi l}^{(3)}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$9.0 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{\varphi e}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$9.5 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{e\varphi}^{\tau\mu}$	$1.9 \cdot 10^{-6} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$		$1.1 \cdot 10^{-5} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$1.3 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$
$C_{le}^{\mu e e\tau}$	$4.7 \cdot 10^{-4} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$			
$C_{le}^{\mu\mu\mu\tau}$	$2.3 \cdot 10^{-6} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$		$8.0 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{le}^{\mu\tau\tau\tau}$	$1.3 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$			
$C_{ee}^{\mu\mu\mu\tau}$			$2.8 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{ll}^{\mu\mu\mu\tau}$			$2.8 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	

Effective coefficients and energy scale

The result of C_T at one loop can schematically be written as:

$$C_T^{(1)} = -\frac{v}{\sqrt{2}} \left(C_{e\gamma} \left(1 + e^2 c_{e\gamma}^{(1)} \right) + \sum_{i \neq e\gamma} e^2 c_i^{(1)} \, C_i \right) .$$

In general, the coefficients $c_{e\gamma}^{(1)}$ and $c_i^{(1)}$ contain UV singularities, *i.e.* a renormalisation of $C_{e\gamma}$ is required.

Such procedure makes the scale dependence explicit via the *anomalous dimensions* of the coefficient.

At the end of the day, the renormalised effective coefficients and the C_{TL} and C_{TR} are running quantities.

A scale dependent limit

MEG sets a limit on $\mu \to e\gamma$ at the $\lambda = m_\mu$ scale; we combine it with the information on the interacting current to obtain:

$$\left.\frac{\sqrt{|C_{TL}(\lambda)|^2 + |C_{TR}(\lambda)|^2}}{\Lambda^2}\right|_{\lambda \ll \Lambda} \leq 4.3 \cdot 10^{-14}\,[\text{GeV}]^{-1}\,.$$

In this formula there are two scale dependencies:

- Λ: this is the scale $\gg \Lambda_{EW}$ at which the theory is defined, according to the decoupling theorem.
- λ: this is the scale at which the coefficient is probed by the experiment.

Next step: connecting low and high energy scales.

From $\lambda = m_\mu$ to $\lambda = \Lambda_{EW}$

In the assumption that $C_{e\gamma}$ is the dominant coefficient in the energy range $m_\mu < \lambda < m_Z \sim m_H$, its running below the EW scale is QED driven:

$$16\pi^2 \frac{\partial C_{e\gamma}}{\partial \log \lambda} \simeq e^2 \left(10 + \frac{4}{3} \sum_q e_q^2(\lambda) \right) C_{e\gamma}.$$

Applying this to the limit on $C_{e\gamma}^{\mu e}(m_\mu)$ and $C_{e\gamma}^{e\mu}(m_\mu)$, one obtains:

$$\sqrt{\frac{|C_{e\gamma}^{\mu e}(m_Z)|^2 + |C_{e\gamma}^{e\mu}(m_Z)|^2}{2}} < 1.8 \cdot 10^{-16} \frac{\Lambda^2}{[\text{GeV}]^2}.$$

This is the limit that must be used to determine the constraints on the remaining effective coefficients at the scale Λ.

Renormalisation Group Equations

If one considers only the gauge contributions and the top-Yukawa coupling, the evolution of the coefficient $C_{e\gamma}$ is described by a coupled SoDE:

$$16\pi^2 \frac{\partial \boxed{C^{\mu e}_{e\gamma}}}{\partial \log\lambda} \simeq \left(\frac{47e^2}{3} + \frac{e^2}{4c_W^2} - \frac{9e^2}{4s_W^2} + 3Y_t^2\right) \boxed{C^{\mu e}_{e\gamma}} + 6e^2\left(\frac{c_W}{s_W} - \frac{s_W}{c_W}\right)\boxed{C^{\mu e}_{eZ}} + 16eY_t \boxed{C^{(3)}_{\mu ett}},$$

$$16\pi^2 \frac{\partial \boxed{C^{\mu e}_{eZ}}}{\partial \log\lambda} \simeq -\frac{2e^2}{3}\left(\frac{2c_W}{s_W} + \frac{31s_W}{c_W}\right)\boxed{C^{\mu e}_{e\gamma}} + 2e\left(\frac{3c_W}{s_W} - \frac{5s_W}{c_W}\right)Y_t \boxed{C^{(3)}_{\mu ett}} + \left(-\frac{47e^2}{3} + \frac{151e^2}{12c_W^2} - \frac{11e^2}{12s_W^2} + 3Y_t^2\right)\boxed{C^{\mu e}_{eZ}},$$

$$16\pi^2 \frac{\partial \boxed{C^{(3)}_{\mu ett}}}{\partial \log\lambda} \simeq \frac{7eY_t}{3}\boxed{C^{\mu e}_{e\gamma}} + \frac{eY_t}{2}\left(\frac{3c_W}{s_W} - \frac{5s_W}{3c_W}\right)\boxed{C^{\mu e}_{eZ}} + \left(\frac{2e^2}{9c_W^2} - \frac{3e^2}{s_W^2} + \frac{3Y_t^2}{2} + \frac{8g_S^2}{3}\right)\boxed{C^{(3)}_{\mu ett}} + \frac{e^2}{8}\left(\frac{5}{c_W^2} + \frac{3}{s_W^2}\right)\boxed{C^{(1)}_{\mu ett}},$$

$$16\pi^2 \frac{\partial \boxed{C^{(1)}_{\mu ett}}}{\partial \log\lambda} \simeq \left(\frac{30e^2}{c_W^2} + \frac{18e^2}{s_W^2}\right)\boxed{C^{(3)}_{\mu ett}} + \left(-\frac{11e^2}{3c_W^2} + \frac{15Y_t^2}{2} - 8g_S^2\right)\boxed{C^{(1)}_{\mu ett}}.$$

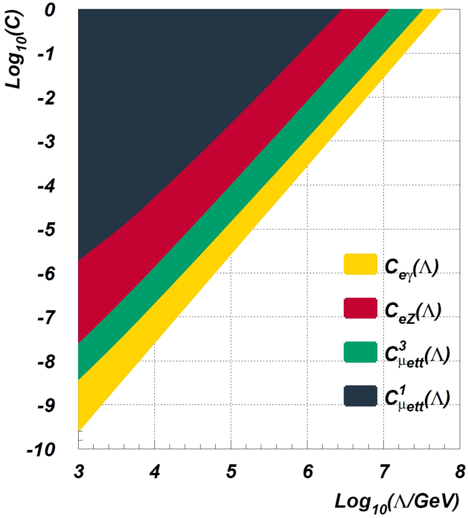
Evolution and bounds from low energy

GMP and A. Signer, JHEP **1410** (2014) 014

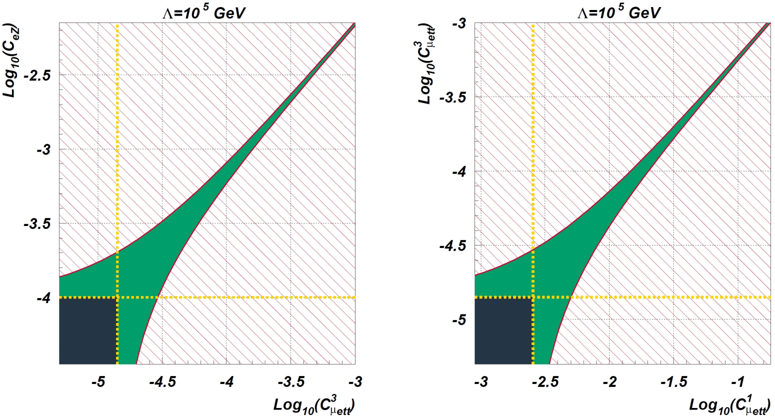
A remarkable set of different constraints on coefficients defined at the decoupling scale Λ!

Behaviour is not completely linear: solutions are not analytically simple.

Bounds on $C_{\mu ett}^{(1,3)}$!



Effects of correlation in the RGE analysis



Cancellations can represent a delicate issue:
naturalness is not a strong argument in effective scenarios!

Limits for coefficients defined at the Λ scale (1)

If no correlation is assumed, one obtains the following limits:

3-P Coefficient	at $\Lambda = 10^3$ GeV	at $\Lambda = 10^5$ GeV	at $\Lambda = 10^7$ GeV
$C_{e\gamma}^{\mu e}$	$2.7 \cdot 10^{-10}$	$2.9 \cdot 10^{-6}$	$3.1 \cdot 10^{-2}$
$C_{eZ}^{\mu e}$	$2.5 \cdot 10^{-8}$	$1.0 \cdot 10^{-4}$	$7.1 \cdot 10^{-1}$
$C_{\mu ett}^{(3)}$	$3.6 \cdot 10^{-9}$	$1.4 \cdot 10^{-5}$	$9.8 \cdot 10^{-2}$
$C_{\mu ett}^{(1)}$	$1.9 \cdot 10^{-6}$	$2.5 \cdot 10^{-3}$	n/a
$C_{\mu ecc}^{(3)}$	$4.8 \cdot 10^{-7}$	$1.9 \cdot 10^{-3}$	n/a
$C_{\mu ecc}^{(1)}$	$2.6 \cdot 10^{-4}$	$3.3 \cdot 10^{-1}$	n/a

TABLE 5: Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^5, 10^7$ GeV.

Limits from MEG are applied at a fixed scale $\lambda = m_Z$.

Limits for coefficients defined at the Λ scale (2)

If no correlation is assumed, one obtains the following limits:

$\tau \to e\gamma$			
3-P Coefficient	at $\Lambda = 10^3$ GeV	at $\Lambda = 10^4$ GeV	at $\Lambda = 10^5$ GeV
$C_{e\gamma}^{\tau e}$	$2.5 \cdot 10^{-6}$	$2.6 \cdot 10^{-4}$	$2.8 \cdot 10^{-2}$
$C_{eZ}^{\tau e}$	$2.3 \cdot 10^{-4}$	$1.3 \cdot 10^{-2}$	$9.5 \cdot 10^{-1}$
$C_{\tau ett}^{(3)}$	$3.4 \cdot 10^{-5}$	$1.9 \cdot 10^{-3}$	$1.4 \cdot 10^{-1}$
$C_{\tau ett}^{(1)}$	$1.8 \cdot 10^{-2}$	$5.0 \cdot 10^{-1}$	n/a
$C_{\tau ecc}^{(3)}$	$4.6 \cdot 10^{-3}$	$2.5 \cdot 10^{-1}$	n/a
$C_{\tau ecc}^{(1)}$	~ 2.4	n/a	n/a

TABLE 6: Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^4, 10^5$ GeV.

Limits from BaBar are applied at a fixed scale $\lambda = m_Z$.

Limits for coefficients defined at the Λ scale (3)

If no correlation is assumed, one obtains the following limits:

$\tau \to \mu\gamma$			
3-P Coefficient	at $\Lambda = 10^3$ GeV	at $\Lambda = 10^4$ GeV	at $\Lambda = 10^5$ GeV
$C_{e\gamma}^{\tau\mu}$	$3.0 \cdot 10^{-6}$	$3.1 \cdot 10^{-4}$	$3.2 \cdot 10^{-2}$
$C_{eZ}^{\tau\mu}$	$2.8 \cdot 10^{-4}$	$1.5 \cdot 10^{-2}$	~ 1.1
$C_{\tau\mu tt}^{(3)}$	$4.0 \cdot 10^{-5}$	$2.2 \cdot 10^{-3}$	$1.6 \cdot 10^{-1}$
$C_{\tau\mu tt}^{(1)}$	$2.1 \cdot 10^{-2}$	$5.9 \cdot 10^{-1}$	n/a
$C_{\tau\mu cc}^{(3)}$	$5.4 \cdot 10^{-3}$	$3.0 \cdot 10^{-1}$	n/a
$C_{\tau\mu cc}^{(1)}$	~ 2.8	n/a	n/a

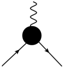
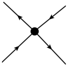
TABLE 8: Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^4, 10^5$ GeV.

Limits from BaBar are applied at a fixed scale $\lambda = m_Z$.

The good old k plot

Due to the **extremely-low** accessible **branching ratios**, CLFV muon channels can strongly **constrain** new physics models and scales.

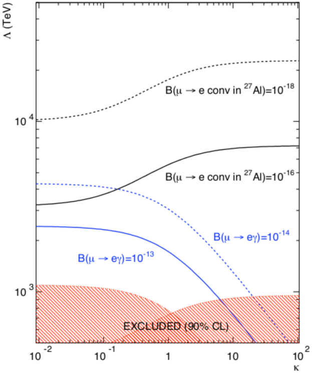
Model independent Lagrangian:

$$\boxed{\frac{m_\mu}{(\kappa+1)\Lambda^2} \times}$$  $$\boxed{+ \frac{\kappa}{(\kappa+1)\Lambda^2} \times}$$ 

Dipole term (*e.g.* SUSY)	Contact term (*e.g.* Z', LQ)
$\mu \to e\gamma$	

$$\mu \to eee$$
$$\mu - e \text{ conversion}$$

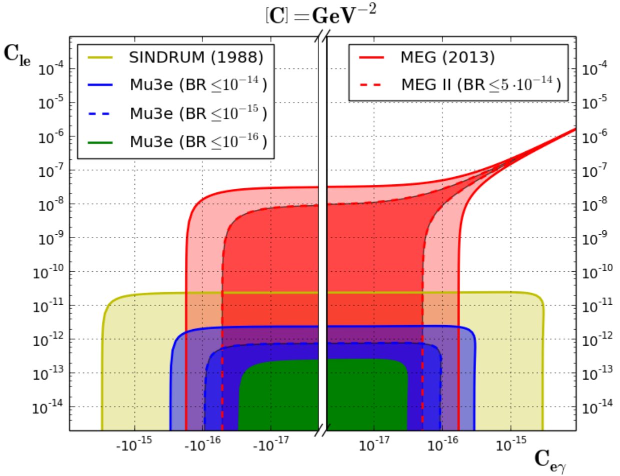
Sensitive to high-mass New Physics!



de Gouvea et Vogel, *ArXiv e-prints* (2013), 1303.4097

An alternative to the k plot (in preparation)

Below the EW-scale, only two non-zero operators contribute to both $\mu \to e\gamma$ and $\mu \to 3e$ (only in CDR, toy plot):



Conclusion

- $\surd$ We are all testing the Standard Model of Particle Physics.
- $\surd$ If new particles are above the scale of our theory, they can be either discovered at HE experiments, or produce signals that we can interpret in terms of effective operators.
- $\surd$ Different observables are related to (possibly) different combinations of effective coefficients.
- $\surd$ A consistent EFT approach can gives us information about the correlations among observables at different scales.

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giovanni-marco.pruna(at)psi.ch

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