Recent Developments in Neutrino-Nucleus Scattering (THEORY)

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Introduction

In *all v*-oscillation experiments, neutrinos are detected through neutrino-nucleus scattering

neutrino detectors (^{16}O , ^{12}C , ^{40}Ar , ...)

Next-generation exp. \rightarrow leptonic \mathcal{O} , mass hierarchy

 ν -nucleus scattering needs to be understood more accurately (~ 5%)







Wide kinematical region with different characteristic

➔ Combination of different expertise is necessary



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This talk : Very recent and selected developments in neutrino-nucleus scattering in QE and RES regions are reviewed (Sorry, many other recent works !)

QE (quasi-elastic)

Neutrino-nucleus scattering in QE region



- Crucially relevant to T2K ($E_v \sim 1 \text{ GeV}$)
- E_{ν} is reconstructed with lepton kinematics assuming QE kinematics and initial nucleon is at rest
- Contamination of CCQE-like event is problematic

INC : initial nucleon correlations FSI : final state interactions

Mechanisms







Genuine QE

meson exchange current, 2p-2h (QE-like)

General FSI, np-nh (QE-like)

Lots of recent works on QE-like mechanisms (Martini et al., Nieves et al., Amaro et al. ...)

Very Recent Update

Ab initio approach to electron inclusive scattering on ¹²C in QE region Lovato et al., PRC 91, 062501(R) (2015); PRL **117**, 082501 (2016)

- The best approach in non-relativistic regime (except for computational cost)
- The work is about electron scattering but the same method should work as well for neutrino scattering
- The method can be tested by a large amount of electron scattering data

$$H |\Psi_i\rangle = E_i |\Psi_i\rangle \qquad \qquad H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

$$H |\Psi_i\rangle = E_i |\Psi_i\rangle \qquad \qquad H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i < j} \underline{v_{ij}}_i + \sum_{i < j < k} \underline{V_{ijk}}_i + \dots$$

Argonne v18 Illinois 7

$$H |\Psi_i\rangle = E_i |\Psi_i\rangle \qquad \qquad H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i < j} \frac{v_{ij}}{m} + \sum_{i < j < k} \frac{V_{ijk}}{k} + \dots$$

GFMC (Green's Function Monte Carlo) method

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$$H |\Psi_i\rangle = E_i |\Psi_i\rangle \qquad \qquad H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

Electromagnetic current (constrained by current conservation, $\partial_{\mu}j^{\mu} = 0$, and data)



Response functions

 $J_{\rm FM} = J_{\rm 1b} + J_{\rm 2b}$

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_{f} \langle \Psi_0 | J_{\alpha}^{\dagger}(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | J_{\beta}(\mathbf{q}) | \Psi_0 \rangle \delta(\omega - E_f + E_0)$$

Cross section is written in terms of $R_{lphaeta}$

EM transverse response of ¹²C



Data : Jourdan, NPA 603, 117 (1996)

Conclusions

• Excellent agreement with data in QE region

• PWIA: genuine QE \rightarrow



Difference between PWIA and GFMC 1b \rightarrow importance of FSI

- Large 2b (MEC) enhancement
- Benchmark for other many-body methods

Ab initio approach to heavier nuclei is computationally difficult

RES (resonance)

Neutrino-nucleus scattering in resonance region



Theoretical description



Relevance to T2K ($E_v \sim 1 \text{ GeV}$) $\Delta(1232) \text{ excitation} \rightarrow \pi \text{ emission} \rightarrow \pi \text{ absorption}$ CCQE-like event Relevance to DUNE ($E_v = 2 \sim 4 \text{ GeV}$) higher resonance excitations $\rightarrow 1\pi$, 2π emissions

Initial nucleon correlations (INC)

 $P(\vec{p},E)$: spectral function (Fermi gas, shell model)

• Final state interactions (FSI)

hadron transport model (GiBUU, cascade)

• Medium modifications (MM)

Pauli blocking , width broadening modified hadron properties

Neutrino-nucleus scattering in resonance region



Theoretical description



to be discussed from now

Relevance to T2K ($E_v \sim 1 \text{ GeV}$) Δ (1232) excitation $\rightarrow \pi \text{ emission} \rightarrow \pi \text{ absorption}$ CCQE-like event Relevance to DUNE ($E_v = 2 \sim 4 \text{ GeV}$) higher resonance excitations $\rightarrow 1\pi$, 2π emissions

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Very Recent Update

- Elementary neutrino-nucleon reaction model from Δ (1232) up to boundary with DIS region SXN, Kamano, Sato, Lee, PRD 92, 074024 (2015)
- Possibly important FSI effect in extracting elementary $\nu N \rightarrow \mu^{-} \pi N$ cross sections from deuterium bubble chamber data

Wu, Sato, and Lee, PRC 91, 035203 (2015)

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Development of *vN*-reaction model for whole RES region

SXN, Kamano, Sato, Lee, PRD 92, 074024 (2015)

Strategy

- Dynamical coupled-channels (DCC) model for γN , $\pi N \rightarrow \pi N$, $\pi \pi N$, ηN , $K\Lambda$, $K\Sigma$
- Extension to $vN \rightarrow l X$ ($X = \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma$)

Important improvements over previous models (Breit-Wigner, tree-level non-res)

- Channel-couplings required by unitarity
- 2π production mechanisms with resonance contributions
- Interference between res and non-res axial amplitudes under control

 \leftarrow consistency between πN interaction and axial current required by PCAC

DCC (Dynamical Coupled-Channel) model

Matsuyama et al., Phys. Rept. 439, 193 (2007) Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

$$T_{ab} = V_{ab} + \sum_{c} V_{ac} G_{c} T_{cb}$$

$$\{a,b,c\} = \pi N, \ \eta N, \ \pi \pi N, \ \pi \Delta, \sigma N, \rho N, \ K\Lambda, \ K\Sigma$$

By solving the LS equation, coupled-channel unitarity is fully taken into account

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$$T_{ab} = V_{ab} + \sum_{c} V_{ac} G_{c} T_{cb}$$



In addition, γN , $W^{\pm}N$, ZN channels are included perturbatively



DCC analysis of meson production data

Kamano, SXN, Lee, Sato, PRC 88 (2013)

Fully combined analysis of γN , $\pi N \rightarrow \pi N$, ηN , $K\Lambda$, $K\Sigma$ data

~ 23,000 data points are fitted

by adjusting parameters (N^* mass, $N^* \rightarrow MB$ couplings, cutoffs)

Data for electron scattering on proton and neutron are analyzed by adjusting $\gamma^* N \rightarrow N^*$ coupling strength

Very small portion of analysis results will be presented next

Partial wave amplitudes of π N scattering



Partial wave amplitudes of π N scattering



$\gamma p \rightarrow \pi^0 p$

$d\sigma/d\Omega$ for W < 2.1 GeV



Vector current (Q²=0) for 1π Production is well-tested by data

Inclusive electron-proton scattering



Data: JLab E00-002 (preliminary)

- Reasonable fit to data for application to neutrino interactions
- Important 2π contributions for high W region

Similar analysis of electron-neutron scattering data has also been done

DCC vector currents has been tested by data for whole kinematical region relevant to neutrino interactions of $E_v \le 2 \text{ GeV}$

Neutrino-induced reactions

SXN, Kamano, Sato, Lee, PRD 92, 074024 (2015)

DCC model for vector current

Vector current for neutrino processes is obtained by analyzing electron reaction ($Q^2 \neq 0$) data on both proton and neutron targets, and by isospin separation

Detailed discussion in Kamano's talk today

DCC model for axial current

Because neutrino reaction data are scarce, axial current cannot be determined phenomenologically \rightarrow Chiral symmetry and PCAC (partially conserved axial current) are guiding principle

PCAC relation $\langle X' | q \cdot A | X \rangle \sim i f_{\pi} \langle X' | T | \pi X \rangle$



both πN interaction and axial current constructed consistently with PCAC

> Interference among resonances and background can be uniquely fixed within DCC model

 Q^2 -dependence \rightarrow dipole form is assumed with $M_A \sim 1 \text{ GeV}$

Comparison with single pion data



DCC model prediction is consistent with data

ANL Data : PRD **19**, 2521 (1979) BNL Data : PRD **34**, 2554 (1986)

- DCC model has flexibility to fit data $(ANN^*(Q^2))$
- Data should be analyzed with nuclear effects (Wu et al., PRC91, 035203 (2015))

Comparison with double pion data



Fairly good DCC predication

ANL Data : PRD **28**, 2714 (1983) BNL Data : PRD **34**, 2554 (1986)

First dynamical model for 2 π production in resonance region





• $\Delta(1232)$ dominates for $v_{\mu}p \rightarrow \mu^{-}\pi^{+}p$ (*I*=3/2) for $E_{\nu} \leq 2 \text{ GeV}$

Non-resonant mechanism and Higher N*s becomes important

towards $E_v \approx 2$ GeV for $v_\mu n \Rightarrow \mu \pi N$

- ightarrow essential to understand interference pattern among them
- \rightarrow DCC model can do this; consistency between π interaction and axial current

Very Recent Update

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 Possibly important FSI effect in extracting elementary v N → μ⁻πN cross sections from deuterium bubble chamber data
Wu, Sato, and Lee , PRC 91, 035203 (2015)
Model for γd , $W^{\pm} d \rightarrow \pi N N$



- $\gamma N, W^{\pm}N \rightarrow \pi N$ amplitude $\pi N \rightarrow \pi N$ amplitude T_{NN} , deuteron w.f.
- ← SL model (PRC 54 (1996), PRC 67 (2003))
- ← SL model
- ← CD-Bonn potential (PRC 63 (2001))

SL model is for Δ region and includes πN channel only

$$\gamma d \rightarrow \pi N N$$

Purpose : test the soundness of the model

Wu, Sato, and Lee, PRC 91, 035203 (2015)



- Model prediction is reasonably consistent with data
- Large NN (small πN) rescattering effect for π⁰ production orthogonality between deuteron and pn scattering wave functions
- Small rescattering effect for π^- production

 $v_{\mu}d \rightarrow \mu^{-}\pi^{+}n p$ cross sections

Wu, Sato, and Lee, PRC 91, 035203 (2015)



Conclusions

• Large *NN* (small πN) rescattering effect

orthogonality between deuteron and pn scattering wave functions

• ANL and BNL data did not consider FSI

 \rightarrow calling for reanalysis with FSI taken into account



Remarkable progress in neutrino-nucleus interaction theory in 2015-2016

QE

First ab initio calculation of electromagnetic response of ¹²C Lovato et al. (2016)

- Large 2b (MEC) enhancement
- Benchmark for other many-body models

RES

First coupled-channel calculation of neutrino-induced meson productions SXN et al. (2015)

- First two pion production model for whole RES region
- res and non-res interference are under control by maintaining PCAC

Possibly important FSI effect on pion production in v-deuteron reactions Wu et al. (2015)

- Large *NN* rescattering effect
- ANL and BNL bubble chamber data need reanalysis with FSI taken into account



Lots of recent works on QE-like mechanisms



Martini et al.'s answer based on mean field + RPA approach (PRC 80, 065501 (2009), ...)



np-nh (mostly 2p-2h) explains M_A puzzle

Lots of subsequent works

- Valencia model (mean field +RPA)
 Nieves et al., PRC 83 (2011), ...
- (phenomenological) Superscaling + MEC
 Amaro et al., PLB 696 (2011) , ...
 - ... and more

Integral transform technique

 $E_{\alpha\beta}(\tau,\vec{q}) = \int d\omega \, e^{-\omega\tau} R_{\alpha\beta}(\omega,\vec{q}) \qquad \text{(Euclidean response)}$

With the completeness of final states, $E_{\alpha\beta}$ can be expressed by ground-state expectation value that can be handled with GFMC method

$$E_{\alpha\beta}(\tau,\vec{q}) = \left\langle \Psi_0 \left| J_{\alpha}^{\dagger}(\vec{q}) e^{-(H-E_0)\tau} J_{\beta}(\vec{q}) \right| \Psi_0 \right\rangle$$

Inversion : $E_{\alpha\beta}(\tau, \vec{q}) \Rightarrow R_{\alpha\beta}(\omega, \vec{q})$ ill-posed problem

Maximum Entropy Method (MEM) can help

Jarrell and Gubernatis, Phys. Rept. 269, 133 (1996)

MEM can reconstruct (infer) information from incomplete and noisy data

Neutrino-nucleus scattering for v-oscillation experiments



 μ Neutrino flux

 v_{μ} survival probability (two-flavor case)

$$P(\nu_{\mu} \rightarrow \nu_{\mu}) = 1 - \sin^2 2\theta \sin^2 \left(\frac{1.27 \,\Delta m_{21}^2 L}{E_{\nu}}\right)$$

heta : mixing angle

$$\Delta m_{21}^2 \ (\text{eV}^2) = \ m_2^2 - m_1^2$$

L (km) : distance between v source and detector E_v (GeV) : neutrino energy

Next-generation exp. \rightarrow leptonic $\mathcal{Q}P$, mass hierarchy

 ν -nucleus scattering needs to be understood more precisely (~ 5%)

Neutrino-nucleus interactions need to be known for neutrino flux measurement

and then oscillation parameters ($heta_{
m ij}$, $\Delta m_{
m ij}{}^2$, $\delta_{
m CP}$) are determined

Elementary amplitude for *v*-induced 1π production

resonant only

Lalalulich et al. (2005), (2006)



+ non-resonant (tree-level)

Rein et al.(1981),(1987); Lalakulich et al. (2010); Serot et al.(2012) Hernandez et al. (2007), (2010), (2016) ; Alam et al. (2016)









+ rescattering (πN unitarity)

Sato, Lee (2003), (2005)

+



All elementary process models fit axial N- Δ (1232) coupling strength to bubble chamber data





In $\Delta(1232)$ region, all models can be made (more or less) consistent with the bubble chamber data

> but this is not the end of the story about elementary amplitude model !

Summary : FSI effects on v-deuteron reactions in Δ (1232) region

• Model for photo- and neutrino-induced reactions on the deuteron developed

impulse, $N\!N\,$ and $\pi\,N\,$ rescattering mechanisms

• Model predictions reasonably describe $\gamma d \rightarrow \pi NN$ data (soundness checked)

NN rescattering significantly reduces cross sections (orthogonality)

• Significant FSI effects on $\nu d \rightarrow \mu^- \pi^+ n p$ differential cross sections at certain kinematics

NN rescattering significantly reduces cross sections (orthogonality)

• $vd \rightarrow \mu^- \pi^+ n p$ total cross sections need to be calculated to analyze ANL and BNL bubble chamber data \rightarrow ongoing SXN, Kamano, Sato

Summary : DCC model for vN reactions in RES region

Start with DCC model for γN , $\pi N \rightarrow \pi N$, $\pi \pi N$, ηN , KA, $K\Sigma$

and $e^{-}-p$ & $e^{-}-n'$ reactions for $W \leq 2 \text{ GeV}$, $Q^2 \leq 3 (\text{GeV}/c)^2$

→ Development of vector current by isospin separation

and axial current for vN interaction; PCAC is maintained

Conclusion

- DCC model prediction is consistent with single pion production data
- Two pion production model for RES region developed for the first time
- Δ , N^* s, non-resonant are all important in few-GeV region (for $v_{\mu} n \rightarrow \mu^- X$)
 - → essential to understand interference pattern among them
 - \rightarrow DCC model can do this; consistency between π interaction and axial current

Neutrino interaction data in resonance region

 $v_{\mu}p \rightarrow \mu \bar{x}^{+}p$ (u) 1.4 + ANL 1.2 + BNL - GENIE 0.8 0.8 0.6 0.6 0.6 0.6 0.4 - 0.2 0.2 0.2 0.2 0.4 - 0.2 0.2 0.4 - 0.2 0.2 0.4 - 0.2 0.4 - 0.2

- Data to fix nucleon axial current ($g_{AN\Delta}$)
- Discrepancy between BNL & ANL data
- Recent reanalysis of original data
 - \rightarrow discrepancy resolved (!?)

PRD 90, 112017 (2014)



- Final state interaction (FSI) changes charge, momentum, number of π
- Cross section shape is worse described with FSI
- MINERvA data (PRD92,092008(2015)) favor FSI $\langle E_v \rangle = 4.0 \text{ GeV}, W < 1.4 \text{ GeV}$

More data are coming \rightarrow better understanding of neutrino-nucleus interaction

DCC (Dynamical Coupled-Channel) model

Matsuyama et al., Phys. Rep. **439**, 193 (2007) Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation

$$T_{ab} = V_{ab} + \sum_{c} V_{ac} G_{c} T_{cb}$$



essential for three-body unitarity

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Coupled-channel Lippmann-Schwinger equation

$$T_{ab} = V_{ab} + \sum_{c} V_{ac} G_{c} T_{cb}$$

Gc =

for stable channels



for unstable channels

Relation between neutrino and electron (photon) interactions

Charged-current (CC) interaction (e.g. $v_{\mu} + n \rightarrow \mu^{-} + p$)

$$L^{cc} = \frac{G_F V_{ud}}{\sqrt{2}} [J_{\lambda}^{cc} \ell_{cc}^{\lambda} + h.c.] \qquad J_{\lambda}^{cc} = V_{\lambda} - A_{\lambda} \qquad \ell_{cc}^{\lambda} = \overline{\psi}_{\mu} \gamma^{\lambda} (1 - \gamma_5) \psi_{\nu}$$

Electromagnetic interaction (e.g. $\gamma^{(*)} + p \rightarrow p$)

$$L^{em} = e J_{\lambda}^{em} A_{em}^{\lambda} \qquad \qquad J_{\lambda}^{em} = V_{\lambda} + V_{\lambda}^{IS}$$

V and *V*^{*IS*} in *J*^{*em*} can be separately determined by analyzing photon ($Q^2=0$) and electron reaction ($Q^2\neq 0$) data on both proton and neutron targets, because:

$$= - < n | V_{\lambda} | n > \qquad = < n | V_{\lambda}^{IS} | n >$$

Matrix element for the weak vector current is obtained from analyzing electromagnetic processes

$$= 2$$

Kamano's talk on Tuesday

Predicted $\pi N \rightarrow \pi \pi N$ total cross sections with our DCC model



Single π production in electron-proton scattering

Purpose : Determine Q^2 – dependence of vector coupling of p- N^* : $VpN^*(Q^2)$

 $\sigma_T + \varepsilon \sigma_L$ for $Q^2=0.40 (\text{GeV}/c)^2$ and W=1.1-1.68 GeV



 $p(e,e'\pi^0)p$

 $p(e,e'\pi^+)n$

 $\cos \theta_{\pi}^{*}$

 $\cos \theta_{\pi}^{*}$

DCC model for axial current

$Q^2 \neq 0$ $F_A(Q^2)$: axial form factors

non-resonant mechanisms

$$F_A(Q^2) = \left(\frac{1}{1+Q^2/M_A^2}\right)^2$$
 $M_A = 1.02 \text{ GeV}$

resonant mechanisms

$$F_A(Q^2) = \left(\frac{1}{1+Q^2/M_A^2}\right)^2$$

More neutrino data are necessary to fix axial form factors for ANN^*

Neutrino cross sections will be predicted with this axial current

Cross section for $v_{\mu} N \rightarrow \mu X$



- $\pi N \& \pi \pi N$ are main channels in few-GeV region
- DCC model gives predictions for all final states
- ηN , KY cross sections are $10^{-1} 10^{-2}$ smaller

Cross section for $v_{\mu} N \rightarrow \mu X$



- $\pi N \& \pi \pi N$ are main channels in few-GeV region
- DCC model gives predictions for all final states
- ηN , KY cross sections are $10^{-1} 10^{-2}$ smaller

$d\sigma/dW dQ^2$ (×10⁻³⁸ cm²/GeV²)

 $E_{v} = 2 \text{ GeV}$

 $v_{\mu}p \rightarrow \mu^{-}\pi^{+}p$





$d\sigma/dW dQ^2$ (×10⁻³⁸ cm²/GeV²)

 $E_{v} = 2 \text{ GeV}$





W (GeV)

W (GeV)

Q^2 – dependence

 $v_{\mu}p \rightarrow \mu \pi^{+} p$



Resonance region (single nucleon)





Multi-channel reaction

- 2π production is comparable to 1π
- η , K productions (v case: background of proton decay exp.)

Neutrino interaction data in resonance region

- Data to fix nucleon axial current ($g_{AN\Delta}$)
- Discrepancy between BNL & ANL data
- Recent reanalysis of original data
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PRD 90, 112017 (2014)



- Final state interaction (FSI) changes charge, momentum, number of π
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More data are coming \rightarrow better understanding of neutrino-nucleus interaction

"Δ" resonances (I=3/2)

Kamano, Nakamura, Lee, Sato, PRC 88 (2013)



Analysis of electron-proton scattering data

Purpose : Determine Q^2 – dependence of vector coupling of p- N^* : $VpN^*(Q^2)$

- Data : * 1π electroproduction
 - * Empirical inclusive inelastic structure functions σ_T , σ_L Christy et al, PRC 81 (2010)



Database

- $p(e,e'\pi^0)p$
- *p(e,e'π*+)*n*
- both

region where inclusive $\sigma_T \& \sigma_L$ are fitted

Analysis of electron-'neutron' scattering data

Purpose : Vector coupling of neutron- N^* and its Q^2 -dependence : $VnN^*(Q^2)$ (I=1/2) I=3/2 part has been fixed by proton target data

- Data : * 1π photoproduction (Q^2 =0)
 - * Empirical inclusive inelastic structure functions σ_T , σ_L ($Q^2 \neq 0$)
 - ← Christy and Bosted, PRC 77 (2010), 81 (2010)

Done

DCC vector currents has been tested by data for whole kinematical region relevant to neutrino interactions of $E_V \leq 2$ GeV

Formalism

Cross section for $vN \rightarrow lX$ ($X = \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma$)

$$\theta \to 0 \qquad \qquad \frac{d\sigma}{dE_\ell d\Omega_\ell} = \frac{G_F^2}{2\pi^2} E_\ell^2 \left(\frac{2W_1 \sin^2 \theta}{2} + W_2 \cos^2 \frac{\theta}{2} \pm W_3 \frac{E_\nu + E_\ell}{m_N} \sin^2 \frac{\theta}{2} \right)$$

$$Q^2 \rightarrow 0 \qquad W_2 = \frac{Q^2}{\vec{q}^2} \sum \left[\frac{1}{2} \left(|\langle J^x \rangle|^2 + |\langle J^y \rangle|^2 \right) + \frac{Q^2}{\vec{q}_c^2} \left| \left\langle J^0 + \frac{\omega_c}{Q^2} q \cdot J \right\rangle \right|^2 \right]$$

CVC & PCAC
$$\langle q \cdot J \rangle = \langle q \cdot V \rangle - \langle q \cdot A \rangle = i f_{\pi} m_{\pi}^2 \langle \hat{\pi} \rangle$$

LSZ & smoothness
$$\langle X|\hat{\pi}|N\rangle = \frac{\sqrt{2\omega_c}}{m_\pi^2} \mathcal{T}_{\pi N \to X}(0) \sim \frac{\sqrt{2\omega_c}}{m_\pi^2} \mathcal{T}_{\pi N \to X}(m_\pi^2)$$

Finally
$$F_2\equiv \omega W_2=rac{2f_\pi^2}{\pi}\sigma_{\pi N o X}$$
 $\sigma_{\pi N o X}$ is from our DCC model

Results



- Prediction based on model well tested by data (first $vN \rightarrow \pi\pi N$)
- πN dominates for $W \leq 1.5$ GeV
- $\pi\pi N$ becomes comparable to πN for $W \ge 1.5$ GeV
- Smaller contribution from ηN and $KY O(10^{-1}) O(10^{-2})$
- Agreement with SL (no PCAC) in Δ region

Comparison with Rein-Sehgal model



Comparison in whole kinematical region will be done after axial current model is developed

F₂ from RS model



SL model applied to v-nucleus scattering

 1π production

$$\nu_{e} + {}^{12}\text{C} \rightarrow e^{-} + X (E_{\nu} = 1 \text{ GeV}) \qquad e^{-} + {}^{12}\text{C} \rightarrow e^{-} + X (E_{e} = 1.1 \text{ GeV})$$

Szczerbinska et al. (2007)

SL model applied to v-nucleus scattering

coherent π production



Nakamura et al. (2010)
Eta production reactions

$$\pi^- p \to \eta n$$



KY production reactions

 $d\sigma/d\Omega$ (µb/sr)



 $\pi N \to \pi \pi N$

(parameters had been fitted to $\pi N \rightarrow \pi N$) Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC79 025206 (2009)



$d\sigma/d\Omega$ (µb/sr)

 $\gamma p \rightarrow \eta p$

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Vector current (Q²=0) for η Production is well-tested by data



 $\gamma N \to \pi \pi N$

(parameters had been fitted to $\pi N, \gamma N \rightarrow \pi N$) Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC80 065203 (2009)



DCC model for neutrino interaction

$$vN \rightarrow lX$$
 (X = πN , $\pi \pi N$, ηN , KA, K Σ)
at forward limit $Q^2=0$

Kamano, Nakamura, Lee, Sato, PRD 86 (2012)



$$\frac{d\sigma}{dE_\ell d\Omega_\ell} = \frac{G_F^2}{2\pi^2} E_\ell^2 W_2$$

via PCAC $F_2 \equiv \omega W_2 = \frac{2f_\pi^2}{\pi}\sigma_{\pi N \to X}$

 $\sigma_{\pi N \twoheadrightarrow X}$ is from our DCC model



F₂(Q²=0) from DCC model and PCAC



DCC model keeps good consistency with PCAC

Comparison with LPP model

LPP model : Lalakulich et al, PRD 74 (2006)



- Large difference beyond $\Delta(1232)$ region
- Importance of consistency between axial-current and πN interaction

Analysis result (single π)

 $Q^2 = 0$

$d\sigma / d\Omega$ ($\gamma n \rightarrow \pi p$) for W=1.1-2.0 GeV



Analysis result (inclusive *e*⁻-*d*)



Data: NP Proc. Suppl. 159, 163 (2006)

- Our calculation : $[\sigma(e^--p) + \sigma(e^--n)]/2$
- Too sharp resonant peaks \rightarrow fermi motion smearing, other nuclear effects needed
- Reasonable starting point for application to neutrino interactions

For application to neutrino interactions

Analysis of electron scattering data

→ $VpN^*(Q^2)$ & $VnN^*(Q^2)$ fixed for several Q^2 values

→ Parameterize $VpN^*(Q^2)$ & $VnN^*(Q^2)$ with simple analytic function of Q^2

$$I=3/2 \qquad : \quad VpN^*(Q^2) = VnN^*(Q^2) \qquad \Rightarrow \text{ CC, NC}$$

$$I=1/2 \text{ isovector part} : \quad (VpN^*(Q^2) - VnN^*(Q^2))/2 \qquad \Rightarrow \text{ CC, NC}$$

$$I=1/2 \text{ isoscalar part} : \quad (VpN^*(Q^2) + VnN^*(Q^2))/2 \qquad \Rightarrow \text{ NC}$$

DCC vector currents has been tested by data for whole kinematical region relevant to neutrino interactions of $E_v \le 2$ GeV

Analysis result (single π)

 Q^2 =1.76 (GeV/c)²

 $\sigma_T + \varepsilon \sigma_L$ for W=1.10-2.01 GeV

 $p(e,e'\pi^+)n$







Analysis result (single π)

 Q^2 =2.91-3.00 (GeV/c)²

 $\sigma_T + \varepsilon \sigma_L$ for W=1.10-1.67 GeV



$p(e,e'\pi^+)n$



