A Novel Technique for Bayesian Model Selection

Dark Energy

-- From WMAP to Planck --

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Kavli Institute for Cosmology Cambridge
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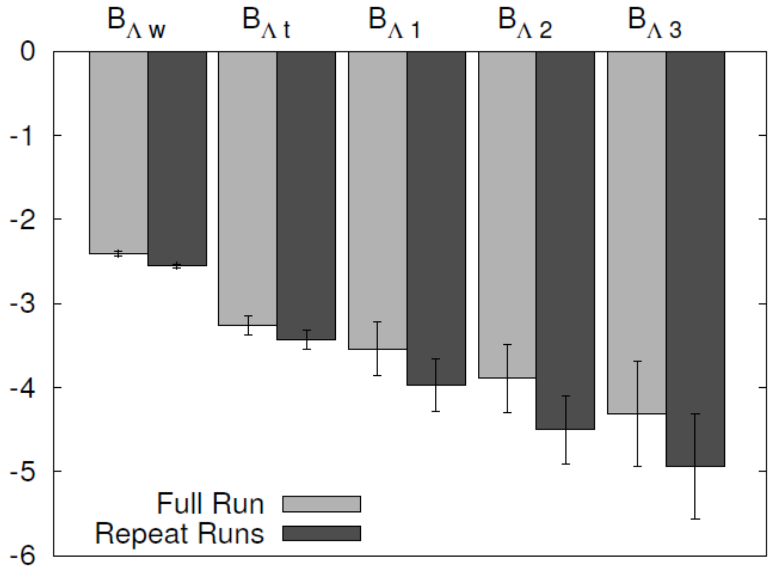
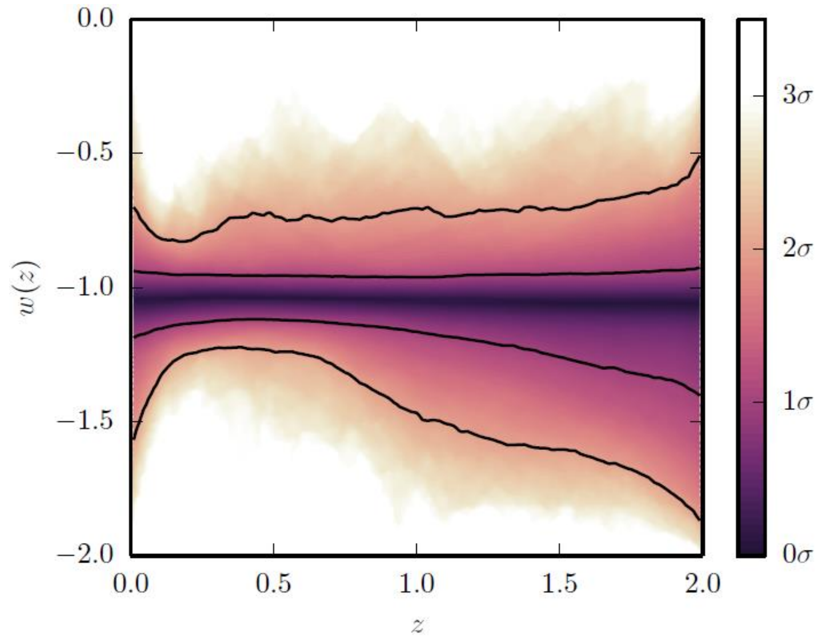
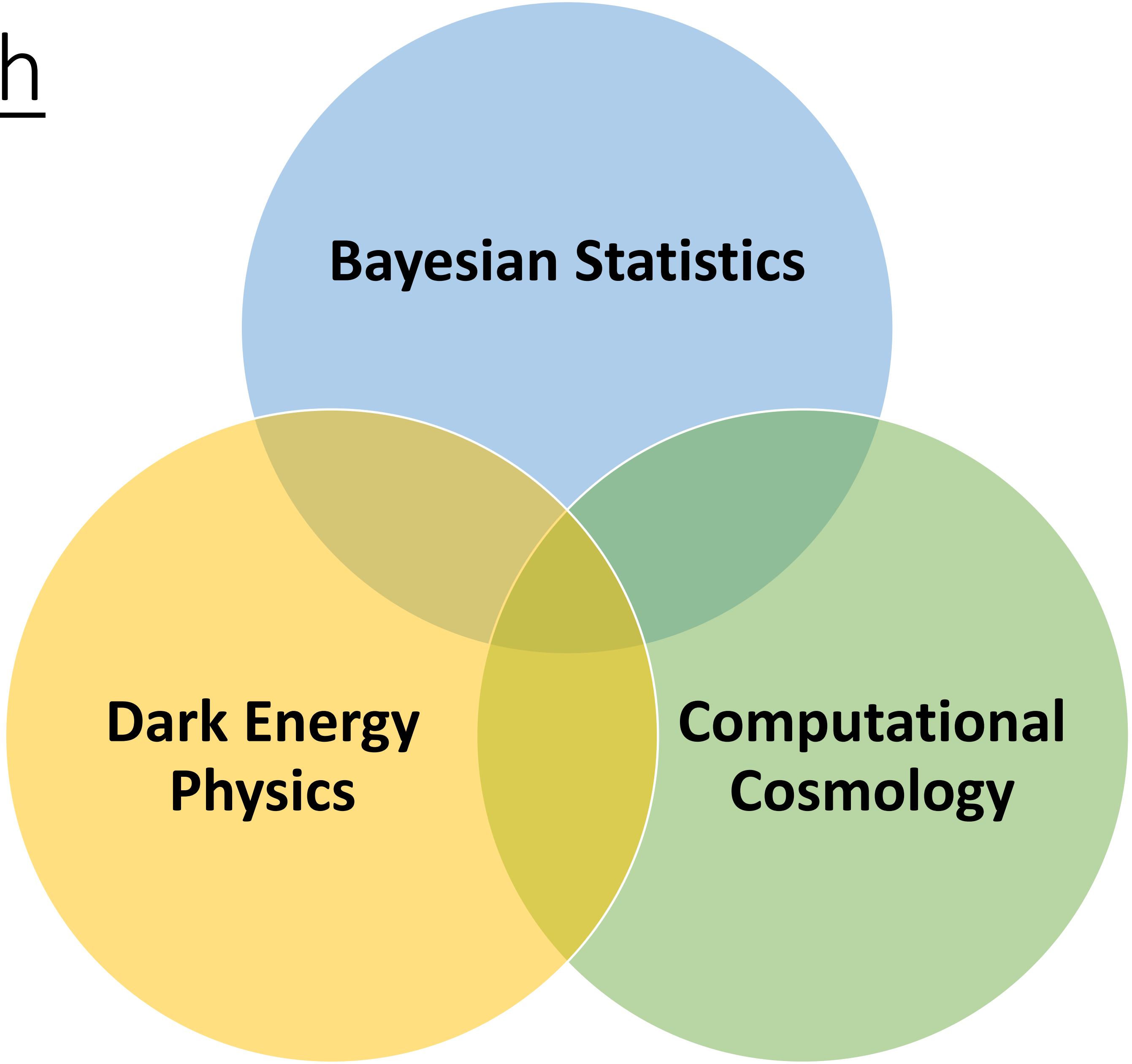


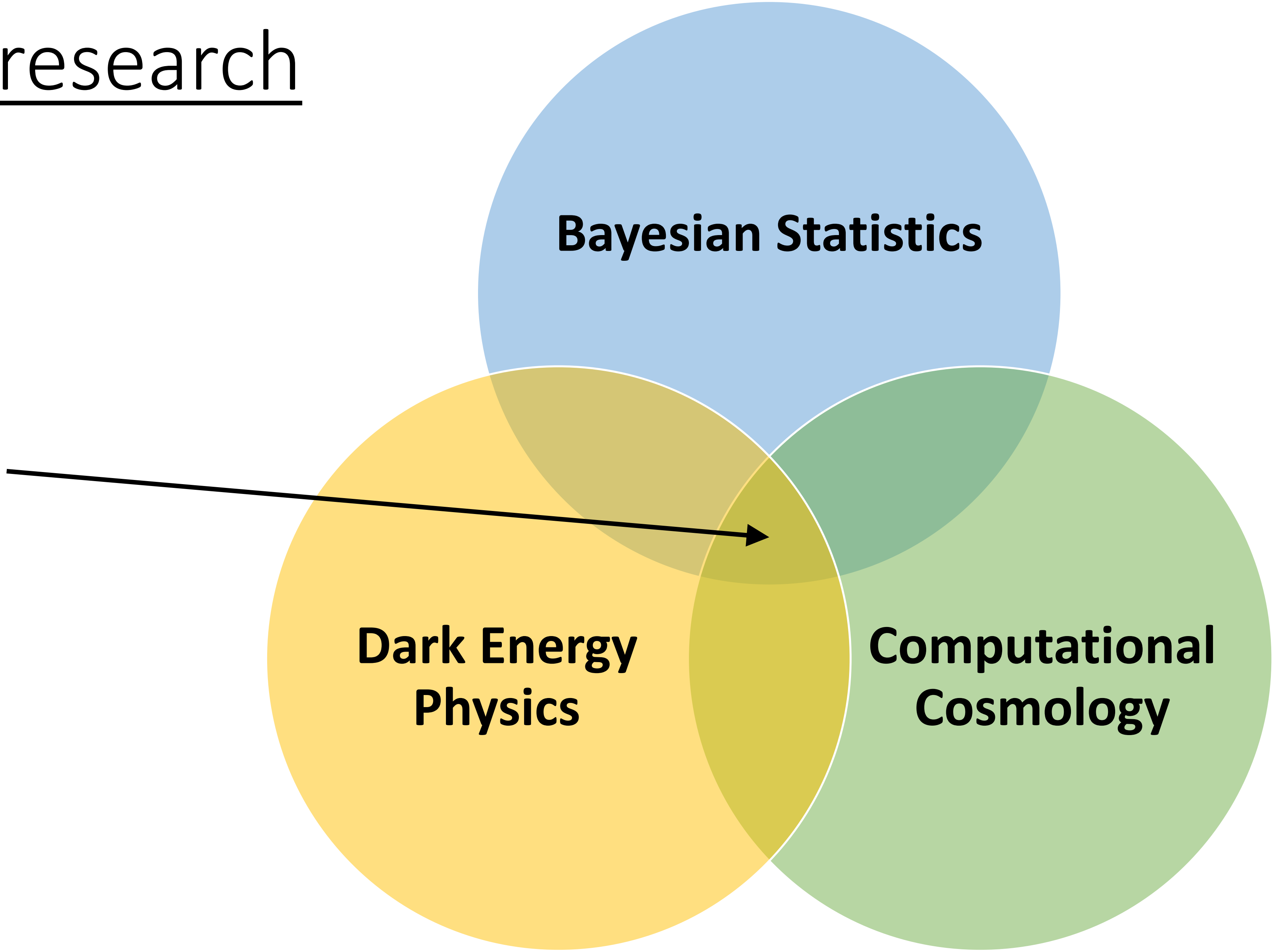


Where is this research taking place?

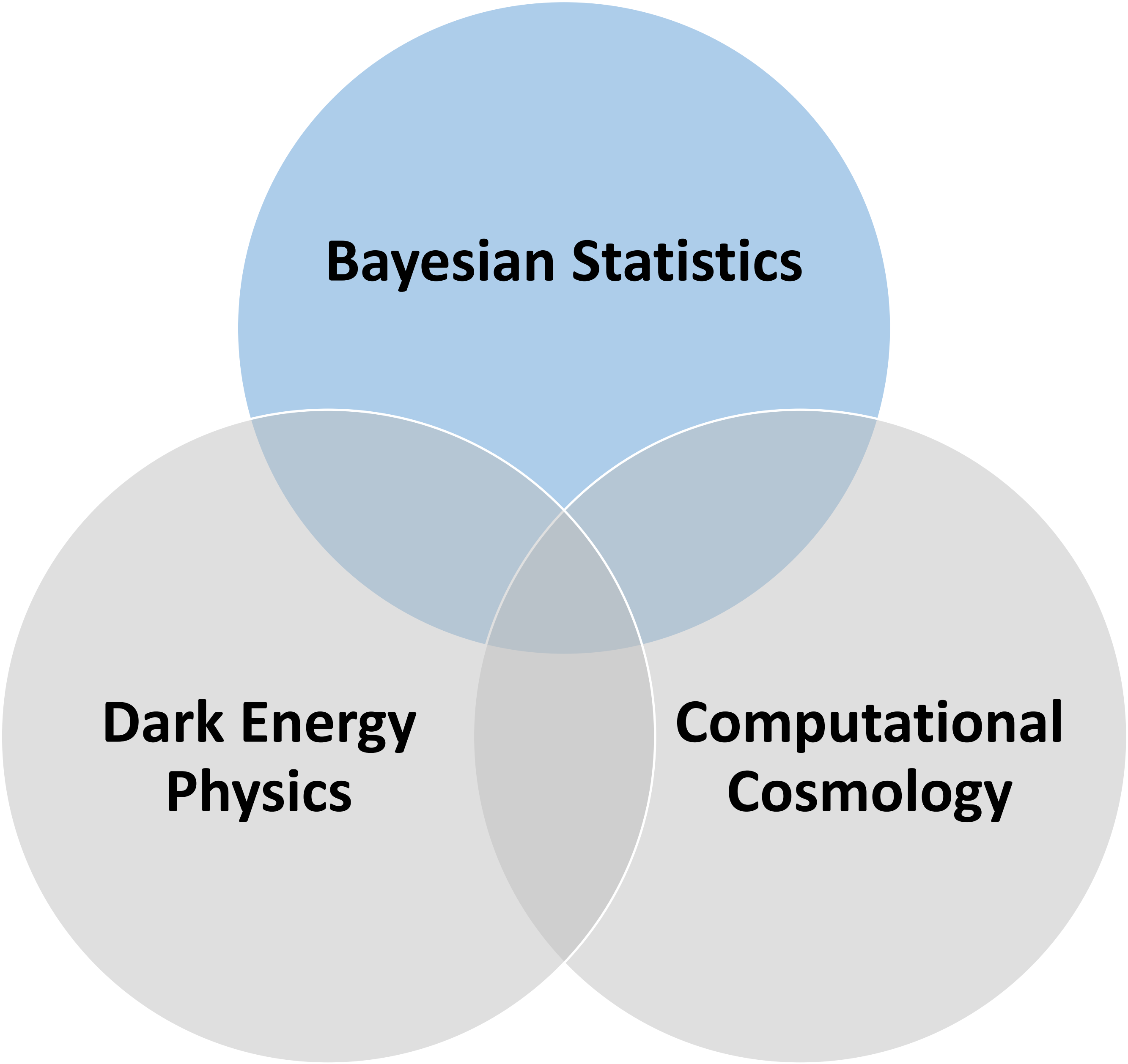


Where is this research taking place?

Here!



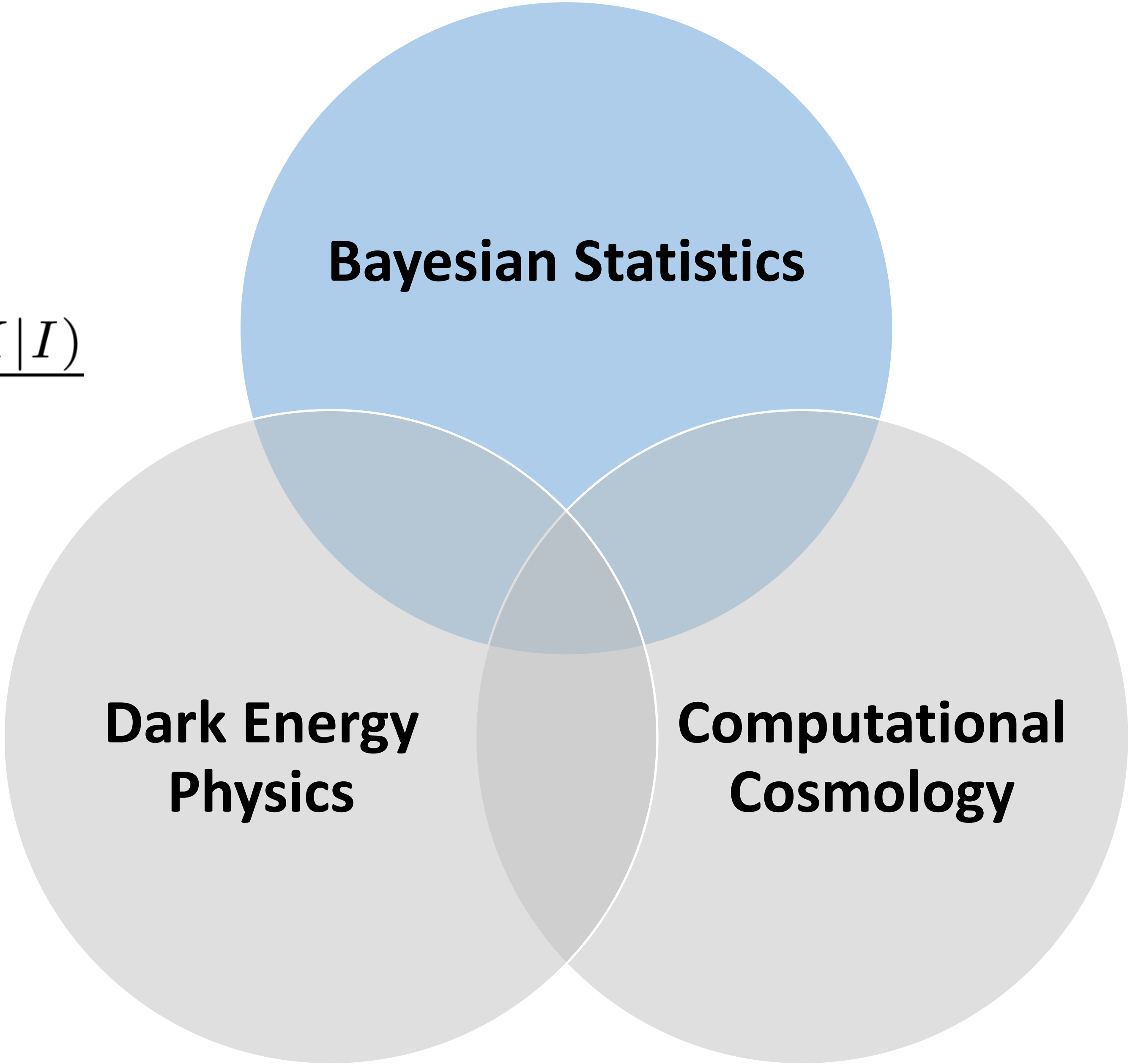
Bayesian Statistics



Bayesian Statistics

Bayes Theorem:

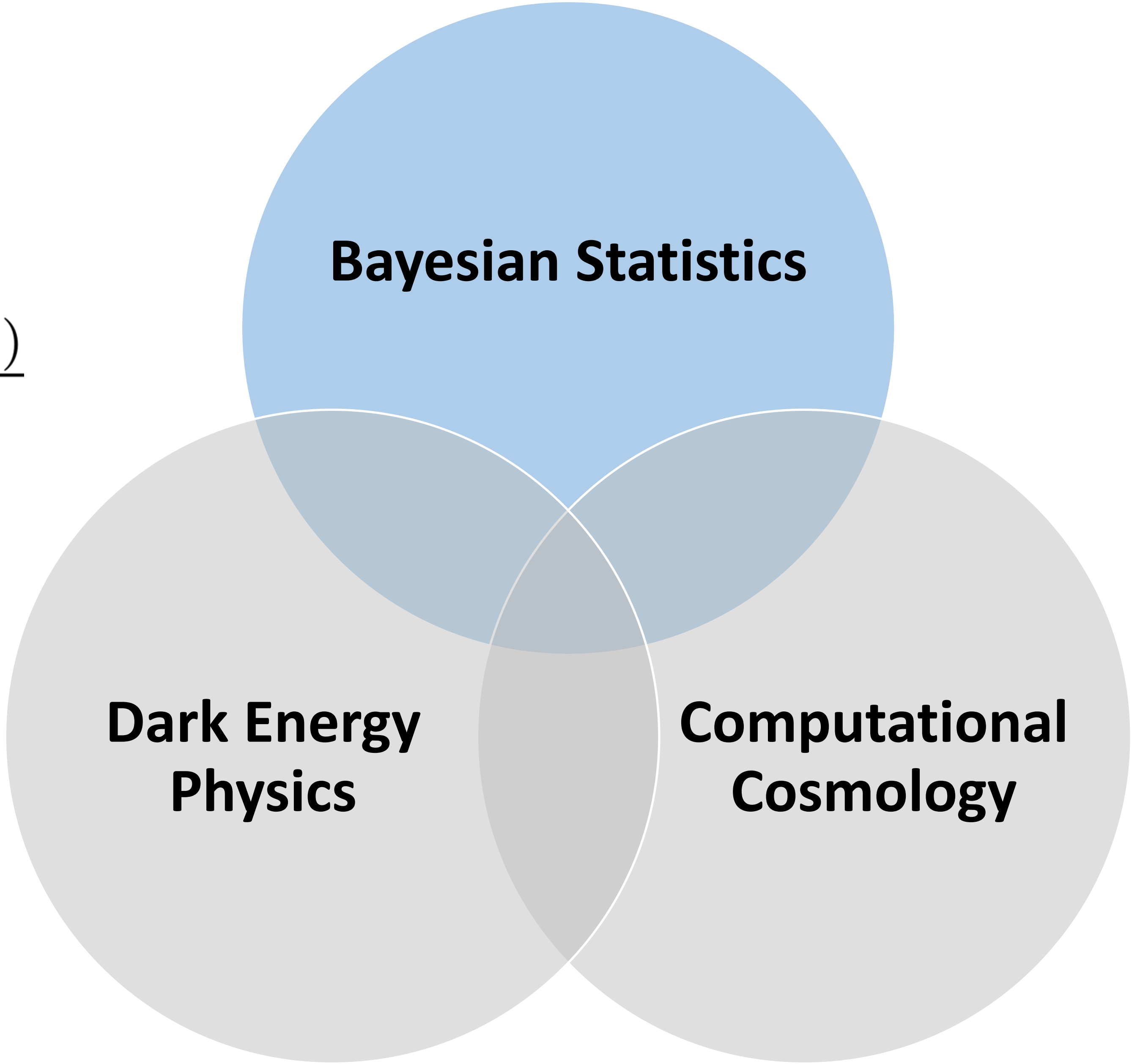
$$Pr(X|Y,I) = \frac{Pr(Y|X,I) \times Pr(X|I)}{Pr(Y|I)}$$



Bayesian Statistics

(1)Parameter Estimation:

$$Pr(\theta|\mathcal{D}, I) = \frac{Pr(\mathcal{D}|\theta,I)\times Pr(\theta|I)}{Pr(\mathcal{D}|I)}$$



Bayesian Statistics

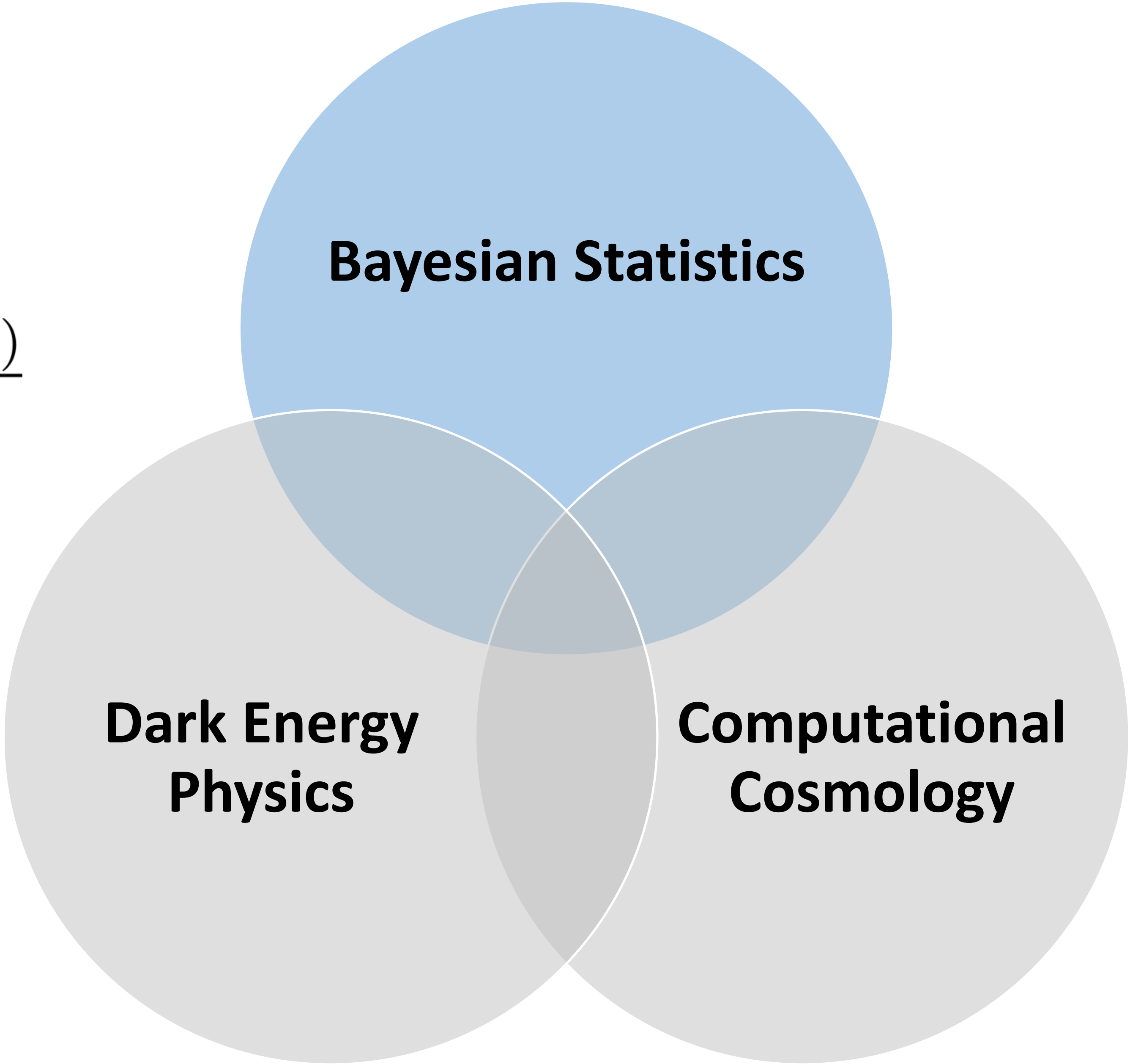
(1)Parameter Estimation:

$$Pr(\theta|\mathcal{D},I) = \frac{Pr(\mathcal{D}|\theta,I)\times Pr(\theta|I)}{Pr(\mathcal{D}|I)}$$

(2)Model Selection

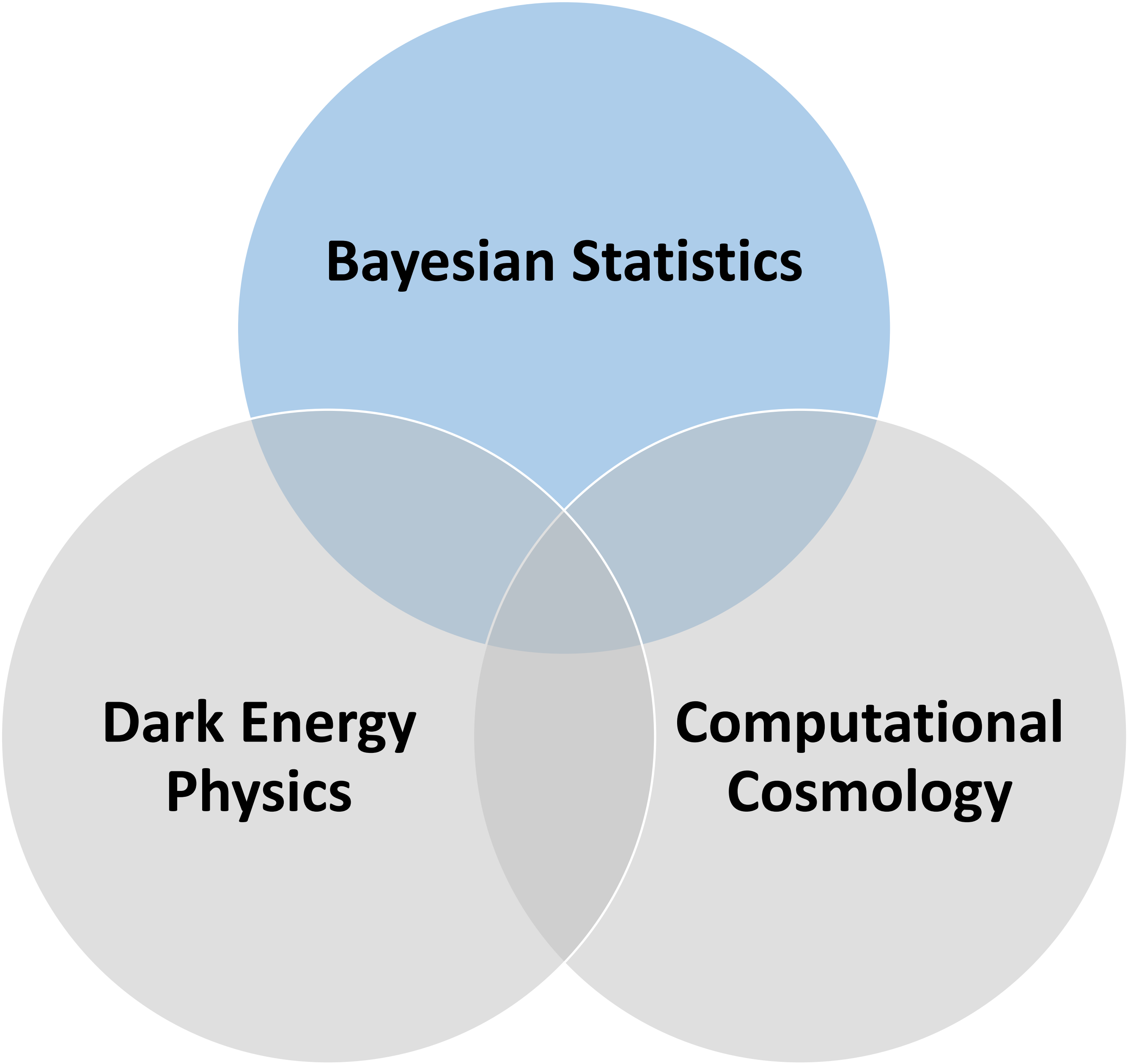
$$\mathcal{B}_{ij} = \ln(\frac{Pr(\mathcal{M}_j|\mathcal{D})}{Pr(\mathcal{M}_i|\mathcal{D})}) = \ln(\frac{\mathcal{Z}_i}{\mathcal{Z}_j})$$

$\mathcal{B}_{ij} = \text{Bayes Factor}$
$\mathcal{M}_i = \text{Model } i$
$\mathcal{D} = \text{Data}$
$\mathcal{Z} = Pr(\mathcal{D}|I) = \text{'Evidence'}$



Bayesian Statistics

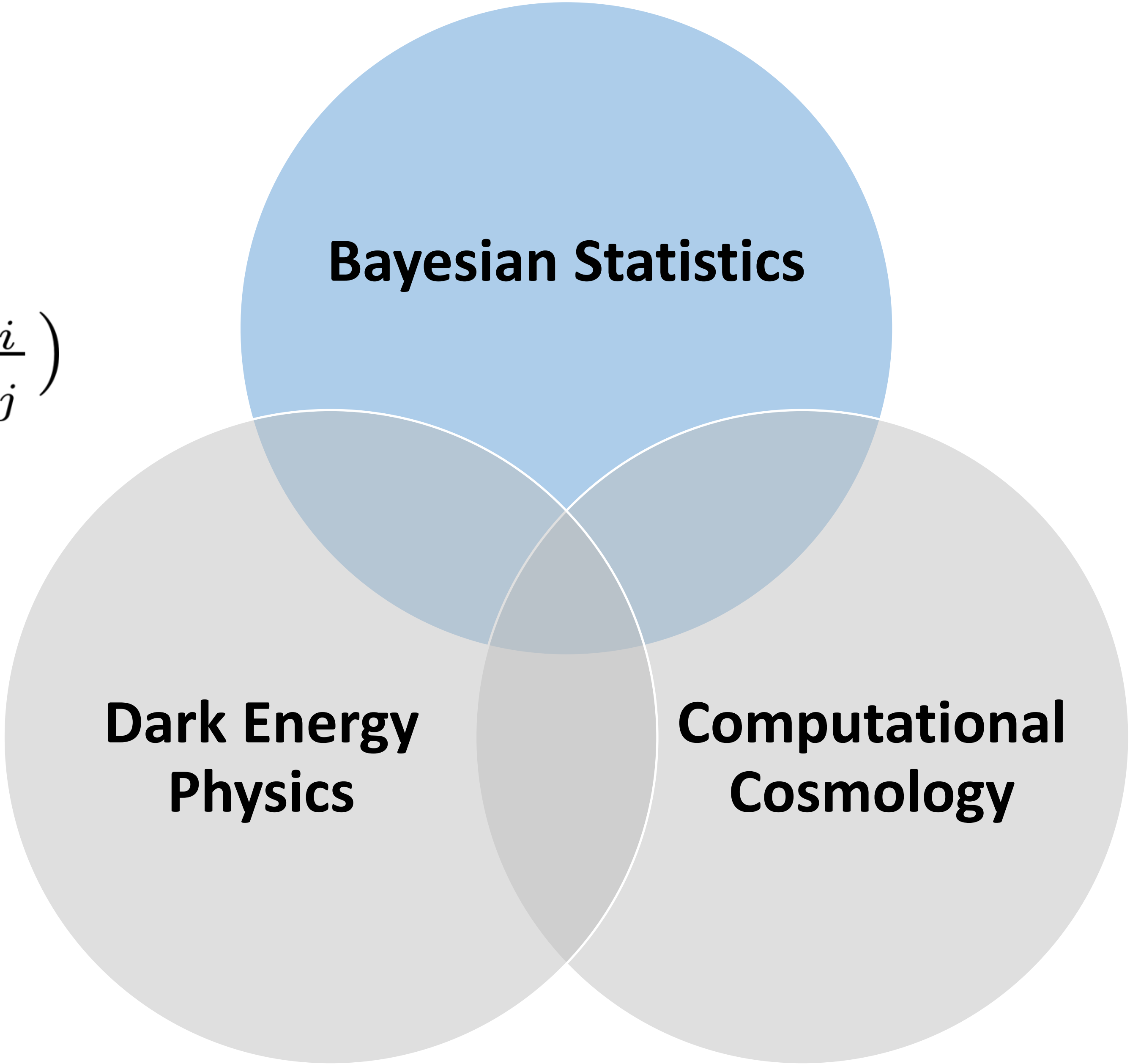
$$\mathcal{Z} = \int_{all\ \theta} \mathcal{L}(\theta)\pi(\theta)\ d\theta$$



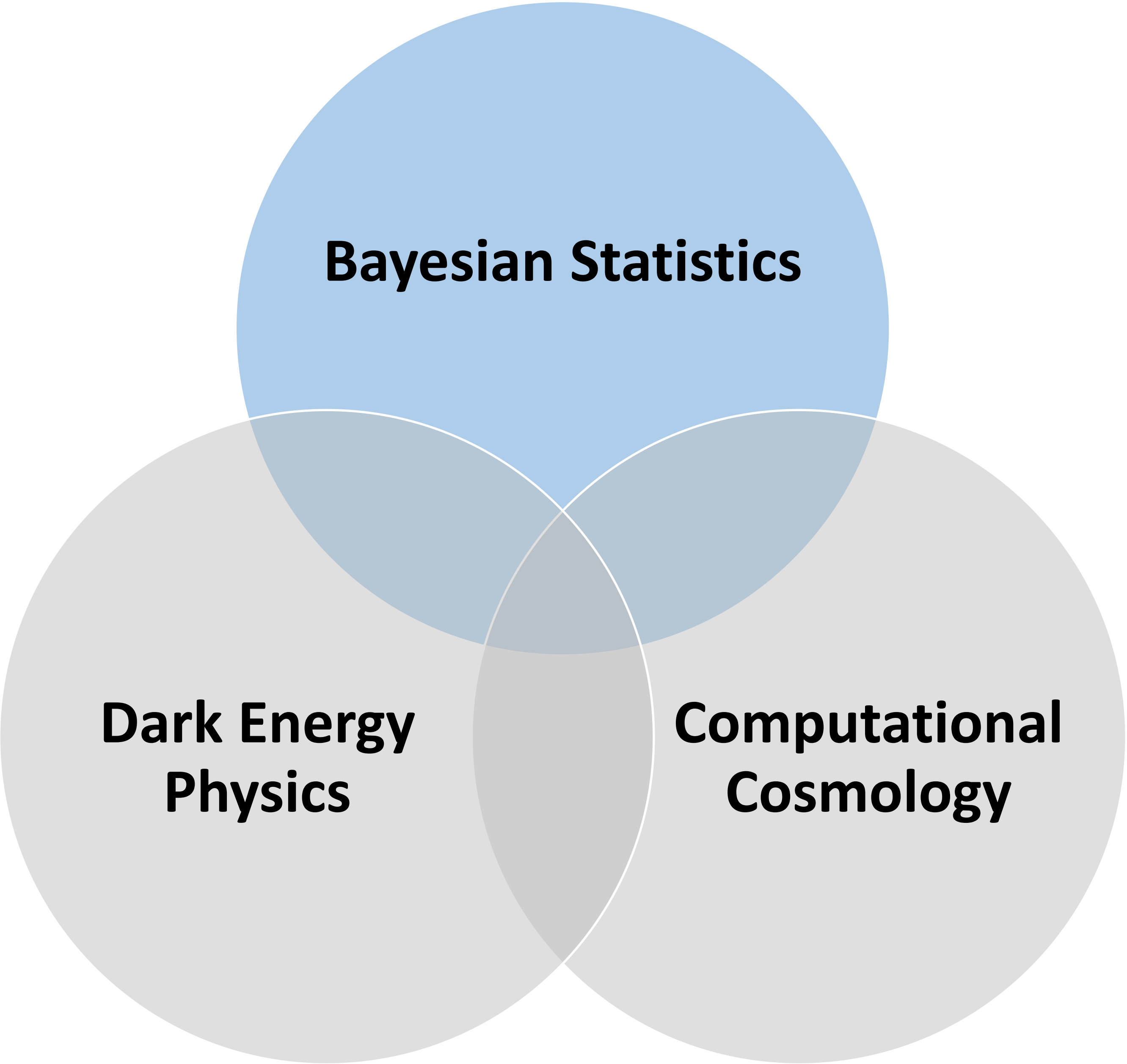
Bayesian Statistics

$$\mathcal{B}_{ij} = \ln\left(\frac{Pr(\mathcal{M}_j|\mathcal{D})}{Pr(\mathcal{M}_i|\mathcal{D})}\right) = \ln\left(\frac{\mathcal{Z}_i}{\mathcal{Z}_j}\right)$$

Bayes factor	Interpretation
$0.0 \leq \mathcal{B}_{ij} < 1.0$	Not significant
$1.0 \leq \mathcal{B}_{ij} < 2.5$	Slight
$2.5 \leq \mathcal{B}_{ij} < 5.0$	Strong
$5.0 \leq \mathcal{B}_{ij}$	Decisive

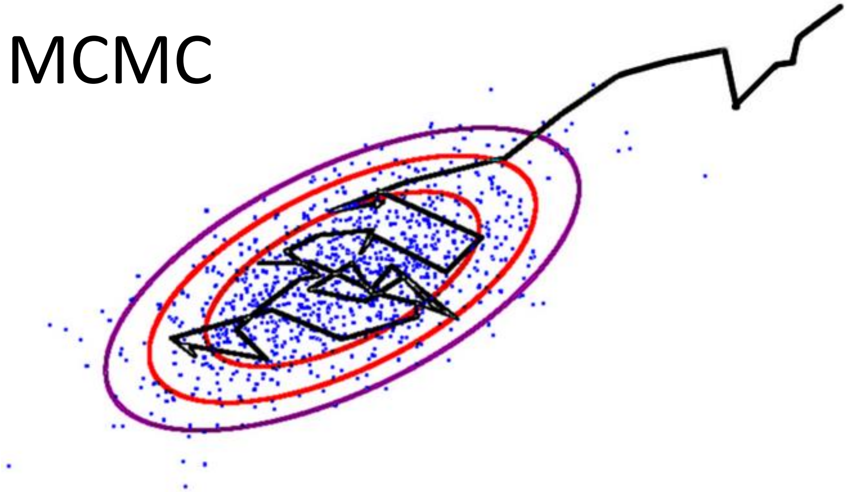


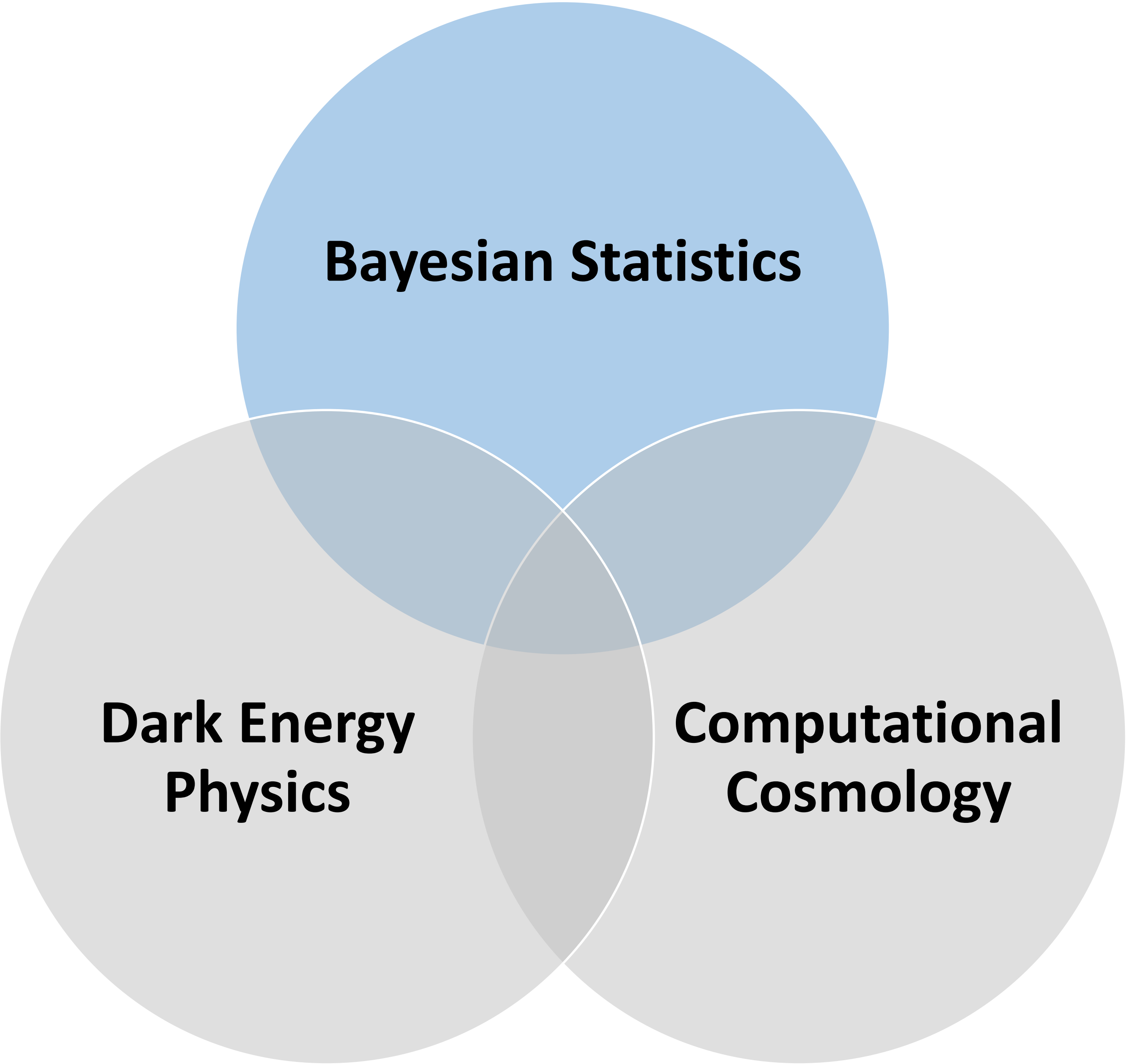
Bayesian Samplers



Bayesian Samplers

MCMC





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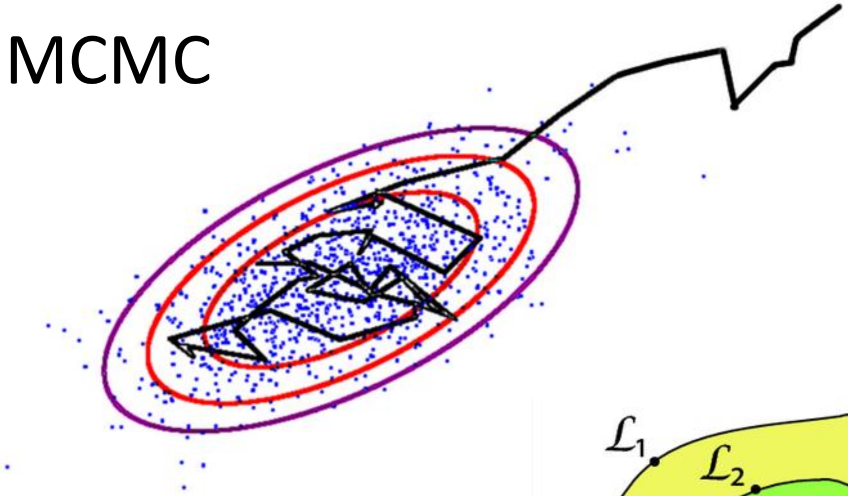
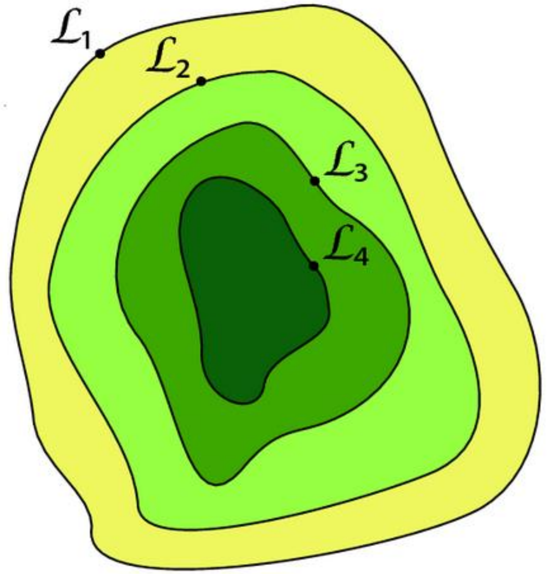
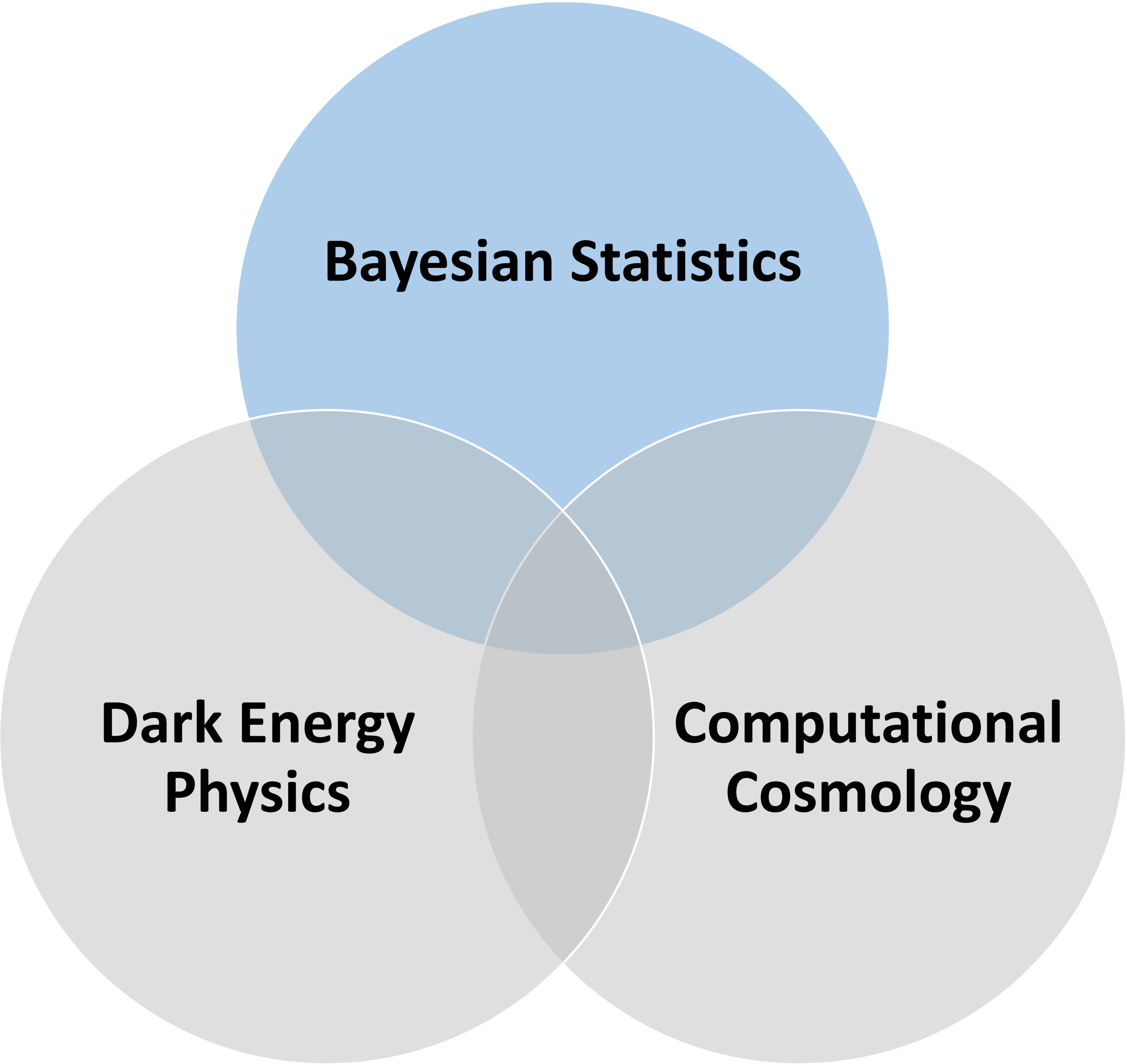
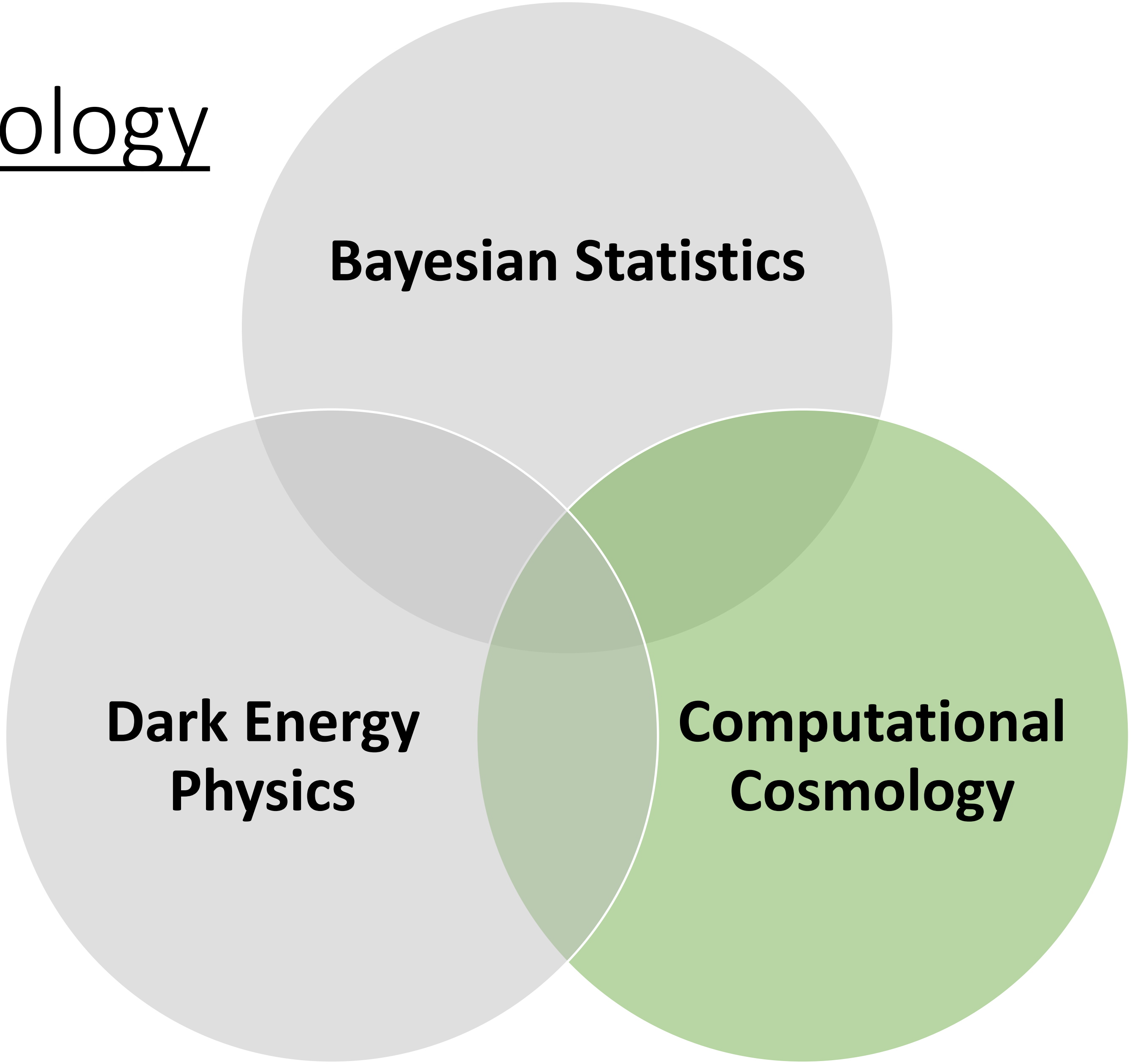


Nested Sampling



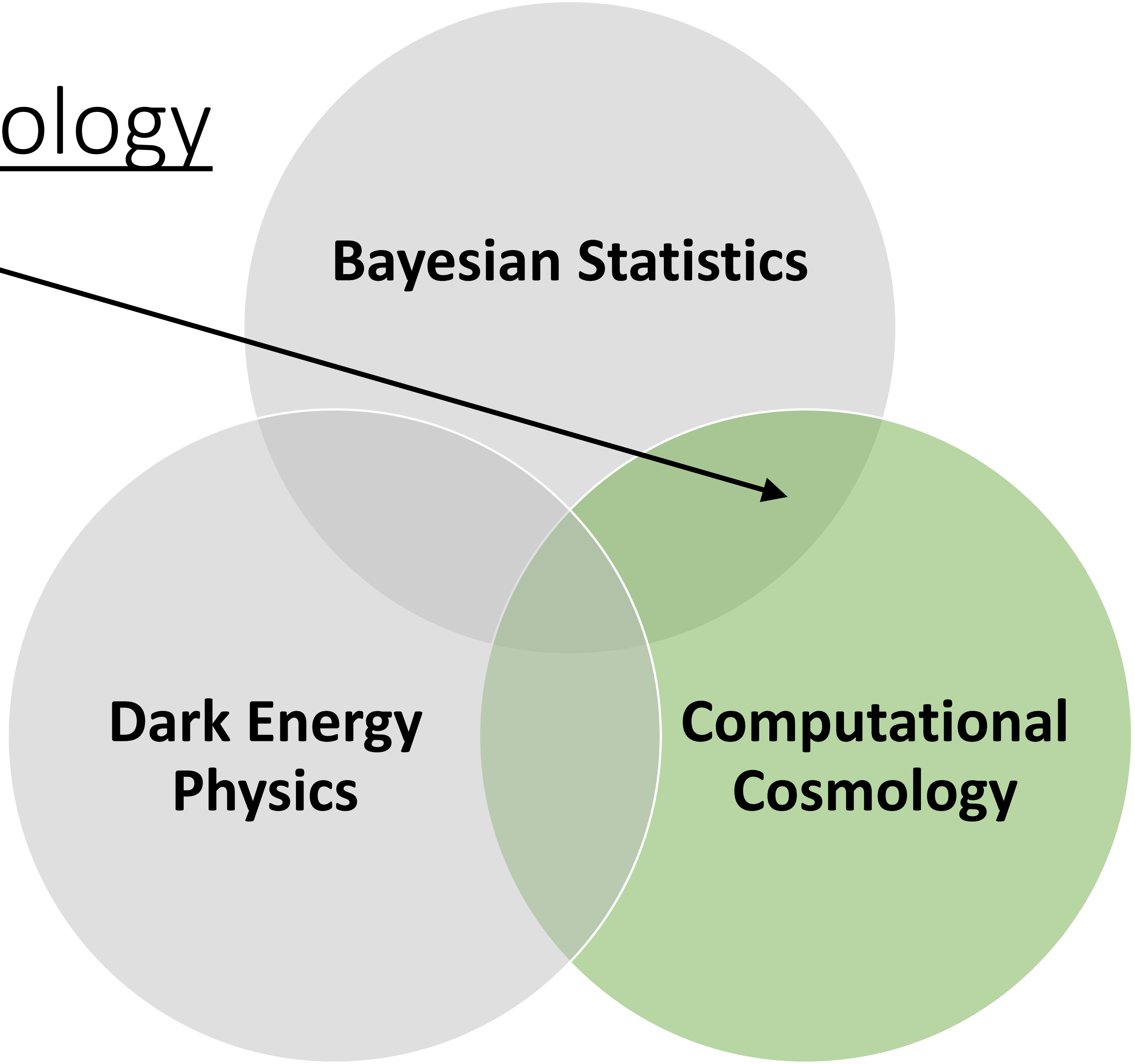


Computational Cosmology



Computational Cosmology

COSMOMC [1]

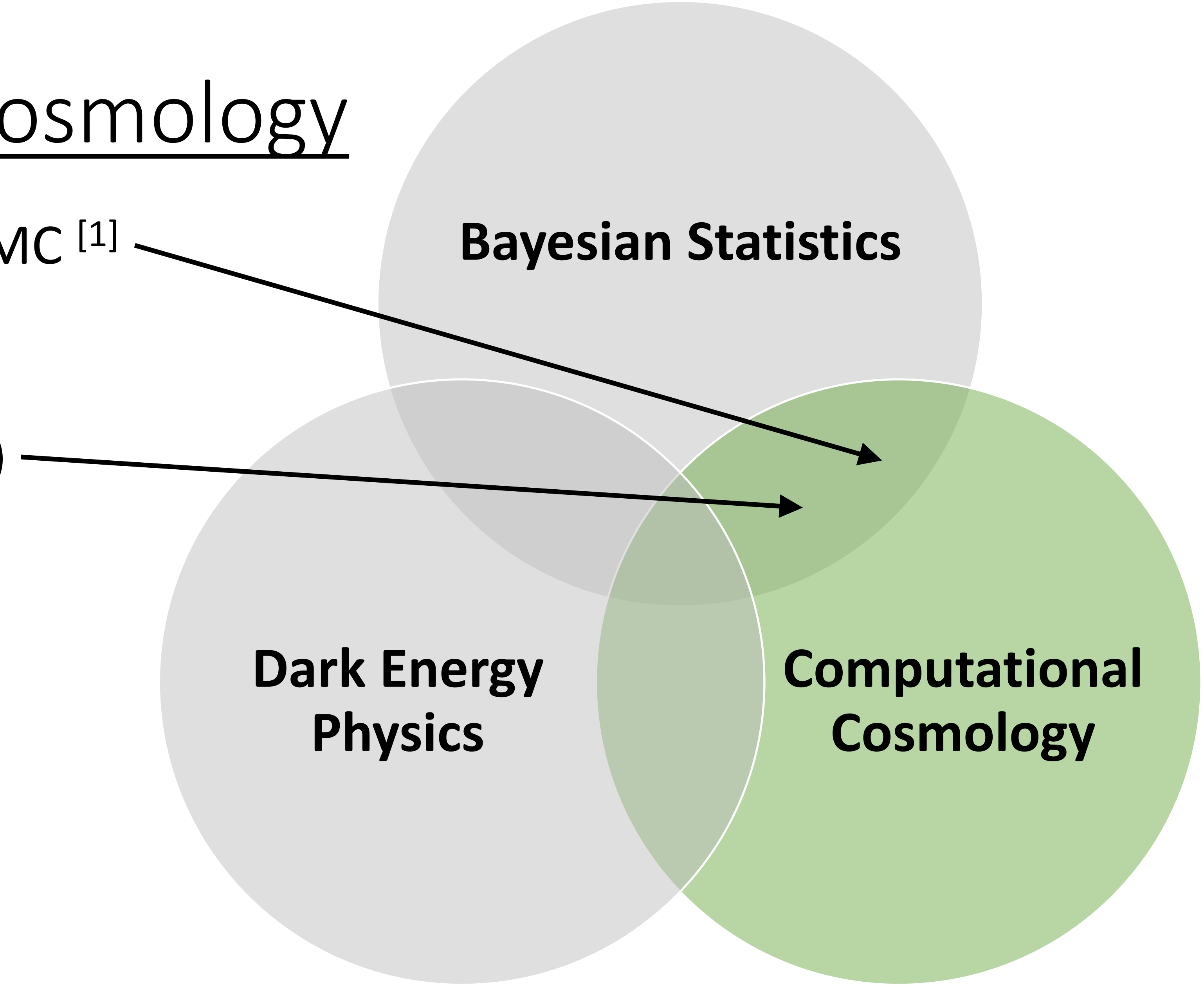


[1] Lewis A., Bridle S., 2002, Phys. Rev. D, 66, 103511

Computational Cosmology

COSMOMC [1]

Likelihood codes (the data)



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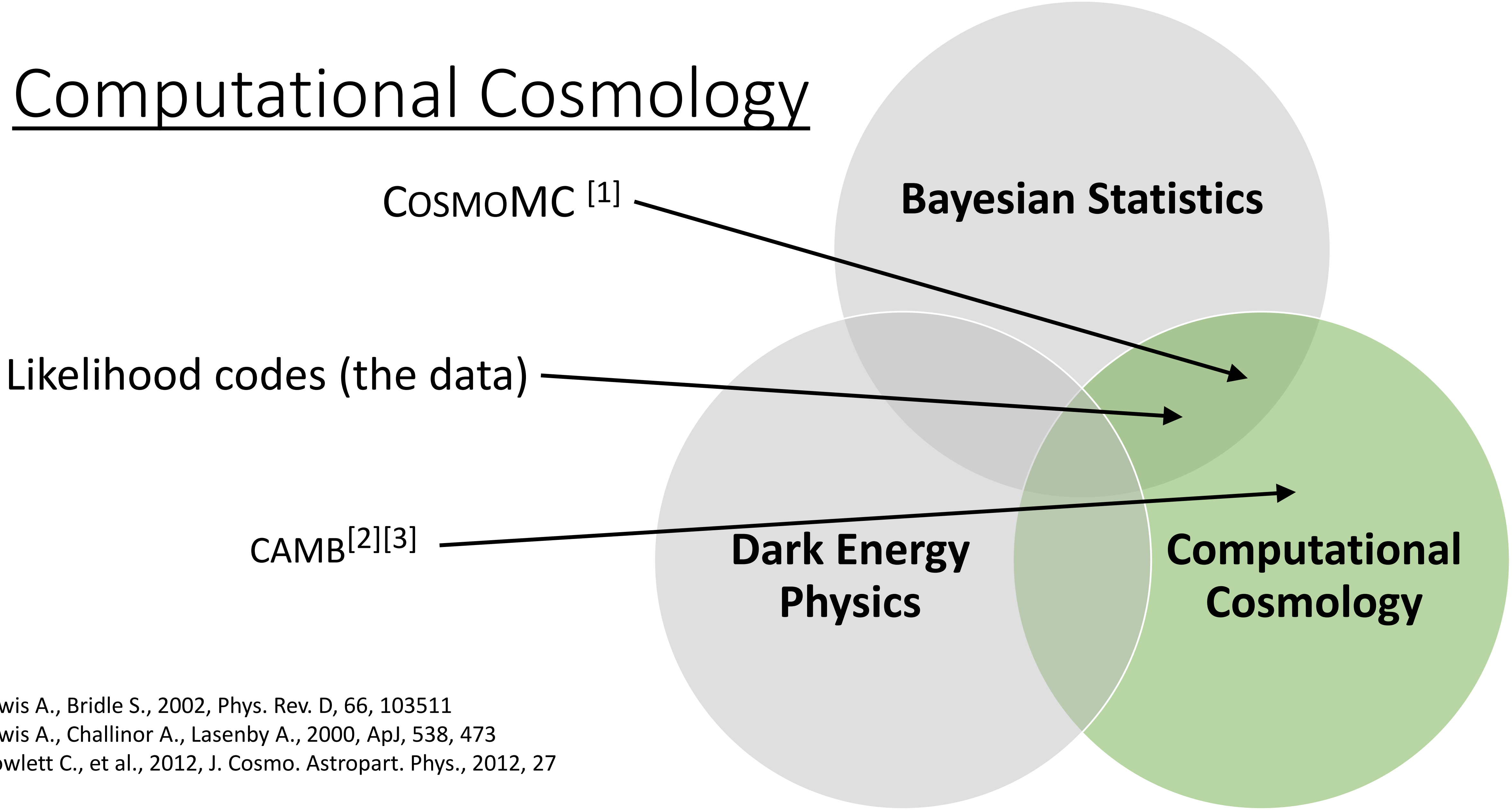
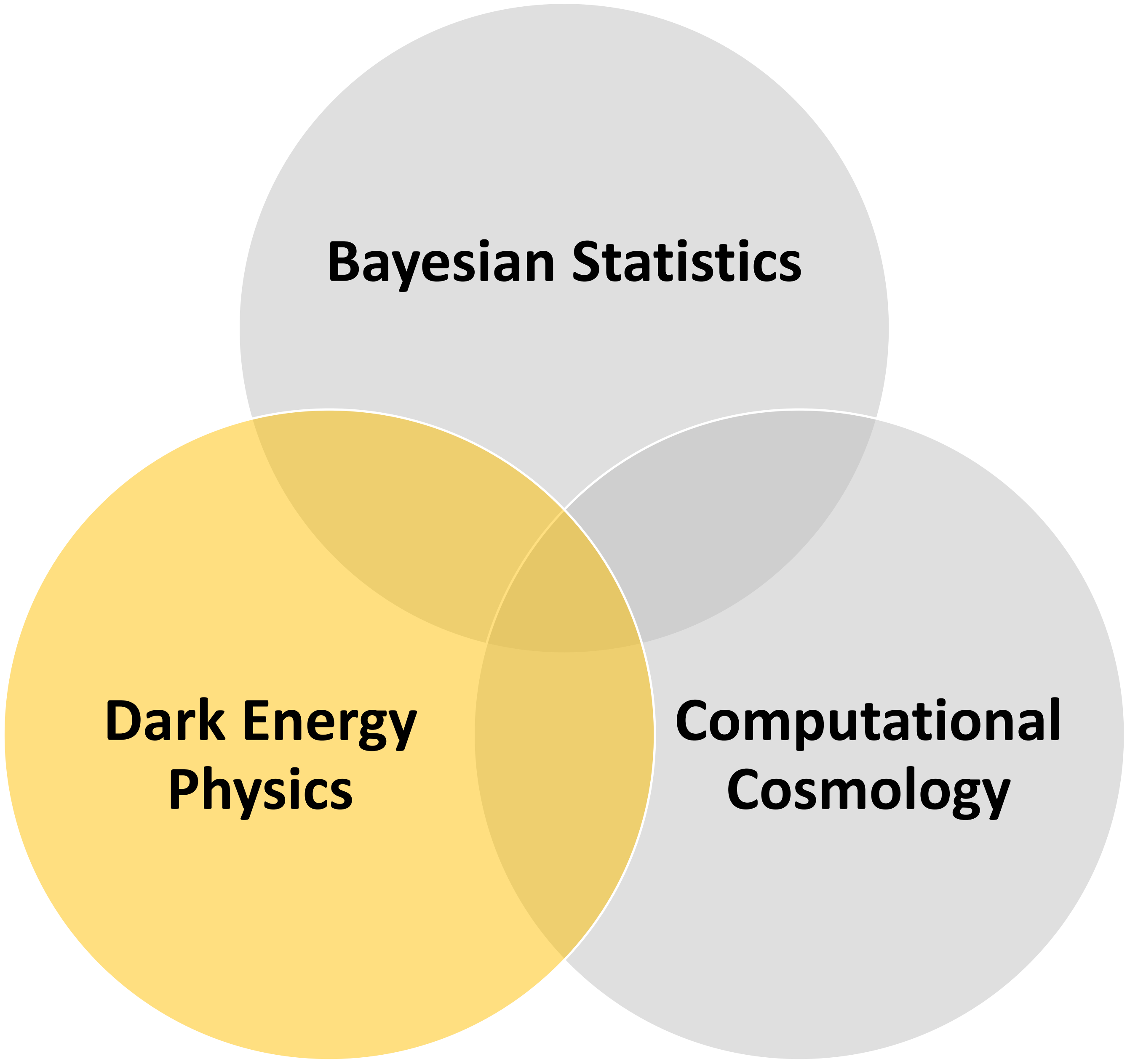
Computational Cosmology



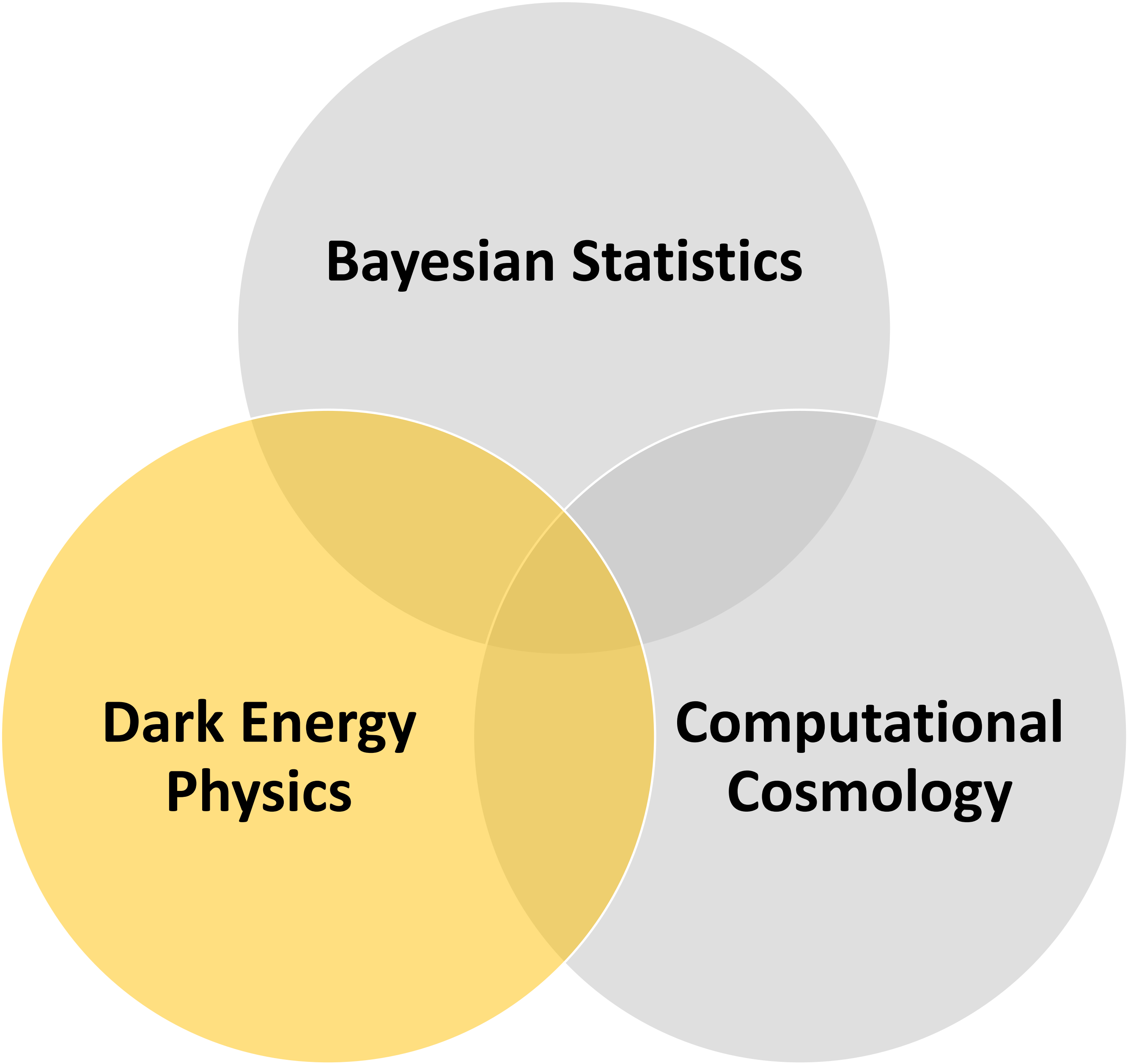
[1] Lewis A., Bridle S., 2002, Phys. Rev. D, 66, 103511
[2] Lewis A., Challinor A., Lasenby A., 2000, ApJ, 538, 473
[3] Howlett C., et al., 2012, J. Cosmo. Astropart. Phys., 2012, 27

Dark Energy Physics



Dark Energy Physics

LCDM?
Quintessence?
Phantom?



Dark Energy Physics

LCDM?
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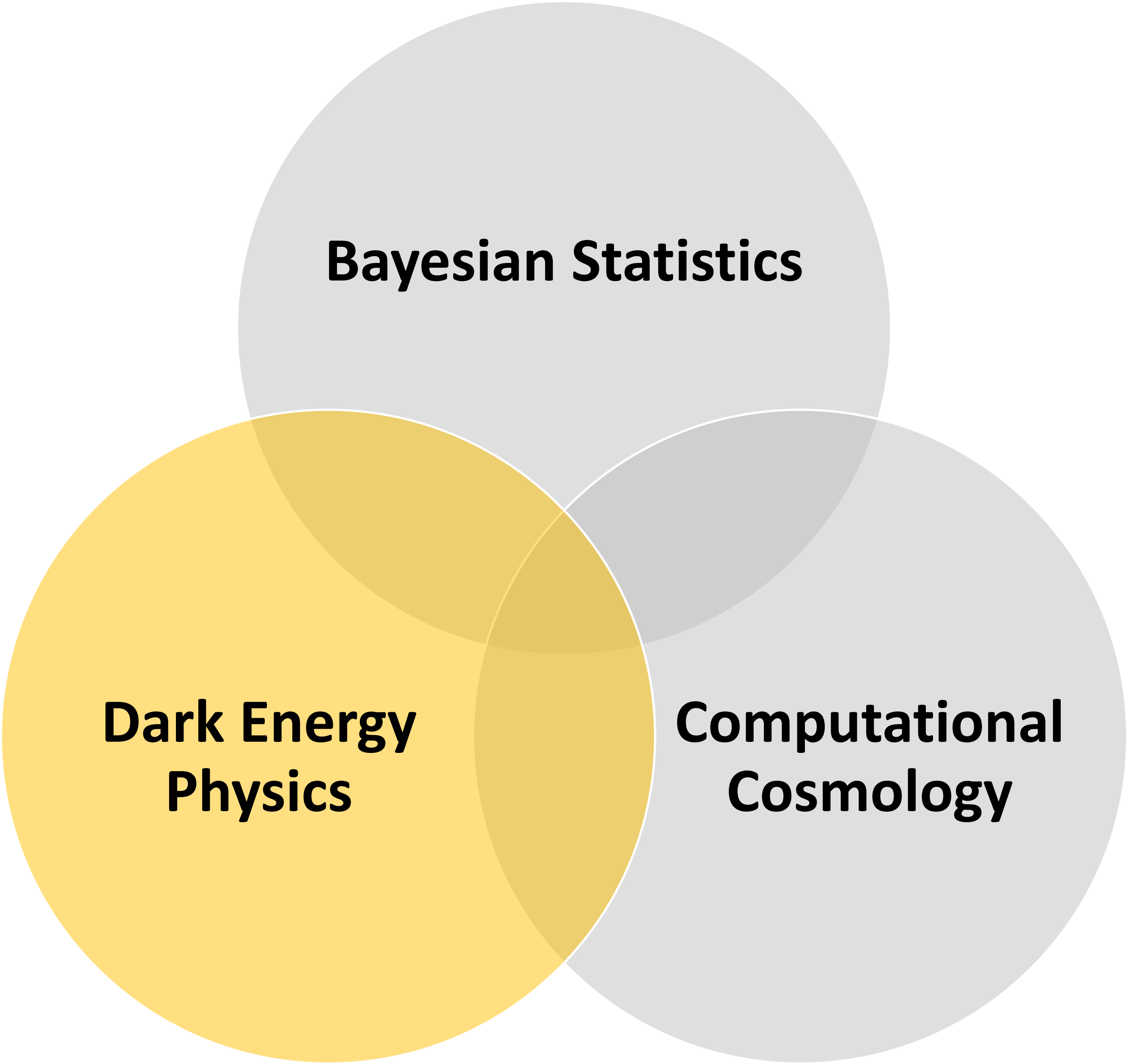
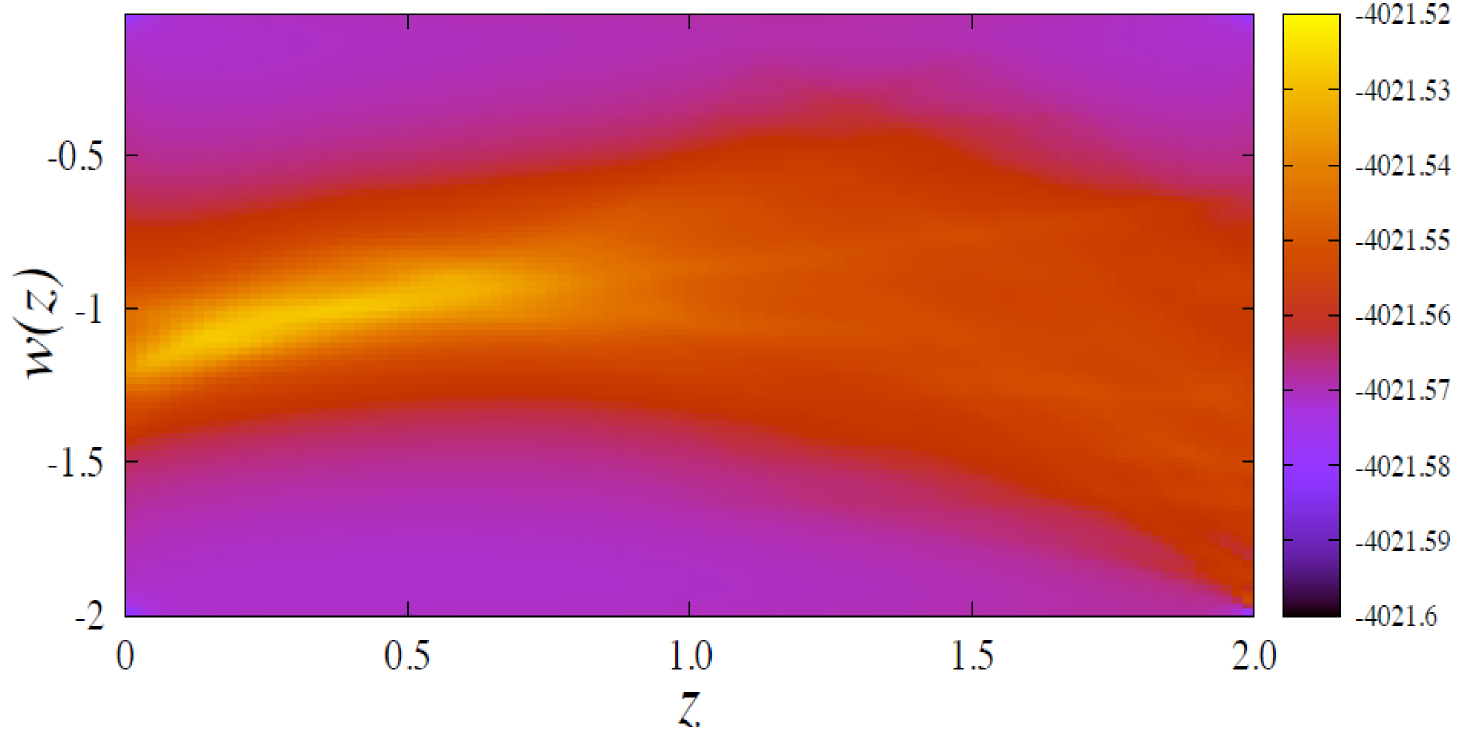
We investigate $w(z)$

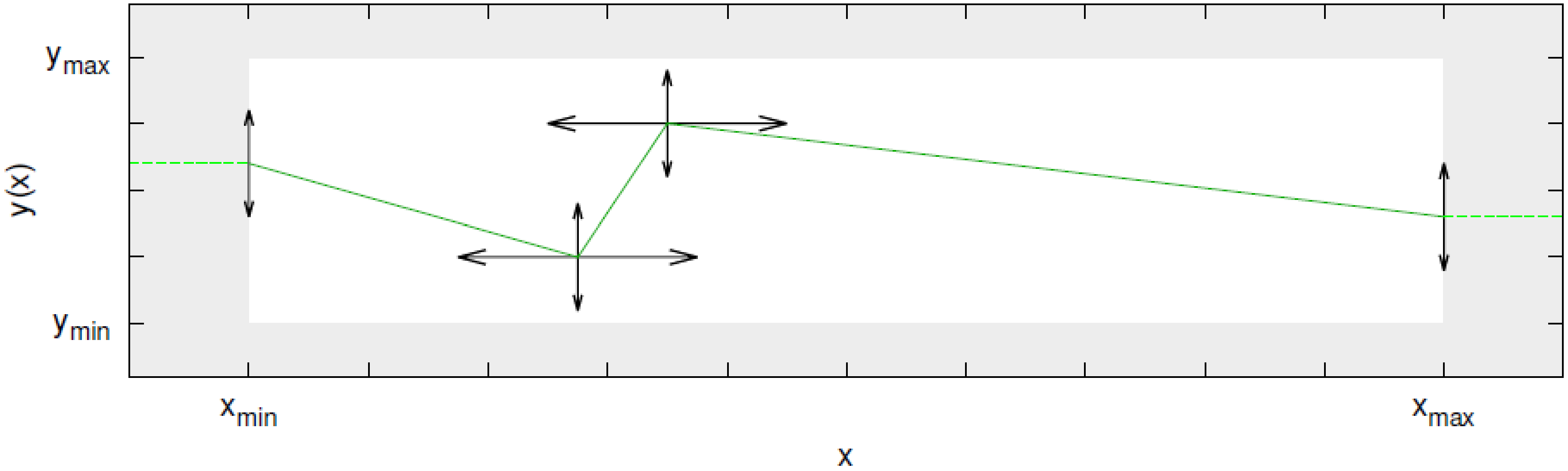


The Problem we are Investigating



J.A. Vázquez, M. Bridges, M.P. Hobson and A.N. Lasenby (2012) “Reconstruction of the dark energy equation of state”. JCAP, 09, 20. (arxiv 1205.0847)

Previous Results - WMAP



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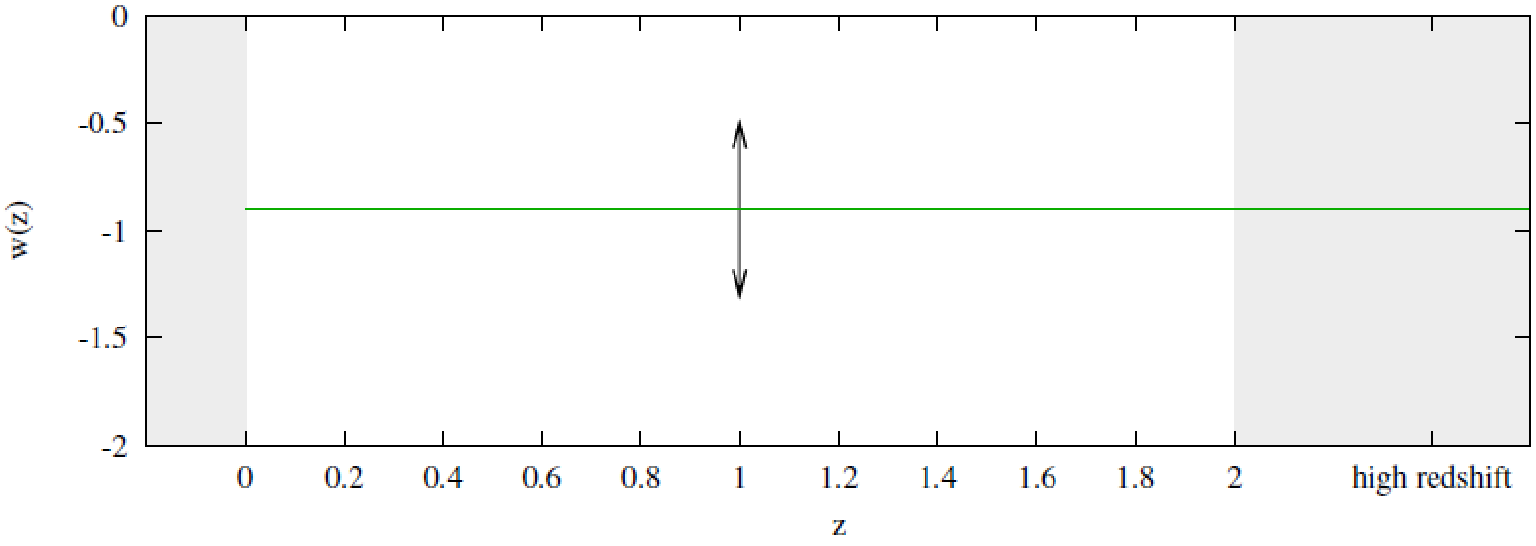

Model (number of nodes)	Description
wCDM (1)	Constant w(z)
tCDM (2)	Tilted w(z)
1CDM (3)	Sudden change
2CDM (4)	Kinked model
3CDM (5)	Spikes/Kinks, etc

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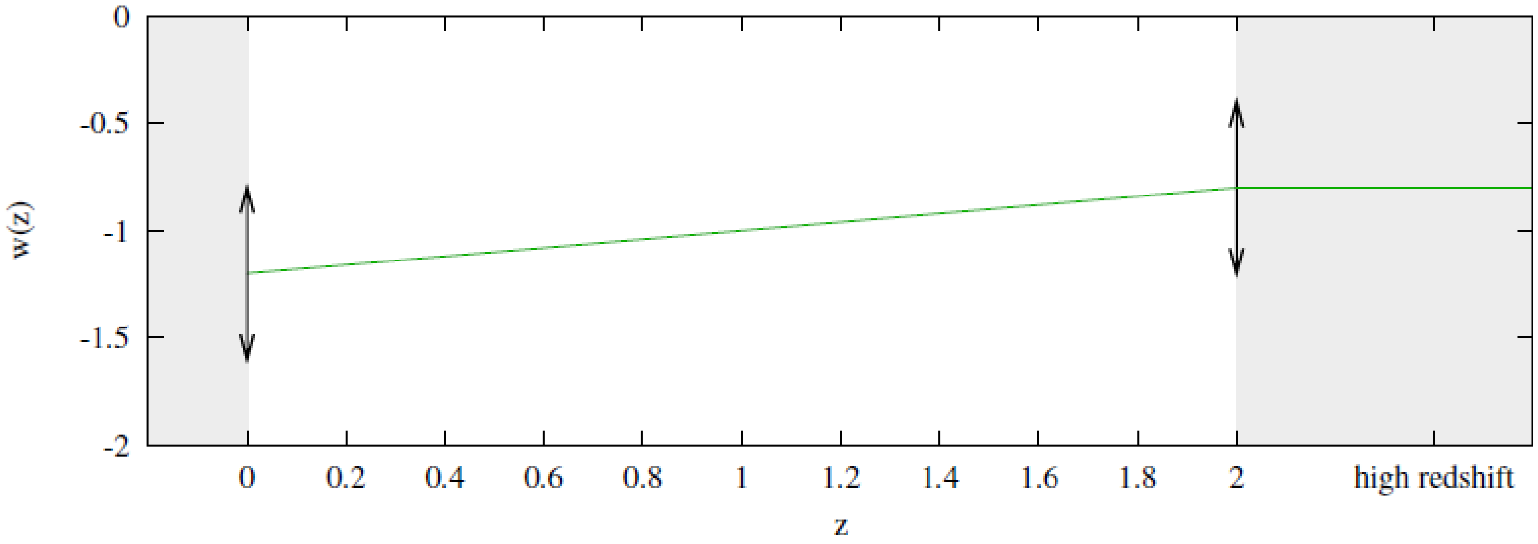

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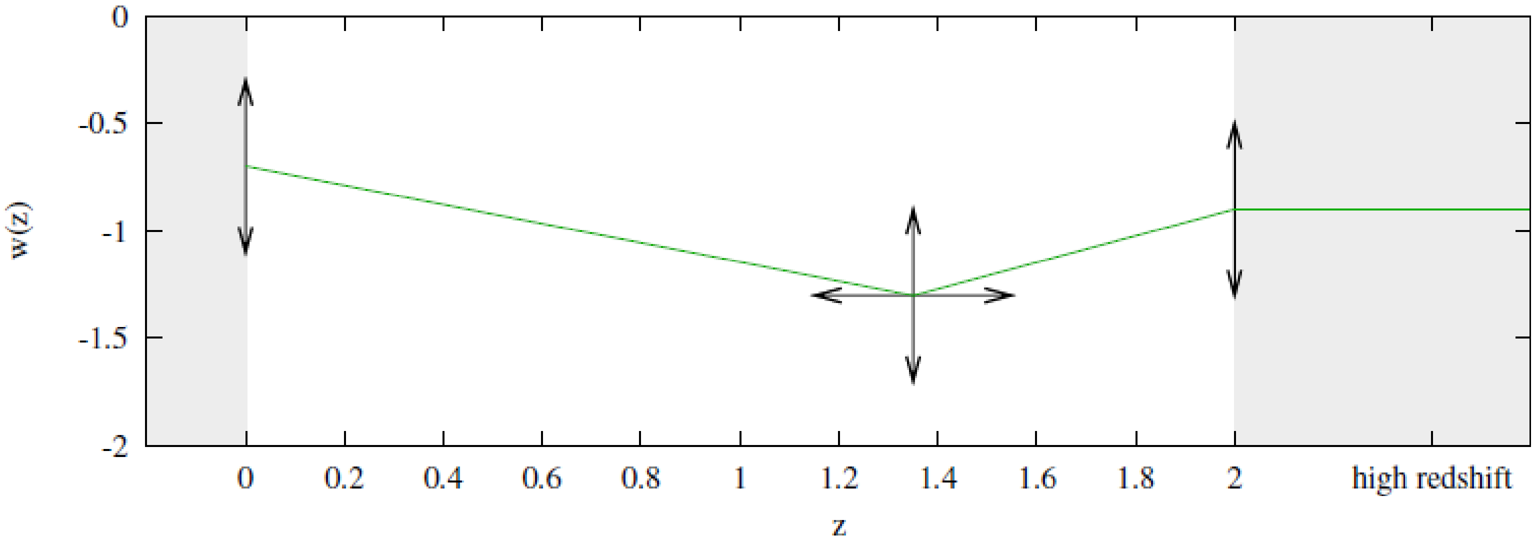

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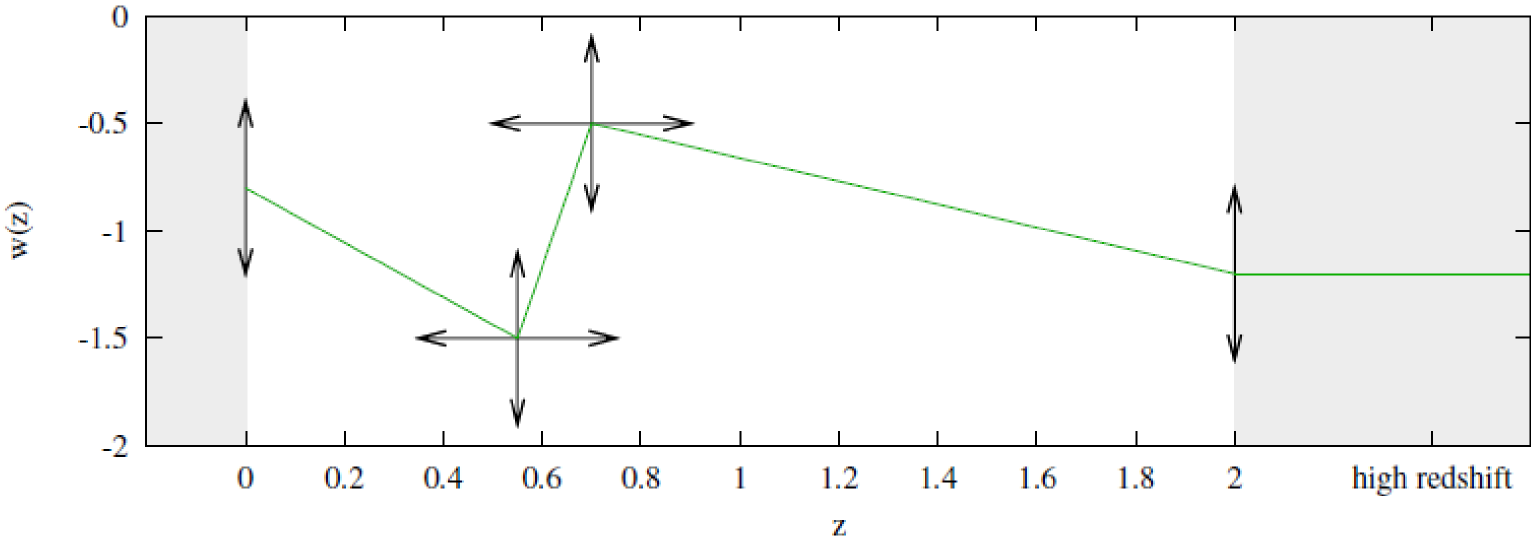

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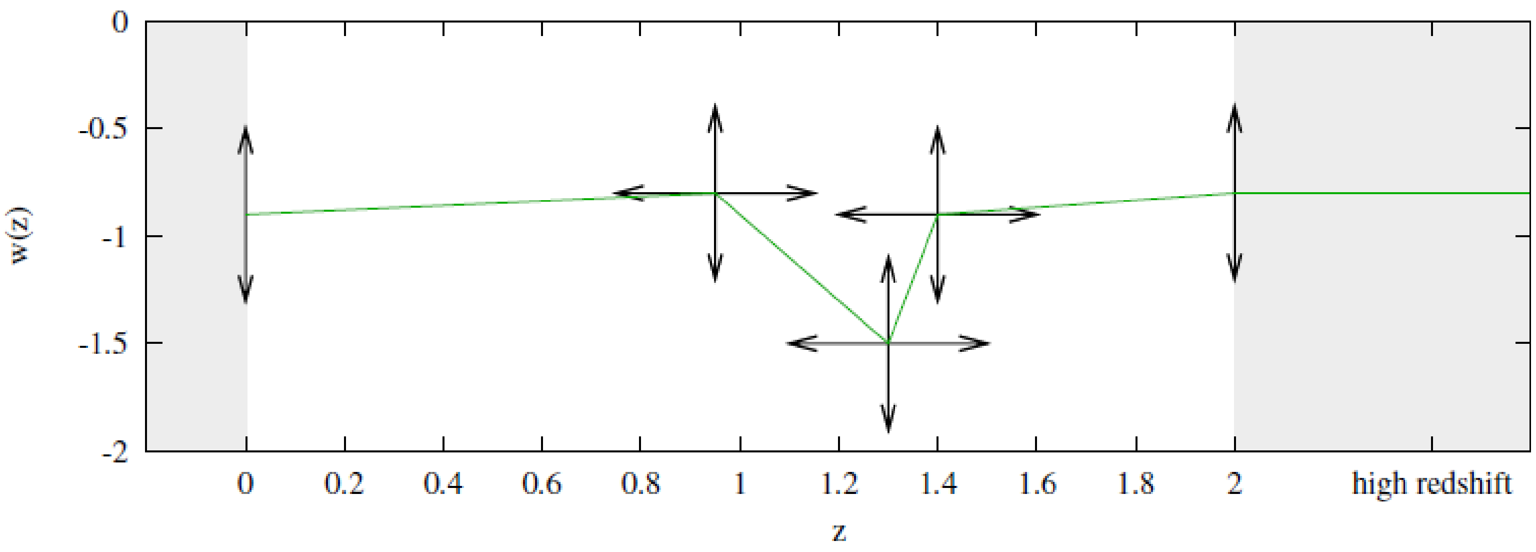

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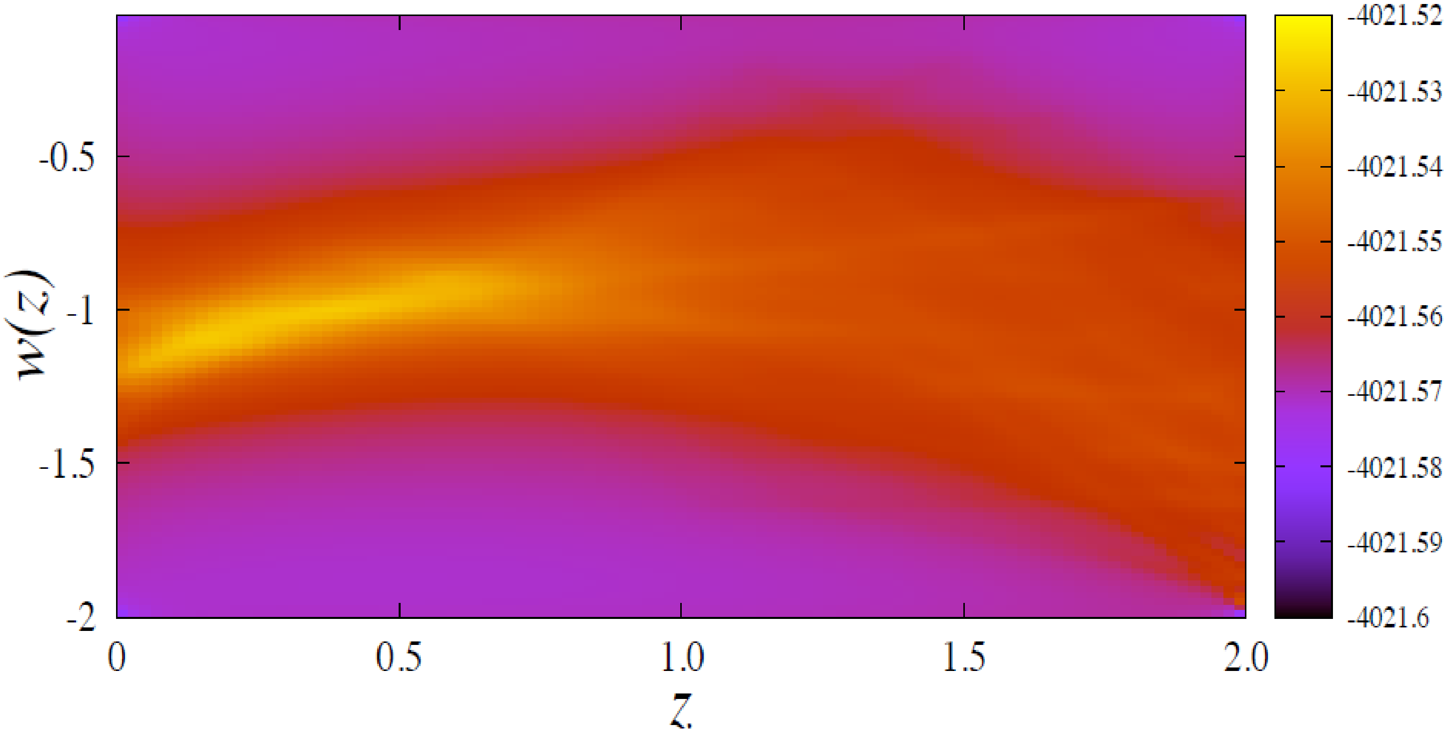
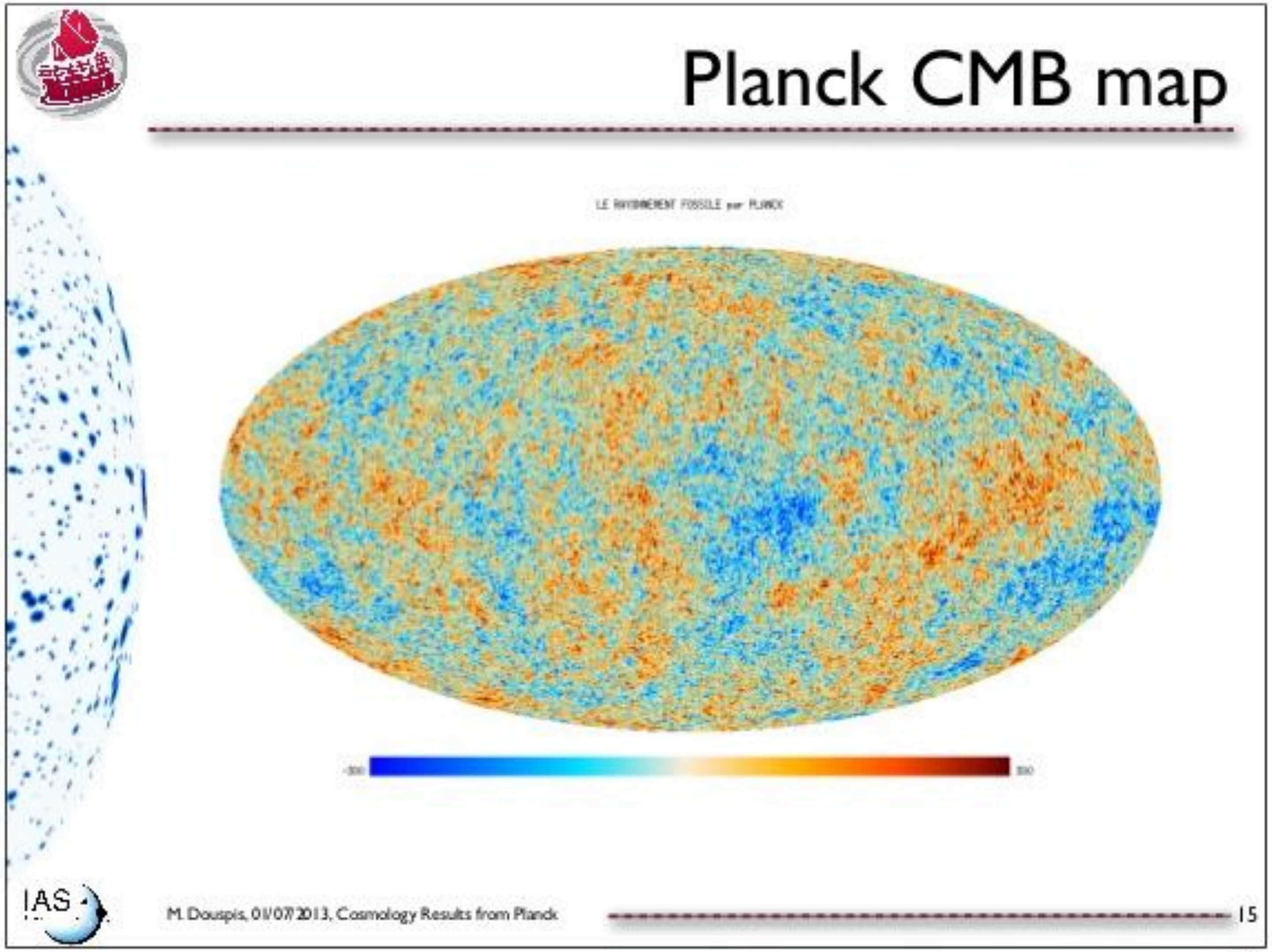


Model (number of nodes)	Bayes Factor compared to LCDM
wCDM (1)	-2.19 +/- 0.35
tCDM (2)	-2.34 +/- 0.35
1CDM (3)	-1.27 +/- 0.35
2CDM (4)	-0.81 +/- 0.35
3CDM (5)	-0.95 +/- 0.35

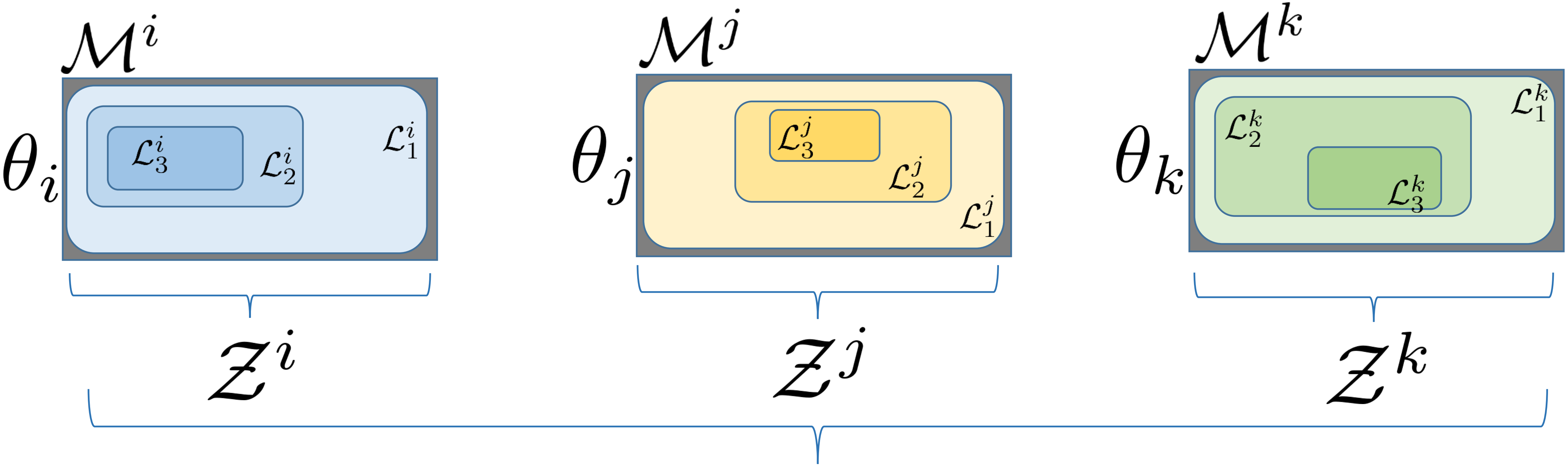
The Problem to Solve



Parameter	Prior Range	Prior Type
$\Omega_b h^2$	$[0.019, 0.025]$	uniform
$\Omega_c h^2$	$[0.095, 0.145]$	uniform
$100\theta_{MC}$	$[1.03, 1.05]$	uniform
τ	$[0.01, 0.4]$	uniform
n_s	$[0.885, 1.04]$	uniform
$\ln(10^{10} A_s)$	$[2.5, 3.7]$	uniform
A^{PS}_{100}	$[0, 360]$	uniform
A^{PS}_{143}	$[0, 270]$	uniform
A^{PS}_{217}	$[0, 450]$	uniform
A^{CIB}_{143}	$[0, 20]$	uniform
A^{CIB}_{217}	$[0, 80]$	uniform
A^{tSZ}_{143}	$[0, 10]$	uniform
$r^{PS}_{143\times 217}$	$[0, 1]$	uniform
$r^{CIB}_{143\times 217}$	$[0, 1]$	uniform
γ^{CIB}	$[-2, 2]$	uniform
c_{100}	$[0.98, 1.02]$	uniform
c_{217}	$[0.95, 1.05]$	uniform
$\xi^{tSZ-CIB}$	$[0, 1]$	uniform
A^{kSZ}	$[0, 10]$	uniform
β^1_1	$[-20, 20]$	uniform
$w(z_1)$	$[-2, -0.01]$	uniform
$w(z_2)$	$[-2, -0.01]$	uniform
$w(z_3)$	$[-2, -0.01]$	uniform
$w(z_4)$	$[-2, -0.01]$	uniform
$w(z_5)$	$[-2, -0.01]$	uniform
z_2	$[0.01, 2.0]$	sorted-uniform
z_3	$[0.01, 2.0]$	sorted-uniform
z_4	$[0.01, 2.0]$	sorted-uniform
n	$[\Lambda, w, t, 1, 2, 3]$	uniform

‘Vanilla’ Method

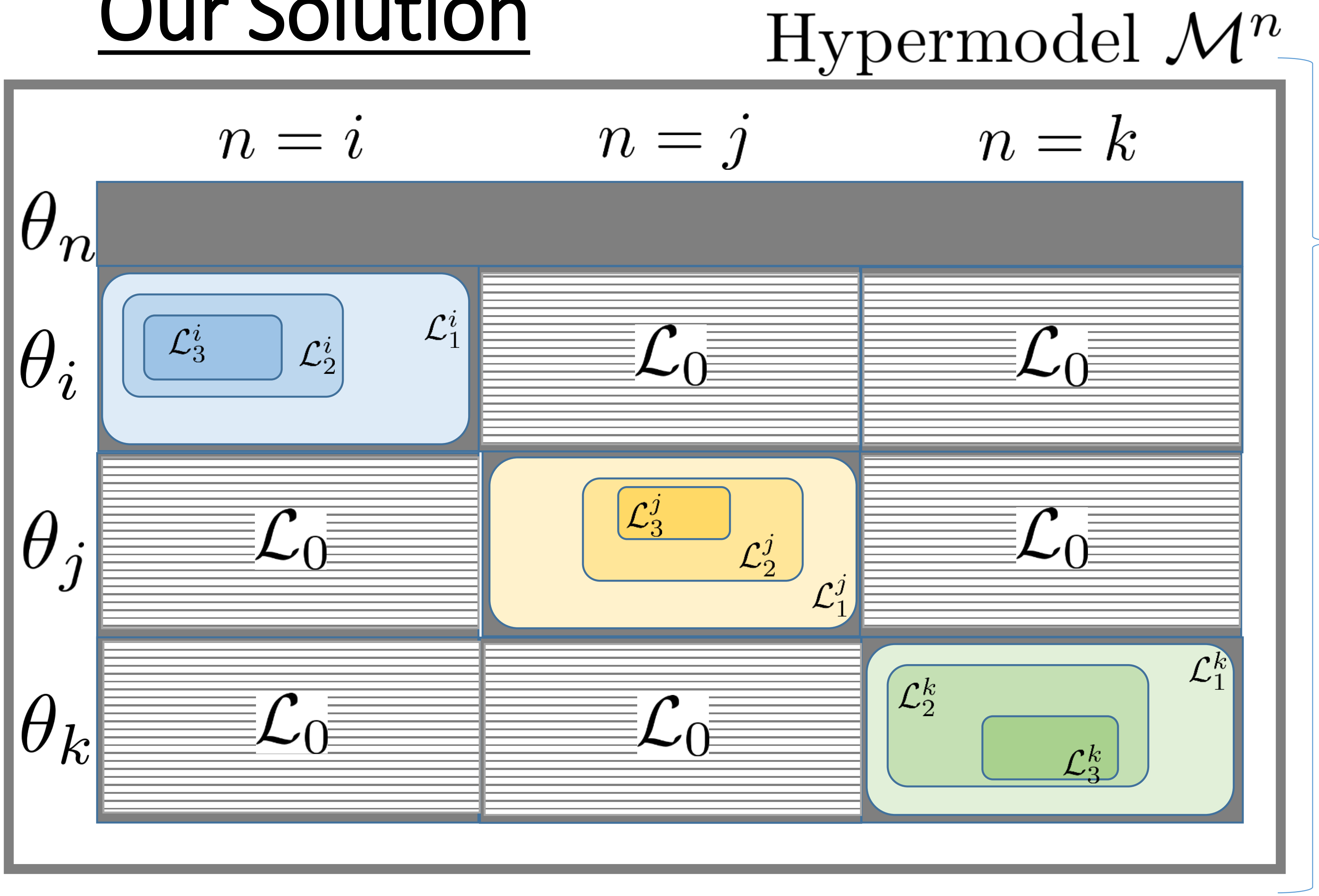
- Calculate Bayes factors with Evidences



$$\mathcal{B}_{ij} = \ln(\frac{\mathcal{Z}^j}{\mathcal{Z}^i})$$

$$\mathcal{B}_{ik} = \ln(\frac{\mathcal{Z}^k}{\mathcal{Z}^i})$$

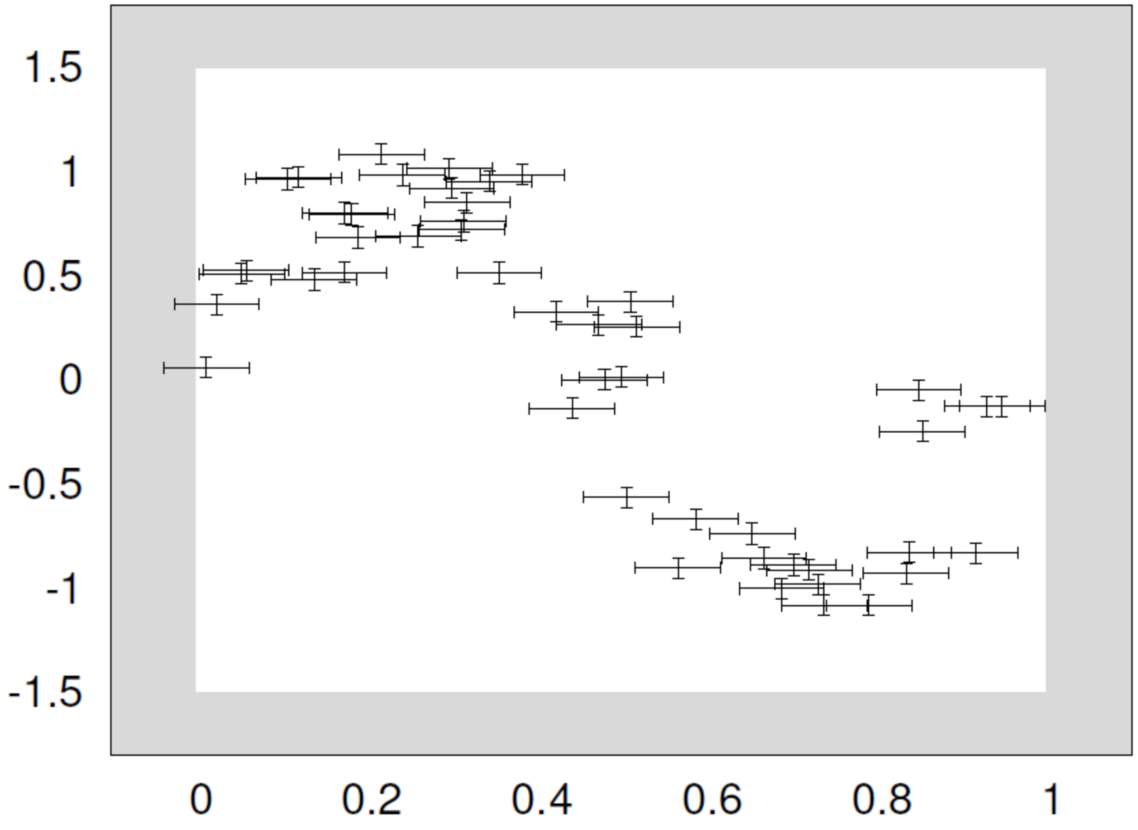
Our Solution

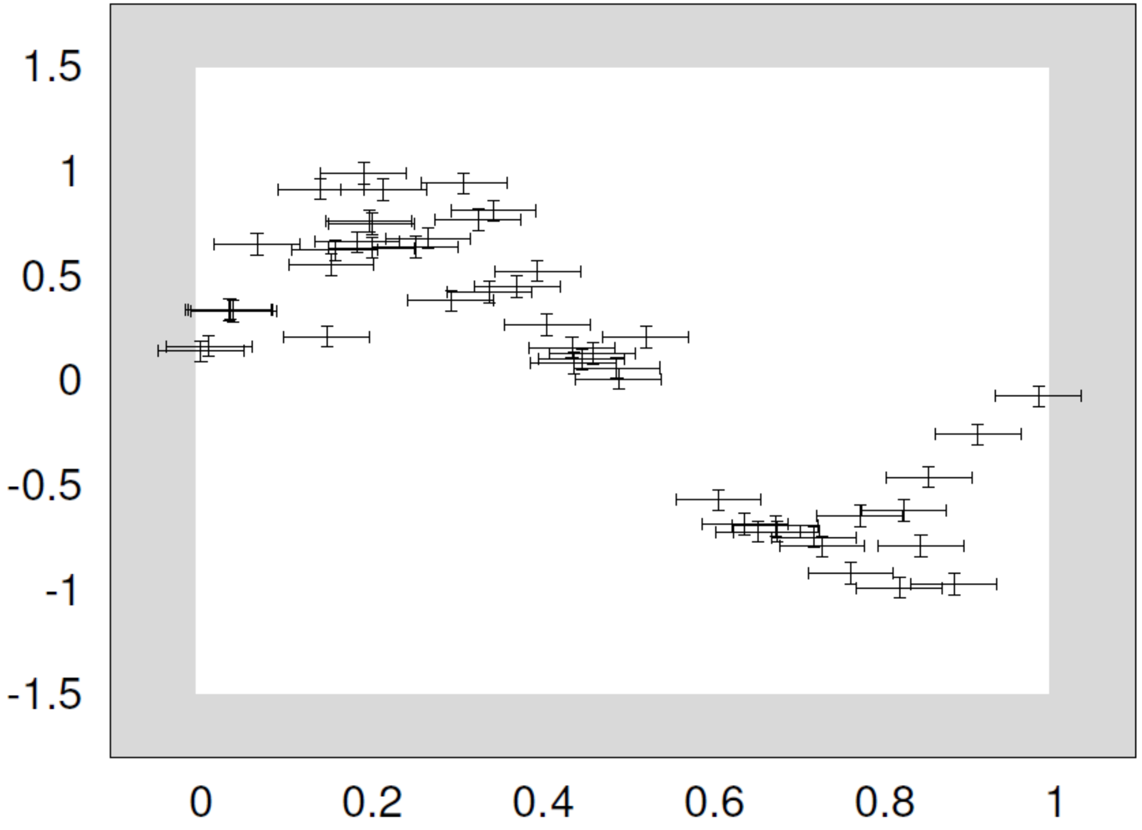


Gives us:
$Post(n) = Pr(n|\mathcal{D}, \theta)$

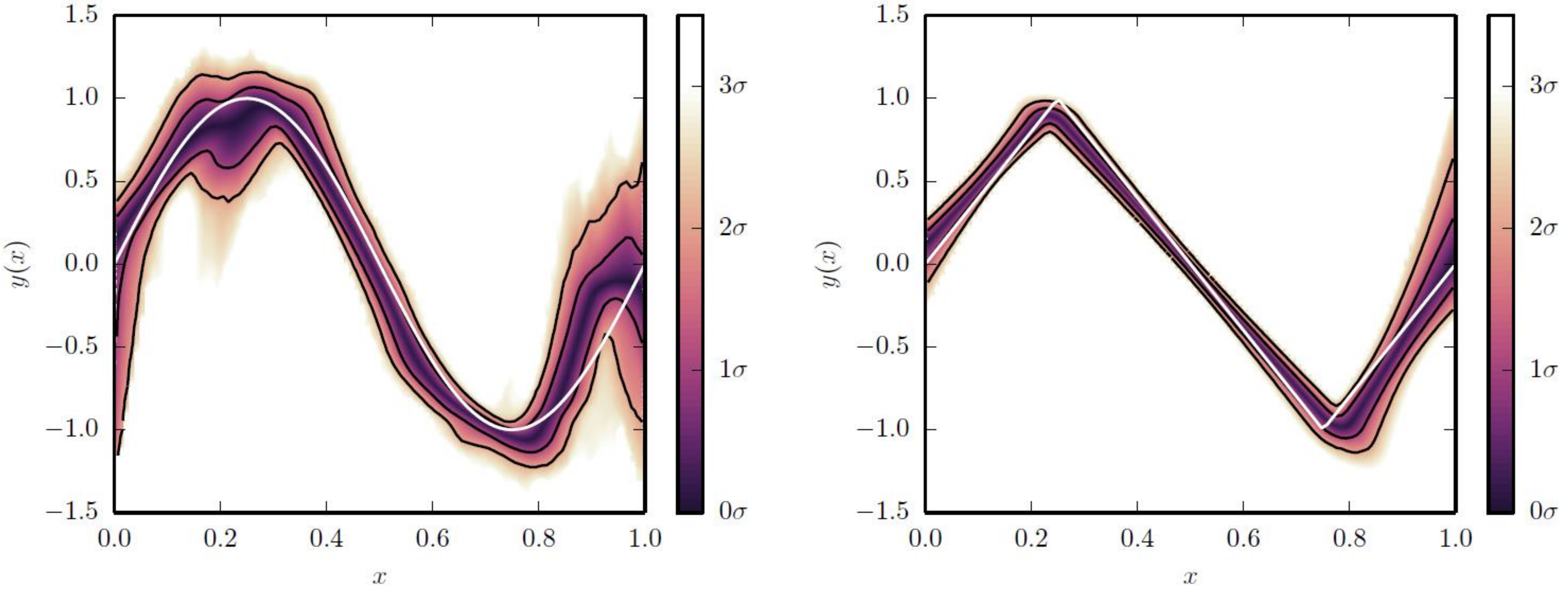
From which we get
$\mathcal{B}_{ij} = \ln(\frac{Post(n=j)}{Post(n=i)})$

Our Solution – Testing





Our Solution – Works!



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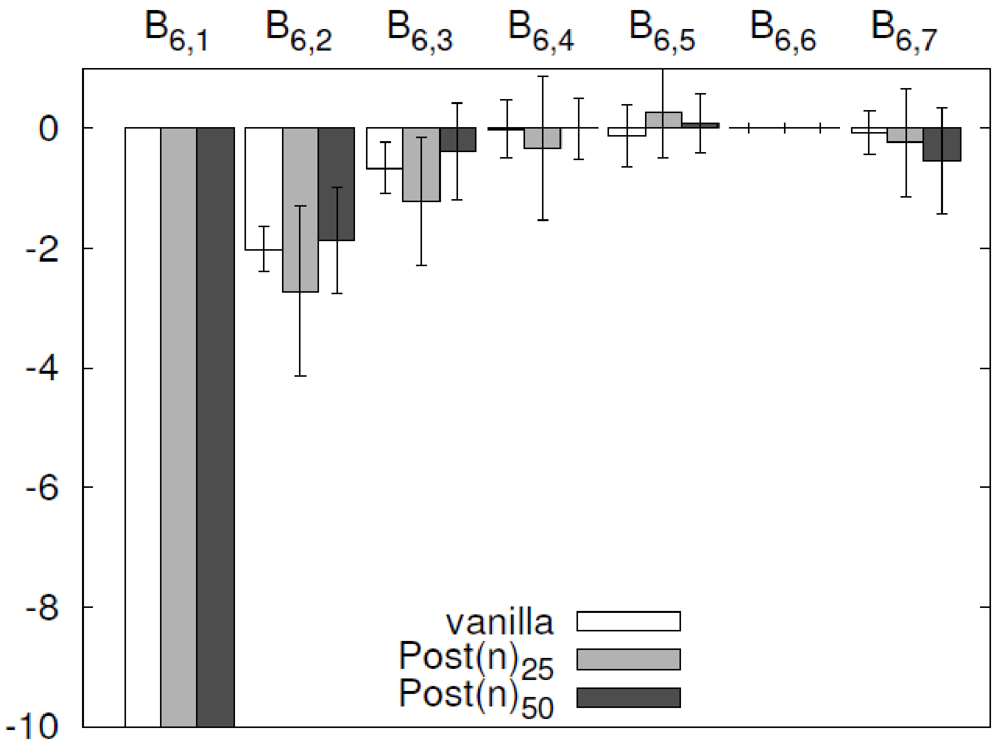
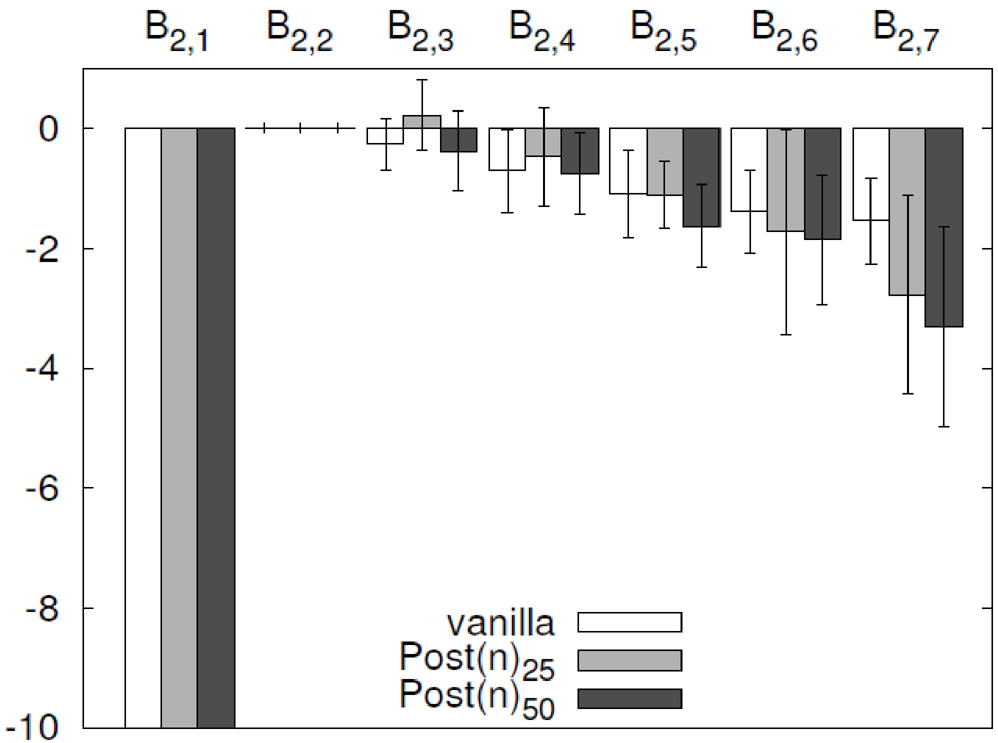
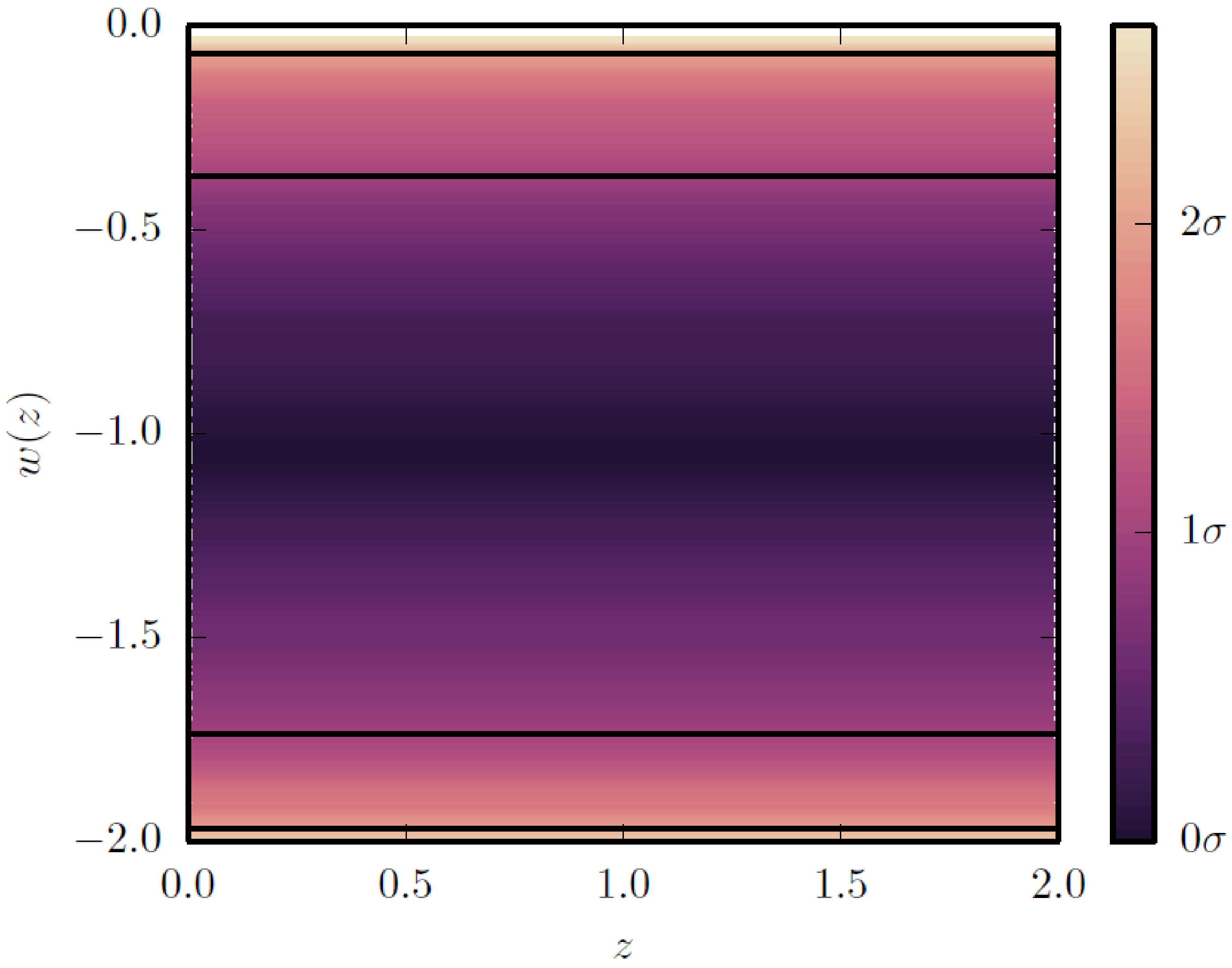




So let's do some Physics – finally!



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Planck 2013 CMB data

SNIa Union 2.0

BAO (BOSS DR11)

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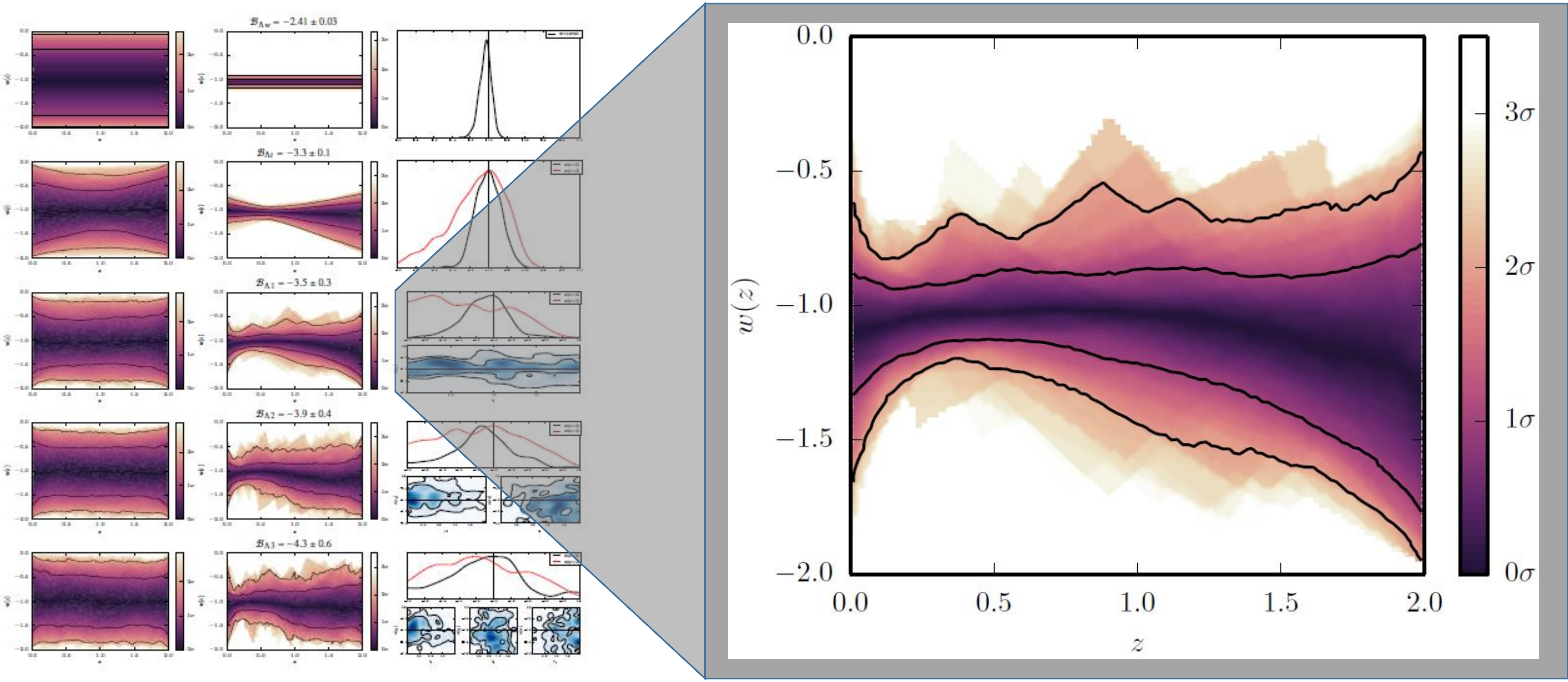

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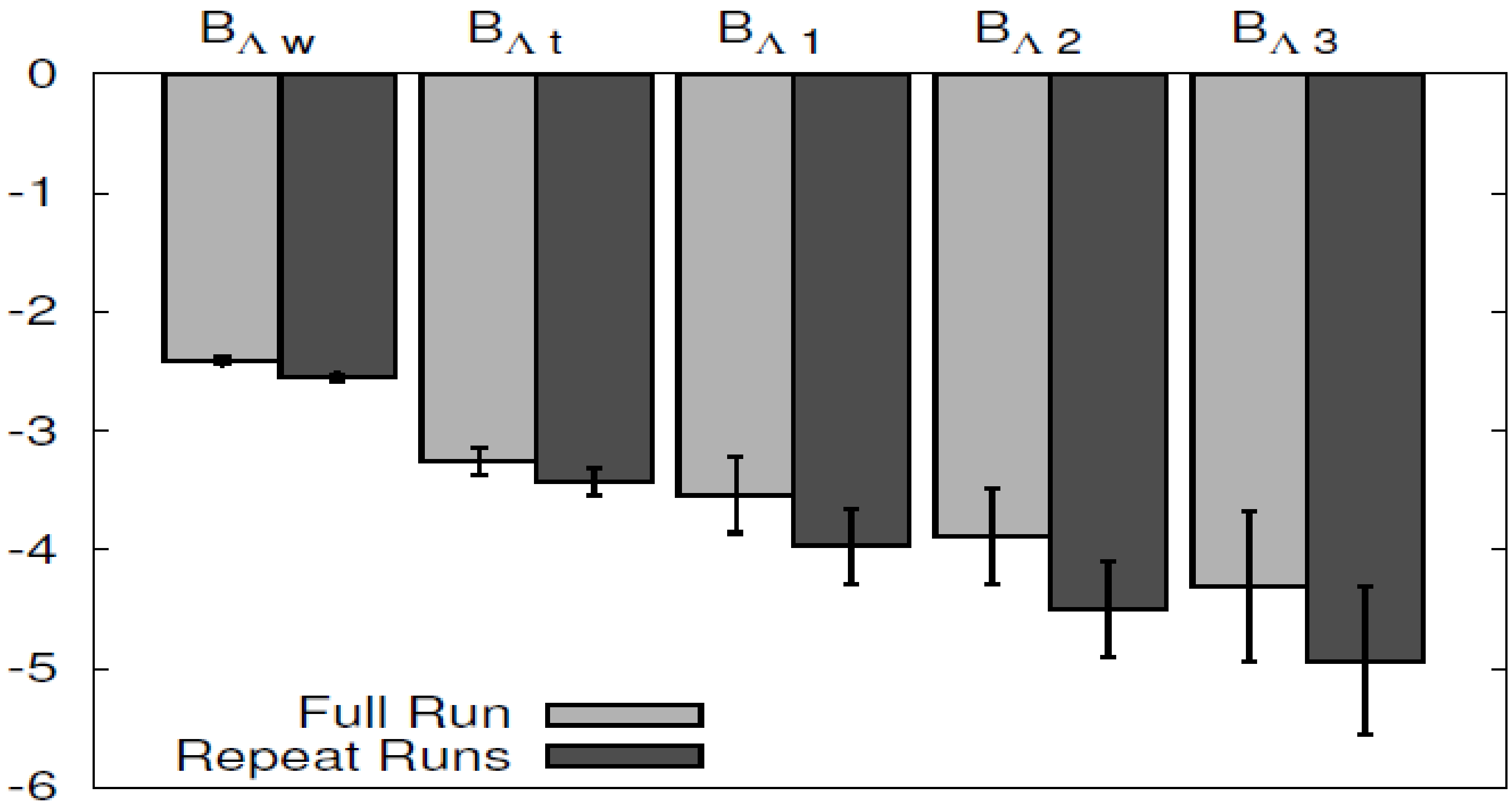

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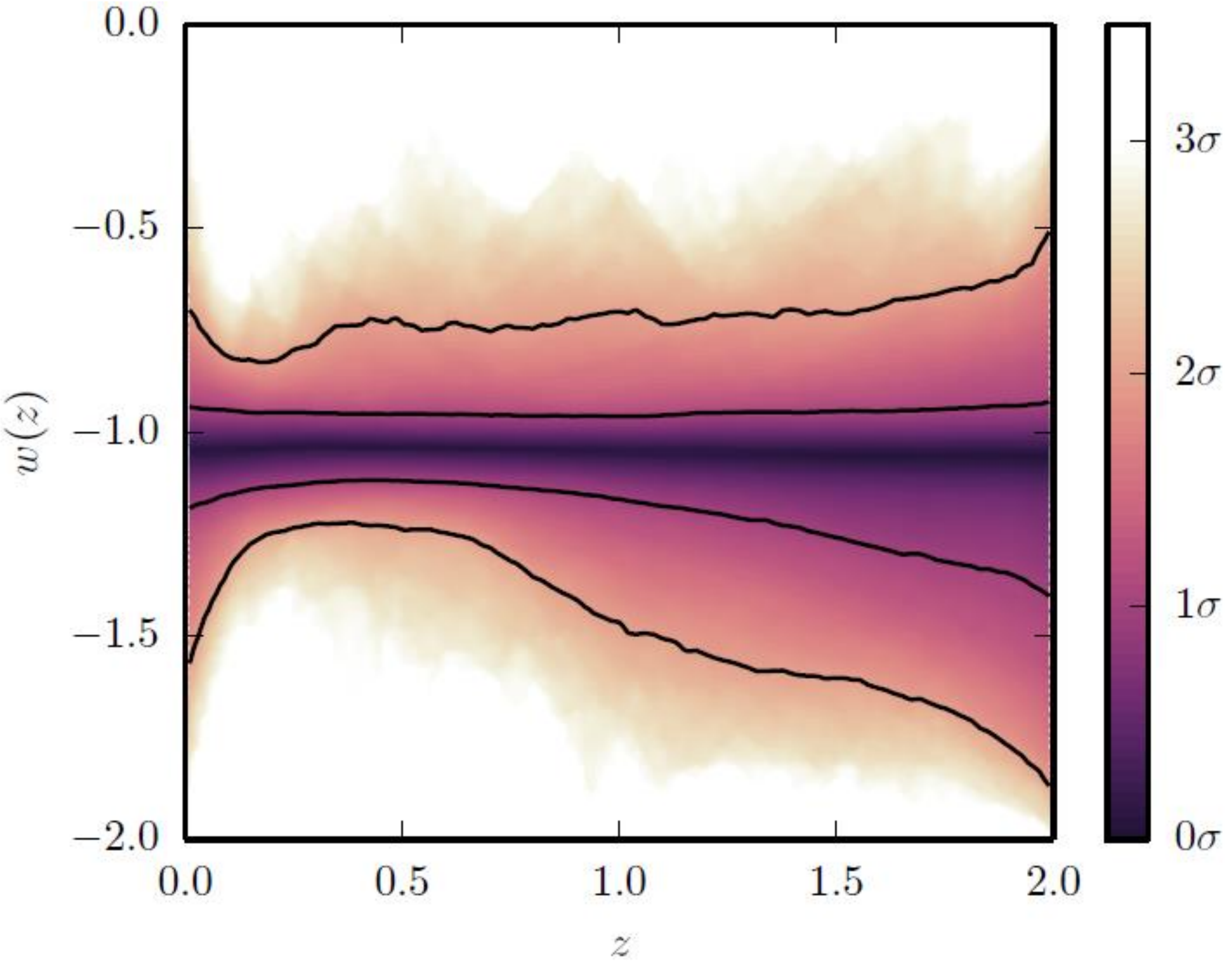
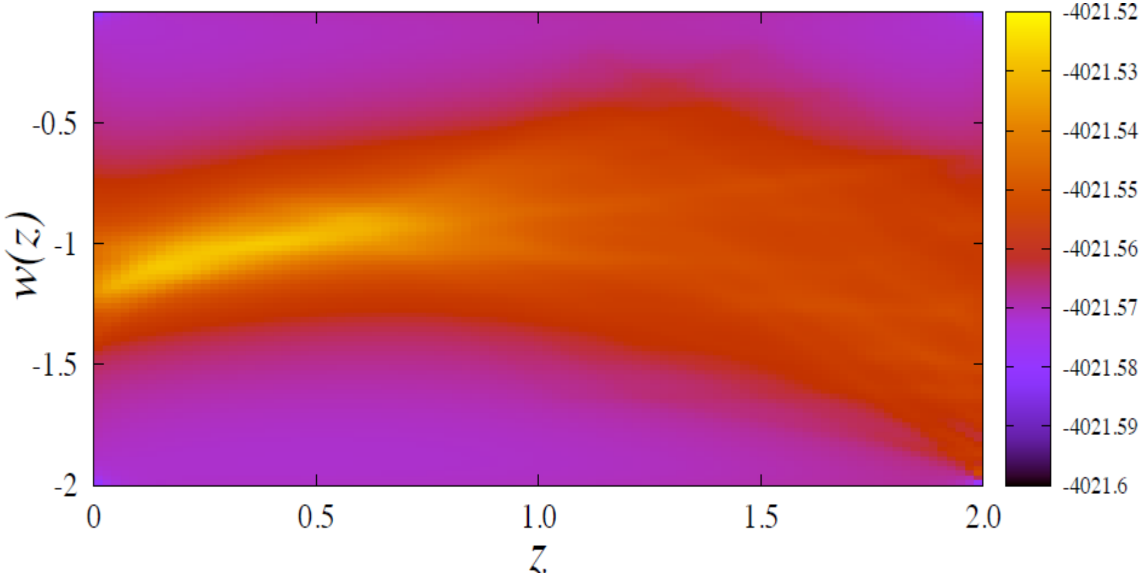
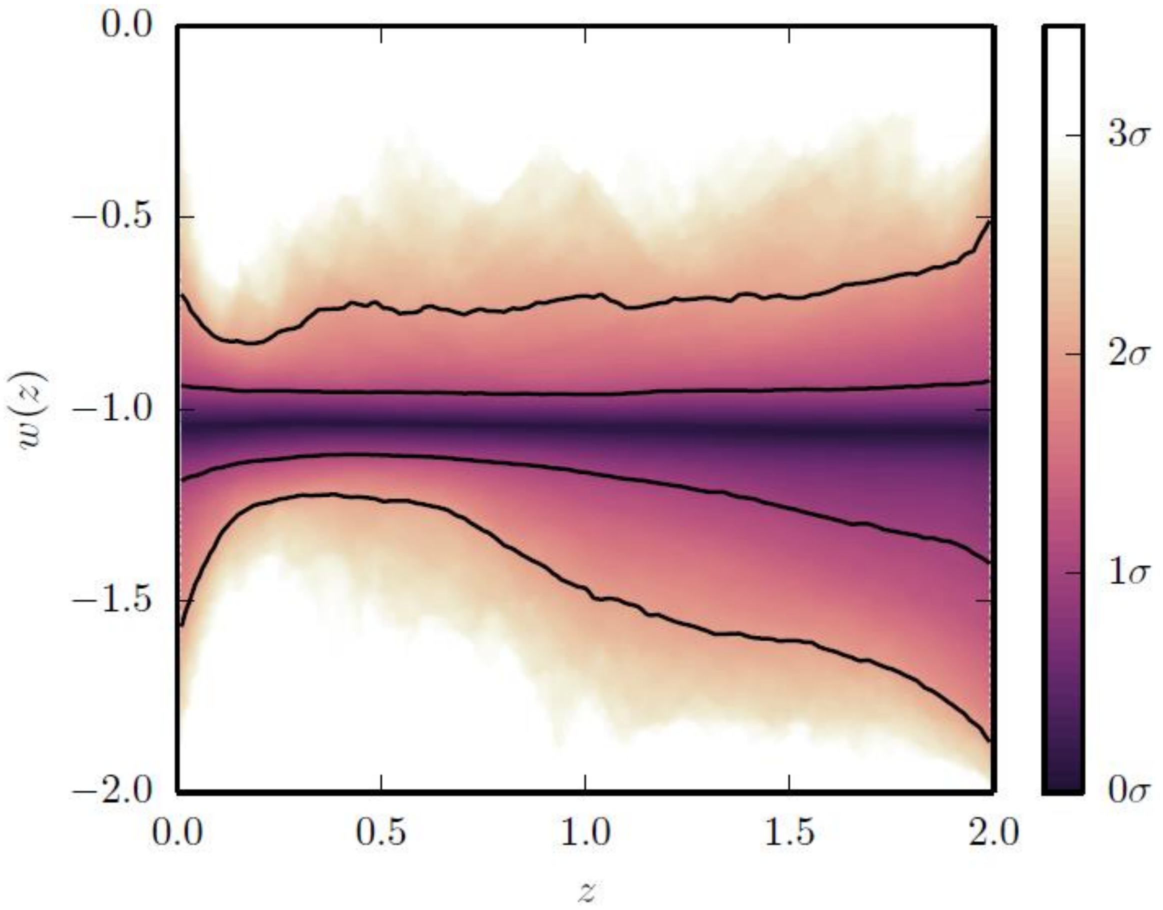


Final Conclusions

WMAP era Datasets



Planck 2013 era Datasets



Thanks

To my supervisors!

To Will! (POLYCHORD)

To Rich and Do Young!

Cambridge University, and STFC for funding!

S. Hee, W. Handley, M. Hobson and A. Lasenby (2015) "Bayesian model selection without evidences: application to the dark energy equation-of-state". Arxiv: 1506.09024, submitted to MNRAS.