Testing Isotropy and Statistics of the CMB with Planck

Andrei Frolov on behalf of Planck Collaboration

Rencontres du Vietnam:
Cosmology - 50 years after CMB discovery

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1 Instrument and Mission Overview
2 Variance Asymmetry
3 Peak Statistics & Cold Spot
4 Stacking & Polarization
5 Conclusions
Planck 2015: What’s New?

- **More data:** 48/29 months of LFI/HFI observations, enabling further checks
- **Improved data processing:** systematics removal, calibration, beam reconstruction
- **Improved foreground model:** larger sky-fraction used for analysis
- **More robust to systematics:** based on half-mission cross power spectra
- The 2015 analysis includes polarization
More data: 48/29 months of LFI/HFI observations, enabling further checks

Improved data processing: systematics removal, calibration, beam reconstruction

Improved foreground model: larger sky-fraction used for analysis

More robust to systematics: based on half-mission cross power spectra

The 2015 analysis includes polarization
CMB Intensity Map
- Smoothed to 1 degree resolution
- High-pass filtered with $l=20-40$ cosine filter
- Galactic plane replaced with constrained Gaussian realization
Red curve is the prediction based on the best fit TT in base $\Lambda$CDM.
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$\ell_k \equiv k D_{\text{rec}}$

- $\mathcal{P}_R$ samples
- $\mathcal{P}_t$ samples
- mean $\mathcal{P}_R$
- mean $\mathcal{P}_t$
- fiducial model $\mathcal{P}_R$
- fiducial model $\mathcal{P}_t$

free $r$; TT + low-z

$10^{10} \mathcal{P}_{R,t}$

$k [\text{Mpc}^{-1}]$
Primordial Spectrum Reconstruction

$$\ell_k \equiv kD_{\text{rec}}$$

$$(\ell_k)^{-1} = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^0$$

$$(k)^{-1} = 10^1, 10^2, 10^3, 10^4$$

Fixed $r_{0.05} = 0.1$; TT, TE, EE + low-z

$P_R$ samples $P_t$ samples

Mean $P_R$ $P_t$

Fiducial model $P_R$ $P_t$

$10^{10} P_{R,t}$
Running Spectral Index is Not a Good Fit!

- free $r$ samples
- $r_{0.05} = 0.1$ mean
- $r_{0.05} = 0.01$ mean
- $\frac{dn_s}{d\ln k}$ best fit
- $\tau = 0.04$

$\Delta D_\ell(\mu K^2)$ vs $\ell$
mean-subtracted and inverse-variance-weighted local-variance map for 8° discs in Commander component-separated CMB map
Local Variance Dipole Modulation

variance dipole amplitude $0.052 \pm 0.016$, direction $(l, b) = (210^\circ, -26^\circ)$
(no high-pass filter applied)
Going after localized anomalies...
Let’s look at peaks!
Estimating observable from a noisy data:

$$\overline{o(\vec{x})} = h(\vec{x}) \ast s(\vec{x}) + \epsilon(\vec{x}) \quad \Rightarrow \quad \hat{s}(\vec{x}) = g(\vec{x}) \ast \overline{o(\vec{x})}$$

observable  transfer  signal  noise

In Fourier domain, optimal Wiener filter is:

$$G = \frac{\overline{H} \cdot S}{|H|^2 \cdot S + N} \sim \frac{\overline{H}}{N} \cdot S$$

Take a shortcut - whiten data using isotropic CMB+noise model!

$$G \sim C_\ell^{-\frac{1}{2}} \cdot S$$

Whiten and filter, search for peaks!
**CMB Data Analysis Pipeline**

- SMICA
- Whiten
- Mask
- Filter
- Find Peaks

**Planck 2014 release** [SSG84 filter at 240′ FWHM]
Planck 2014 release [SSG84 filter at 240′ FWHM]
CMB Data Analysis Pipeline

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Planck 2014 release [SSG84 filter at 240' FWHM]
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Testing CMB Peak Statistics

- Peak CDF
- Gaussian CDF
- Deviation
- Simulations

SSG84 filter at 240′ FWHM

957 peaks

hottest at \((-0.00 + 3.59)\sigma\)
coldest at \((-0.00 - 4.26)\sigma\)
Bond and Efstathiou (1987)

\[
\frac{n_{\text{max}} + n_{\text{min}}}{n_{\text{pk}}} \left( \frac{x}{\sigma} > \nu \right) = \sqrt{\frac{3}{2\pi}} \gamma^2 \nu \exp\left(-\frac{\nu^2}{2}\right) + \frac{1}{2} \text{erfc}\left[\frac{\nu}{\sqrt{2-\frac{4}{3}\gamma^2}}\right]
\]

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Testing CMB Peak Statistics

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Kolmogorov deviation from FFP8 peak CDF

957 peaks

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Gaussian peak fit, \(\gamma = 0.82\)

SSG84 filter at 240' FWHM
Kolmogorov deviation from FFP8 peak CDF

- **Gaussian peak fit,** $\gamma = 0.82$
- **SSG84 filter at 240’ FWHM**

- **957 peaks**
  - hottest at $(-0.00 + 3.59)\sigma$
  - coldest at $(-0.00 - 4.26)\sigma$

- **957 peaks**
SSG84 Filter Sweep

Planck 2014 release [SSG84 filter at 120' FWHM]
Planck 2014 release [SSG84 filter at 240° FWHM]
SSG84 Filter Sweep

Planck 2014 release [SSG84 filter at 400' FWHM]
Planck 2014 release [SSG84 filter at 800′ FWHM]
Planck 2014 release [SSG84 filter at 1200′ FWHM]
Significance evaluated by counting simulations which exceed observed value –
For full details see Isotropy and Statistics paper.
Significance of Cold Spot

- Whitened Savitzky-Golay
- Mexican Hat Wavelet

Filter kernel size [degrees FWHM]

PTE in Gaussian field

Effective confidence level [$\nu$]

Significance evaluated by counting simulations which exceed observed value –

For full details see Isotropy and Statistics paper.
Cold Spot is Fairly Cold!

Gaussian peak fit, $\gamma = 0.83$

SSG84 filter at 800′ FWHM

64 peaks

hottest at $+2.68\sigma$

coldest at $-4.12\sigma$
Asymmetry in Peak Distributions

Threshold

Kolmogorov-Smirnov deviation

peak distributions are also different in two hemispheres!
(pre-whitened GAUSS filter at 40′ full-width half-max)
How does a neighbourhood of a peak look like?
Let’s do some stacking!
Three key elements:

A  What to stack? (cosmic field $u$)
B  Where to stack? (selection of patches, e.g., peaks)
C  How to stack? (patch orientations)

“where” and “how” give constrained parameter(s) $q$;

$$
\begin{array}{ll}
\text{What} & \text{WMAP & Planck 2013} \\
T, Q, U, Q_r, U_r & T, Q, U, Q_r, U_r, E, B, Q_T, U_T, \zeta_{dv}, \ldots \\
\text{Where} & \text{Planck 2014} \\
T \text{ peaks} & T, E, B, Q^2 + U^2, Q_T^2 + U_T^2, \zeta_{dv} \ldots \text{ peaks} \\
\text{How} & \text{oriented and unoriented} \\
\text{unoriented} & \\
\end{array}
$$

For Gaussian fields,

$$
\langle u | q; \text{peak, orientation} \rangle = \langle u q^\dagger \rangle \langle q q^\dagger \rangle^{-1} \langle q | \text{peak, orientation} \rangle.
$$
Planck 2014: Stacking Temperature

$T$ on hot spots

24645 patches on $T$ maxima, random orientation, threshold $\nu=0$

resolution: FWHM 15 arcmin
Peaks are selected above a threshold $|T_{\text{peak}}| > \nu \sqrt{\langle T^2 \rangle}$ ($\nu = 0$ here).

Full statistics in Isotropy and Statistics paper!

$T$ on cold spots

24582 patches on $T$ minima, random orientation, threshold $\nu=0$

Full statistics in Isotropy and Statistics paper!
Planck 2014: Stacking Polarization

$Q_r$ on hot spots

33214 patches on $T$ maxima, random orientation, threshold $\nu=0$

$Q_r$ on cold spots

33126 patches on $T$ minima, random orientation, threshold $\nu=0$

resolution: FWHM 15 arcmin

Peaks are selected above a threshold $|T_{\text{peak}}| > \nu \sqrt{\langle T^2 \rangle}$ ($\nu = 0$ here).

Full statistics in Isotropy and Statistics paper!
flat-sky polar coor. $(\varpi, \phi)$:

$$\varpi = 2 \sin \frac{\theta}{2}$$

$$Q_r = -Q \cos 2\phi - U \sin 2\phi$$

$$U_r = -U \cos 2\phi + Q \sin 2\phi$$
Oriented Stacking of Polarization

$E$ on oriented $T$ peaks
33216 patches on $T$ maxima, oriented, threshold $\nu=0$

$Q$ on oriented $Q_T^2 + U_T^2$ peaks
58099 patches on $P_T$ maxima, oriented, threshold $\nu=0$

Planck 2014 (peak threshold $\nu = 0$; resolution FWHM 15 arcmin)
Stacking on Polarization Peaks

$Q_r$ on unoriented $E$ peaks

99529 patches on $E$ maxima, random orientation, threshold $\nu=0$

$Q$ on oriented $Q^2 + U^2$ peaks

196910 patches on $P$ maxima, oriented, threshold $\nu=0$

Planck 2014 (peak threshold $\nu = 0$; resolution FWHM 15 arcmin)
Stacking Polarized Dust

Planck 2014 Component Separated Commander Dust Map

Dust Component, $T < 25\mu K$

43 patches on $P$ maxima, oriented, threshold $\nu = 1, I \leq 25\mu K$

$Q$ stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. $U$ vanishes in the centre).

Patch size: $\varpi \leq 7^\circ$; threshold $\nu = 1$

$T$ map FWHM $2^\circ$; $Q, U$ maps FWHM 15 arcmin.

CMB Component

33536 patches on $P$ maxima, oriented, threshold $\nu = 1$
Planck 2014 Component Separated Commander Dust Map

Dust Component, $T < 35\mu K$

274 patches on $P$ maxima, oriented, threshold $\nu = 1$, $I \leq 35\mu K$

$Q$ stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. $U$ vanishes in the centre).

Patch size: $\varpi \leq 7^\circ$; threshold $\nu = 1$

$T$ map FWHM $2^\circ$; $Q, U$ maps FWHM 15 arcmin.
Stacking Polarized Dust

Planck 2014 Component Separated Commander Dust Map

Dust Component, $T < 45 \mu K$

809 patches on $P$ maxima, oriented, threshold $\nu = 1$, $I \leq 45 \mu K$

CMB Component

33536 patches on $P$ maxima, oriented, threshold $\nu = 1$

$Q$ stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. $U$ vanishes in the centre).

Patch size: $\pm 7^\circ$; threshold $\nu = 1$

$T$ map FWHM $2^\circ$; $Q, U$ maps FWHM 15 arcmin.
Stacking Polarized Dust

**Planck 2014 Component Separated Commander Dust Map**

**Dust Component, $T < 55\mu K$**

1855 patches on $P$ maxima, oriented, threshold $\nu = 1$, $I \leq 55\mu K$

**CMB Component**

33536 patches on $P$ maxima, oriented, threshold $\nu = 1$

$Q$ stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. $U$ vanishes in the centre).

Patch size: $\omega \leq 7^\circ$; threshold $\nu = 1$

$T$ map FWHM $2^\circ$; $Q$, $U$ maps FWHM 15 arcmin.
Planck 2014 Component Separated Commander Dust Map

**Dust Component, \( T < 95 \mu K \)**

6673 patches on \( P \) maxima, oriented, threshold \( \nu = 1, I \leq 95 \mu K \)

**CMB Component**

33536 patches on \( P \) maxima, oriented, threshold \( \nu = 1 \)

\( Q \) stacked on \( Q^2 + U^2 \) oriented peaks (oriented s.t. \( U \) vanishes in the centre).

Patch size: \( \varpi \leq 7^\circ \); threshold \( \nu = 1 \)

\( T \) map FWHM 2°; \( Q, U \) maps FWHM 15 arcmin.
Planck 2014 Component Separated Commander Dust Map

Dust Component, $T < 115\mu K$

8531 patches on $P$ maxima, oriented, threshold $\nu = 1$, $I \leq 115\mu K$

$Q$ stacked on $Q^2 + U^2$ oriented peaks (oriented s.t. $U$ vanishes in the centre).

Patch size: $\omega \leq 7^\circ$; threshold $\nu = 1$

$T$ map FWHM 2$^\circ$; $Q, U$ maps FWHM 15 arcmin.
Conclusions

- A lot more and better processed and analyzed data.
- As in 2013, base $\Lambda$CDM continues to be a good fit to the Planck data, including polarization.
- Polarization has a degeneracy lifting capability often comparable to BAO.
- No convincing evidence for any simple extensions. Scalar fluctuations consistent with pure adiabatic modes with a featureless tilted spectrum.
- 2015 statistics: mostly Gaussian, but with similar anomalies than 2013. Many new methods explored, including of novel oriented stacking and peak statistics methods.
- Stacking and peak statistics give a complimentary approach for probing hemispherical asymmetry and component separation tests.
2015 papers and data are released!

+ more to come...
The End.
The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada.
Generalized Savitzky-Golay filter kernel:

\[ F_{n,k}(x) = \left( \sum_{i=0}^{n/2} a_i x^{2i} \right) (1 - x^2)^k \]

Orthogonal to polynomials up to order \( n \):

\[
\int_0^1 x F_{n,k}(x) \, dx = 1, \quad \int_0^1 x^{i+1} F_{n,k}(x) \, dx = 0
\]

Savitzky and Golay (1964)

locate peaks in noisy spectra – topcite in Analytical Chemistry!
Filter Kernels in Harmonic Space

real space  [compact support]  harmonic space  [low-pass filter]
First derivative vanishes on the peak. Need to use the 2nd derivatives.

Intuitively (flat-sky limit):
\[ Q_T \equiv \nabla^{-2}(\partial_y^2 - \partial_x^2)T, \quad U_T \equiv -2\nabla^{-2}(\partial_x \partial_y)T \]

Slightly non-intuitive (on the sphere):
\[ Q_T(n) \pm iU_T(n) \equiv \sum_{l,m} \left[ \int T(n') Y^*_l m(n') \, d^2n' \right] \pm 2 Y_{lm}(n) \]

Orient the patch such that \( U_T \) vanishes in the centre.
\[ \langle u|q; \text{peak, orientation}\rangle(\psi, \phi) \text{ decomposes to } \cos m\phi, \quad m = 0, 2, 4. \]