COSMOLOGICAL PARAMETER ESTIMATION USING THE LINE CORRELATION FUNCTION

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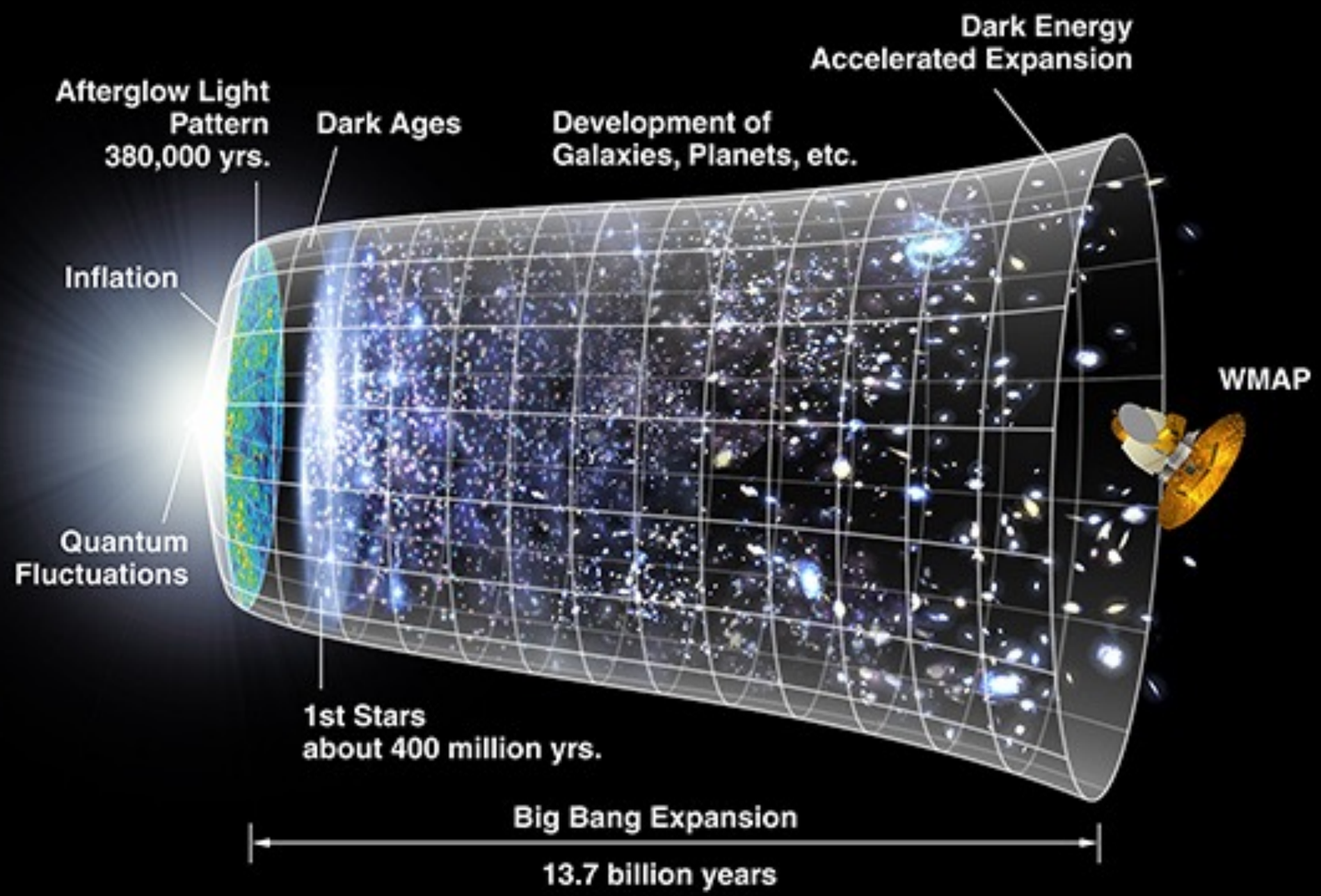


NASA/WMAP Science Team

CORRELATION STATISTICS

We define the density contrast in real space as $\delta(\mathbf{x}) = \dfrac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}}$

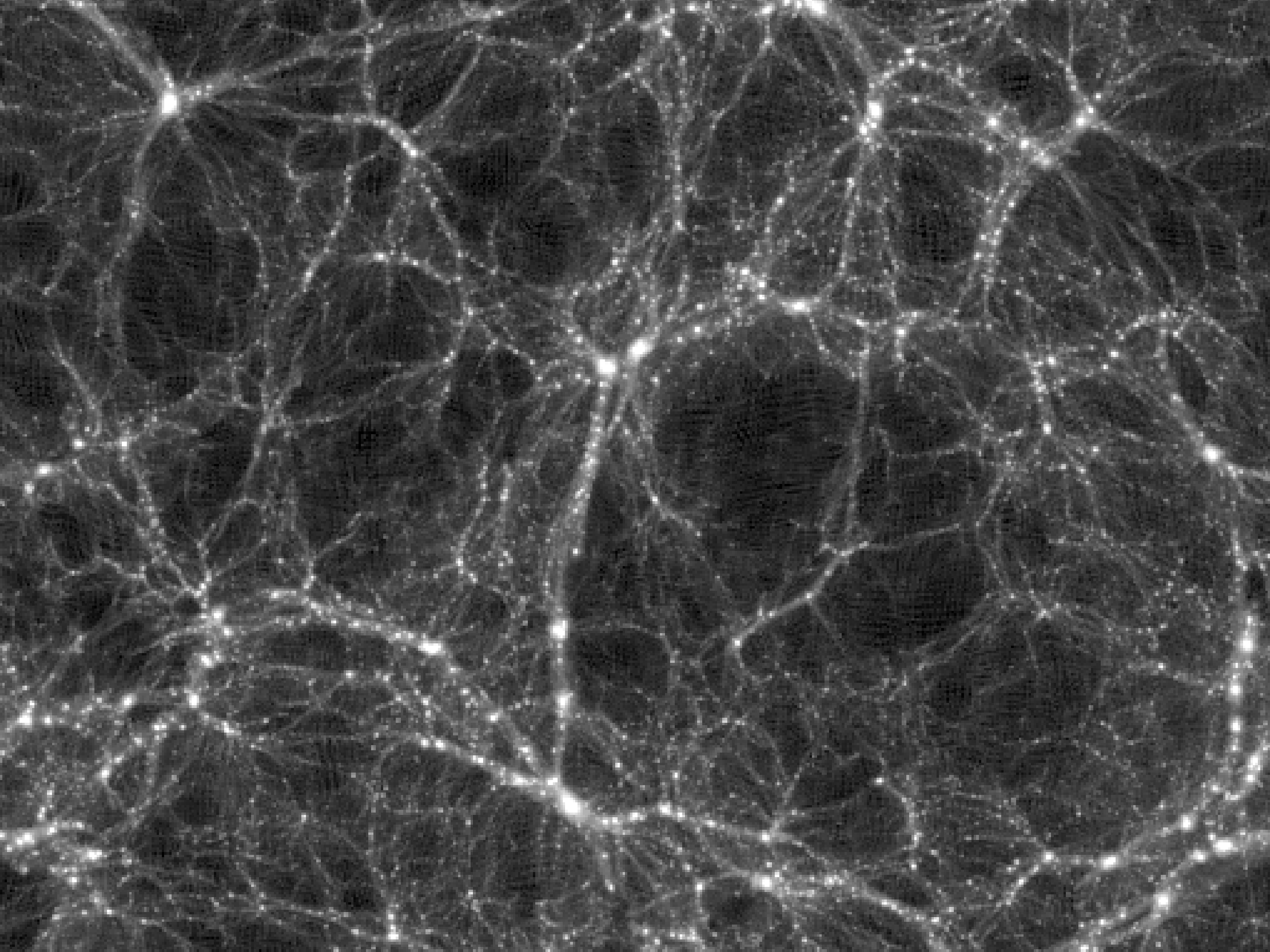
Then we can define the two- and three-point correlation statistics

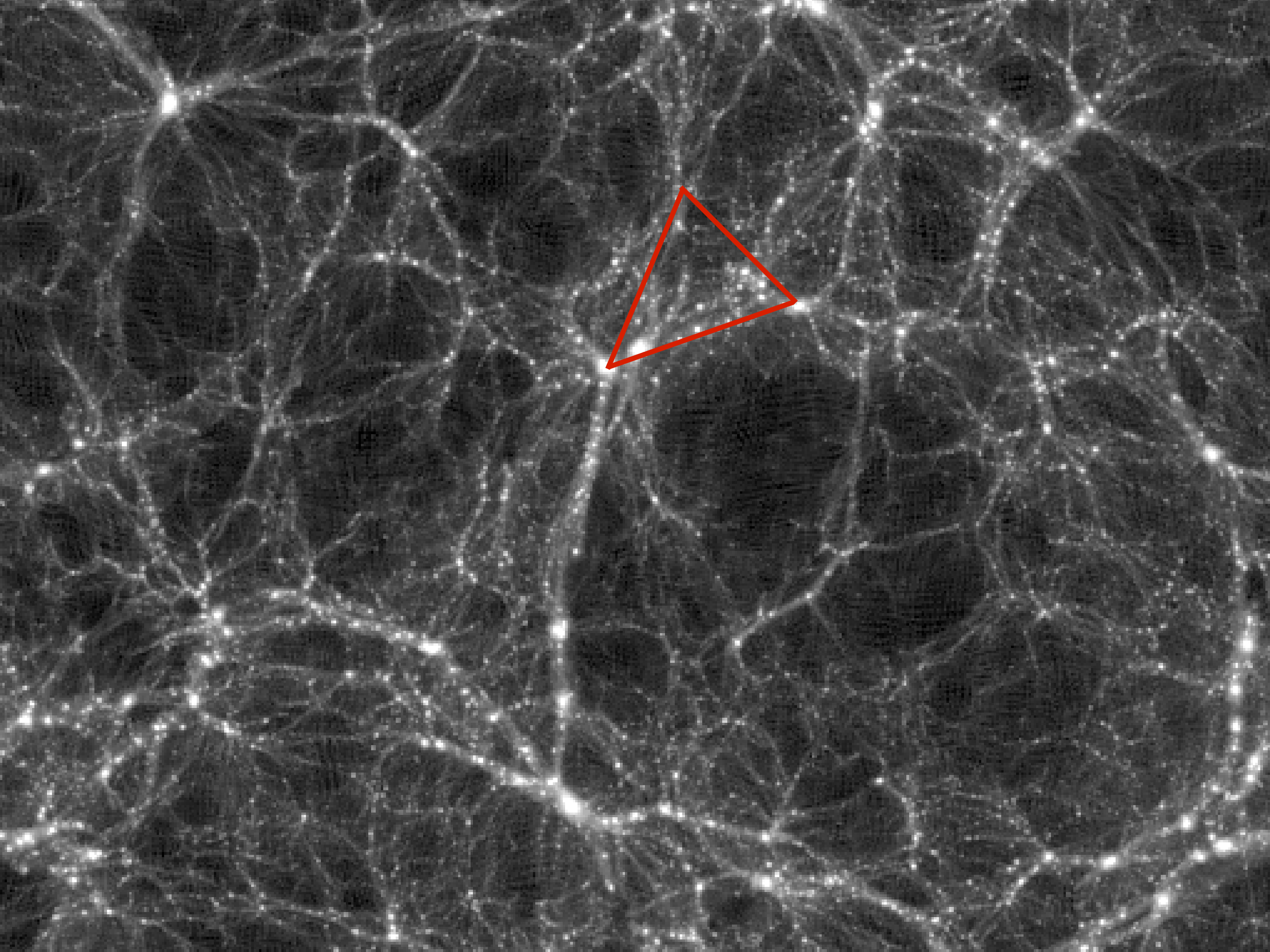
$$\langle \delta(\mathbf{s})\delta(\mathbf{s}+\mathbf{r})\rangle_c = \xi(r)$$

$$\langle \delta(\mathbf{s})\delta(\mathbf{s}+\mathbf{r}_1)\delta(\mathbf{s}+\mathbf{r}_2)\rangle_c = \zeta(\mathbf{r}_1, \mathbf{r}_2)$$

The power spectrum is simply the Fourier transform of $\xi(r)$

$$\xi(r) = 4\pi \int dk\, k^2\, P(k)\, \frac{\sin(kr)}{kr}$$

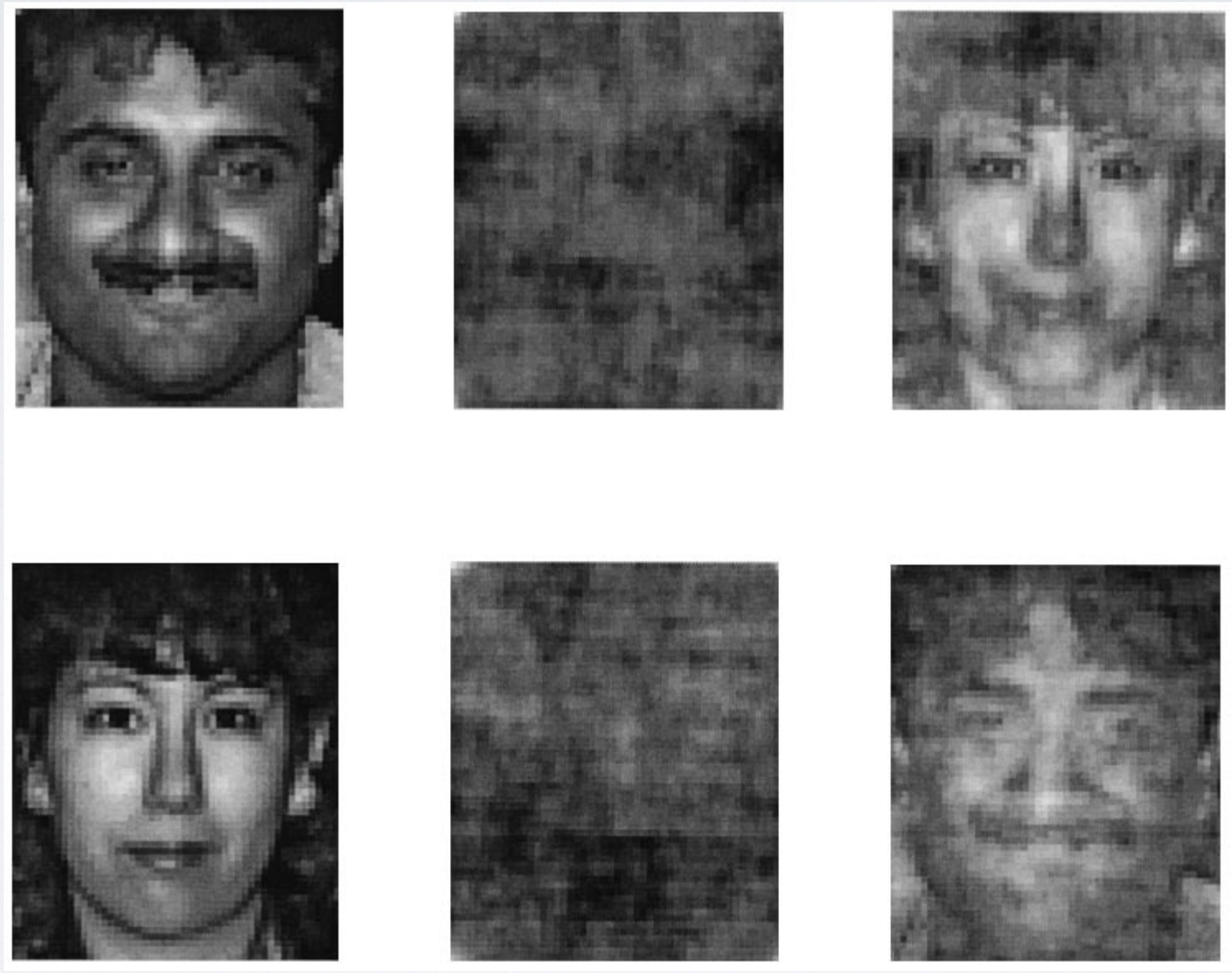




INTRODUCING THE PHASE-CORRELATION

- A powerful discriminator of filamentary structure
- A cheap way to incorporate information from higher-order correlation functions
- Inherently a probe of non-linear gravitational evolution: dark matter properties, modified gravity
- New cosmological parameter estimation
- Constraints on primordial non-Gaussianity

PHASE INFORMATION



Original | Phases Shuffled | Phases Swapped

Bartlett et al. (2002)

CORRELATION STATISTICS

We define the density contrast in real space as $$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}}$$

We take the Fourier transform $$\delta(\mathbf{k}) = \frac{1}{\hat{\mathrm{V}}} \int \mathrm{d}^3\mathbf{x}\, \delta(\mathbf{x}) \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}$$

$$= |\delta_{\mathbf{k}}| e^{i\theta_{\mathbf{k}}}$$

The power spectrum $$\langle \delta(\mathbf{k}) \delta(\mathbf{k}') \rangle_c = P(k) \delta_D(\mathbf{k} + \mathbf{k}')$$

The bispectrum $$\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \rangle_c = B(k_1, k_2, k_3) \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

THE PHASE

The phase is defined as: $$\epsilon(\mathbf{k}) = \frac{\delta(\mathbf{k})}{|\delta_{\mathbf{k}}|} = e^{i\theta_{\mathbf{k}}}$$

From which we can define the analogous correlation functions, e.g.

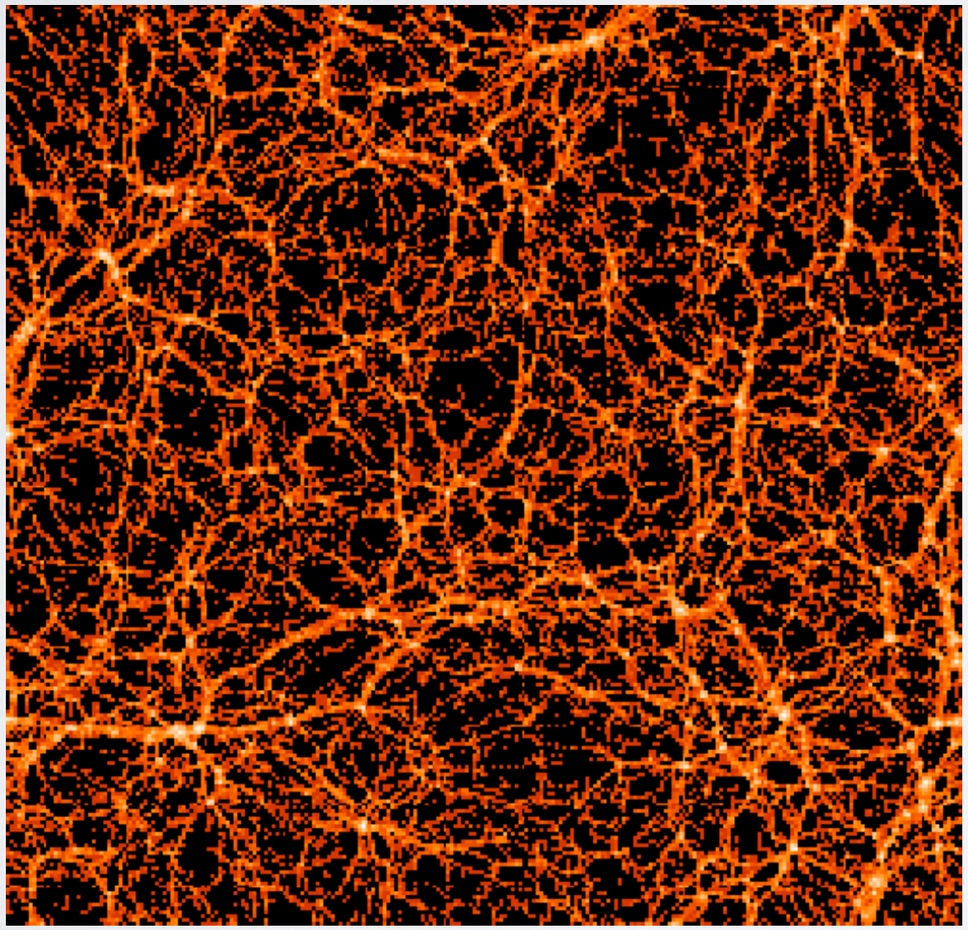
$$\langle \epsilon(\mathbf{k}_1)\epsilon(\mathbf{k}_2)\epsilon(\mathbf{k}_3)\rangle_c = \beta(k_1, k_2, k_3)\delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

We can then Fourier transform to get the three-point correlation in real space, the so called line correlation

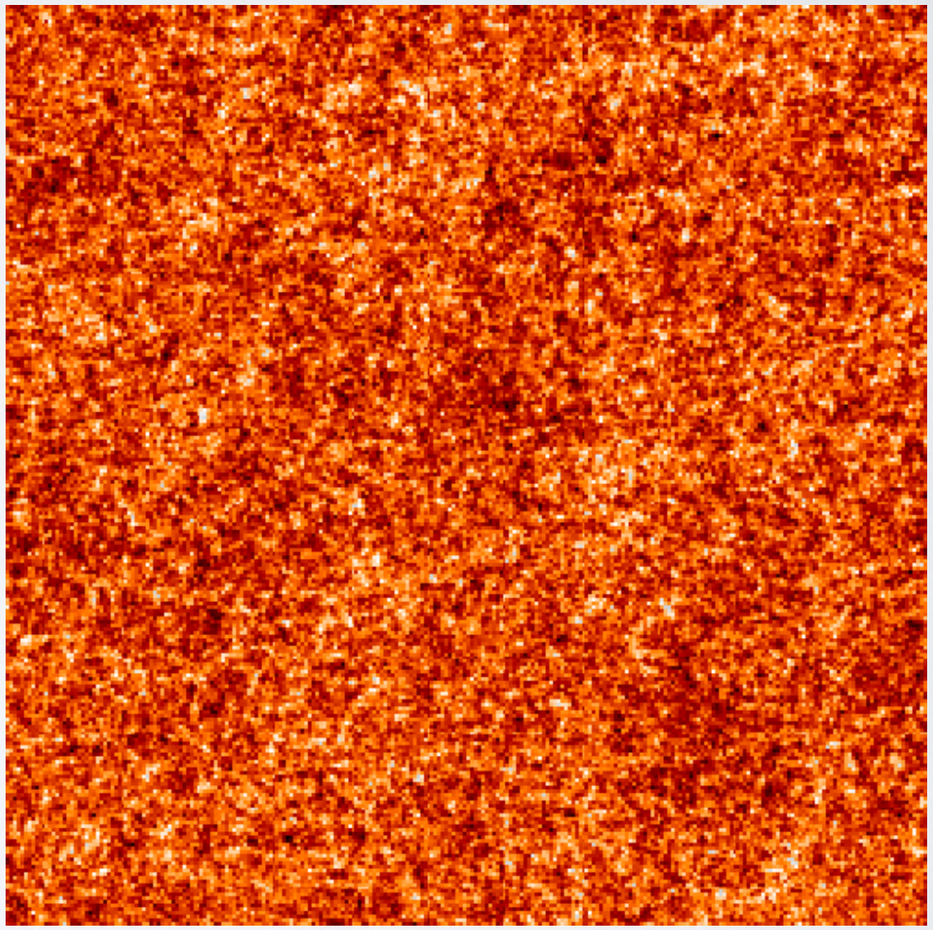
$$\ell(r) \propto \langle \epsilon(\mathbf{s})\epsilon(\mathbf{s}+\mathbf{r})\epsilon(\mathbf{s}-\mathbf{r})\rangle_c$$

A FURTHER DEMONSTRATION OF PHASE INFORMATION

Original Simulation

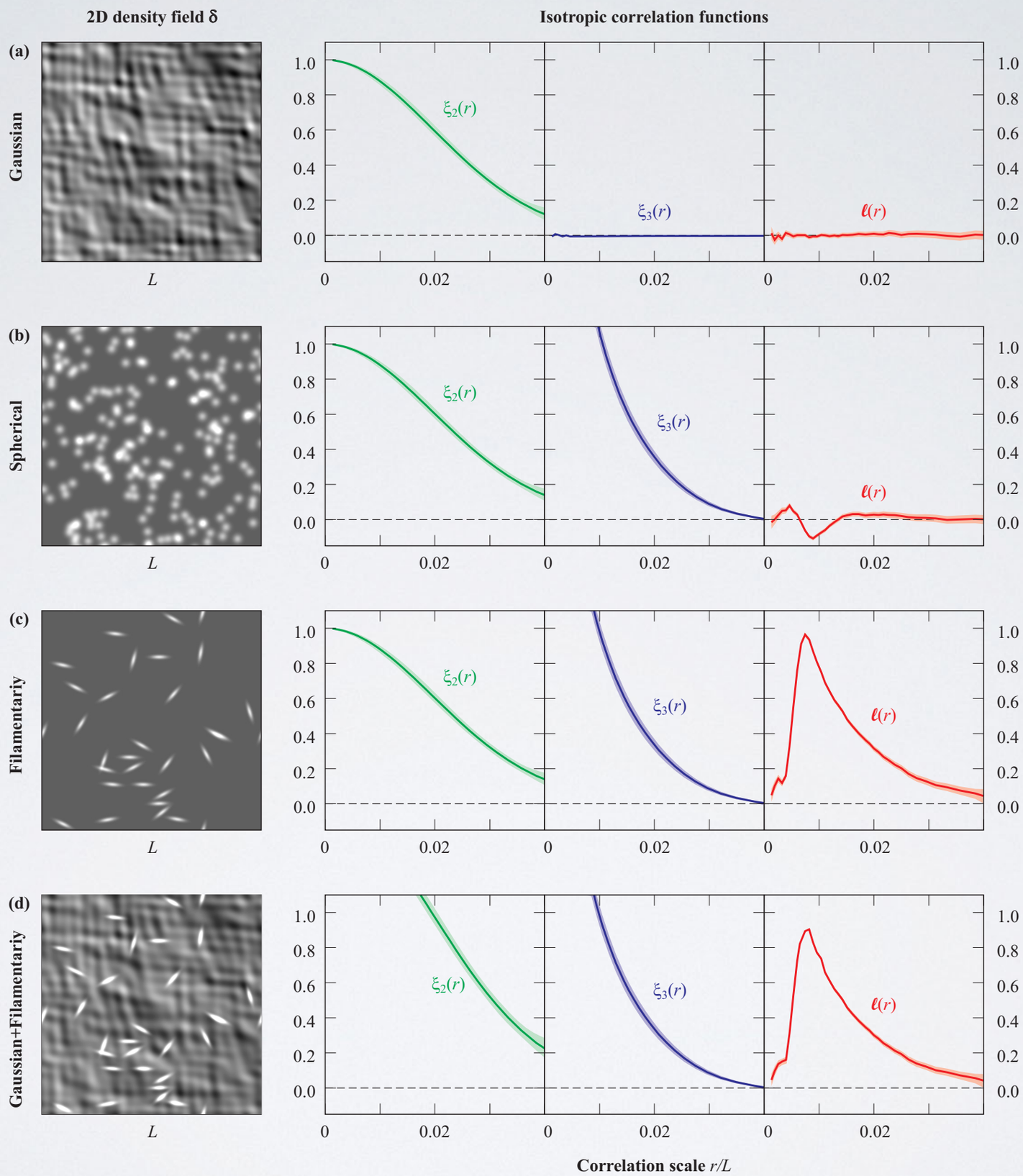


Phases Shuffled

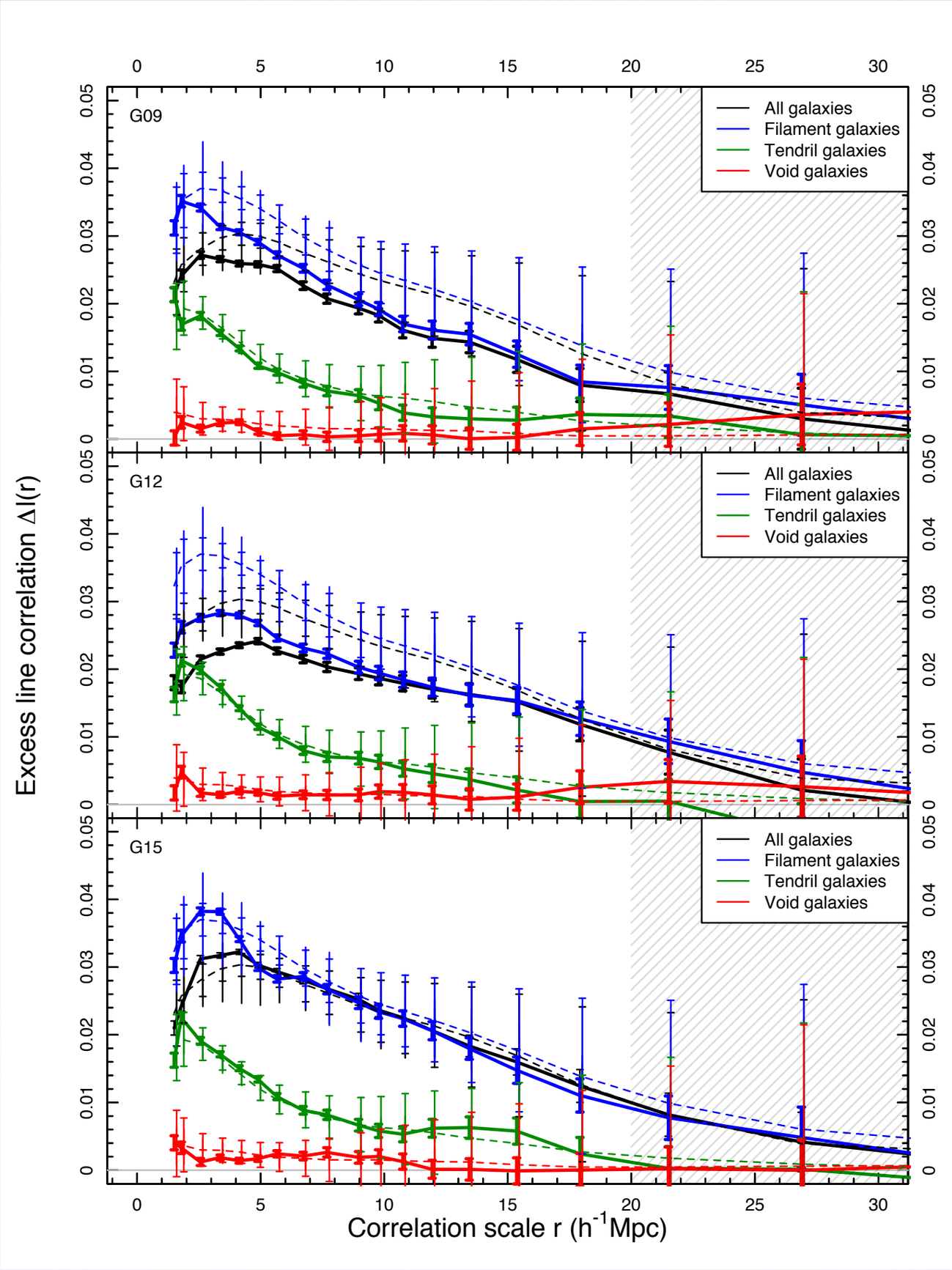


Two fields have the same power spectrum but different morphology

Coles & Chiang (2000)



Obreschkow et al. (2013)



Alpaslan et al. (2014)

ANALYTIC CALCULATION

Starting with a probability distribution function, we can expand in the mildly non-linear regime (Edgeworth expansion)

$$\mathcal{P}[\delta] = \exp\left[\sum_{N=3}^{\infty} \frac{(-1)^N}{N!} \int d^3\mathbf{k}_1 \ldots d^3\mathbf{k}_N \langle \delta(\mathbf{k}_1) \ldots \delta(\mathbf{k}_N) \rangle_c \frac{\partial}{\partial \delta(\mathbf{k}_1)} \cdots \frac{\partial}{\partial \delta(\mathbf{k}_N)}\right] \mathcal{P}_G[\delta]$$

ANALYTIC CALCULATION

Starting with a probability distribution function, we can expand in the mildly non-linear regime (Edgeworth expansion)

$$\mathcal{P}[\delta] = N_G \exp\left(-\frac{1}{2}\int d^3\mathbf{k}\frac{\delta(\mathbf{k})\delta(-\mathbf{k})}{P(k)}\right)$$
$$\left\{1 + \frac{1}{3!}\int d^3\mathbf{p}\, d^3\mathbf{q}\ \frac{B(\mathbf{p},\mathbf{q},-\mathbf{p}-\mathbf{q})\delta(-\mathbf{p})\delta(-\mathbf{q})\delta(\mathbf{p}+\mathbf{q})}{P(p)P(q)P(|\mathbf{p}+\mathbf{q}|)}\right\}$$

ANALYTIC CALCULATION

We rewrite in terms of the phase and then take expectations

$$\mathcal{P}[\delta] \Rightarrow \mathcal{P}[|\delta|, \theta]$$

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ANALYTIC CALCULATION

We rewrite in terms of the phase and then take expectations

$$\langle \epsilon_{\mathbf{k}_1} \epsilon_{\mathbf{k}_2} \epsilon_{\mathbf{k}_3} \rangle = \delta^K_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3} \left(\frac{\sqrt{\pi}}{2} \right)^3 \frac{(2\pi)^{3/2} B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{\sqrt{V P(\mathbf{k}_1) P(\mathbf{k}_2) P(\mathbf{k}_3)}}$$

PERTURBATION THEORY

In Eulerian perturbation theory we can express the bispectrum as

$$B(k_1, k_2, k_3) = F_2(\mathbf{k}_1, \mathbf{k}_2, z) P_L(k_1, z) P_L(k_2, z) + \text{cyc}$$

$$F_2(\mathbf{k}_1, \mathbf{k}_2, z) = \frac{1}{2}(1+\epsilon) + \frac{\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) + \frac{1}{2}(1-\epsilon)(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)^2$$

$$\epsilon(z) \equiv \frac{3}{7}\left(\frac{\rho_m(z)}{\rho_{\text{tot}}(z)}\right)^{-\frac{1}{143}}$$

PERTURBATION THEORY

In Eulerian perturbation theory we can express the bispectrum as

$$B(k_1, k_2, k_3) = F_2^{\rm eff}(\mathbf{k}_1, \mathbf{k}_2, z) P(k_1, z) P(k_2, z) + {\rm cyc}$$

$$F_2^{\rm eff}(\mathbf{k}_1, \mathbf{k}_2, z) = \frac{5}{7} a(n_1, k_1) a(n_2, k_2) + \frac{\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2}{2} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) b(n_1, k_1) b(n_2, k_2)$$
$$+ \frac{2}{7} (\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)^2 c(n_1, k_1) c(n_2, k_3)$$

$$n \equiv \frac{\partial \ln P_L(k)}{\partial \ln k}$$

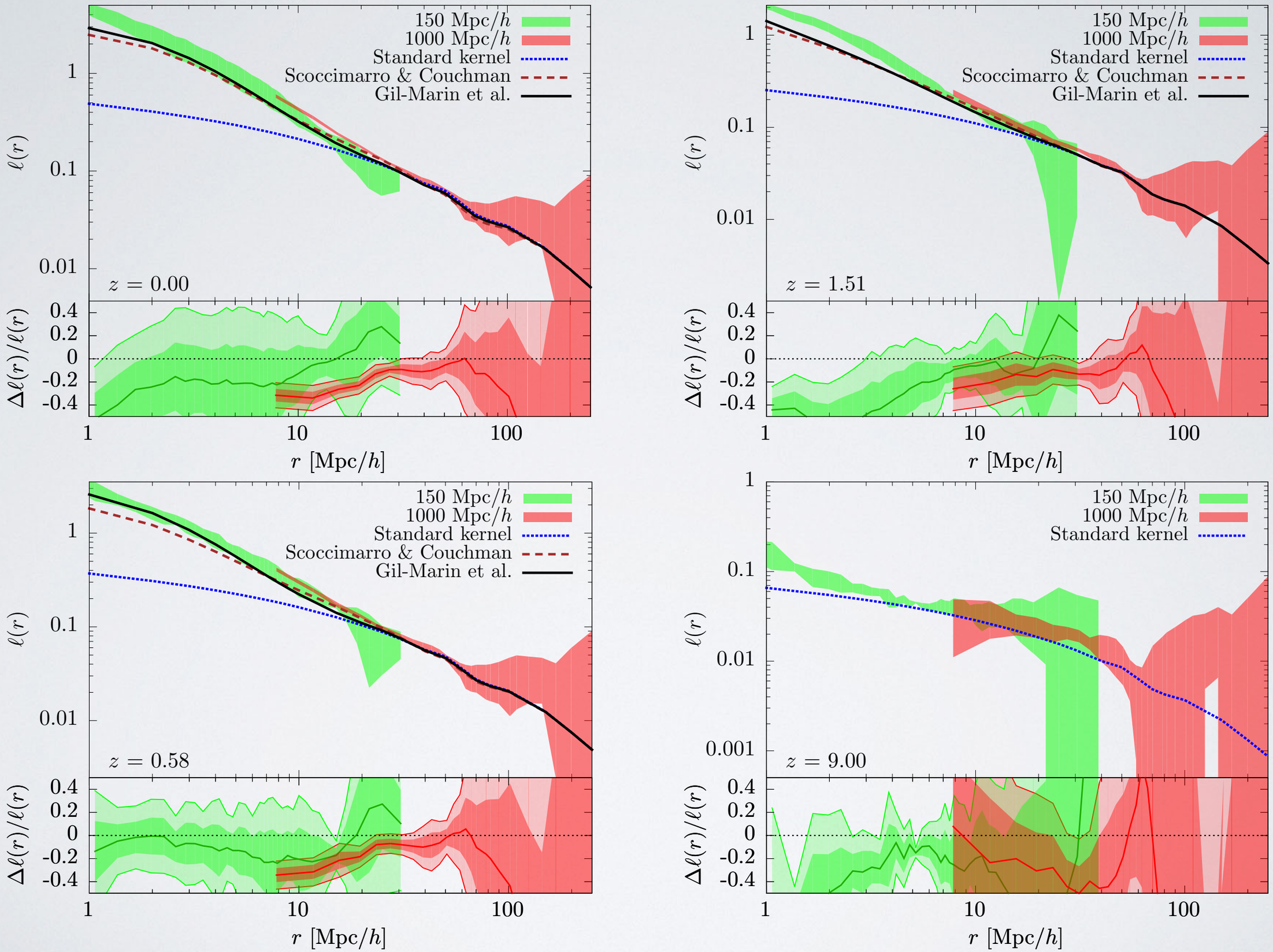
Scoccimarro & Couchman (2001)
Gil-Marin et al. (2012)

PERTURBATION THEORY

We can then recast the expression for the three-point phase correlation as

$$\langle \epsilon(\mathbf{k}_1)\epsilon(\mathbf{k}_2)\epsilon(\mathbf{k}_3)\rangle = \left(\frac{\sqrt{\pi}}{2}\right)^3 \sqrt{\frac{(2\pi)^3}{V}}\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$
$$\times\ 2\left[F_2(\mathbf{k}_1,\mathbf{k}_2)\sqrt{\frac{P(k_1,\eta)P(k_2,\eta)}{P(k_3,\eta)}} + \text{cyc.}\right]$$

So we can calculate the correlation function in this limit requiring only the power spectrum


150 Mpc/h
1000 Mpc/h
Standard kernel
Scoccimarro & Couchman
Gil-Marin et al.
ℓ(r)
Δℓ(r)/ℓ(r)
r [Mpc/h]
z = 0.00
z = 1.51
z = 0.58
z = 9.00

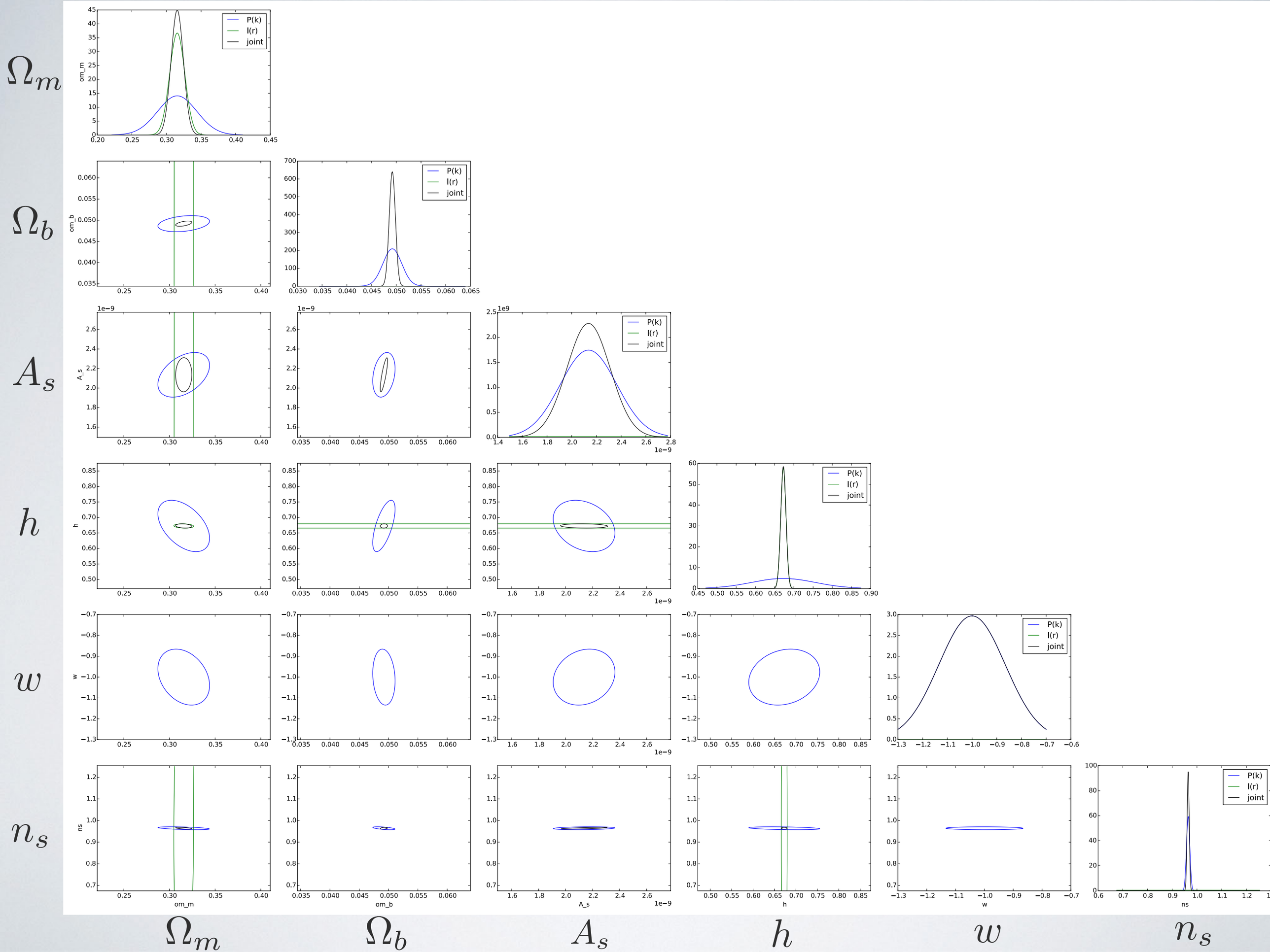
INTERMISSION

PARAMETER ESTIMATION

Fisher Information gives the optimal parameter estimation

$$F_{\alpha\beta} = \sum_{i,j} \frac{\partial X_i}{\partial \alpha} \left[C_{ij}\right]^{-1} \frac{\partial X_j}{\partial \beta}$$

Include only the Gaussian contribution in the covariance matrix as a first approximation


Ω_m
Ω_b
A_s
h
w
n_s
P(k)
I(r)
joint
om_m
om_b
A_s
h
w
ns
1e−9

SUMMARY

- There is information in the phase which is not encapsulated in the power spectrum
- Correlation statistics of the phase are a new measure of structure with multiple applications
- I have presented an analytical expression which, within the framework of cosmological perturbation theory, is dependent only on the power spectrum

The first two types of terms in the bispectrum covariance matrix are

$$C^B_{ij} = s_B \frac{V_f}{V_B}\delta_{ij}P(k_{i_1})P(k_{i_2})P(k_{i_3}) + \delta_{i_1 j_1}\frac{V_f}{V_B(i)V_B(j)}\int_{k_{i1}} d^3\mathbf{q}_1 \ldots \int_{k_{i3}} d^3\mathbf{q}_3$$
$$\times \int_{k_{j2}} d^3\mathbf{p}_2 \int_{k_{j3}} d^3\mathbf{p}_3 \delta_D(\mathbf{q}_{123})\delta_D(\mathbf{q}_1+\mathbf{p}_{23})B(\mathbf{q}_1,\mathbf{p}_2,\mathbf{p}_3)B(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) + \mathrm{cyc}$$

The equivalent terms in the covariance matrix of the phase-only spectra equivalent to the bispectrum are

$$C^\beta_{ij} = s_B \frac{V_f^4}{V_B}\delta_{ij} + \delta_{i_1 j_1}\frac{V_f^4}{V_B(i)V_B(j)}\left(\frac{\sqrt{\pi}}{2}\right)^6 \int_{k_{i1}} d^3\mathbf{q}_1 \ldots \int_{k_{i3}} d^3\mathbf{q}_3 \int_{k_{j2}} d^3\mathbf{p}_2 \int_{k_{j3}} d^3\mathbf{p}_3$$
$$\times \delta_D(\mathbf{q}_{123})\delta_D(\mathbf{q}_1+\mathbf{p}_{23})\frac{B(\mathbf{q}_1,\mathbf{p}_2,\mathbf{p}_3)}{\sqrt{P(q_1)P(p_2)P(p_3)}}\frac{B(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3)}{\sqrt{P(q_1)P(q_2)P(q_3)}} + \mathrm{cyc}$$

Notice that the Gaussian variance is now independent of the power spectrum

QUANTIFYING THE INFORMATION

One measure we can explore is the signal-to-noise ratio

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)^2_P = \sum_{k_i,k_j<k_{\mathrm{max}}} P(k_i)[C^P]^{-1}_{ij}P(k_j)$$

We can extend this to a joint measurement of the power spectrum and the bispectrum $D = \{P_1, P_2, \ldots, P_N, B_1, B_2, \ldots, B_M\}$

$$\left(\frac{S}{N}\right)^2 = \sum_{k_i,k_j<k_{\mathrm{max}}} D_i[C^{P+B}]^{-1}_{ij}D_j \qquad C = \begin{pmatrix} C^P & C^{PB} \\ C^{PB} & C^B \end{pmatrix}$$