Electroweak, Strong interactions and Higgs fields as components of gravity in noncommutative spacetime

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Motivations

1. Search for the original structure of spacetime
2. The idea of noncommutative spacetime
3. The continuity between two concepts

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Content

The weakness of extra dimensions

- 1921-1948, Kaluza-Klein theory: $\mathcal{M}^4 \times \mathcal{S}^1$ Zero-mode spectrum: Gravity + Electromagnetism + Brans-Dicke scalar.
- 1968, the theory can be extended to include the nonabelian gauge fields. 1971, spinor dimensions can be added to include supersymmetries.
- 1980's, the theory can be extended to include an arbitrary number of compact internal space dimensions.
- The multi-dimensional theories are usually too large, including unobserved fields and infinite tower of fields, leading to inconsistencies at quantum level.

2.1 Basic building blocks (1)

Alain Connes: NCG is a new structure of spacetime. NCG is constructed out of three building blocks called "spectral triplet". In a special case of $\mathcal{M}^d \times Z^2$

i) The Hilbert space which is a direct sum of left and right-handed chiral spinors $\mathcal{H}_v \oplus \mathcal{H}_w$. So the wave function in this Hilbert space can be represented by

$$\Psi = \left[\begin{array}{c} \psi_v \\ \psi_w \quad , \end{array} \right] \tag{1}$$

ii) The algebra $\mathcal{A} = \mathcal{A}_v \oplus \mathcal{A}_w$, $\mathcal{A}_i = \mathcal{C}^\infty(\mathcal{M})^\lceil$, $\mathcal{M}^d$ is the d-dimensional spin manifold. The elements of $\mathcal{A}$ are represented as the diagonal 0-form matrices $\mathcal{F}$ as follows

2.1 Basic building blocks (2)

$$\mathcal{F} = \begin{bmatrix} F_v & 0 \\ 0 & F_w \end{bmatrix}, F_i \in \mathcal{A}_i \tag{2}$$

iii) Dirac operator $\mathcal{D} = D + \Theta$, D is the d-dimensional Dirac operator. Thus, $\mathcal{D} = D^2 = D\Theta + \Theta D = \Theta^2 = 0$. Represented as

$$\begin{aligned} \mathcal{D} = D + \Theta &= DX^m\partial_m + DX^d\partial_d = DZ\partial_z + D\bar{Z}\partial_{\bar{z}} \\ DX^m = \begin{bmatrix} \gamma^m & 0 \\ 0 & \gamma^m \end{bmatrix}, \quad \partial_d\mathcal{F} &= \partial_z\mathcal{F} = -\partial_{\bar{z}}\mathcal{F} = -m(F_w - F_v) \end{aligned} \tag{3}$$

The action of $\mathcal{D}$ on the 0-forms $\mathcal{F}$ can be defined as follows

$$\mathcal{D}\mathcal{F} = \begin{bmatrix} \gamma^m\partial_m F_v & m\gamma^d(F_w - F_v) \\ m\gamma^d(F_v - F_w) & \gamma^m\partial_m F_v \end{bmatrix}, (\gamma^d)^2 = 1 \tag{4}$$

Higgs fields as components of gauge fields

Gauge potential 1-form

$$\begin{aligned} A &= DX^m A_m(x) + DX^d \Phi(x) = \begin{bmatrix} \gamma^m a_{vm}(x) & \gamma^d H(x) \\ -\gamma^d \bar{H}(x) & \gamma^m a_{wm}(x) \end{bmatrix} \\ A_m(x) &= \begin{bmatrix} a_{vm}(x) & 0 \\ 0 & a_{wm}(x) \end{bmatrix}, \Phi(x) = \begin{bmatrix} \bar{H}(x) & 0 \\ 0 & H(x) \end{bmatrix} \end{aligned} \quad (5)$$

$a_{v,wm} = a^i_{v,wm} T^i_{v,w}$, $T^i_{v,w}$: generators of the gauge groups $G_{v,w}$
The gauge Lagrangian with the field strength $F = dA + A \wedge A$

$$\mathcal{L}_g = -\frac{1}{4} < F, F >= -\frac{1}{4} F_{mn} F^{mn} + \frac{1}{2} \nabla^m \bar{H} \nabla_m H + V(H, \bar{H}) \quad (6)$$

Quartic Higgs potential is obtained when $DZ \wedge D\bar{Z} \neq 0$

Refined structure

Nguyen Ai Viet,[1] MRST 1994; [2](Dedicated to the memory of E. Wigner)Acta Physica Hungaricae, Heavy Ion Physics **1** (1995), 263:

$$
\begin{aligned}
D_Q = DZ\partial_z + D\bar{Z}\partial_{\bar{z}} \quad &, \quad D_Q F = \begin{bmatrix} 0 & m\theta(f_2 - f_1) \\ m\bar{\theta}(f_1 - f_2) & 0 \end{bmatrix} \\
DZ = \begin{bmatrix} 0 & \theta \\ 0 & 0 \end{bmatrix} \quad &, \quad D\bar{Z} = \begin{bmatrix} 0 & 0 \\ \bar{\theta} & 0 \end{bmatrix} \\
\partial_z F = m(f_2 - f_1) \quad &, \quad \partial_{\bar{z}} F = m(f_1 - f_2)
\end{aligned} \tag{7}
$$

Standard Model: with L, R quarks and leptons. The model has less parameters, being more predictive.
The model predicts: Top quark mass 172 GeV, Higgs mass: 241 GeV.

a. A brief history

- Question: What is the gravity in NC spacetime?
- Chamseddine, Felder, Fröhlich (1993, CMP): No-go.
- Landi, Viet, Wali (1994, PL): Kaluza-Klein's zero modes.
- Viet-Wali(1996a, 1996b, JIMP) Discretized version K-K theory with bigravity, bivector and biscalar. In each pair: massless and massive. New features: cosmological constant, non-linear scalars, massive gravity.
- Coupling with chiral spinors (Viet-Wali, 1997, PRD 2000, AIP Conf). Unified spacetime with Kähler metrics (Viet 1995, 1996, 2000, 2014).
- Nonabelian gauge fields and unified theory of all interactions by NCG (Viet-Du 2015)

b. General model (Viet-Wali, 1996b IJMPA)

1.General curved basis $DX^M; M = m, \dot{d}$ and the local orthonormal basis $\mathcal{E}^A, A = a, \dot{d}$ with the vielbein $E^A_M(x)$

$$\mathcal{E}^A = DX^M \mathcal{E}^A_M(x) \quad , \quad DX^M = \mathcal{E}^A \mathcal{E}^A_M(x) \tag{8}$$

The most general forms

$$\mathcal{E}^a_m = \mathcal{E}^a_m(x) \ , \ \mathcal{E}^a_{\dot{5}} = 0 \ , \ \mathcal{E}^{\dot{5}}_m = A_m(x) \ , \ \mathcal{E}^a_{\dot{5}} = \Phi(x) \tag{9}$$

2. The Cartan structure equations

$$\mathcal{T}^A = D\mathcal{E}^A + \mathcal{E}^B \wedge \Omega^A{}_B \tag{10}$$

$$\mathcal{R}^{AB} = D\Omega^{AB} + \Omega^A{}_C \wedge \Omega^{CB} \tag{11}$$

3. R_{d+1} contains d-dimensional bigravity, bivector, biscalar. Massive and massless. Instead of the torsion free condition, a minimal set of constraints is used (Nguyen Ai Viet 1999, 2014)

c.Special "ansatz"

- Landi-Viet-Wali (1994, PLB): K-K's zero modes. Nonabelian gauge does not have quartic potential.

$$\mathcal{E}^a_m = e^a_m(x)\ ,\ \mathcal{E}^a_5 = 0\ ,\ \mathcal{E}^{\dot{5}}_m = a_m(x)\ ,\ \mathcal{E}^a_5 = \phi(x) \qquad (12)$$

- Viet-Wali (1996a, IJMPA) $T^A = 0$ Scalars $\beta(x)$ and $\alpha(x)$ with nonlinear potential (sixth order or third order), the mass of $\beta(x)$ is predicted as:

$$\begin{aligned} e^a_{1m}(x) &= \beta(x) e^a_{2m}(x) \quad , \quad \mathcal{E}^a_5 = 0 \\ a_{1m}(x) &= \alpha(x) a_{2m}(x) \quad , \quad \phi_1(x) = \phi_2 \alpha^{-1}(x) \end{aligned} \qquad (13)$$

- Viet-Du (2015, to be published) Zero modes with nonabelian gauge fields

$$a_\mu = a^i_\mu(x) T^i, \phi_1(x) = -\phi_2(x) = \phi(x) \qquad (14)$$

3.1 Spacetime with two extra discrete dimensions

- Combining the idea of NC gauge theories and NC gravity.
- Now we can "see" the NC spacetime of $\mathcal{M}^4 \times Z^2 \times Z^2$
- The dimension reduction is carried out in two steps.
- In the first step: 6-th dim gravity is reduced to 5-dim gravity and the nonabelian gauge theory of $SU(2) \times U(1)$.
- In the second step: 5-dim gravity is reduced to 4-dim gravity and the nonabelian gauge theory of $SU(3)$. At the same time, 5-dim $SU(2) \times U(1)$ theory is reduced to 4-dim $SU(2) \times U(1)$ gauge theory and Higgs fields with a quartic potential.
- The color gauge theory does not have their Higgs.
- The theory is torsion free and metric compatible as GR is.

3.2 Final action

- Electroweak, strong interactions and Higgs fields are parts of gravity in a noncommutative four sheeted spacetime.
- After the dimensional reduction, $\phi_1 = \phi_2 = 1$

$$\begin{aligned} R_6 &= R_5 - \frac{m_1^2 g_{EW}^2}{4} F^i_{MN} F^{iMN} = R_4 - \frac{m_2^2 g_S^2}{4} G^a_{\mu\nu} G^{a\mu\nu} \\ &- \frac{m_1^2 g_{EW}^2}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{m_1^2 g_{EW}^2}{2v^2} \nabla_\mu \bar{H} \nabla^\mu H + V(\bar{H}, H) \\ S &= \int d^4x \sqrt{detg} \frac{1}{16\pi G} R_6 \end{aligned} \tag{15}$$

- $g_S = 4\sqrt{\pi G}/m_2 \;\; , \;\; g_{EW} = 4\sqrt{\pi G}/m_1$

4.1 Bigravity and massive gravity

Bigravity is also obtained as a natural component of gravity in our NC spacetime. The case $\phi_1 = \phi_2 = 1; a_1 = a_2 = 0$ leads to the Einstein-Hilbert action

$$\begin{aligned} \mathcal{L}(5)_{EH} &= \frac{1}{16\pi G} Tr \int d^4x(\sqrt{-detg_1}R(g_1) + \sqrt{-detg_2}R(g_2) \\ &+ \ m^2\mathcal{L}_m(g_1, g_2)) \\ \mathcal{L}_m &= \lambda + \mathcal{L}_{1m}(h,g) + \mathcal{L}_{2m}(h,g) + \mathcal{L}_{4m}(h,g) \\ \Lambda &= \frac{m^2}{16\pi G}\lambda > 0 \end{aligned} \tag{16}$$

With four copies of spacetime, one can assume 4-gravity in the most general case.

Summary

- New structure of spacetime: NCG + two discrete extra dimensions. Modified spacetime leads to modified gravity.
- All the known interactions and Higgs fields are components of gravity in NC spacetime.
- Less parameters leading to phenomenological predictions
- Bigravity and multigravity are components of gravity in NC spacetime
- Scalar of sixth order interaction.
- The theory has finite content
- Cosmological implications are possible
- Standard Model coupled with gravity.
- Four copied spacetime and quaternion NCG (Balakrisha-Viet-Gürsey-Wali, 1991, PRD)

Connes spacetime

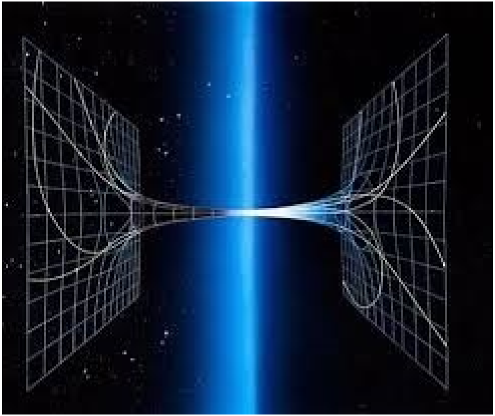

Figure : Two sheeted spacetime

Four parallel Universes

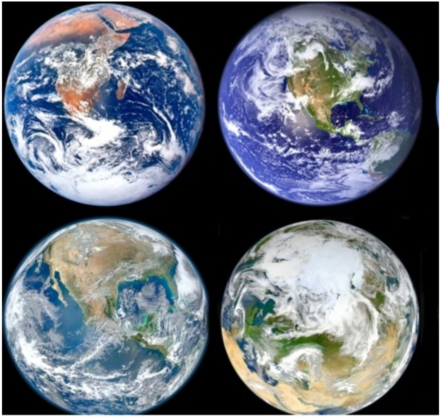

Figure : Four copied spacetime

How does the elephant look like?

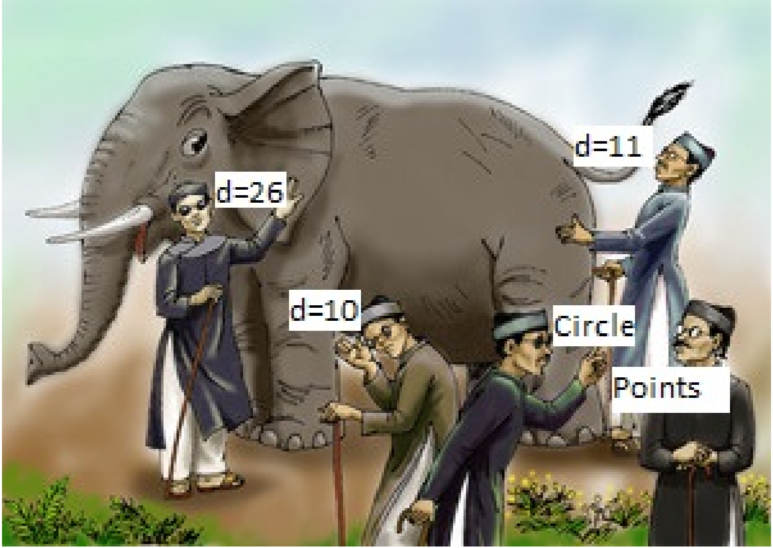

Figure : Blind fortune tellers