A NEW WAY OF DEALING WITH THE NEUTRINO COMPONENT IN COSMOLOGY

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Rencontres du Vietnam
Cosmology fifty years after CMB discovery
August, 19th 2015

The Newtonian description of structure growth

- The behavior of CDM perturbations is governed by the continuity and Euler equations:

$$\frac{\partial\delta(\mathbf{x},t)}{\partial t}+\frac{1}{a}[(1+\delta(\mathbf{x},t))u_i(\mathbf{x},t)]_{,i}=0,$$

$$\frac{\partial u_i(\mathbf{x},t)}{\partial t}+\frac{\dot{a}}{a}u_i(\mathbf{x},t)+\frac{1}{a}u_j(\mathbf{x},t)u_i(\mathbf{x},t)_{,j}=-\frac{1}{a}\Phi(\mathbf{x},t)_{,i}-\frac{(\rho(\mathbf{x},t)\sigma_{ij}(\mathbf{x},t))_{,j}}{a\rho(\mathbf{x},t)}$$

- Single-flow approximation: $\cancel{\frac{(\rho(\mathbf{x},t)\sigma_{ij}(\mathbf{x},t))_{,j}}{a\rho(\mathbf{x},t)}}.$

Illustration of the emergence of shell-crossing

The Newtonian description of structure growth

- In the single-flow approximation, the Euler equation reads

$$\frac{\mathrm{d}(a u_i(x^i,t))}{\mathrm{d}t} = -\frac{\partial \Phi(x^i,t)}{\partial x^i}.$$

➡ The velocity field is a gradient.

➡ It is entirely characterized by its divergence

$$\theta(x^i,t) = \frac{1}{aH}\frac{\partial u_i(x^i,t)}{\partial x^i}.$$

➡ In reciprocal space, the system can be rewritten compactly with the help of the variable

$$\Psi_a(\mathbf{k},\eta) \equiv (\delta(\mathbf{k},\eta), -\theta(\mathbf{k},\eta)).$$

The Newtonian description of structure growth

- The resulting equation is

$$\boxed{\frac{\partial \Psi_a(\mathbf{k},\eta)}{\partial \eta} + \Omega_a{}^b(\eta)\Psi_b(\mathbf{k},\eta) = \gamma_a{}^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1,\eta)\Psi_c(\mathbf{k}_2,\eta)},$$

with $\gamma_a{}^{bc}(\mathbf{k}_a,\mathbf{k}_b) = \gamma_a{}^{cb}(\mathbf{k}_b,\mathbf{k}_a)$,

$$\gamma_2{}^{22}(\mathbf{k}_1,\mathbf{k}_2) = \int \mathbf{d}^3\mathbf{k}_1\mathbf{d}^3\mathbf{k}_2\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{|\mathbf{k}_1+\mathbf{k}_2|^2(\mathbf{k}_1.\mathbf{k}_2)}{2\mathrm{k}_1^2\mathrm{k}_2^2},$$

$$\gamma_2{}^{21}(\mathbf{k}_1,\mathbf{k}_2) = \int \mathbf{d}^3\mathbf{k}_1\mathbf{d}^3\mathbf{k}_2\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{(\mathbf{k}_1+\mathbf{k}_2).\mathbf{k}_1}{2\mathrm{k}_1^2}$$

and $\gamma = 0$ otherwise.

- This compact equation of motion has a formal solution

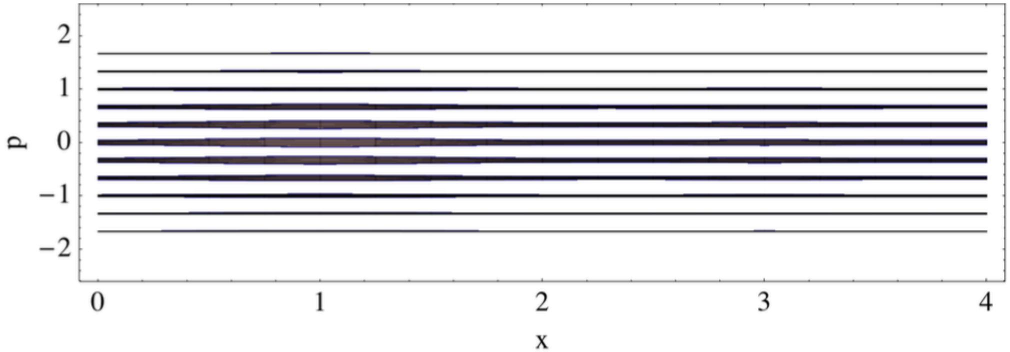
$$\Psi_a(\mathbf{k},\eta) = g_a{}^b(\eta)\Psi_b(\mathbf{k},\eta_0) + \int_{\eta_0}^{\eta} d\eta' g_a{}^b(\eta,\eta')\gamma_b{}^{cd}(\mathbf{k}_1,\mathbf{k}_2)\Psi_c(\mathbf{k}_1,\eta')\Psi_d(\mathbf{k}_2,\eta').$$

initial time

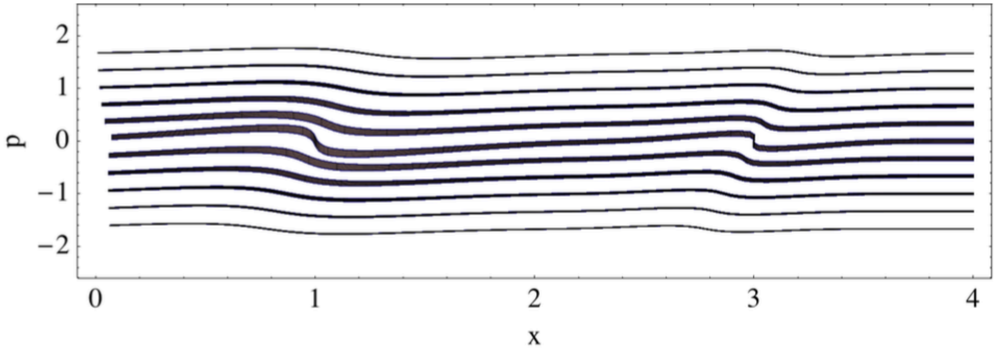
Green function

Towards a relativistic generalization of cosmological perturbation theory

- Discretized phase space at initial time:



- Discretized phase space at a later time:



- One alternative modeling of massive neutrinos beyond the linear regime: arXiv 1408.2995 (Blas et al.).

Towards a relativistic generalization of cosmological perturbation theory

- Our density field is $n_c(\eta, x^i) = \int \mathrm{d}^3 p_i \; f(\eta, x^i, p_i)$.
- By definition, $T_{\mu\nu}(\eta, x^i) = \int \mathrm{d}^3 p_i \, (-g)^{-1/2} \frac{p_\mu p_\nu}{p^0} \, f(\eta, x^i, p_i)$,

$$J_\mu(\eta, x^i) = -\int \mathrm{d}^3 p_i \, (-g)^{-1/2} \frac{p_\mu}{p^0} \, f(\eta, x^i, p_i).$$

- In the single-flow approximation,

our momentum field

$$f(\eta, x^i, p_i) = n_c(\eta, x^i) \delta_{\mathrm{D}} \left[p_i - P_i(\eta, x^i) \right].$$

After decoupling, the Einstein equations read

$$G_{\mu\nu}(\eta, x^i) = 8\pi G \sum_{\text{species, flows}} \frac{P_\mu(\eta, x^i) P_\nu(\eta, x^i)}{(-g)^{1/2} P^0(\eta, x^i)} n_c(\eta, x^i).$$

Towards a relativistic generalization of cosmological perturbation theory

- The Vlasov equation gives the first equation of motion:

$$\frac{\partial}{\partial \eta} n_c + \frac{\partial}{\partial x^i}\left(\frac{P^i}{P^0} n_c\right) = 0,$$

where $P^i = g^{ij} P_j$ and P^0 is defined so that $P^\mu P_\mu = -m^2$.

- In a single-flow fluid, $T^{\mu\nu} = -P^\mu J^\nu$.

energy-momentum tensor

particle four-current

Combined conservation laws impose

$$P^\nu \partial_\nu P_i = \frac{1}{2} P^\sigma P^\nu \partial_i g_{\sigma\nu}.$$

Towards a relativistic generalization of cosmological perturbation theory

- The equations of motion corresponding to subhorizon scales are:

$$\mathcal{D}_\eta n_c + \partial_i (V_i n_c) = 0,$$

$$\mathcal{D}_\eta P_i + V_j \partial_j P_i = \tau_0 \partial_i A + \tau_j \partial_i B_j - \frac{1}{2}\frac{\tau_j \tau_k}{\tau_0} \partial_i h_{jk},$$

initial momentum of the flow

with $\tau_0 = -\sqrt{m^2 a^2 + \tau_i^2}$, $\mathcal{D}_\eta = \frac{\partial}{\partial \eta} - \frac{\tau_i}{\tau_0}\frac{\partial}{\partial x^i}$

and $V_i = -\frac{P_i - \tau_i}{\tau_0} + \frac{\tau_i}{\tau_0}\frac{\tau_j (P_j - \tau_j)}{(\tau_0)^2}.$

peculiar velocity

Towards a relativistic generalization of cosmological perturbation theory

- On subhorizon scales, it is possible to show that the comoving momentum field is a gradient.

→ It is entirely characterized by its divergence.

→ It can be treated like the velocity field of CDM.

- By analogy with CDM, we introduce for each flow

$$\theta_{\tau_i}(\eta, x^i) = -\frac{1}{ma\mathcal{H}}\frac{\partial P_i(\eta, x^i; \tau_i)}{\partial x^i}, \quad \delta_{\tau_i}(\eta, x^i) = -\frac{n_c(\eta, x^i; \tau_i)}{n_c^{(0)}(\tau_i)} - 1$$

and for *N* flows:

$$\Psi_a(\mathbf{k}) = (\delta_{\tau_1}(\mathbf{k}), \theta_{\tau_1}(\mathbf{k}), \ldots, \delta_{\tau_n}(\mathbf{k}), \theta_{\tau_n}(\mathbf{k}))^T.$$

Towards a relativistic generalization of cosmological perturbation theory

- The resulting equations is

$$\boxed{\partial_\eta \Psi_a(\mathbf{k}) + \Omega_a^{\ b}\,\Psi_b(\mathbf{k}) = \gamma_a^{\ bc}(\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2)}.$$

➡ The relativistic equation is formally the same as the equation of motion describing CDM.

➡ It can be exploited using the standard non-relativistic formalism.

- For more details, see arXiv 1311.5487, 1411.0428 and 1503.05707 (Dupuy and Bernardeau, published in *JCAP*).