Excited Initial States in Inflation

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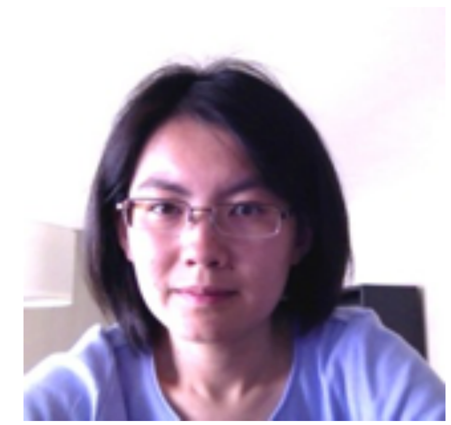
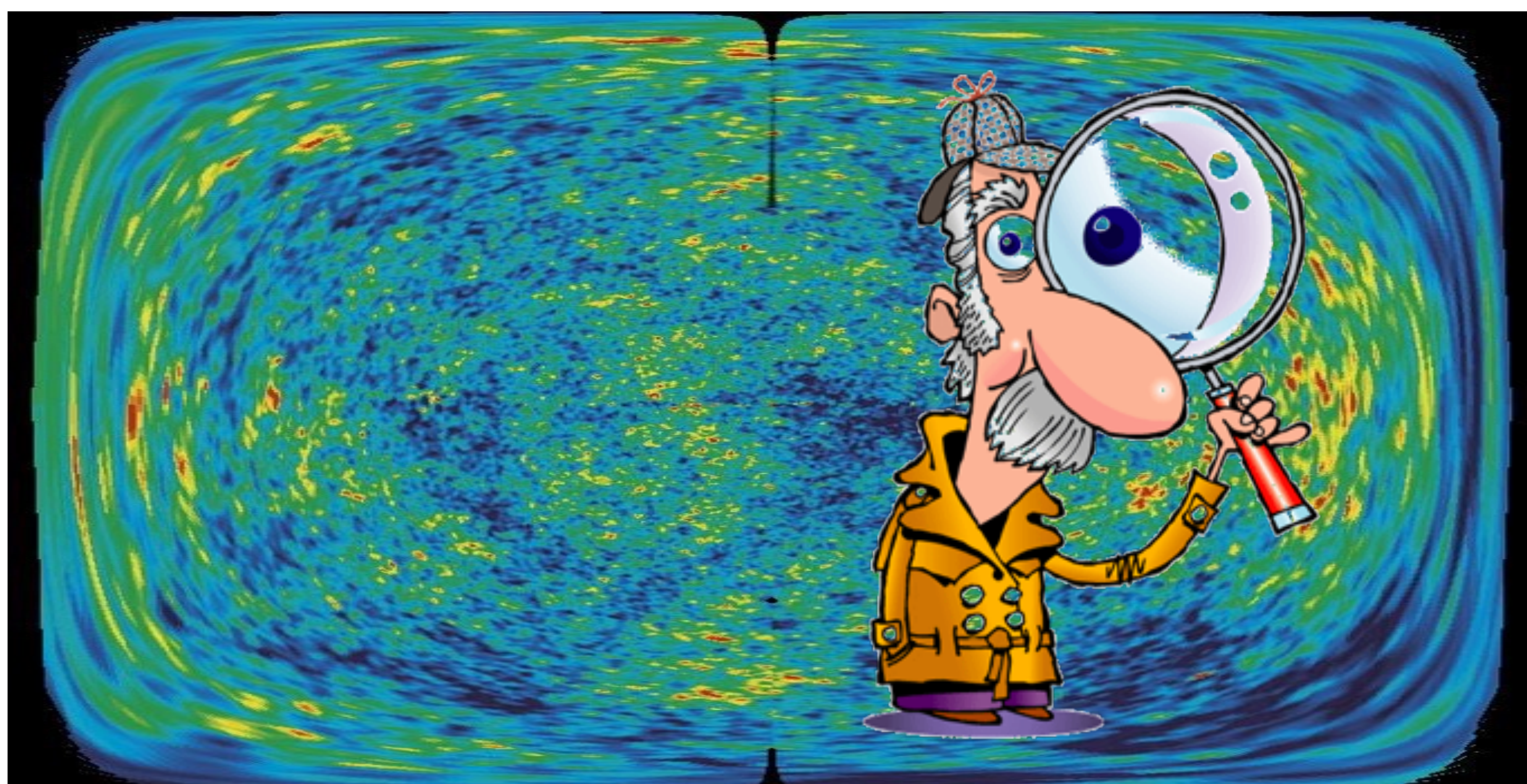
Partly a review but own contributions based on …

Based on: N.~Agarwal, R.~Holman and AJT
``What is the shape of the initial state?,'' 1305.3615
N.~Agarwal, R.~Holman, AJT and J.~Lin,
``Effective field theory and non-Gaussianity from general inflationary states,'' 1212.1172
R.~Holman and AJT ``Enhanced Non-Gaussianity from Excited Initial States,'' 0710.1302





Some Slides generously Donated by Nishant Agarwal

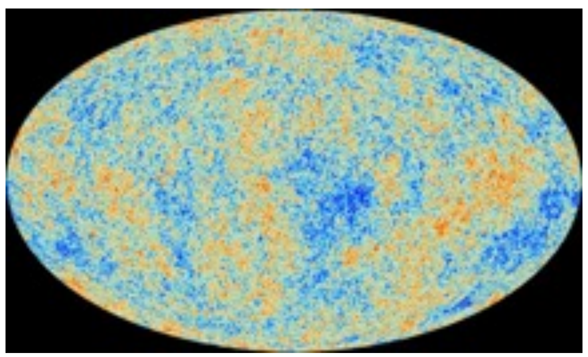
Quantum Fluctuations in the Inflaton Field lead to Fluctuations in the Spacetime Metric

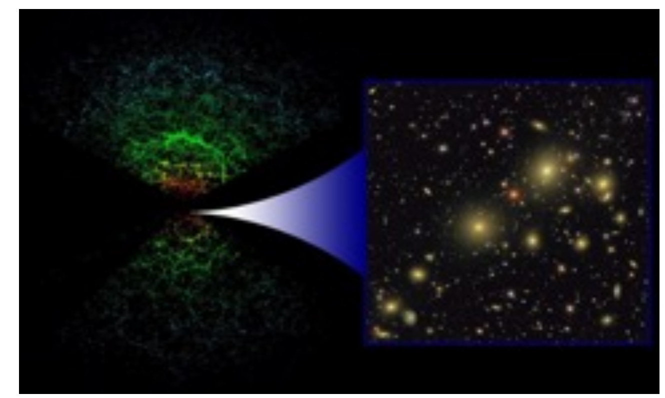
Scalars

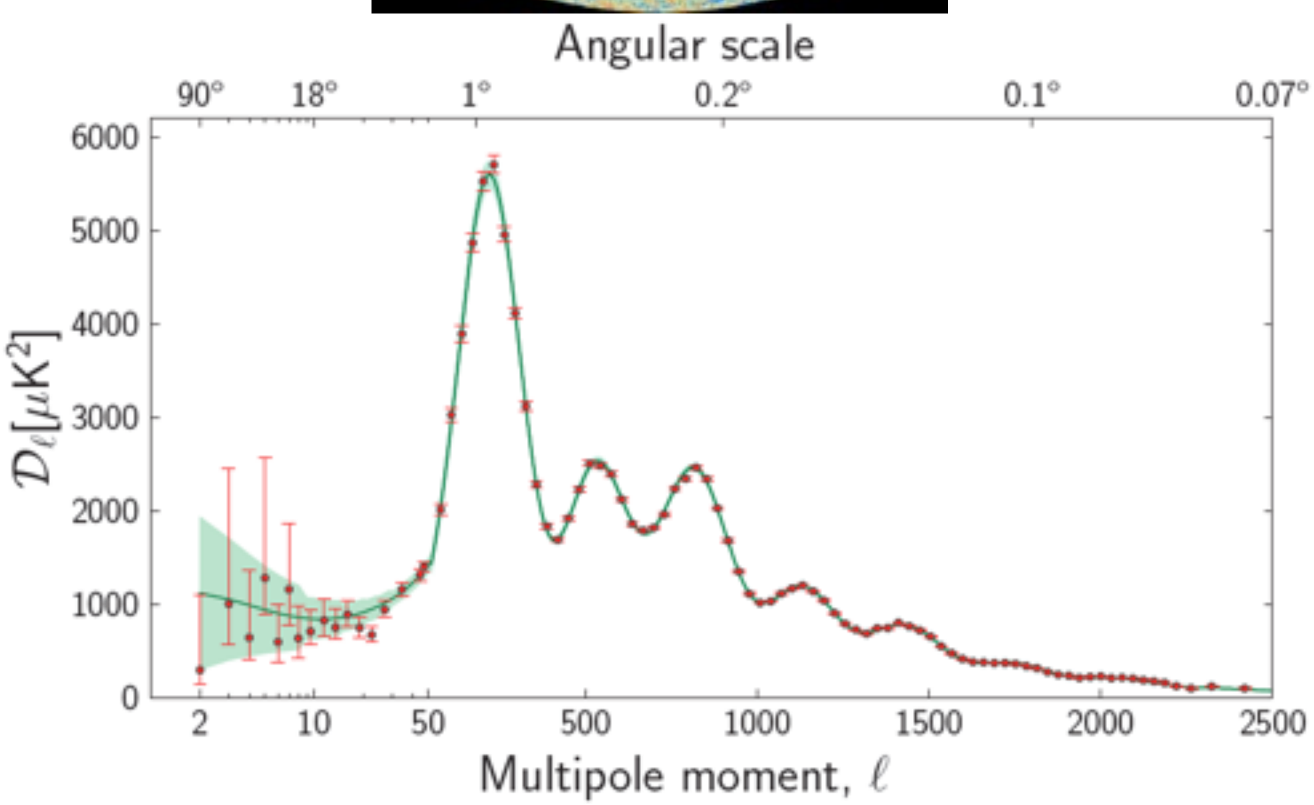
$$g_{ij}(t,x) = e^{2\zeta(t,x)} a^2(t) \delta_{ij}$$

δT_{CMB}

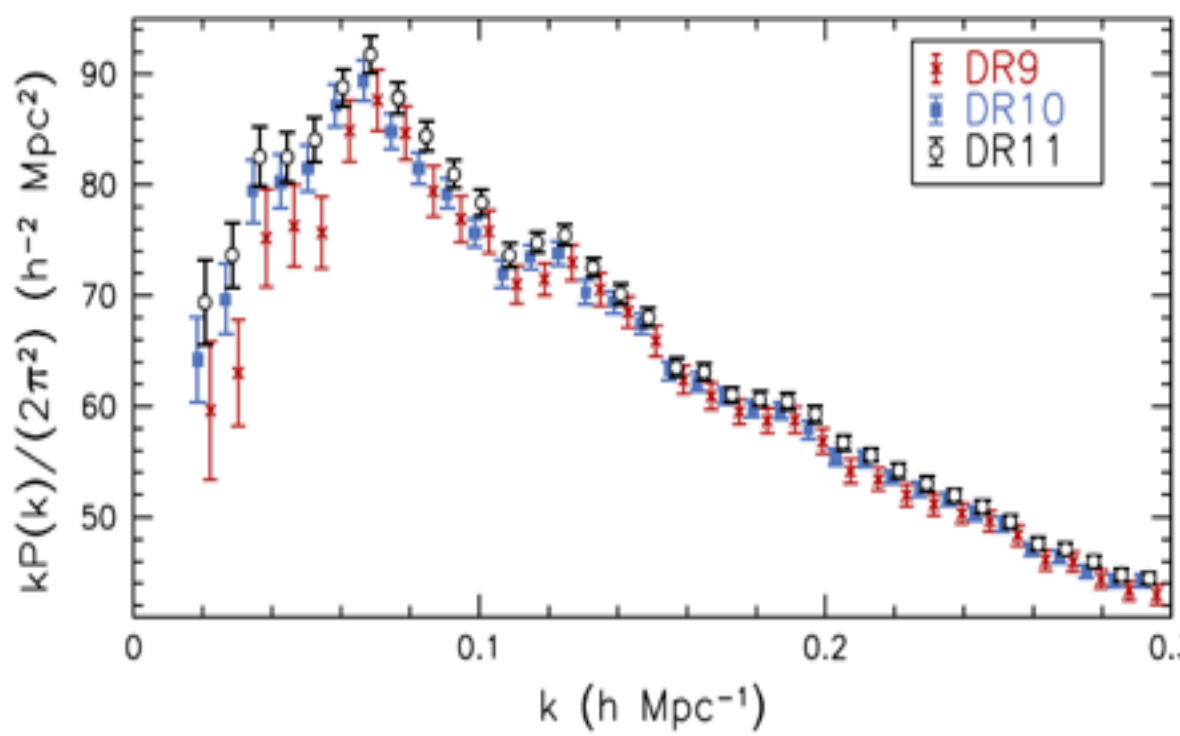
$\delta \rho_{\mathrm{matter}}$







Planck



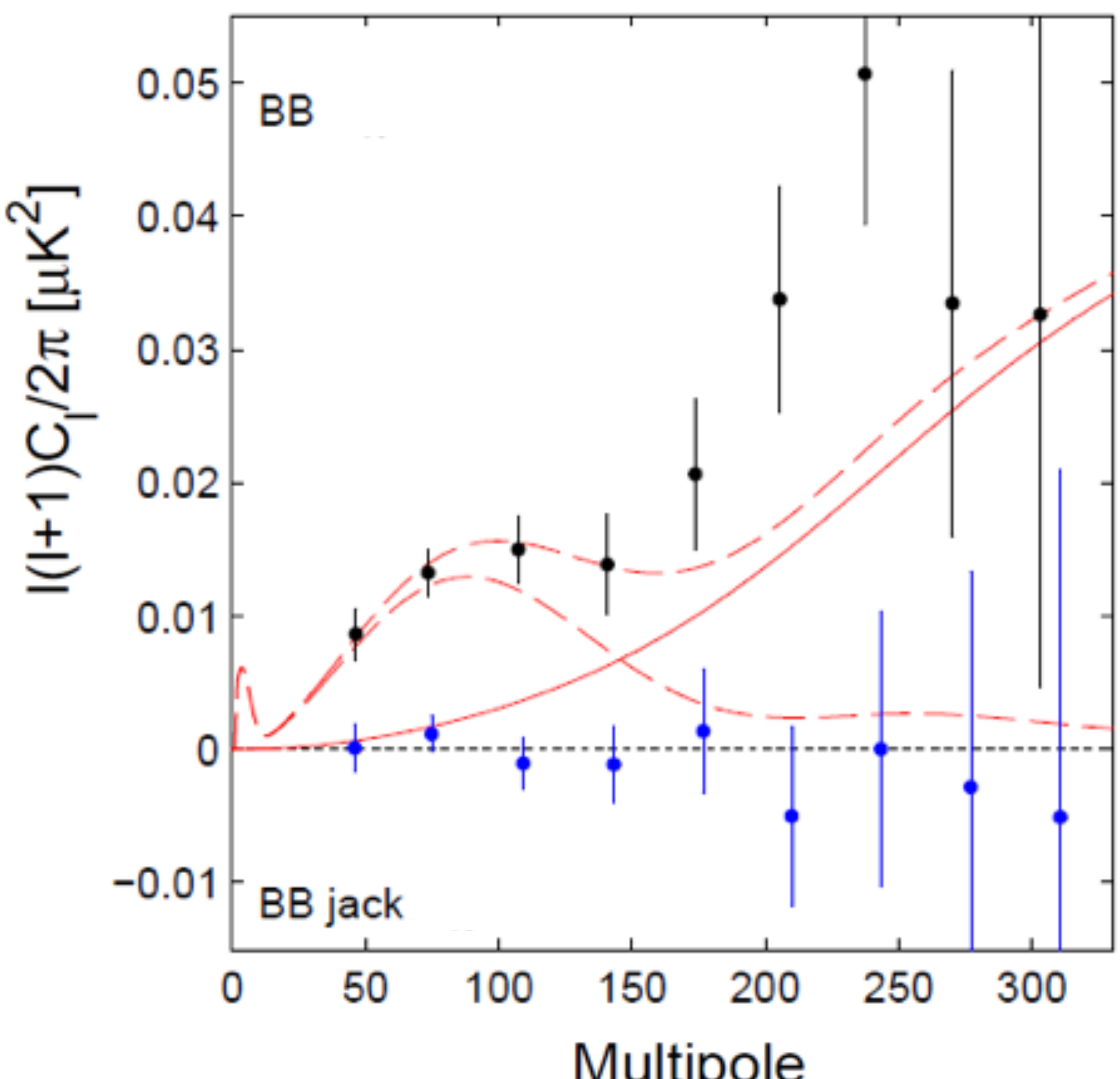
SDSS

Quantum Fluctuations in the Inflaton Field lead to Fluctuations in the Spacetime Metric

Tensors

$$\delta g_{ij}(t,x) = a^2(t) h_{ij}(t,x);\ \ i \neq j$$

moral: quantize gravity too!



BICEP2

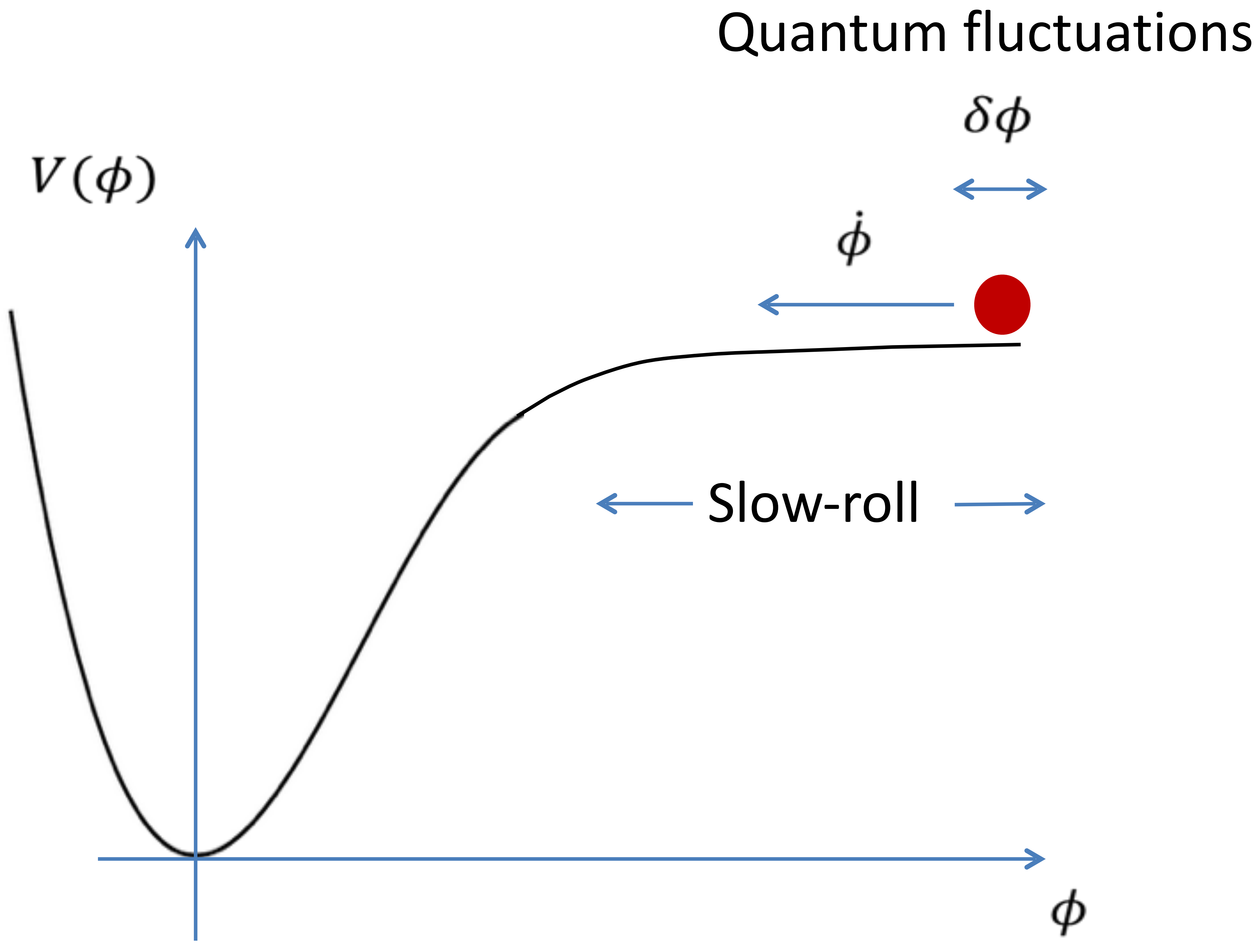
Textbook Inflation

- Our **textbook** picture of Inflation is described by some Field Theory with a finite number of (typically) scalar fields coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}G^{IJ}(\phi)\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$

e.g. supergravity type Lagrangian

Textbook Inflation



Inflation and Fundamental Physics

- Our **textbook** picture of Inflation is described by some Field Theory with a finite number of (typically) scalar fields coupled to gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} G^{IJ}(\phi) \partial_\mu \phi^I \partial_\nu \phi^J - V(\phi^I) \right]$$

- However we can only really expect that this is some LEEFT valid during the epoch of inflation while the inflationary perturbations are being generated

- In a real UV completion, there may be many more fields of all spins and masses contributing

Low Energy Effective Field Theories (LEEFTs)

- A given UV complete theory may contain any number (infinite?) of massive states with $m_H \gg H_{inf}$. How do these contribute to inflation?

- General principle of LEEFT, integrate out Heavy Particles/ Fields $H^a(x)$

$$e^{iS_{LEEFT}(g_{\mu\nu},\phi^I)} = \int DH^a \, e^{iS_{UV}(g_{\mu\nu},\phi^I,H^a)}$$

Renormalized

$$S_{LEEFT}(g_{\mu\nu},\phi^I) = S^R_{UV}(g_{\mu\nu},\phi^I,0) + \Delta S(g_{\mu\nu},\phi^I)$$

Low Energy Effective Field Theories (LEEFTs)

- Corrections from heavy corrections are encoded an an Infinite number of irrelevant (non-renormalizable) operators whose contribution is suppressed by inverse powers of m_H

- Schematically

$$\Delta\mathcal{L} = \sum_{nmp} c_{nmp} \frac{1}{m_H^{n+m+p-4}} \nabla^n R^m \phi^p$$

Stands for all possible scalar contractions of Riemann curvature, covariant derivatives and powers of fields

Low Energy Effective Field Theories (LEEFTs)

$$\Delta\mathcal{L} = \sum_{nmp} c_{nmp} \frac{1}{m_H^{n+m+p-4}} \nabla^n R^m \phi^p$$

In simple single field models inflaton scalar vev takes values above scales of (suspected) new physics (e.g. Planck scale)!

$$\phi_{\text{initial}} \geq m_H \gg H$$

m_H maybe e.g. SUSY breaking scale, GUT scale, string scale, Planck scale ….

This is the origin of the difficulty in constructing large field models in supergravity/string theory

sensitivity of inflation to `quantum gravity' effects

c.f. Eva Silverstein talk

However EFT for fluctuations is not breaking down

Quantum Fluctuations remain below cutoff
Theory is weakly coupled!

$$\delta\phi \sim H \ll m_H$$

$$\delta\dot{\phi} \sim H^2 \ll m_H^2$$

similarly backreaction is under control:

$$\langle \delta T_0^0 \rangle \sim \langle \delta\dot{\phi}^2 \rangle \sim H^4 \ll H^2 M_P^2 \sim \rho_{\rm inflation}$$

One option: Ignore how background evolution gets corrected, and construct LEEFT for perturbations around a given background

Construct LEEFT for Perturbations - Goldstone boson

C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, 2008

Since the background is time-dependent, natural to construct EFT of Goldstone mode associated with spontaneously broken time translations

technically speaking this is **Stueckelberg field** not Goldstone since symmetry is gauged (time diffeomorphism)

$$\phi(t,x) = \bar{\phi}(t) + \delta\phi = \bar{\phi}(t + \pi(t,x))$$

$\pi(t,x)$ is the goldstone mode

$\pi(t,x)$ transforms nonlinearly under time diffs

$$t \to t + \xi^0(t,x) \qquad\qquad \pi \to \pi + \xi^0 \partial_t \pi + \xi^0$$

Goldstone mode is closely connected with curvature perturbation which is conserved at long wavelengths

$$\zeta \sim H\pi$$

At high energies, interactions of Goldstone mode *dominate* over those of gravity

Can take a decoupling limit where focus on Goldstone mode

guaranteed to exist by nonlinearly realized diffs

$$S_\pi = \int d^4x\sqrt{-g}\left[-\frac{M_{Pl}^2\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) + M_{Pl}^2\dot{H}\left(1-\frac{1}{c_s^2}\right)\left(\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 \ldots\right]$$

This high energy limit can still be of sufficiently low energy to describe modes as they cross the horizon!

$$\epsilon/c_s^2 \ll 1$$

At high energies, interactions of Goldstone mode *dominate* over those of gravity

$$S_\pi = \int d^4x\sqrt{-g}\left[-\frac{M_{Pl}^2\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) + M_{Pl}^2\dot{H}\left(1-\frac{1}{c_s^2}\right)\left(\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 \ldots\right]$$

High energy corrections to this Lagrangian will come suppressed by powers of m_H with time dependent coefficients

$$\Delta S_\pi \sim \int d^4x\sqrt{-g}\sum_{mnp}\frac{1}{m_H^{m+n+p-4}}c_{mnp}(t)M_{Pl}^p(-\dot{H})^{p/2}a^{-n}\partial_t^m\partial_i^n\pi^p$$

all powers of fields and derivatives of fields

e.g. $\dot{\pi}^{27}(\nabla^3\pi)^{101}$

subject to constraint of nonlinearly realized diffeomorphism invariance

Gaussian fluctuations

To quadratic order action for scalar fluctuations looks = massless scalar on spacetime with scale factor z

$$S = \frac{1}{2}\int d\tau d^3x z^2 \left(\zeta'^2 - (\nabla\zeta)^2\right)$$

Exhibits symmetry!

$$\zeta \to \zeta + c$$

For constant equation of state w: $z \propto a$

If background is close to de Sitter, then power spectrum at long wavelengths is close to scale invariant:

Determines 2-pt function of temperature fluct's in CMB

$$\langle \delta T(x)\delta T(y)\rangle$$

+ matter power spectrum

$$\langle \delta\rho(x)\delta\rho(y)\rangle$$

evaluate at horizon crossing

$$\langle \zeta(x)\zeta(y)\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^3}\frac{H^2(k)}{4\epsilon^2 M_P^2}e^{ik.(x-y)}$$

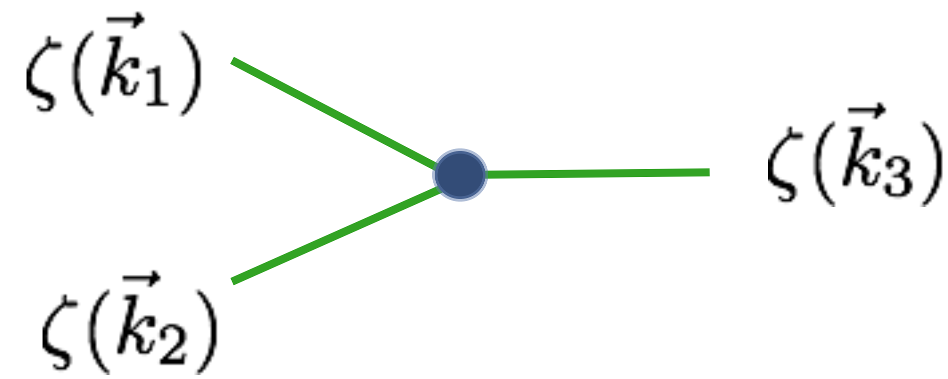
scale invariant

Non-Gaussianities measure inflaton interactions

nG's are to cosmologists what scattering experiments are to particle physicists: Measure strength of interactions!

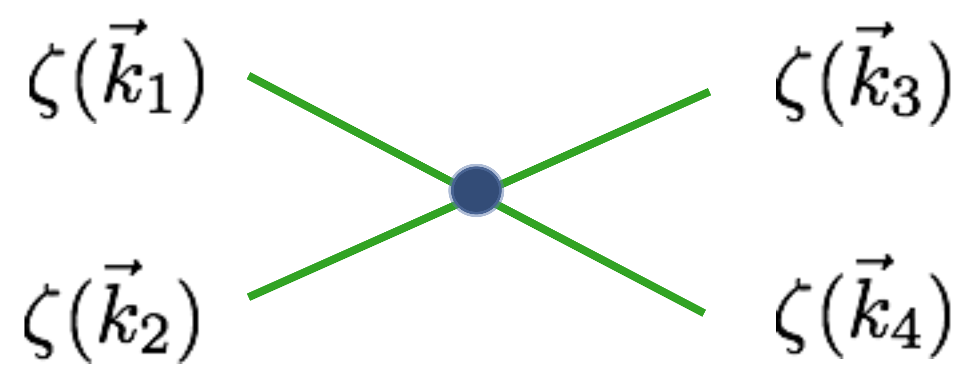
3pt point interactions:

$$S_I = \int d^4x \left(\frac{\dot{\phi}^2}{H^2}\right) f_{NL} H^{-1} \dot{\zeta}^3$$



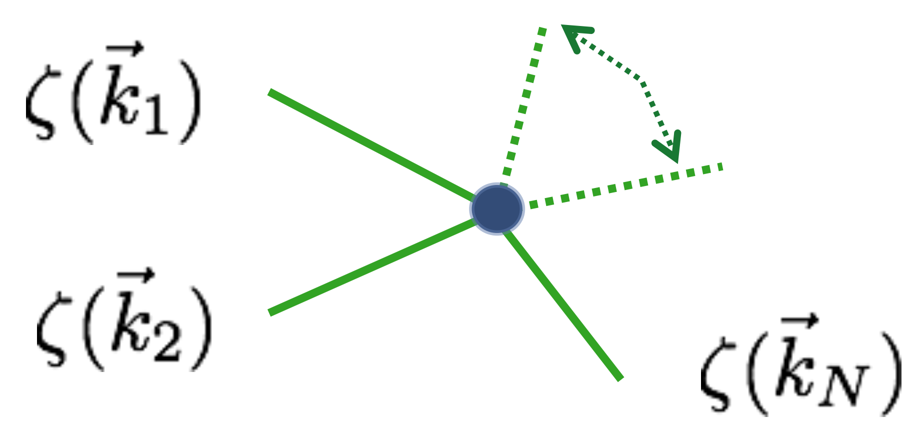
4pt point interactions:

$$S_I = \int d^4x \left(\frac{\dot{\phi}^2}{H^2}\right) \tau_{NL} H^{-2} \dot{\zeta}^4 + \dots$$



Npt point interactions:

$$S_I = \int d^4x \left(\frac{\dot{\phi}^2}{H^2}\right) f_{NL}^{(N)} H^{2-N} \dot{\zeta}^N + \dots$$



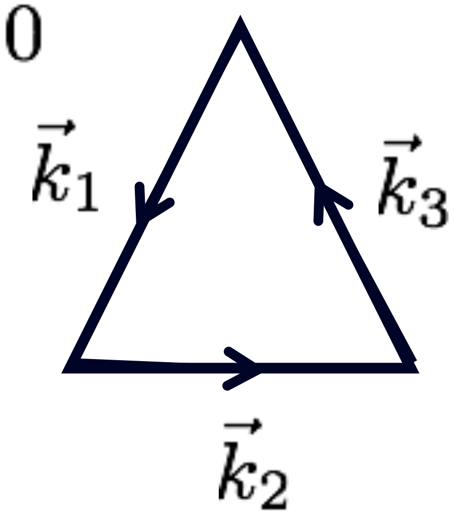
Shapes from interactions

For slow roll models: non-gaussianity is suppressed by slow roll parameters but can be larger for models with non-minimal kinetic terms

$$f_{NL} = \frac{\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)}$$

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$

horizon crossing:
equilateral

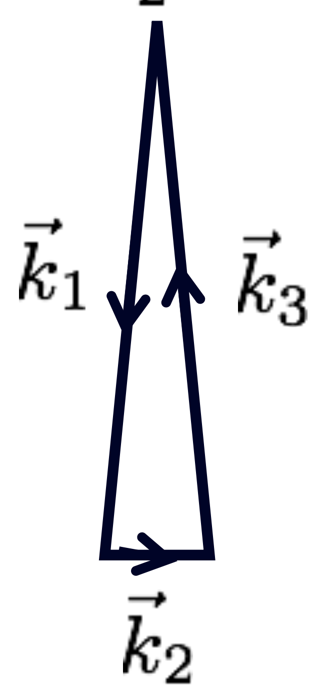


$|\vec{k}_1| \sim |\vec{k}_2| \sim |\vec{k}_3|$ Single field models

Squeezed limit

c.f. Filippo Vernizzi talk

superhorizon:
local /squeezed



$|k_i| \sim 0$

Curvaton models

conversion mechanism occurring after horizon crossing!

Non-minimal horizon:
orthogonal Senatore, Smith, Zaldariagga '09

However this is not the end of the story

Three Key Assumptions
(beyond standard hierarchy of scales)

1. Heavy Fields start out in Vacuum

2. Effect of Generation of Heavy Particle is ignored (it is suppressed but nevertheless always present)

3. Light Fields (Inflaton and Graviton) start out in Vacuum

Instead of this (in-out)

$$e^{iS_{LEEFT}(g_{\mu\nu},\pi)} = \int DH^a e^{iS_{UV}(g_{\mu\nu},\pi,H^a)}$$

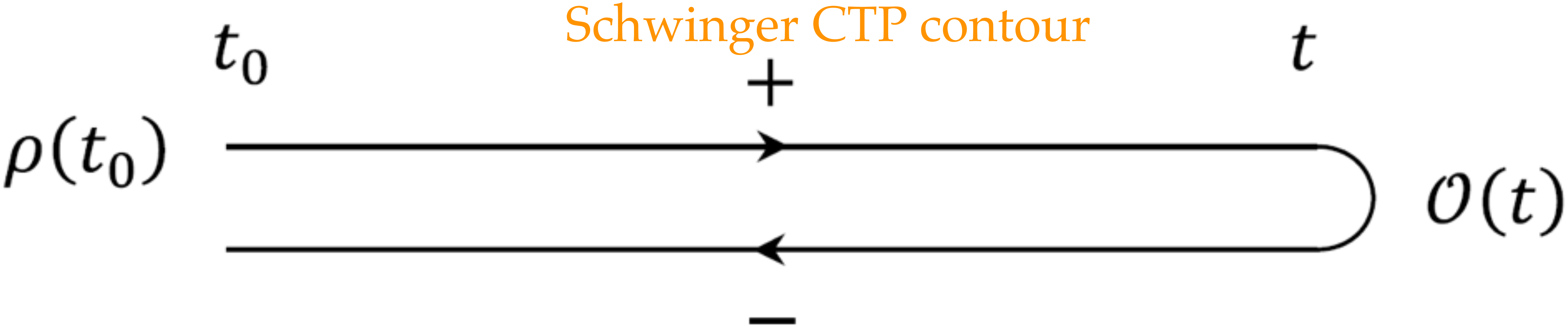
we should have done this (in-in)

$$e^{iS_\pi(g^+_{\mu\nu},\pi^+)-iS_\pi(g^-_{\mu\nu},\pi^-)+i\Delta S_\pi(g^+_{\mu\nu},\pi^+,g^-_{\mu\nu},\pi^-)} =$$

$$\int_{H_a^+(t_f)=H_a^-(t_f)} DH_a^+ \int DH_a^- e^{iS_{UV}(g^+_{\mu\nu},\pi^+,H_a^+)-iS_{UV}(g^-_{\mu\nu},\pi^-,H_a^-)} \rho(g^+,\pi^+,H^+;g^-\pi^-,H^-;t_0)$$

initial density matrix for UV theory

Schwinger CTP contour



Schwinger-Mahanthappa-Keldysh/Feynman-Vernon Influence Functional

$$e^{iS_\pi(g^+_{\mu\nu},\pi^+)-iS_\pi(g^-_{\mu\nu},\pi^-)+i\Delta S_\pi(g^+_{\mu\nu},\pi^+,g^-_{\mu\nu},\pi^-)} =$$

$$\int_{H_a^+(t_f)=H_a^-(t_f)} DH_a^+ \int DH_a^- e^{iS_{UV}(g^+_{\mu\nu},\pi^+,H_a^+)-iS_{UV}(g^-_{\mu\nu},\pi^-,H_a^-)}\rho(g^+,\pi^+,H^+;g^-\pi^-,H^-;t_0)$$

There are now interactions between the two branches of the Schwinger CTP contour

$$\Delta S_\pi(g^+_{\mu\nu},\pi^+,g^-_{\mu\nu},\pi^-)$$

These encode 1. and 3. (the possibility that the light and heavy modes are not in vacuum through)

$$\rho(g^+,\pi^+,H^+;g^-\pi^-,H^-;t_0)$$

and also 2, through the boundary conditions in the path integral that

$$H_a^+(t_f)=H_a^-(t_f)$$

Influence Functional

In particular

$$\Delta S_\pi(g^+_{\mu\nu}, \pi^+, g^-_{\mu\nu}, \pi^-) = -\Delta S^*(g^-_{\mu\nu}, \pi^-, g^+_{\mu\nu}, \pi^+)$$

(Encodes Unitarity)

includes the important effects of dissipation and diffusion - which are famously responsible for the fluctuation-dissipation theorem in time independent systems

$e^{i\Delta S_\pi(g^+,\pi^+,g^-,\pi^-)}$ is known as the **Feynman-Vernon influence** functional

FVIF = Path integral analogue of Density Matrix

It is also possible to account for dissipation by noise terms (Langevin), and coupling to other operators but this is less elegant

e.g. Nacir et al 2012,
Dissipative effects in the Effective Field Theory of Inflation

But wait, doesn't Inflation wipe out all remnants of Initial State?

Raison d'etre of inflation is to solve initial conditions problems

Horizon	$$\frac{a\lambda}{H^{-1}}$$	increases exponentially
Monopole	$$\frac{\rho_0/a^3}{\rho_{\text{inflaton}}}$$	decreases exponentially
Flatness	$$\frac{\kappa/a^2}{H^2}$$	decreases exponentially

But, Inflation is less successful at wiping out quantum initial conditions!

$$S = \int d^4x \frac{1}{2} z^2 \left(\zeta'^2 - c_s^2 (\nabla\zeta)^2\right)$$

$$\zeta = \psi + H\frac{\delta\phi}{\dot\phi} \qquad\qquad z \sim a$$

Subhorizon scales

$$\zeta \sim \frac{1}{z} e^{\pm i c_s k \eta} \qquad\qquad c_s k > \frac{z'}{z}$$

Superhorizon scales

$$\zeta \sim \text{constant} \qquad\qquad c_s k < \frac{z'}{z}$$

Inflation is less successful at wiping out quantum initial conditions!

For the quantum fluctuations:

$$\zeta_{\text{latetimes}} = \zeta_{\text{horizon crossing}} = e^{-N_*}\zeta_{\text{initial}}$$

Only the e-folds before horizon crossing contribute!

$$N_* = \text{number of e-folds before a given mode crosses horizon}$$

Once modes cross the horizon -
permanently imprinted on late time primordial fluctuations

How many e-folds?

In simplest models of inflation (e.g. chaotic)
inflation lasts sufficiently much longer than necessary to solve horizon problem

$$N_{\text{total}} = 70 - 90$$

$$V(\phi) = \frac{1}{n!}\lambda\phi^n \qquad n \geq 2 \qquad N_* = 15 - 35$$

$$\zeta_{\text{h.c.}} \sim 10^{-5} \sim e^{-11.5}$$

N.B. large field models,
ones that give potentially observable tensor modes

But these are not typically the models that typically get realized in realistic string/supergravity constructions

String theory tends to favor small field models, not many e-folds of inflation

Implication:

If inflation is short, feasible to expect imprint of pre-inflationary physics initial state in late time observables

$$\langle \zeta(k)\zeta(k')\rangle = (2\pi)^3 P_\zeta(k)\delta^{(3)}(k+k')$$

Either show up in power spectrum

$$P_\zeta(k) = A k^{n_s(k)-3}$$

Running or oscillations in spectral index

$$n_s(k) = n_s(k_*) + n_s'(k_*)(k-k_*) + \ldots$$

Or in non-gaussianities (higher point functions)

$$\langle \zeta^3\rangle, \langle \zeta^4\rangle, \langle \zeta^5\rangle \ldots$$

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = \frac{5}{6} f_{NL}(k_1,k_2,k_3)\left(P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_1)P_\zeta(k_3) + P_\zeta(k_2)P_\zeta(k_3)\right)$$

Nontrivial 3-pt function

nG's generated from initial state interactions

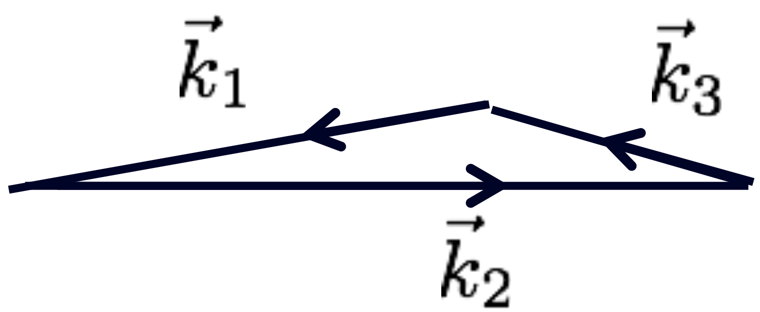
AJT+Holman 2007
Chen et al 2007

Even if initial state is gaussian but not Bunch-Davies, interactions in early stages of inflation will generate nG's

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}(\tau)\rangle = -2\mathcal{R}e\int_{\tau_i}^{\tau} d\tau' i\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}(\tau)H_I(\tau')\rangle$$

$$\sim \beta_{k_1}\frac{A(k)}{k_2+k_3-k_1} + \text{permutations}$$

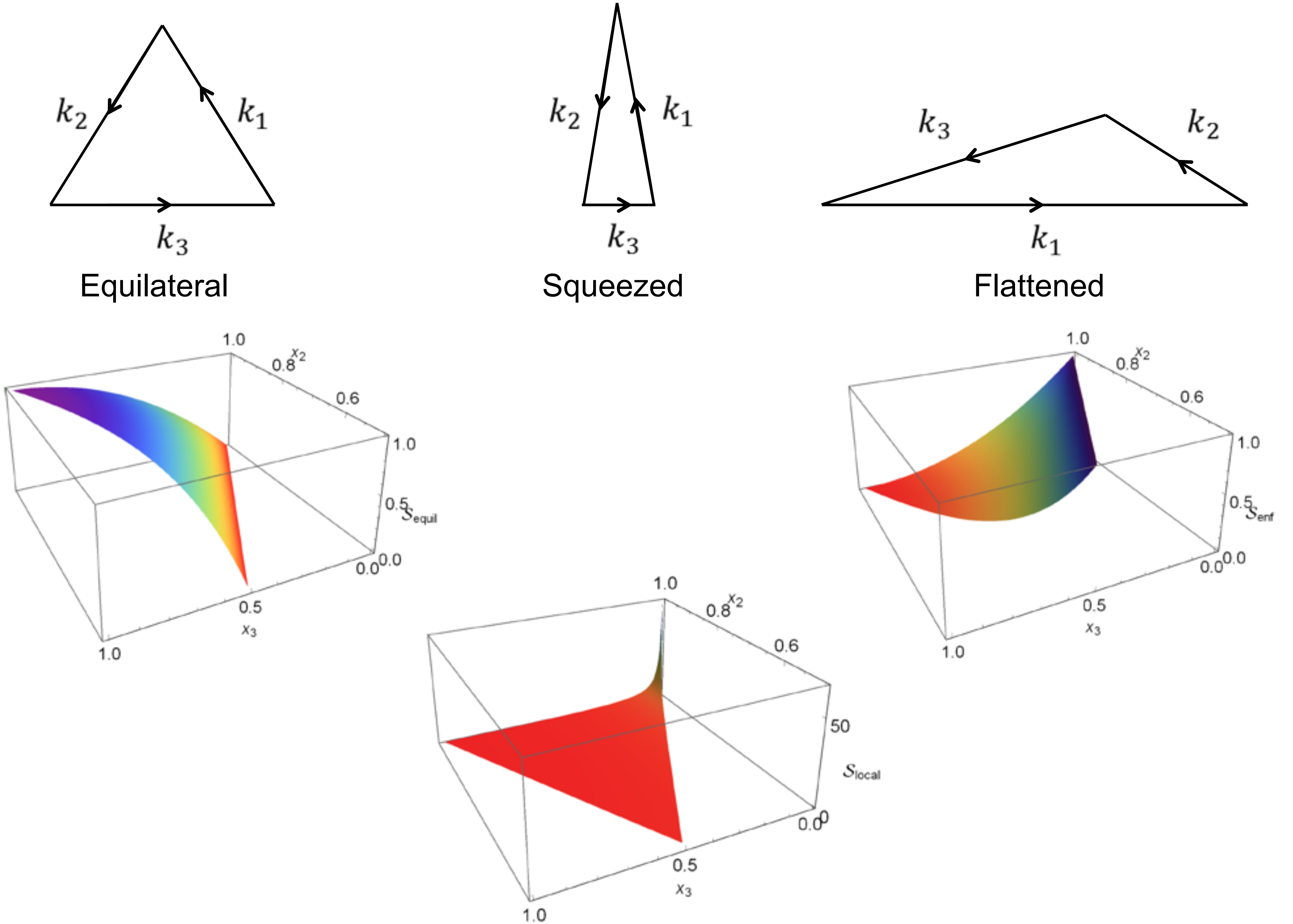
Resonance type interaction shows up in flattened/folded triangles (energy conservation)



$$\delta\theta \sim \frac{H}{M_*}$$

For minimal slow-roll inflation effect is measure suppressed

$$\frac{\delta f_{NL}}{f_{NL}} \sim \beta_k \frac{M_*}{H}\times\frac{H}{M_*} \sim \beta_k \ll 1$$



k_2
k_1
k_3
Equilateral
k_2
k_1
k_3
Squeezed
k_3
k_2
k_1
Flattened
1.0
x_2
0.8
0.6
1.0
0.5
S_{equil}
0.0
0.0
0.5
x_3
1.0
1.0
x_2
0.8
0.6
50
S_{local}
0.0
0
0.5
x_3
1.0
1.0
x_2
0.8
0.6
1.0
0.5
S_{enf}
0.0
0.0
0.5
x_3
1.0

More General Initial States

However there are far more to initial states than Bogoliubov transforms

e.g. Even a Free Theory can have non-Gaussianities!!!!

$$\psi(\zeta(\vec{x}),\tau) = e^{-\int d^3x d^3y \zeta(\vec{x}) K(\vec{x},\vec{y},\tau)\zeta(\vec{y})} \times e^{-\int d^3x d^3y \int d^3z \zeta(\vec{x}) H(\vec{x},\vec{y},\vec{z},\tau)\zeta(\vec{y})\zeta(\vec{z}) + \ldots}$$

Directly contributes to f_{NL}

Shape dependence of $H(x, y, z, \tau)$ is completely undetermined by inflation since it has to do with (unknown) pre-inflationary physics

More Generally, remnants from Bubble Collisions, initial background anisotropies/inhomgenities (can include modification of initial state for tensors), remnants from heavy states which have decayed into inflaton quanta, entanglement between light and heavy modes, remnant of a prior thermalised state (excite all N-point functions), pre-inflationary phase transitions, remnants of string scale physics, quantum gravity, pre-big-bang state ….

General Initial States

Agarwal, Holman, AJT, Lin 2013

$$Z[J^+, J^-, t_0] = \int D\pi^+ \int D\pi^- e^{iS_\pi(\pi^+) - iS_\pi(\pi^-) + i\Delta S_\pi(\pi^+, \pi^-, t_0) + i\int d^4x (J^+\pi^+ - J^-\pi^-)}$$

I'm neglecting the metric for simplicity as appropriate in decoupling limit

We then expand the FVIF (localized at t_0) in powers of π

$$\Delta S_\pi(\pi^+, \pi^-, t_0) = \int dt\delta(t - t_0) \sum_{n=2}^{\infty} \int d^{3n}x\, F^{(n)}_{\alpha_1\alpha_2\ldots\alpha_n}(x_1, \ldots, x_n; t_0)\pi^{\alpha_1}(x_1)\ldots\pi^{\alpha_n}(x_n)$$

$$\alpha_i = \pm$$

$F^{(2)}_{\alpha_1\alpha_2}(x_1, x_2; t_0)$ accounts for modified Propagators (two-point functions)

$F^{(n)}_{\alpha_1\alpha_2,\ldots\alpha_n}(x_1, \ldots, x_n; t_0)$ accounts for modified interactions

The 3-point function of $\pi(x)$ (equivalently $\zeta(x)$) can be obtained from

$$\langle \pi^+_{k_1} \pi^+_{k_2} \pi^+_{k_3} \rangle (t) =$$
$$\langle \pi^+_{k_1}(t) \pi^+_{k_2}(t) \pi^+_{k_3}(t) \left(1 + i\Delta S^{(3)}(\pi^+, \pi^-; t_0)\right) \left[1 + iS^{(3)}_\pi(\pi^+) - iS^{(3)}_\pi(\pi^-)\right] \rangle_{\mathrm{Gaussian}}$$

$$\xrightarrow{\text{Wick contractions}}$$ Integrals of products of Green's functions

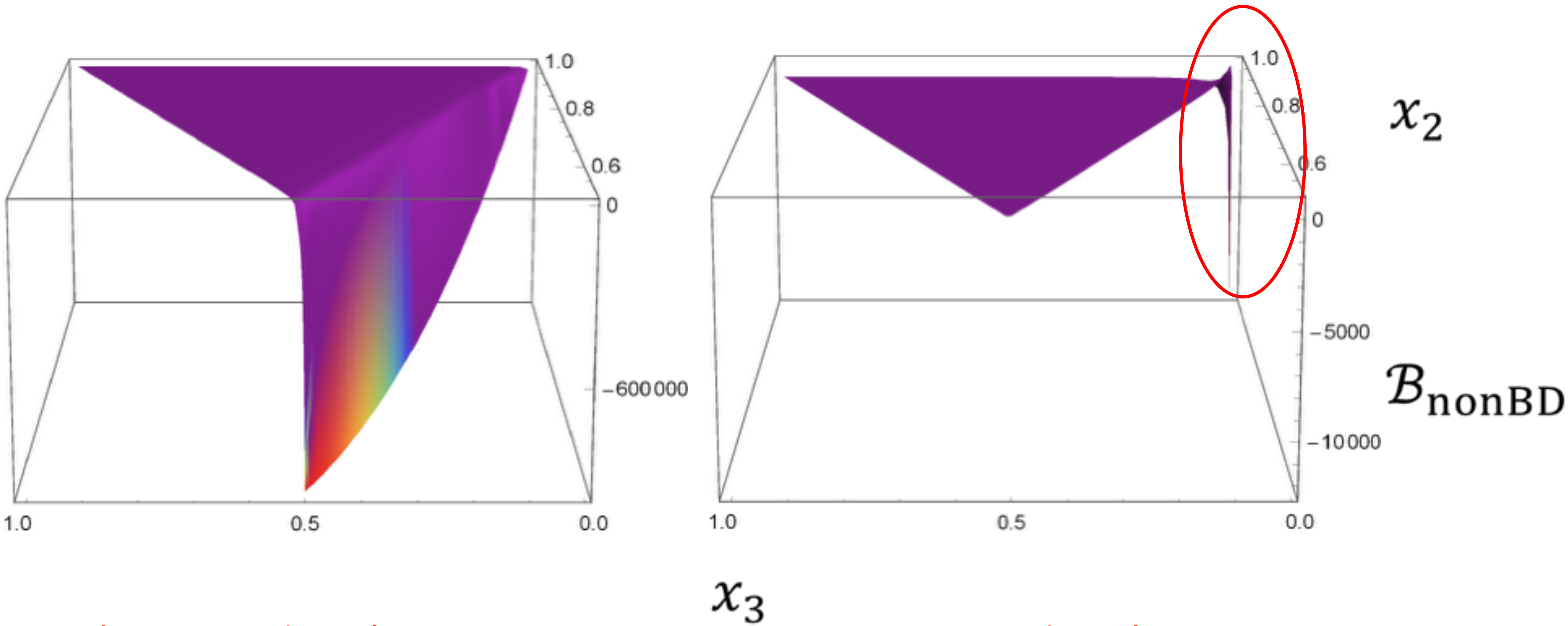
where Wick contractions are computed using modified propagators influenced by $F^{(2)}_{\alpha_1 \alpha_2}(x_1, x_2; t_0)$

$\alpha_i = \pm$

$$\Delta S^{(3)}(\pi^+, \pi^-; t_0) = \int dt \delta(t - t_0) \int d^3x_1 \int d^3x_2 \int d^3x_3 F^{(3)}_{\alpha_1, \alpha_2, \alpha_3}(x_1, x_2, x_3; t_0) \pi^{\alpha_1}(x_1) \pi^{\alpha_2}(x_2) \pi^{\alpha_3}(x_3)$$

Examples:

- Bispectrum for general initial states (for $c_s < 1$)



Flattened enhancement

Squeezed enhancement

Ade et al Planck 2015 results. XVII. Constraints on primordial non-Gaussianity, 1502.01592

$$B^{\mathrm{NBD-sin}}(k_1,k_2,k_3) = \frac{2A^2 f_{\mathrm{NL}}^{\mathrm{NBD-sin}}}{(k_1k_2k_3)^2}\left(e^{-\omega\tilde{k}_1}+e^{-\omega\tilde{k}_2}+e^{-\omega\tilde{k}_3}\right) \times \sin(\omega K+\phi)\,,$$

$$B^{\mathrm{NBD}i-\mathrm{sin}}(k_1,k_2,k_3) = \frac{2A^2 f_{\mathrm{NL}}^{\mathrm{NBD}i-\mathrm{sin}}}{(k_1k_2k_3)^2}(f_i(k_1;k_2,k_3) \times \sin(\omega\tilde{k}_1)/\tilde{k}_1 + 2\,\mathrm{perm.})\,,$$

Table 24. Constraints on models with excited initial states (non-Bunch-Davies models), as well as warm inflation. See Sect. 2 for further explanation and the labelling of these classes of NBD models. Note that the NBD, NBD1, and NBD2 models contain free parameters, so here we quote the maximum significance found over the available parameter range; the maximum for T and T+E can occur at different parameter values (on which the error bars are also dependent).

	SMICA		SEVEM		NILC		Commander	
Flattened-type model	$A \pm \sigma_A$	S/N	$A \pm \sigma_A$	S/N	$A \pm \sigma_A$	S/N	$A \pm \sigma_A$	S/N
Flat model T-only	49 ± 65	0.8	57 ± 65	0.9	47 ± 65	0.7	19 ± 65	0.3
Flat model T+E	44 ± 37	1.2	70 ± 37	1.9	33 ± 37	0.9	47 ± 37	1.3
Non-Bunch-Davies T-only	42 ± 82	0.5	53 ± 82	0.6	26 ± 82	0.3	17 ± 82	0.2
Non-Bunch-Davies T+E	61 ± 47	1.3	76 ± 47	1.6	43 ± 47	0.9	58 ± 47	1.2
NBD sine T-only	567 ± 341	1.7	513 ± 341	1.5	588 ± 341	1.7	604 ± 341	1.8
NBD sine T+E	−387 ± 206	−1.9	−485 ± 218	−2.2	−425 ± 206	−2.1	−417 ± 210	−2.0
NBD1 cos flattened T-only	−10 ± 22	−0.5	−4 ± 22	−0.2	−8 ± 22	−0.4	−9 ± 22	−0.4
NBD1 cos flattened T+E	−20 ± 19	−1.1	−10 ± 19	−0.5	−19 ± 19	−1.0	−14 ± 19	−0.8
NBD2 cos squeezed T-only	10 ± 17	0.6	10 ± 17	0.6	8 ± 17	0.5	−2.5 ± 17	−0.1
NBD2 cos squeezed T+E	−3 ± 5	−0.5	−0.8 ± 5.5	−0.1	−4 ± 5	−0.8	−3.8 ± 5.5	−0.7
NBD1 sin flattened T-only	−25 ± 22	−1.1	−27 ± 22	−1.2	−18 ± 22	−0.8	−33 ± 23	−1.4
NBD1 sin flattened T+E	48 ± 30	1.6	49 ± 33	1.5	35 ± 31	1.1	26 ± 34	0.8
NBD2 sin squeezed T-only	−2.0 ± 1.4	−1.4	−1.4 ± 1.4	−1.0	−1.6 ± 1.4	−1.1	−1.3 ± 1.4	−0.9
NBD2 sin squeezed T+E	−0.8 ± 0.4	−1.9	−0.5 ± 0.4	−1.2	−0.6 ± 0.4	−1.4	−0.5 ± 0.4	−1.2
NBD3 non-canonical T-only (×10 3)	−5.9 ± 6.7	−0.9	−6.0 ± 6.8	−0.9	−5.4 ± 6.8	−0.8	−5.5 ± 6.7	−0.8
NBD3 non-canonical T+E (×10 3)	−8.7 ± 5.0	−1.7	−6.2 ± 5.2	−1.2	−7.5 ± 5.2	−1.5	−9.4 ± 5.2	−1.8
WarmS inflation T-only	−23 ± 36	−0.6	−26 ± 36	−0.7	−32 ± 36	−0.9	−24 ± 36	−0.7
WarmS inflation T+E	−14 ± 23	−0.6	−28 ± 23	−1.2	−21 ± 23	−0.9	−17 ± 23	−0.7

Looking for non-Gaussianity in all the right places: A new basis for non-separable bispectra

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Non-Gaussianity in the distribution of inflationary perturbations, measurable in statistics of the cosmic microwave background (CMB) and large scale structure fluctuations, can be used to probe non-trivial initial quantum states for these perturbations. The bispectrum shapes predicted for generic non-Bunch-Davies initial states are non-factorizable ("non-separable") and are highly oscillatory functions of the three constituent wavenumbers. This can make the computation of CMB bispectra, in particular, computationally intractable. To efficiently compare with CMB data one needs to construct a separable template that has a significant similarity with the actual shape in momentum space. In this paper we consider a variety of inflationary scenarios, with different non-standard initial conditions, and how best to construct viable template matches. In addition to implementing commonly used separable polynomial and Fourier bases, we introduce a basis of localized piecewise spline functions. The spline basis is naturally nearly orthogonal, making it easy to implement and to extend to many modes. We show that, in comparison to existing techniques, the spline basis can provide better fits to the true bispectrum, as measured by the cosine between shapes, for sectors of the theory space of general initial states. As such, it offers a useful approach to investigate non-trivial features generated by fundamental properties of the inflationary Universe.

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Optimal CMB estimators for bispectra from excited states

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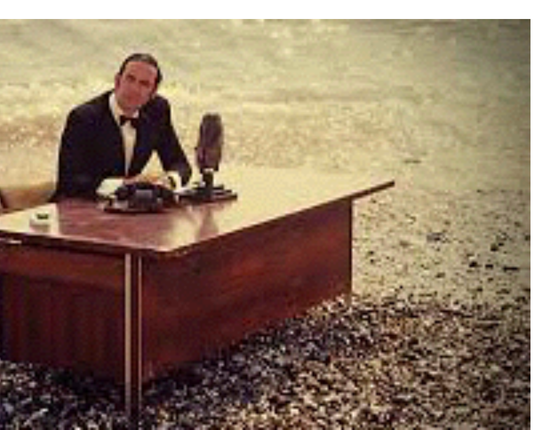
We propose optimal estimators for bispectra from excited states. Two common properties of such bispectra are the enhancement in the collinear limit, and the prediction of oscillating features. We review the physics behind excited states and some of the choices made in the literature. We show that the enfolded template is a good template in the collinear limit, but does poorly elsewhere, establishing a strong case for an improved estimator. Although the detailed scale dependence of the bispectra differs depending on various assumptions, generally the predicted bispectra are either effectively 1 or 2-dimensional and a simple Fourier basis suffices for accurate reconstruction. For an optimal CMB data analysis, combining all n-point functions, the choice for the excited state needs to be the same when computing power spectrum, bispectrum and higher order correlation functions. This has not always been the case, which could lead to wrong conclusions. We calculate the bispectrum for different choices previously discussed for the power spectrum, setting up a consistent framework to search for evidence of excited states in the CMB data.

1505.05882

Pipe Dream

Most interesting shape of primordial non-Gaussianity is one that does not come from equiliateral, squeezed, orthogonal, nor of the flattened/ folded forms considered since then it would come from pre-inflationary non-Gaussian initial conditions `the quantum gravity regime'

Equilateral - Physics at Horizon Crossing
Squeezed - Physics at Super Horizon Scales
Orthogonal - Mix of previous two
Flattened (Folded) - Physics at Sub-Horizon Scales



Something Completely Different -
Pre-inflationary physics

Let us imagine the initial state
has some non-gaussian feature

We want to address:
How big will the non-gaussian
feature appear by the end of
inflation?

Alternatively we could ask,
how much inflation do we need
to completely remove
sensitivity to initial state?

Backreaction bound on 2-pt function

Most direct bound comes from backreaction:

Energy density of the excited state cannot be bigger than inflaton energy density at beginning of inflation

Define excited initial state by Bogoliubov transformation

$$\left[\alpha_k a_k + \beta_k a_k^\dagger\right] |\psi\rangle = 0 \qquad\qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

annihilation/creation operators of adiabatic/Bunch-Davies vacuum

$$\langle\psi|T^0{}_0|\psi\rangle - \langle 0|T^0{}_0|0\rangle = \int \frac{d^3k}{(2\pi)^3} k|\beta_k|^2 \ll H^2 M_P^2$$

implies $$|\beta_k| \leq \frac{H M_P}{M_*^2}$$

physical length scale of CMB modes at beginning of inflation

Backreaction bound on 3-pt function

Agarwal, Holman, AJT, Lin 2013

Similar bound exists for 3-pt function

$$\langle\psi|T^0{}_0|\psi\rangle - \langle 0|T^0{}_0|0\rangle \ll H^2 M_P^2$$

where l.h.s. picks up contributions from cubic terms in action

$$T^0{}_0 \sim \zeta'^3$$

This condition amounts to statement $\langle\dot{\zeta}^3(\vec{x})\rangle \lesssim H^3$ at beginning of inflation

since $\zeta \sim \dfrac{1}{a}$ $\qquad \langle\zeta_H^3\rangle \lesssim \left(\dfrac{H}{M_*}\right)^6$

which implies $\qquad f_{NL} \sim \dfrac{\langle\zeta_H^3\rangle}{\langle\zeta_H^2\rangle^2} \lesssim \epsilon^2 \dfrac{H^2 M_P^4}{M_*^6}$

Backreaction bound on 3-pt function

Consistency of the Goldstone EFT imposes restriction on size of initial non-gaussianity regardless of origin

$$|M_{\rm pl}^2 \dot H (1 - c_s^{-2}) \langle \dot\pi^3 - \dot\pi (\partial_i \pi)^2 / a^2 \rangle| \ll \epsilon H^2 M_{\rm pl}^2$$

Generalization of usual backreaction bound

$$\langle T_0^0 \rangle \sim \frac{M_{\rm pl}^2}{c_s^2} \dot H \langle \dot\pi^2 \rangle \ll \epsilon H^2 M_{\rm pl}^2$$

This imposes bound on observable non-gaussianity at late times

$$f_{NL} \ll 10^{20} e^{-6N_*} c_s^2$$

$$f_{NL} \ll e^{4(7.7-N_*)} c_s^2$$

e.g. $c_s = 10^{-2}$

$$f_{NL} \ll e^{4(5.4-N_*)}$$

Summary

General Initial States can:

- lead to enhancements in the bispectrum in the squeezed in flattened limits
- lead to stronger (non slow-roll-suppressed enhancements in models with $c_s < 1$
- give rise to entirely different shapes on N-point functions which have not so far been considered dependent on pre-inflationary physics
- although I have focused on scalar 3-point function, all scalar and tensor N-point functions can receive NBD contributions
- however ultimately all such effects will be wiped out if inflation lasts long enough due to Backreaction Constraint