Aspects of non-Gaussian and... Gaussian primordial perturbations

Filippo Vernizzi IPhT, CEA Saclay and CERN

ICISE, Quy Nhon - August 19, 2015

Outline

Primordial non-Gaussianity See also A. Tolley (next talk), E. Silverstein (Thursday) and B. Racine (on Friday) !

Cosmic Microwave Background bispectrum from "primordial Gaussianity"

Consistency relations of Large Scale Structure



Outline

Primordial non-Gaussianity See also A. Tolley (next talk), E. Silverstein (Thursday) and B. Racine (on Friday) !

Cosmic Microwave Background bispectrum from "primordial Gaussianity"

Consistency relations of Large Scale Structure









Planck 2015: our cosmic collider



Planck 2015: our cosmic collider



All there is for Gaussian fluctuations

Inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right]$$

$$\frac{\dot{H}}{-\frac{\dot{H}}{H^2} \ll 1} \qquad \phi$$

Scalar fluctuations

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right]$$



Each Fourier mode is a quantum harmonic oscillator with time dependent spring "constant"

$$S = \int d^4x \,\epsilon \, a^3 \, M_{\rm Pl}^2 \left[\dot{\zeta}^2 - \frac{1}{a^2} (\partial_i \zeta)^2 \right]$$

 $\phi = \phi(t)$ $g_{ij} = a^2(t)e^{2\zeta}\delta_{ij}$

Scalar fluctuations

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right]$$



Each Fourier mode is a quantum harmonic oscillator with time dependent spring "constant"



• Inside Hubble radius: vacuum I.C.

· Outside Hubble radius: fluctuations freeze-in



$$S = \int d^4x \epsilon a^3 M_{\rm Pl}^2 \left[\dot{\zeta}^2 - \frac{1}{a^2} (\partial_i \zeta)^2 + \frac{2}{H} \dot{\zeta} (\partial_i \zeta)^2 + \dots \right]$$

Discriminative power between different early universe models



$$S = \int d^4x \epsilon a^3 M_{\rm Pl}^2 \left[\dot{\zeta}^2 - \frac{1}{a^2} (\partial_i \zeta)^2 + \frac{2}{H} \dot{\zeta} (\partial_i \zeta)^2 + \dots \right]$$

Discriminative power between different early universe models





Squeezed limit: Maldacena '02 $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \left[-(n_s - 1) + \mathcal{O}\left(\frac{k_1^2}{k_2^2}\right) \right] P(k_1) P(k_2) , \qquad k_1 \ll k_2 \approx k_3$

The long mode redefines the background (rescaling of the momenta):

$$g_{ij}dx^i dx^j = a^2(t)e^{2\zeta_L(\vec{x})}d\vec{x}^2 = a^2(t)d\tilde{\vec{x}}^2 \quad \Longrightarrow \quad \tilde{k} = ke^{-\zeta_L}$$



Squeezed limit: Maldacena '02 $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \left[-(n_s - 1) + \mathcal{O}\left(\frac{k_1^2}{k_2^2}\right) \right] P(k_1) P(k_2) , \qquad k_1 \ll k_2 \approx k_3$

The long mode redefines the background (rescaling of the momenta):

$$g_{ij}dx^i dx^j = a^2(t)e^{2\zeta_L(\vec{x})}d\vec{x}^2 = a^2(t)d\tilde{\vec{x}}^2 \quad \Longrightarrow \quad \tilde{k} = ke^{-\zeta_L}$$



Squeezed limit: Maldacena '02 $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \left[-(n_s - 1) + \mathcal{O}\left(\frac{k_1^2}{k_2^2}\right) \right] P(k_1) P(k_2) , \qquad k_1 \ll k_2 \approx k_3$

The long mode redefines the background (rescaling of the momenta):

$$g_{ij}dx^i dx^j = a^2(t)e^{2\zeta_L(\vec{x})}d\vec{x}^2 = a^2(t)d\tilde{\vec{x}}^2 \quad \Longrightarrow \quad \tilde{k} = ke^{-\zeta_L}$$



Blue spectrum n_s-1>0

Robust prediction of all single-field models!

$$\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 - V(\phi) \qquad \qquad \epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_{\rm Pl}^2}{2} \left(\frac{V'}{V}\right)^2$$

Single-field slow-roll:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \left[-(n_s - 1) + \epsilon \mathcal{O}\left(\frac{k_1^2}{k_2^2}\right) \right] P(k_1) P(k_2) , \qquad k_1 \ll k_2 \approx k_3$$

Single-field attractor solution

Only gravitational interactions

$$\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 - V(\phi) + \frac{1}{\Lambda^4} (\partial \phi)^4 + \dots$$
 Creminelli '03

Single-field with interactions:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \left[-(n_s - 1) + f_{\mathrm{NL}}^{\mathrm{equilateral}} \cdot \mathcal{O}\left(\frac{k_1^2}{k_2^2}\right) \right] P(k_1) P(k_2) , \qquad k_1 \ll k_2 \approx k_3$$



Example: DBI inflation Alishahiha, Silverstain, Tong '04

 $f_{\rm NL}^{\rm eq} \gtrsim 1$: large self-interactions are important (strong coupling regime)

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{\Lambda^4}(\partial\phi)^4 + \dots$$
 Creminelli '03

Single-field with interactions:

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle = \left[-(n_s - 1) + f_{\mathrm{NL}}^{\mathrm{equilateral}} \cdot \mathcal{O}\left(\frac{k_1^2}{k_2^2}\right) \right] P(k_1) P(k_2) , \qquad k_1 \ll k_2 \approx k_3$$



Example: DBI inflation Alishahiha, Silverstain, Tong '04

 $f_{
m NL}^{
m eq} \gtrsim 1$: large self-interactions are important (strong coupling regime)



$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_{\sigma}^2\sigma^2$$

Multi-field models:

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle = \left[f_{\mathrm{NL}}^{\mathrm{local}} + \mathcal{O}(\epsilon) \cdot \mathcal{O}\left(\frac{k_1^2}{k_2^2}\right) \right] P(k_1) P(k_2) , \qquad k_1 \ll k_2 \approx k_3$$

Examples: multi-field inflation, curvaton, variable decay rate, etc...

 $f_{
m NL}^{
m loc}\gtrsim1$: extra fields, local correlation between long and short modes



$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_{\sigma}^2\sigma^2$$

Multi-field models:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = \left[f_{\mathrm{NL}}^{\mathrm{local}} + \mathcal{O}(\epsilon) \cdot \mathcal{O}\left(\frac{k_1^2}{k_2^2}\right) \right] P(k_1) P(k_2) , \qquad k_1 \ll k_2 \approx k_3$$

Examples: multi-field inflation, curvaton, variable decay rate, etc...

 $f_{
m NL}^{
m loc}\gtrsim1$: extra fields, local correlation between long and short modes



Outline

Primordial non-Gaussianity

Cosmic Microwave Background bispectrum from "primordial Gaussianity"

Consistency relations of Large Scale Structure







Intrinsic nonlinear effects

Even in the absence of primordial non-Gaussianity, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = 0$, the CMB is non-Gaussian!

$$\begin{aligned} \zeta_{in} & \Rightarrow & \Theta = \frac{\delta T}{T} \\ \Theta_{\vec{k}} = T^{(1)}(t,k)\zeta_{\vec{k}} + T^{(2)}(t,k)(\zeta \star \zeta)_{\vec{k}} \end{aligned}$$

Intrinsic nonlinear effects

Even in the absence of primordial non-Gaussianity, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = 0$, the CMB is non-Gaussian!

$$\begin{aligned} \zeta_{in} & \Longrightarrow & \Theta = \frac{\delta T}{T} \\ \Theta_{\vec{k}} = T^{(1)}(t,k)\zeta_{\vec{k}} + T^{(2)}(t,k)(\zeta \star \zeta)_{\vec{k}} \end{aligned}$$

2nd-order effects induce non-Gaussianity:

• Late time: ISW-lensing; Goldberg, Spergel, '99 $f_{\rm NL}^{\rm loc} = 5.7$ Detected by Planck!

(Planck '15) Shape and method	Independent	ISW-lensing subtracted
$\frac{1}{\text{SMICA } (T+E)}$ Local	6.5 ± 5.0	0.8 ± 5.0

Intrinsic nonlinear effects

Even in the absence of primordial non-Gaussianity, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = 0$, the CMB is non-Gaussian!

$$\begin{aligned} \zeta_{in} & \Rightarrow & \Theta = \frac{\delta T}{T} \\ \Theta_{\vec{k}} = T^{(1)}(t,k)\zeta_{\vec{k}} + T^{(2)}(t,k)(\zeta \star \zeta)_{\vec{k}} \end{aligned}$$

2nd-order effects induce non-Gaussianity:

• Late time: ISW-lensing; Goldberg, Spergel, '99 $f_{\rm NL}^{\rm loc} = 5.7$ Detected by Planck!

(Planck '15)		
Shape and method	Independent	ISW-lensing subtracted
SMICA (T+E) Local	6.5 ± 5.0	0.8 ± 5.0

• At recombination: 2nd-order perturbations in the fluid + GR nonlinearities.

$$\delta = \delta^{(1)} + \delta^{(2)} \quad \Rightarrow \quad \begin{array}{l} D[\delta^{(1)}] = 0\\ D[\delta^{(2)}] = S[\delta^{(1)2}] \end{array} \quad \Rightarrow \quad f_{\rm NL} \sim \frac{\langle \delta^{(2)} \delta^{(1)} \delta^{(1)} \rangle}{\langle \delta^{(1)} \delta^{(1)} \rangle^2} \sim \text{few} \end{array}$$

Numerical goals

New numerical codes:

- CosmoLib2nd Z. Huang, FV '12
- SONG Pettinari, Fidler, Chrittenden, Koyama, Wands '13
- Su, Lim, Shellard '12



• Boltzmann code:

Evolve perturbations up to second order by solving Boltzmann and Einstein equations

$$\frac{df_I}{d\eta} = C_I[f_I] , \quad I = \gamma, \nu, b, \text{CDM}$$

$$\& \quad G_{ij} = 8\pi G \sum_{I} T_{ij}^{(I)}$$
$$\Rightarrow \quad \Theta^{(2)}, \Phi^{(2)}, \Psi^{(2)}, \dots$$

• Line-of-sight integral:

Compute CMB bispectrum by integrating the photon temperature along the line of sight

$$\Theta^{(2)}(\eta_0, \hat{n}) = \int_0^{\eta_0} d\eta S^{(2)}(\eta, \vec{x}(\eta), \hat{n})$$
$$\langle \Theta^{(2)}_{l_1 m_1} \Theta^{(1)}_{l_2 m_2} \Theta^{(1)}_{l_3 m_3} \rangle \propto \langle \zeta \zeta \rangle \langle \zeta \zeta \rangle$$

Based on many contributions:

Bartolo, Matarrese, Riotto '04, '06; Bernardeau, Pitrou, Uzan '08; Pitrou '08; Bartolo, Riotto '08; Khatri, Wandelt '08; Senatore, Tassev, Zaldarriaga '08; Nitta et al. '09, Boubekeur, Creminelli, D'Amico, Norena, FV '09, Beneke and Fidler '10,...

and previous codes:

Bernardeau, Pitrou, Uzan '08 (CMBquick2), Khatri, Wandelt '08 (perturbed rec.), Senatore, Tassev, Zaldarriaga '08 (perturbed recombination), Nitta et al. '09 (product of first order)

Take a long mode

Creminelli, Zaldarriaga '04 with Creminelli, Pitrou '11

Single-field inflation: 1 clock, e.g. everything is determined by T.



Local physics is identical in Hubble patches that differ only by super-horizon modes: two observers in different places on LSS will see exactly the same CMB anisotropies (at given T).

Squeezed limit formula



The long mode is inside the horizon and I can compare different patches. Will see a modulation of the 2-point function due to large scale T:

Transverse rescaling of spatial coords \Rightarrow rescaling of angles: $C_l \rightarrow C_l + \zeta(\hat{n} \cdot \nabla_{\hat{n}} C_l)$

Squeezed limit formula



The long mode is inside the horizon and I can compare different patches. Will see a modulation of the 2-point function due to large scale T:

Transverse rescaling of spatial coords \Rightarrow rescaling of angles: $C_l \rightarrow C_l + \zeta(\hat{n} \cdot \nabla_{\hat{n}} C_l)$

• Squeezed limit formula:

 \Rightarrow

$$b_{l_1 l_2 l_3} = C_{l_1}^{T\zeta} \frac{1}{l^2} \frac{d(l^2 C_l^{\text{no ISW}})}{d \ln l} , \qquad \qquad l_1 \ll l \equiv |\vec{l_1} - \vec{l_2}|/2 \\ l_1 \ll l_H \simeq 110$$

with Creminelli, Pitrou '11; Bartolo, Matarrese, Riotto; '11, Lewis '12

This relation can be used as consistency check of Boltzmann codes based on a physical limit

Code/formula comparison

with Huang '12, '13



• Comparison with the analytic formula:

$$b_{l_1 l_2 l_3} = C_{l_1}^{T\zeta} \frac{1}{l^2} \frac{d(l^2 C_l^{\text{no ISW}})}{d \ln l} + \mathcal{O}\left(\frac{l_1^2}{l^2}\right)$$

Observability and contamination



see also Fidler et al. '14 & Pettinari et al. '14 for polarization

Future: use this as a template

$$b_{l_1 l_2 l_3} = C_{l_1}^{T\zeta} \frac{1}{l^2} \frac{d(l^2 C_l^{\text{no ISW}})}{d \ln l}$$

Outline

Primordial non-Gaussianity

Cosmic Microwave Background bispectrum from "primordial Gaussianity"

Consistency relations of Large Scale Structure







Consistency relations of LSS

 $\Phi_L(t,\vec{x}) = \Phi_L(t,\vec{0}) + \vec{x} \cdot \vec{\nabla} \Phi_L(t)|_{\vec{0}} + \text{ physical effects}$

Uniform gravitational field

Consistency relations of LSS

$$\Phi_L(t, \vec{x}) = \Phi_L(t, \vec{0}) + \vec{x} \cdot \vec{\nabla} \Phi_L(t)|_{\vec{0}} + \text{ physical effects}$$

Uniform gravitational field



$$\left\langle \delta(t_1, \vec{x}_1) \cdots \delta(t_n, \vec{x}_n) | \Phi_L \right\rangle = \left\langle \delta(t_1, \vec{\tilde{x}}_1) \cdots \delta(t_n, \vec{\tilde{x}}_n) \right\rangle$$

Assumption: Gaussianity (the long mode does not change the statistics of the short ones) Multiplying by a long mode and averaging over the it...

Consistency relations of LSS

$$\Phi_L(t, \vec{x}) = \Phi_L(t, \vec{0}) + \vec{x} \cdot \vec{\nabla} \Phi_L(t)|_{\vec{0}} + \text{ physical effects}$$

Uniform gravitational field



Kehagias, Riotto, '13; Peloso, Pietroni '13; with Creminelli, Noreña, Simonovic '13; Valageas '13

Consistency relations for LSS

$$\begin{split} \langle \delta_{\vec{q}}(t) \, \delta_{\vec{k}_1}(t_1) \delta_{\vec{k}_2}(t_2) \rangle' \simeq P_{\delta}(q, t) \frac{\vec{q} \cdot \vec{k}}{q^2} \left[\frac{D_{\delta}(t_2)}{D_{\delta}(t)} P\left(k_1, t_1\right) - \frac{D_{\delta}(t_1)}{D_{\delta}(t)} P\left(k_2, t_2\right) \right] \\ \vec{k}_1 = -\vec{k} + \vec{q}/2 \ , \qquad \vec{k}_2 = \vec{k} + \vec{q}/2 \end{split}$$

- Hold nonlinearly in the short modes, after shell crossing, including baryonic physics, bias and everything!
- Vanishes for equal-time correlators $t_1 = t_2$

Since $\vec{k}_1 + \vec{k}_2 = q$ there is no divergent contribution

Cancellation very robust:

- True also in redshift space
- Non-perturbative in the long mode

with Creminelli, Gleyzes, Simonovic '14

Well known in Perturbation Theory but here derivation is much more general

Solely a consequence of the Equivalence Principle and non-Gaussianity

Consistency relations for LSS

$$\begin{split} \langle \delta_{\vec{q}}(t) \, \delta_{\vec{k}_1}^g(t_1) \delta_{\vec{k}_2}^g(t_2) \rangle' \simeq P_{\delta}(q,t) \frac{\vec{q} \cdot \vec{k}}{q^2} \left[\frac{D_{\delta}(t_2)}{D_{\delta}(t)} P_g(k_1,t_1) - \frac{D_{\delta}(t_1)}{D_{\delta}(t)} P_g(k_2,t_2) \right] \\ \vec{k}_1 = -\vec{k} + \vec{q}/2 \ , \qquad \vec{k}_2 = \vec{k} + \vec{q}/2 \end{split}$$

- Hold nonlinearly in the short modes, after shell crossing, including baryonic physics, bias and everything!
- Vanishes for equal-time correlators $t_1 = t_2$

Since $\vec{k}_1 + \vec{k}_2 = q$ there is no divergent contribution

Cancellation very robust:

- True also in redshift space
- Non-perturbative in the long mode

with Creminelli, Gleyzes, Simonovic '14

Well known in Perturbation Theory but here derivation is much more general

Solely a consequence of the Equivalence Principle and non-Gaussianity

Hard to measure

$$\langle \delta_{\vec{q}}(t) \, \delta_{\vec{k}_1}^g(t_1) \delta_{\vec{k}_2}^g(t_2) \rangle' \simeq P_{\delta}(q,t) \frac{\vec{q} \cdot \vec{k}}{q^2} \left[\frac{D_{\delta}(t_2)}{D_{\delta}(t)} P_g(k_1,t_1) - \frac{D_{\delta}(t_1)}{D_{\delta}(t)} P_g(k_2,t_2) \right]$$

• Difficult to measure a short scale correlation over long times



- "Measure" in simulations as a test
- Look for a violation at equal times: divergence for $q \rightarrow 0$

Local non-Gaussianity: scale dependent bias

$$\langle \delta_{\vec{q}} \delta_{\vec{k}_1} \delta_{\vec{k}_2} \rangle_{q \to 0} \sim \frac{f_{\rm NL}^{\rm loc.}}{H^2 q^2} P_{\delta}(q) P_{\delta}(k)$$

Dalal, Doré, Huterer, Shirokov '07

We Violation of Equivalence Principle: fifth force, modified gravity

 $\langle \delta_{\vec{q}} \delta_{\vec{k}_1} \delta_{\vec{k}_2} \rangle_{q \to 0} \sim \epsilon \cdot \frac{k}{q} P_{\delta}(q) P_{\delta}(k)$

with Creminelli, Gleyzes, Hui, Simonovic '14

Baryon acoustic oscillations

Baldauf, Mirbabayi, Simonovic, Zaldarriaga '15

$$\langle \delta_{\vec{q}} \, \delta_{\vec{k}_1}^g \, \delta_{\vec{k}_2}^g \rangle' \simeq P_\delta(q) \frac{\vec{q} \cdot \vec{k}}{q^2} \left[P_g(k_1) - P_g(k_2) \right]$$

Vanish for smooth $P_{g}(k)$ over long mode q (no effect of the long mode on relative displacement). But consider finite q

• Effects of modes comparable to BAO separation on modes of order of BAO width $\langle \delta_{\vec{q}} \, \delta_{\vec{k}_1}^g \, \delta_{\vec{k}_2}^g \rangle' \simeq 2P_{\delta}(q) \frac{k\vec{q} \cdot \vec{\nabla}_{\hat{k}}}{\ell_{\text{BAO}}q^2} P_g^w(k) \sim \frac{1}{\ell_{\text{BAO}}q}$ lBAO

Spread of BAO width under effect of long mode

• Useful for BAO scale reconstruction: argument depends only on EP and Gaussianity (beyond perturbation theory).

Conclusions

* CMB bispectrum from Gaussianity: CMB is affected by intrinsic second order effects at recombination. Firmly established by 2nd-order Boltzmann code and analytic calculation in the squeezed limit.

★ Consistency relations of LSS: EP and Gaussianity fixes correlation functions in the squeezed limit. Can be used to test these assumptions. In our universe they are the main source of broadening of the baryon acoustic peaks.