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Rencontres du Vietnam
Cosmology – 50 years after CMB discovery

Dark Matter at Galactic Scales & MOND

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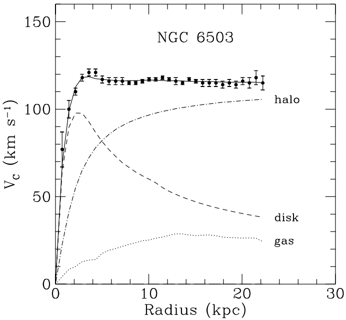
18 août 2015

Evidence for dark matter in Astrophysics

1. Oort [1932] noted that the sum of observed mass in the vicinity of the Sun falls short of explaining the vertical motion of stars in the Milky Way
2. Zwicky [1933] reported that the velocity dispersion of galaxies in galaxy clusters is far too high for these objects to remain bound for a substantial fraction of cosmic time

3. Ostriker & Peebles [1973] showed that to prevent the growth of instabilities in cold self-gravitating disks like spiral galaxies, it is necessary to embed the disk in the quasi-spherical potential of a huge halo of dark matter
4. Bosma [1981] and Rubin [1982] established that the rotation curves of galaxies are approximately flat, contrarily to the Newtonian prediction based on ordinary baryonic matter

Rotation curves of galaxies are approximately flat



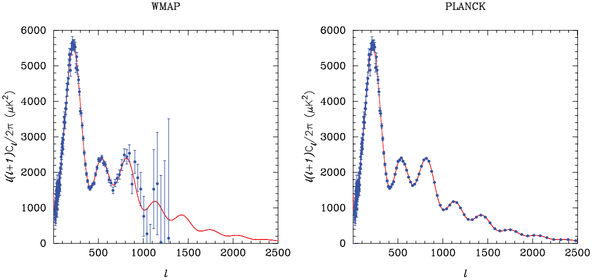
- For a circular orbit we expect

$$v(r) = \sqrt{\frac{G\,M(r)}{r}}$$

- The fact that $v(r)$ is constant implies that beyond the optical disk

$$\boxed{M_{\text{halo}}(r) \simeq r \qquad \rho_{\text{halo}}(r) \simeq \frac{1}{r^2}}$$

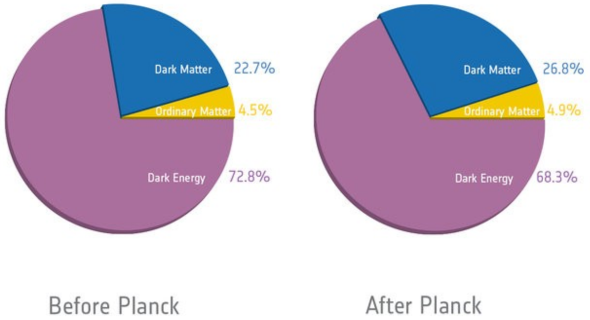
The cosmological concordance model Λ-CDM



This model brilliantly accounts for:

- The mass discrepancy between the dynamical and luminous masses of clusters of galaxies
- The precise measurements of the anisotropies of the cosmic microwave background (CMB)
- The formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
- The fainting of the light curves of distant supernovae

Problem of the dark constituents of the Universe



Λ-CDM assumes General Relativity is the correct theory of gravity but:

- No known particle in the standard model of particle physics could be the particle of dark matter
- Extensions of the standard model of particle physics provide well-motivated but yet to be discovered candidates
- The numerical value of the cosmological constant Λ looks un-natural from a quantum field perspective

Challenges with CDM at galactic scales

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004, Famaey & McGaugh 2012]

1. Unobserved predictions
   - Numerous but unseen satellites of large galaxies
   - Phase-space correlation of galaxy satellites
   - Generic formation of dark matter cusps in galaxies
   - Tidal dwarf galaxies dominated by dark matter
2. Unpredicted observations
   - Correlation between mass discrepancy and acceleration
   - Surface brightness of galaxies and the Freeman limit
   - Flat rotation curves of galaxies
   - Baryonic Tully-Fisher relation for spirals
   - Faber-Jackson relation for ellipticals

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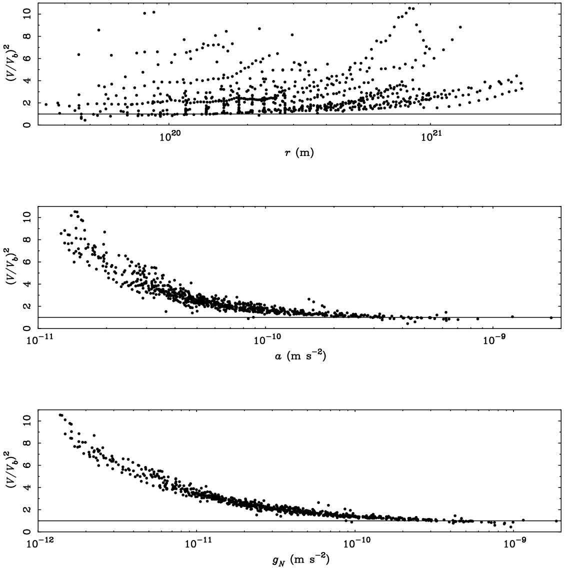
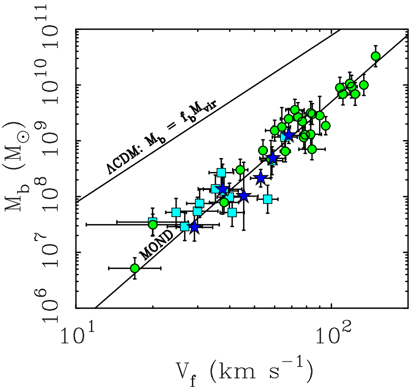
All these challenges are mysteriously solved (sometimes with incredible success) by the MOND empirical formula [Milgrom 1983]

Mass discrepancy versus acceleration [Milgrom 1983]



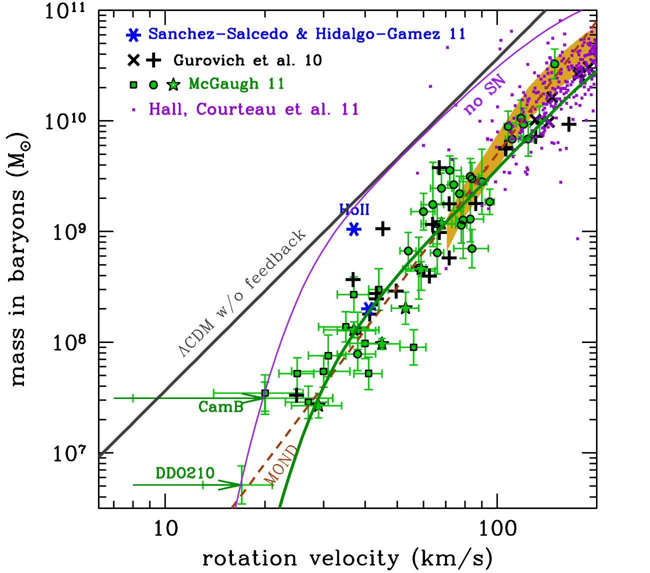
Baryonic Tully-Fisher relation [Tully & Fisher 1977, McGaugh 2011]



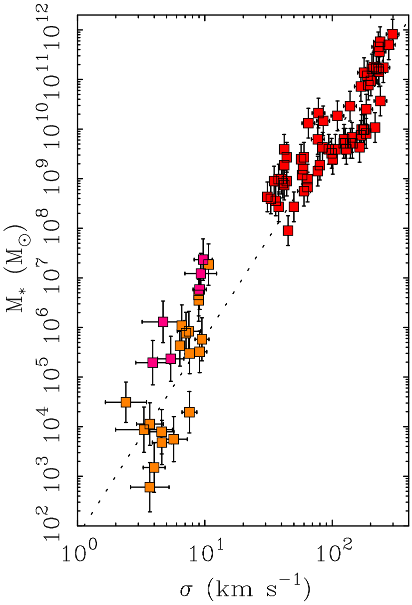
We have approximately $V_f \simeq \left(G\, M_b\, a_0\right)^{1/4}$ where $a_0 \simeq 1.2 \times 10^{-10}$m/s^2 is very close (mysteriously enough) to typical cosmological values

$$\boxed{a_0 \simeq 1.3\, a_\Lambda \quad \text{with} \quad a_\Lambda = \frac{c^2}{2\pi}\sqrt{\frac{\Lambda}{3}}}$$

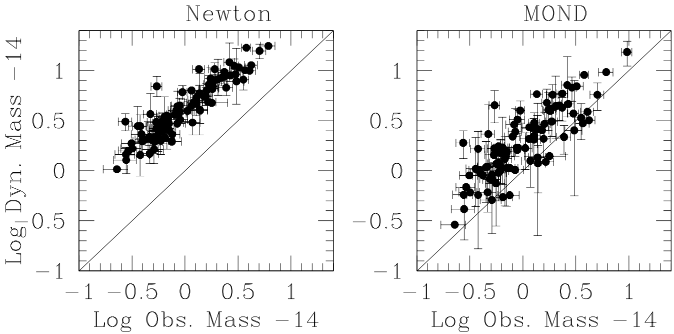
BTF relation fitted with Λ-CDM [Silk & Mamon 2012]



Mass velocity dispersion relation [Faber & Jackson 1976]

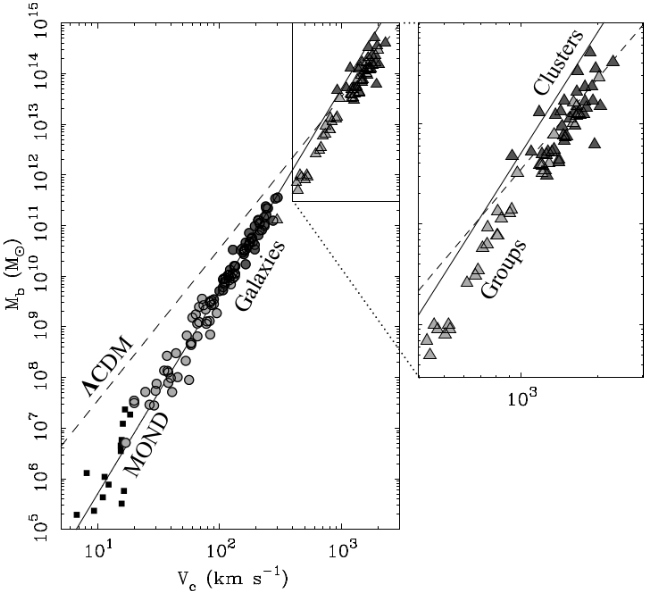


Problem with galaxy clusters [Gerbal, Durret *et al.* 1992, Sanders 1999]



- The mass discrepancy is $\approx 4-5$ with Newton and ≈ 2 with MOND
- The bullet cluster and more generally X-ray emitting galaxy clusters can be fitted with MOND only with a component of baryonic dark matter and/or hot/warm neutrinos [Angus, Famaey & Buote 2008]

Galactic versus cosmological scales

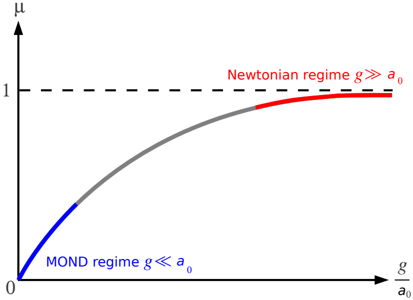


Modified Poisson equation [Milgrom 1983, Bekenstein & Milgrom 1984]

MOND takes the form of a modified Poisson equation

$$\boxed{\nabla \cdot \left[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{fonction MOND}} \boldsymbol{g} \right] = -4\pi\, G\, \rho_{\text{baryon}} \quad \text{avec} \quad \boldsymbol{g} = \nabla U}$$

- The Newtonian regime is recoved when $g \gg a_0$
- In the MOND regime $g \ll a_0$ we have $\boxed{\mu \simeq g/a_0}$



Modified gravity theories

1. Generalized Tensor-Scalar theory (RAQUAL) [Bekenstein & Sanders 1994]
2. Tensor-Vector-Scalar theory (TeVeS) [Bekenstein 2004, Sanders 2005]
3. Generalized Einstein-Æther theories [Zlosnik *et al.* 2007, Halle *et al.* 2008]
4. Khronometric theory [Blanchet & Marsat 2011, Sanders 2011, Barausse *et al.* 2015]
5. Bimetric theory (BIMOND) [Milgrom 2012]

- These theories contain non-standard kinetic terms parametrized by an arbitrary function which is linked *in fine* to the MOND function
- In some cases they have stability problems associated with the fact that the Hamiltonian is not bounded from below [Clayton 2001, Bruneton & Esposito-Farèse 2007]
- Generically they have problems to recover the cosmological model Λ-CDM at large scales and the spectrum of CMB anisotropies [Skordis, Mota *et al.* 2006]

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Dielectric analogy of MOND [Blanchet 2006]

- In electrostratics the Gauss equation is modified by the polarization of the dielectric (dipolar) material

$$\nabla \cdot \Big[\underbrace{(1+\chi_e)\boldsymbol{E}}_{\boldsymbol{D}\text{ field}} \Big] = \frac{\rho_e}{\varepsilon_0} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{E} = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0}$$

- Similarly MOND can be viewed as a modification of the Poisson equation by the polarization of some dipolar medium

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- The MOND function can be written $\mu = 1 + \chi$ where χ appears as a susceptibility coefficient of some dipolar DM medium

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Microscopic description of DDM?

- The DM medium by individual dipole moments $\boldsymbol{p}$ and a polarization field $\boldsymbol{P}$

$$\boldsymbol{P} = n\,\boldsymbol{p} \qquad \text{with} \qquad \boldsymbol{p} = m\,\boldsymbol{\xi}$$

- The polarization is induced by the gravitational field of ordinary masses

$$\boldsymbol{P} = -\frac{\chi}{4\pi G}\,\boldsymbol{g} \qquad \boxed{\rho_{\text{DDM}} = -\boldsymbol{\nabla}\cdot\boldsymbol{P}}$$

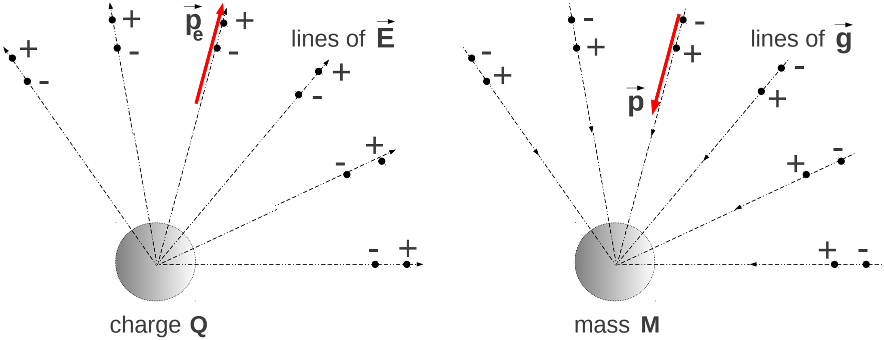
The dipole moments should be made by particles with positive and negative gravitational masses $(m_{\mathrm{i}}, m_{\mathrm{g}}) = (m, \pm m)$

- Because like masses attract and unlike ones repel we have anti-screening of ordinary masses by polarization masses

$$\boxed{\chi < 0}$$

which is in agreement with DM and MOND

Anti-screening by polarization masses



Screening by polarization charges

$$\boxed{\chi_e > 0}$$

Anti-screening by polarization masses

$$\boxed{\chi < 0}$$

Need of a non-gravitational internal force

- The constituents of the dipole will repel each other so we need a non-gravitational force to stabilize the dipolar medium

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{\nabla}(U + \phi) \qquad \frac{\mathrm{d}\underline{\boldsymbol{v}}}{\mathrm{d}t} = -\boldsymbol{\nabla}(U + \phi)$$

- The internal force is generated by the gravitational charge *i.e.* the mass

$$\Delta\phi = -\frac{4\pi G}{\chi}(\rho - \underline{\rho})$$

- The DM medium appears as a polarizable plasma of particles $(m, \pm m)$ oscillating at the natural plasma frequency

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2\boldsymbol{\xi} = 2\boldsymbol{g} \qquad \text{with} \qquad \omega = \sqrt{-\frac{8\pi\, G\, \rho_0}{\chi}}$$

DDM via a bimetric extension of GR [Bernard & Blanchet 2014]

1. To describe relativistically some microscopic DM particles with positive or negative gravitational masses one needs two metrics
   - $g_{\mu\nu}$ obeyed by ordinary particles (including baryons)
   - $f_{\mu\nu}$ obeyed by "dark" particles

2. In addition the DM particles forming the dipole moment should interact via a non-gravitational force field, *e.g.* a (spin-1) "graviphoton" vector field $\mathcal{A}_\mu$ with field strength $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$

3. An initial model was shown to reproduce correctly
   - MOND
   - Λ-CDM and the CMB
   - PPN limit of GR

   but is plagued with ghosts in the gravitational sector

DDM via massive bigravity theory [Blanchet & Heisenberg 2015ab]

1. The matter sector is the same as in the previous model
2. The gravitational sector of the model is based on massive bigravity theory [de Rham, Gabadadze & Tolley 2011; Hassan & Rosen 2012]

$$S = \int \mathrm{d}^4x \Big\{ \sqrt{-g}\left(\frac{M_g^2}{2}R_g - \rho_{\text{bar}} - \rho_g\right) + \sqrt{-f}\left(\frac{M_f^2}{2}R_f - \rho_f\right) + \sqrt{-g_{\text{eff}}}\left[\frac{m^2}{4\pi} + \mathcal{A}_\mu \left(j_g^\mu - \frac{\alpha}{\beta} j_f^\mu\right) + \frac{a_0^2}{8\pi}\mathcal{W}(X)\right]\Big\}$$

3. The ghost-free potential interactions take the particular form of the square root of the determinant of the effective metric [de Rham, Heisenberg & Ribeiro 2014]

$$\boxed{g_{\mu\nu}^{\text{eff}} = \alpha^2 g_{\mu\nu} + 2\alpha\beta\, g_{\mu\rho}X^\rho_\nu + \beta^2 f_{\mu\nu}}$$

with the square-root matrix $X = \sqrt{g^{-1}f}$

4. The graviphoton field $\mathcal{A}_\mu$ is coupled to the effective metric $g_{\mu\nu}^{\text{eff}}$

General structure of the model [Blanchet & Heisenberg 2015ab]

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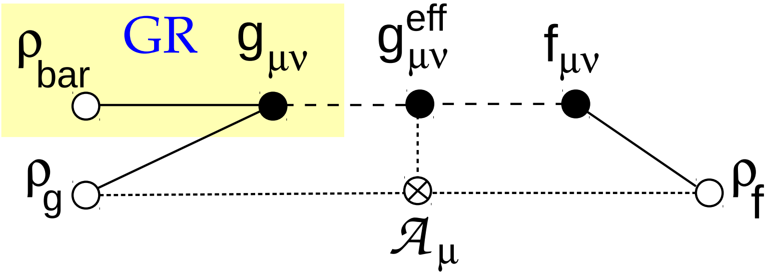

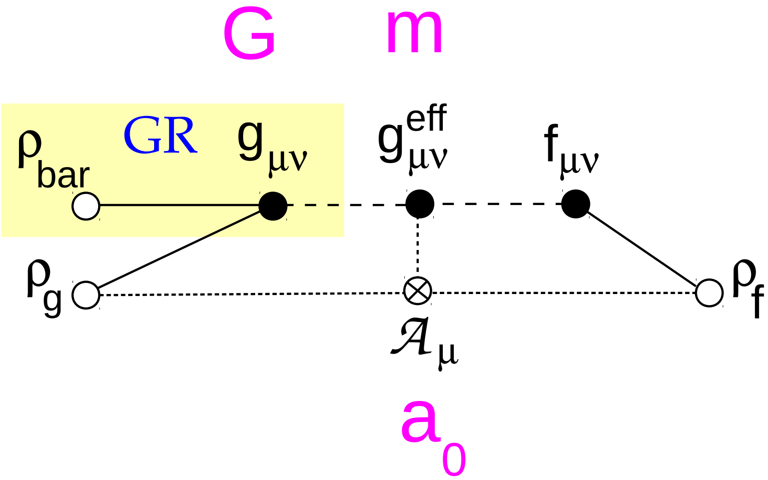

G
m
GR
ρ_{bar}
$g_{\mu\nu}$
$g^{eff}_{\mu\nu}$
$f_{\mu\nu}$
ρ_g
ρ_f
$\mathcal{A}_\mu$
a_0

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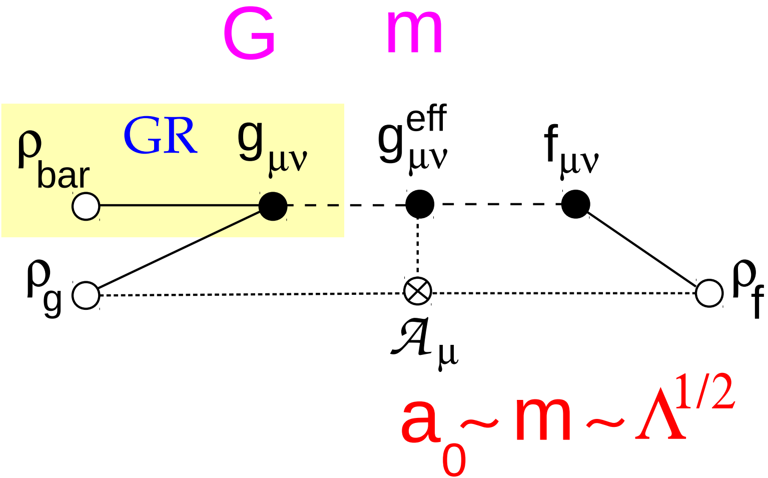

Status of hybrid DM à la MOND (DDM)

- The phenomenology of MOND is explained by a physical mechanism of gravitational polarization
- The DM appears to be a diffuse medium polarizable in the field of ordinary matter and undergoing stable plasma-like oscillations
- The theory has the potential to reproduce the cosmological model Λ-CDM and its successes at cosmological scales
- By construction the model is safe in the gravitational sector below the decoupling limit scale [de Rham, Gabadadze & Tolley 2011; Hassan & Rosen 2012][a]

$$\boxed{\Lambda_3 = \left(m^2 M_{\mathrm{P}}\right)^{1/3}}$$

[a] Because of the coupling of the DM fields ρ_g, ρ_f to the internal vector field $\mathcal{A}_\mu$ a ghost in the decoupling limit is also present in the DM sector. Its mass should be computed and compared to Λ_3

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What about the Solar System scale?

1. In spherical symmetry the MOND equation becomes

$$\mu\left(\frac{g}{a_0}\right) g = g_{\mathrm{N}} \equiv \frac{GM_\odot}{r^2}$$

2. Suppose MOND approaches the Newtonian regime like

$$\mu\left(\frac{g}{a_0}\right) = 1 - k\left(\frac{a_0}{g}\right)^q \quad \text{when} \quad g \to \infty$$

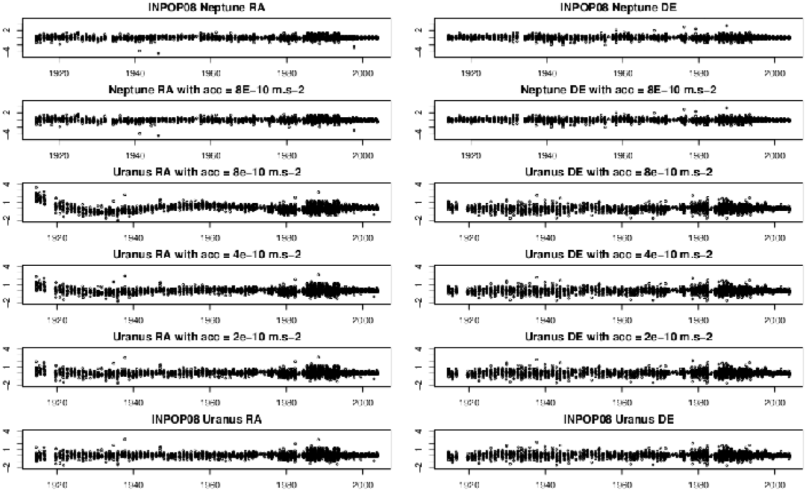
3. With $r_0 = \sqrt{GM_\odot / a_0}$ the MOND transition radius for the Sun

$$g = g_{\mathrm{N}} + k\, a_0 \left(\frac{r}{r_0}\right)^{2q-2}$$

When $q = 1$ this gives a Pioneer-like anomaly

$$a_{\mathrm{P}} = k\, a_0$$

Solar System data and Pioneer anomaly [Fienga *et al.* 2009]



The data exclude a Pioneer-like anomaly at the level $5 \times 10^{-13}\mathrm{m/s}^2$

The external field effect in MOND [Milgrom 1983]

1. Open star clusters in our Galaxy do not show evidence for dark matter despite their typical low internal gravity $g_\mathrm{i} \ll a_0$
2. In the presence of the external Galactic field g_e the MOND equation which is non-linear can be approximated by

$$\mu\left(\frac{|\boldsymbol{g}_\mathrm{i}+\boldsymbol{g}_\mathrm{e}|}{a_0}\right)\boldsymbol{g}_\mathrm{i} \approx \boldsymbol{g}_\mathrm{i}^\mathrm{Newtonian}$$

- When $a_0 \lesssim g_\mathrm{e}$ the sub-system exhibits Newtonian behaviour
- When $g_\mathrm{i} \lesssim g_\mathrm{e} \lesssim a_0$ the system is still Newtonian but with an effective Newton's constant G/μ_e

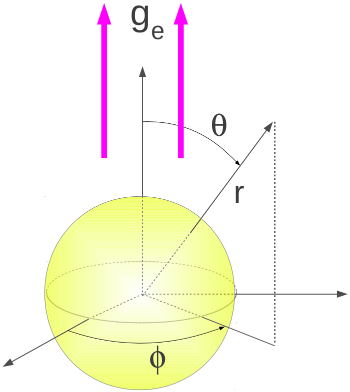
The EFE results from a violation of the strong version of the equivalence principle

The gravitational dynamics of a system is influenced by the external gravitational field in which the system is embedded

Deformation of the Sun's field by the Galactic field

The external field effect is a prediction of the non-linear Poisson equation

$$\nabla \cdot \left[\mu\left(\frac{g}{a_0}\right) \nabla U \right] = -4\pi\, G\, \rho_{\mathrm{b}}$$



The MOND field of the Sun, in the presence of the external field of the Galaxy, is deformed along the direction of the Galactic center

$$U = \boldsymbol{g}_{\mathrm{e}} \cdot \mathbf{x} + \frac{G M_\odot / \mu_{\mathrm{e}}}{r\sqrt{1 + \lambda_{\mathrm{e}} \sin^2\theta}} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

This effect influences the motion of inner planets of the Solar System

Multipole expansion of the MOND field of the Sun

1. The Newtonian physicist measures from the motion of planets the internal gravitational potential $u = U - \boldsymbol{g}_\mathsf{e} \cdot \boldsymbol{x}$ and detects the anomaly

$$\delta u = u - u_\mathsf{N} = G \int \frac{\mathrm{d}^3\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|} \, \rho_\mathsf{pdm}(\mathbf{x}', t)$$

2. Since the phantom dark matter vanishes in the strong-field regime near the Sun δu is an harmonic function and admits the multipole expansion

$$\delta u = \sum_{l=0}^{+\infty} \frac{(-)^l}{l!} \, x^L Q_L$$

where Q_L are trace-free multipolar coefficients

3. This expansion is valid in the region inside the MOND transition radius

$$\boxed{r_0 = \sqrt{\frac{GM_\odot}{a_0}} \approx 7100\,\mathsf{AU}}$$

Effect in the Solar System [Milgrom 2009, Blanchet & Novak 2011]

1. The effect is dominantly quadrupolar and grows with the distance squared

$$u = \frac{GM_\odot}{r} + \frac{1}{2}x^i x^j Q_{ij}$$

2. The quadrupole moment is aligned in the direction of the Galactic center

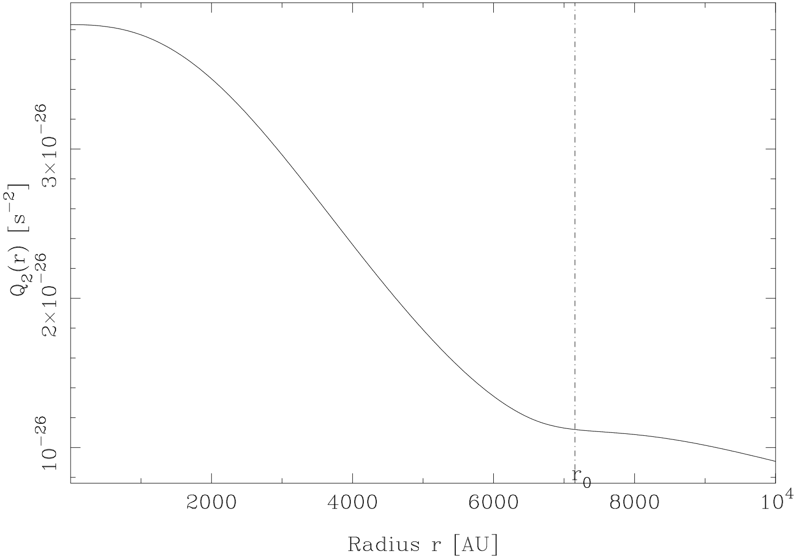
$$Q_{ij} = Q_2 \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)$$

3. The quadrupole moment is computed by solving numerically the MOND equation in the presence of the external galactic field. We find

$$\boxed{2.1 \times 10^{-27}\ \mathrm{s}^{-2} \ \lesssim\ Q_2 \ \lesssim\ 4.1 \times 10^{-26}\ \mathrm{s}^{-2}}$$

depending on the MOND function in use

Quadrupole moment as a function of distance



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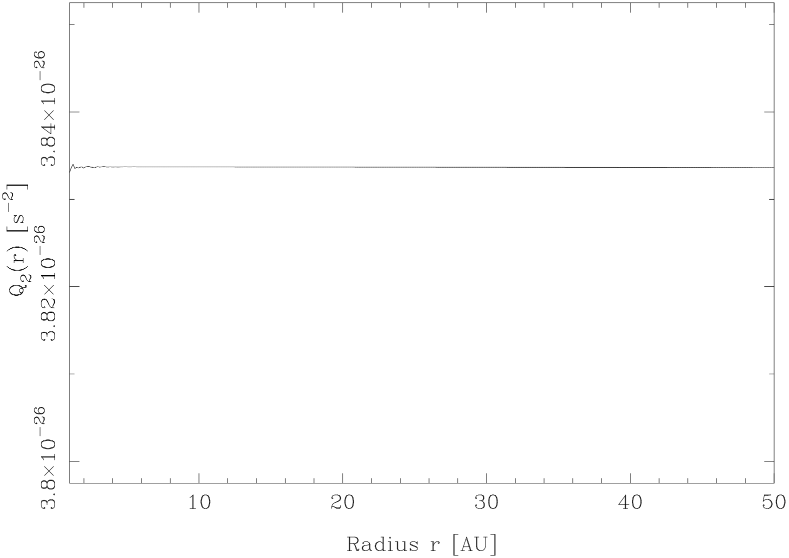

Effect on the dynamics of Solar System planets

The quadrupole effect yields a suplementary precession of the semi-major axis of planets of the Solar System [Blanchet & Novak 2011]

$$\begin{aligned}
\left\langle \frac{\mathrm{d}e}{\mathrm{d}t} \right\rangle &= \frac{5 Q_2 e \sqrt{1-e^2}}{4n} \sin(2\tilde{\omega}) \\
\left\langle \frac{\mathrm{d}\ell}{\mathrm{d}t} \right\rangle &= n - \frac{Q_2}{12n} \Big[7 + 3e^2 + 15(1+e^2)\cos(2\tilde{\omega}) \Big] \\
\left\langle \frac{\mathrm{d}\tilde{\omega}}{\mathrm{d}t} \right\rangle &= \frac{Q_2 \sqrt{1-e^2}}{4n} \Big[1 + 5\cos(2\tilde{\omega}) \Big]
\end{aligned}$$

Comparison with Solar System ephemerides

Predicted values for the orbital precession

Quadrupolar precession rate in mas/cy						
	Mercury	Venus	Earth	Mars	Jupiter	Saturn
μ_1	0.04	0.02	0.16	-0.16	-1.12	5.39
μ_2	0.02	0.01	0.09	-0.09	-0.65	3.12
μ_5	7×10^{-3}	3×10^{-3}	0.03	-0.03	-0.22	1.05
μ_{20}	2×10^{-3}	10^{-3}	9×10^{-3}	-9×10^{-3}	-0.06	0.3

Best published residuals for orbital precession

Postfit residuals for the precession rates in mas/cy						
	Mercury	Venus	Earth	Mars	Jupiter	Saturn
[Pitjeva 2005]	-3.6 ± 5	-0.4 ± 0.5	-0.2 ± 0.4	0.1 ± 0.5	-	-6 ± 2
[Fienga *et al.* 2009]	-10 ± 30	-4 ± 6	0 ± 0.016	0 ± 0.2	142 ± 156	-10 ± 8
[Fienga *et al.* 2010]	0.4 ± 0.6	0.2 ± 1.5	-0.2 ± 0.9	0 ± 0.1	-41 ± 42	0.2 ± 0.7

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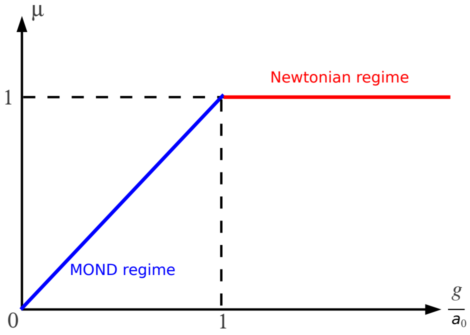
Constraining the MOND function



Solar system dynamics constraints the MOND interpolating function to be essentially exactly one when $g \geqslant a_0$ and exactly linear when $g \leqslant a_0$

Conclusions

1. An important challenge is to reproduce within a single relativistic framework
   - The concordance cosmological model Λ-CDM and its tremendous successes at cosmological scales and notably the fit of the CMB
   - The phenomenology of MOND which is a basic set of phenomena relevant to galaxy dynamics and DM distribution at galactic scales
2. While Λ-CDM meets severe problems when extrapolated at the scale of galaxies, MOND fails on galaxy cluster scales and seems to be marginally excluded by planetary ephemerides in the Solar System due to the external field effect generated by the Galaxy
3. Pure modified gravity theories (extending GR with extra fields) do not seem to be able to reproduce Λ-CDM and the CMB anisotropies at large scales

Conclusions

1. An hybrid approach (DM *à la* MOND) called DDM could meet the challenge
   - The phenomenology of MOND is explained by a physical mechanism of gravitational polarization
   - The DM appears to be a diffuse medium polarizable in the field of ordinary matter and undergoing stable plasma-like oscillations
2. The most promising and elegant route in this hybrid approach is within the framework of massive bigravity theories
   - The cosmology based on massive bigravity should be checked
   - The PPN parameters in the Solar System should be computed
3. Could the Vainshtein mechanism of massive gravity theories resolve the paradox of MOND being marginally excluded in the Solar System?