The discovery of neutrino oscillation has revealed that neutrinos have nonzero masses and that leptons mix.
What We Have Learned
The Three – Neutrino (Mass)$^2$ Spectrum

\[ \Delta m^2_{21} \equiv m^2_2 - m^2_1 \approx 7.5 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{32} \approx 2.4 \times 10^{-3} \text{ eV}^2 \]

There might be more mass eigenstates.
Constraints On the Absolute Scale of Neutrino Mass

\[ \nu_3 \]

\[ \nu_2 \]

\[ \nu_1 \]

\( (\text{Mass})^2 \)

\( 0 \)

\( \Delta m_{\text{big}} \)

\( \Delta m_{\text{little}} \)

How far above zero is the whole pattern?

\[ \sum m(\nu_i) < 0.23 \text{ eV} \]

All \( i \)

Cosmology, under certain assumptions

Tritium beta decay

Oscillation

\[ \sqrt{0.69 m^2(\nu_1) + 0.29 m^2(\nu_2) + 0.02 m^2(\nu_3)} < 2 \text{ eV} \]

Neutrino masses are tiny

\[ \text{Mass}[\text{Heaviest } \nu_i] > \sqrt{\Delta m^2_{\text{big}}} > 0.04 \text{ eV} \]
Leptonic Mixing

A neutrino is often detected indirectly via —

This is what is detected.

\[ e \text{ or } \mu \text{ or } \nu_e \]

Recoil

Target

So, we speak of neutrino “flavor” states \( \nu_e, \nu_\mu, \) and \( \nu_\tau \)
that couple, respectively, to \( e, \mu, \) and \( \tau. \)

As far as we know, each neutrino flavor state couples only to the charged lepton mass eigenstate of the same flavor.
\( \nu_e, \nu_\mu, \) and \( \nu_\tau \) are not the neutrino mass eigenstates but superpositions of the mass eigenstates:

\[
|\nu_\alpha> = \sum_i U^*_{\alpha i} |\nu_i>
\]

Neutrino of flavor \( \alpha = e, \mu, \) or \( \tau \)

Unitary Leptonic Mixing Matrix

The leptonic mixing matrix

\[
U = \begin{pmatrix}
\nu_1 & \nu_2 & \nu_3 \\
\nu_e & U_{e1} & U_{e2} & U_{e3} \\
\nu_\mu & U_{\mu1} & U_{\mu2} & U_{\mu3} \\
\nu_\tau & U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\]
**Leptonic mixing matrix**

\[
U = \begin{pmatrix}
    e & U_{e1} & U_{e2} & U_{e3} \\
    \mu & U_{\mu1} & U_{\mu2} & U_{\mu3} \\
    \tau & U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\]

**Quark mixing matrix**

\[
V = \begin{pmatrix}
    d & s & b \\
    u & V_{ud} & V_{us} & V_{ub} \\
    c & V_{cd} & V_{cs} & V_{cb} \\
    t & V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

They play exactly the same role.
The Quark and Lepton Mixing Matrices

Quark mixing matrix =

Lepton mixing matrix =
The Extra Leptonic CP Phases

Assuming that $V_{\text{quark}}$ and $U_{\text{lepton}}$ are 3 x 3 and unitary, $V_{\text{quark}}$ can contain only 1 CP-violating phase factor, but $U_{\text{lepton}}$ may possibly contain 3.

The extra leptonic phases are physical only if *neutrinos are their own antiparticles*.

The quarks are definitely *not* their own antiparticles, so there are no extra phases in quark mixing.
The Lepton Mixing Matrix $U$

\[
U = \begin{bmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{bmatrix} \times \begin{bmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{bmatrix} \times \begin{bmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[c_{ij} \equiv \cos \theta_{ij}\]
\[s_{ij} \equiv \sin \theta_{ij}\]

**Note big mixing!**

$\theta_{12} \approx 33^\circ$, $\theta_{23} \approx 40\text{-}52^\circ$, $\theta_{13} \approx 8\text{-}9^\circ$ ← Not very small!

The phases violate CP. $\delta$ would lead to $P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$.

But note the crucial role of $s_{13} \equiv \sin \theta_{13}$.

We know essentially nothing about the phases. Only hints.
Important Notice

To determine oscillation probabilities from measured event rates, one must know the un-oscillated fluxes and the interaction cross sections.
Are There Sterile Neutrinos?
Sterile Neutrino
One that does not couple to the SM W or Z boson

A “sterile” neutrino may well couple to some non-SM particles. These particles could perhaps be found at LHC or elsewhere.
Some Hints — First LSND

The LSND experiment at Los Alamos reported a rapid $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation at $L(km)/E(GeV) \sim 1$.

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \sin^2 2\theta \sin^2 \left[1.27\Delta m^2(eV^2) \frac{L(km)}{E(GeV)}\right] \sim 0.26\%$$

From $\mu^+$ decay at rest; $E \sim 30$ MeV

$\sim 1$ eV$^2$ in contrast to

$\Delta m^2_{32} = 2.4 \times 10^{-3}$ eV$^2$
$\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV$^2$

At least 4 mass eigenstates

{from measured $\Gamma(Z \rightarrow \nu \bar{\nu})$} At least 1 sterile neutrino
The Hint From MiniBooNE

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

$78.4 \pm 28.5$ excess events

$\nu_\mu \rightarrow \nu_e$

$162.0 \pm 47.8$ excess events
ICARUS and OPERA, at $L/E \approx 35$ km/GeV, have not seen $\nu_\mu \rightarrow \nu_e$. This disfavors somewhat a $\nu_\mu \rightarrow \nu_e$ interpretation of the low-energy MiniBooNE $\nu_e$ excess.

ICARUS exclusion
Looking to the Future

Open Questions
• What is the absolute scale of neutrino mass?

• Is the physics behind the masses of neutrinos different from that behind the masses of all other known particles?
  • Are neutrinos their own antiparticles?

• Is the spectrum like \( \equiv \) or \( \equiv \) ?
• Do neutrino interactions violate CP?
  Is $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?

• Is CP violation involving neutrinos the key to understanding the matter–antimatter asymmetry of the universe?

• What can neutrinos and the universe tell us about one another?
• Are there more than 3 mass eigenstates?
  • Are there non-weakly-interacting “sterile” neutrinos?

• Do neutrinos break the rules?
  • Non-Standard-Model interactions?
  • Violation of Lorentz invariance?
  • Violation of CPT invariance?
  • Departures from quantum mechanics?
Are Neutrino Masses Different?
Perhaps, neutrino masses have the same source as the quark and charged lepton masses:

**The Standard Model (SM) Brout – Englert – Higgs mechanism for fermion masses.**

Coupling constant

\[ \mathcal{L}_{SM} = y H^0 \nu_L \nu_R \Rightarrow y \langle H^0 \rangle_0 \nu_L \nu_R \equiv m_\nu \nu_L \nu_R \]

SM Higgs field

Vacuum expectation value

\[ \langle H^0 \rangle_0 \equiv \nu = 174 \text{ GeV} \], so \[ y = \frac{m_\nu}{\nu} \sim \frac{0.1 \text{ eV}}{174 \text{ GeV}} \sim 10^{-12} \]

A coupling constant this much smaller than unity leaves many theorists skeptical.
Majorana masses and the See-Saw picture

The See-Saw model is the most popular theory of why neutrinos are so light.

The straightforward (type-I) See-Saw model adds to the SM 3 heavy neutrinos $N_i$, with —

$$
\mathcal{L}_{\text{new}} = -\frac{1}{2} \sum_i m_{N_i} N_i^2_i + \sum_{\alpha=e,\mu,\tau} y_{\alpha i} \left[ \bar{\nu}_{\alpha L} H^0 - \bar{\ell}_{\alpha L} H^- \right] N_{iR} + h.c.
$$

**Large Majorana masses**

**Yukawa coupling matrix**

**SM lepton doublet**

**SM Higgs doublet**

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[An alternative possibility —]

The See-Saw model is the most popular theory of why neutrinos are so light.
In this picture, there is still a coupling of the neutrinos to the SM Higgs field.

In addition, there is a new ingredient: large Majorana masses, whose origin is unknown physics.

Majorana masses cannot come from the standard, linear Yukawa coupling of neutrinos to the SM Higgs field.
Majorana mass terms have the effect —

\[ \nu \times \bar{\nu} \rightarrow \text{Mass} \]

(Or the reverse)

Because they mix neutrino and antineutrino, they do not conserve \( L \equiv #(\text{Leptons}) - #(\text{Antileptons}) \).

There is then no conserved quantum number to distinguish antineutrinos from neutrinos.

**Consequence:** The neutrino mass eigenstates \( \nu_1, \nu_2, \nu_3 \) are their own antiparticles.

\[ \bar{\nu}_i = \nu_i \]

**Majorana neutrinos**
- Presence of Majorana masses
- Non-conservation of $L$
- Self-conjugacy of neutrinos ($\bar{\nu} = \nu$)

— are all signature predictions of the See-Saw picture.

All three predictions would be confirmed by the observation of **neutrinoless double beta decay (0νββ)**

![Delta L = 2](image)

$\Delta L = 2$

does not conserve L.
Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a **Majorana mass term**:

(Schechter and Valle)

$$\bar{\nu} \rightarrow \nu : \text{A (tiny) Majorana mass term}$$

$$\therefore 0\nu\beta\beta \rightarrow \bar{\nu}_i = \nu_i$$
The See-Saw picture leads to —

\[ M_\nu \propto \frac{1}{M_N} \]

\{ Yanagida;
Gell-Mann, Ramond, Slansky;
Mohapatra, Senjanovic;
Minkowski \}
What Is the Mass Ordering?
The Mass Spectrum: $\equiv$ or $\equiv$ ?

Generically, grand unified models (GUTS) favor —

$\equiv$

$\equiv$

GUTS relate the Leptons to the Quarks.

However, *Majorana masses*, with no quark analogues, could turn $\equiv$ into $\equiv$.

The mass ordering is a prime neutrino-mass model discriminator. (Winter at Neutrino 2014)
If the mass spectrum is inverted, and neutrinos have Majorana masses, there is a lower bound on the rate for neutrinoless double beta decay.
Summary

Neutrino oscillation has proved that neutrinos have nonzero masses, and that leptons mix.

The $\nu$ masses may have a quite different origin than the quark and charged lepton masses.

Leptonic mixing is like quark mixing, but with interesting differences.

Surprises may well be coming.