$b \to s$ decays using lattice QCD

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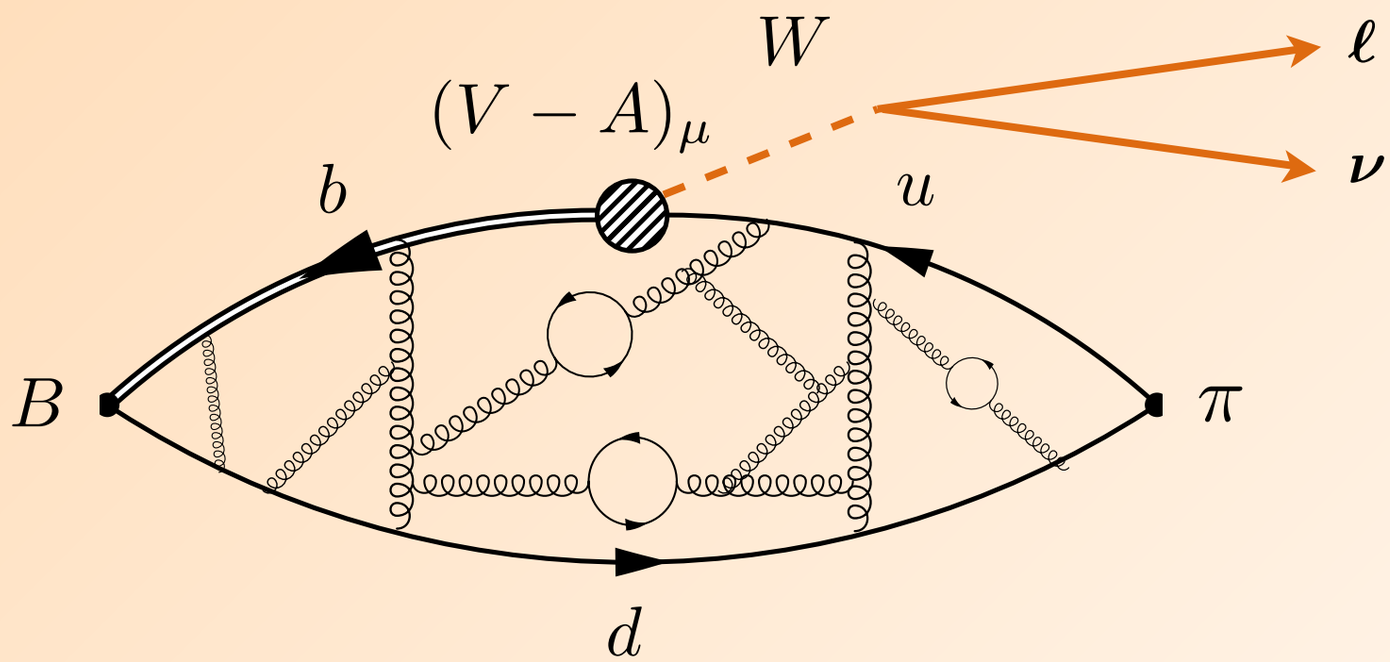
Outline

- **Introduction**
  - Hadronic matrix elements of weak (& BSM) currents
  - Lattice QCD
  - $b \to s$ operators
- LQCD calculation of $B_{(s)} \to K^{(*)}$, $B_s \to \varphi$, $\Lambda_b \to \Lambda$ **form factors**
  - Form factor definitions (for reference)
  - Results
- Consequences for $b \to s\, l^+ l^-$ **observables**
- **Summary**
- **References**

Introduction

Semileptonic decay of hadrons

e.g. $B \to \pi\, l\, \nu$



Observables, such as differential branching fraction, depend in part on hadronic matrix elements of weak (or BSM) operators, e.g.

$$\langle \pi(k)|V_\mu|B(p)\rangle$$

Path integral expression (in imaginary time)

$$\langle \Phi_\pi(z) V_\mu(y) \Phi_B(x)\rangle = \frac{1}{Z}\int [d\psi][d\bar{\psi}][dU]\, \Phi_\pi(z) V_\mu(y) \Phi_B(x)\, e^{-S[\psi,\bar{\psi},U]}$$

Lattice QCD & numerical field theory

$$\langle \Phi_\pi(z) V_\mu(y) \Phi_B(x) \rangle = \frac{1}{Z} \int [d\psi][d\bar{\psi}][dU]\, \Phi_\pi(z) V_\mu(y) \Phi_B(x)\, e^{-S[\psi,\bar{\psi},U]}$$

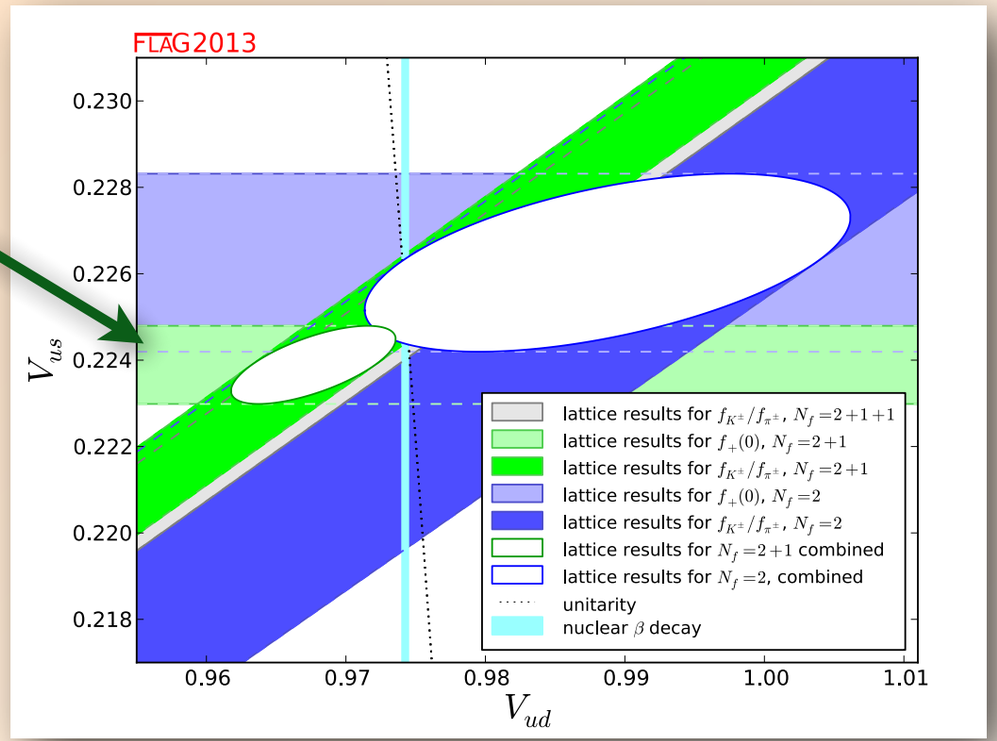
- ✤ Imaginary time formulation: path integrands real, non-negative
- ✤ Discrete lattice points: regulates field theory
- ✤ Sharply peaked path integrand: permits importance sampling

Systematic error	***Controllable limit***	***Theory***
Lattice volume	$L \gg 1/m_\pi$	Chiral pert. th. Brute force
Lattice spacing	$a \ll 1/\Lambda_{\rm QCD}$	Symanzik EFT
Light quark mass	$m_\pi \ll m_\rho, 4\pi f_\pi$	Chiral pert. th. Brute force
Heavy quark mass	$m_Q \gg 1/a$	NRQCD, HQET
	$m_Q < 1/a$	Extra-fine, extra-improvement
	$m_Q \approx 1/a$	Fermilab

CKM m.e. from exclusive decays & LQCD

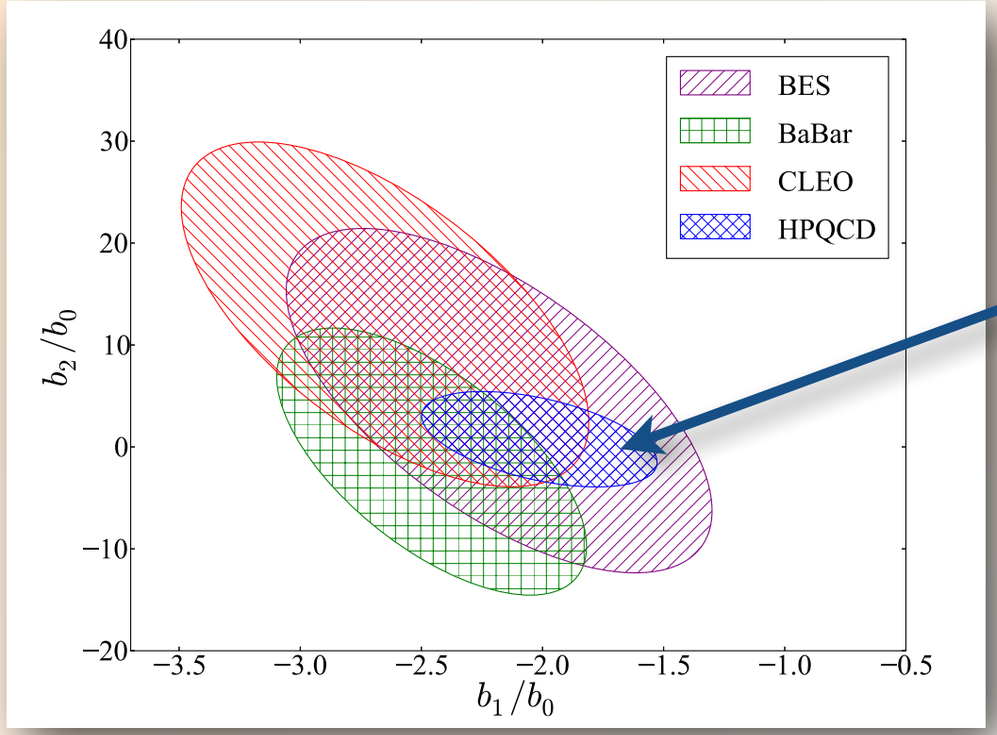
- $|V_{us}|$ from $K \to \pi l \nu$, $l \nu$
- $|V_{cd}|$ from $D \to \pi l \nu$
- $|V_{cs}|$ from $D \to K l \nu$
- $|V_{ub}|$ from $B \to \pi l \nu$
- $|V_{cb}|$ from $B \to D^{(*)} l \nu$

LQCD + expt determine $|V_{us}|$ to < 1%



FLAG, arXiv:1310.8555v2

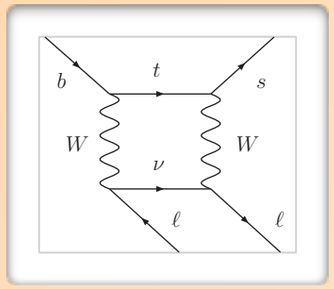
$D \to K l \nu$ f.f. shape parameters from expt vs. LQCD



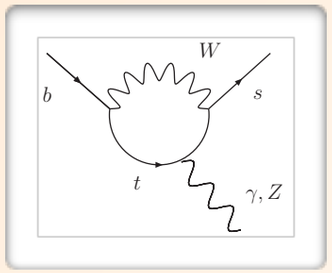
HPQCD, arXiv:1305.1462

Low energy description of $b \to s$ decays

$$\mathcal{H}_{\rm eff}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i (C_i \mathcal{O}_i + C_i' \mathcal{O}_i')$$



In the Standard Model, i = 1, ..., 10, S, P with known Wilson coefficients C_i. Beyond SM, chirality-flipped operators are allowed and the $C_i^{(\prime)}$ depend on the model of new physics



Most important short-distance effects in $b \to sll$ come from 2-quark operators:

$$\mathcal{O}_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \, \bar{\ell}\gamma_\mu \ell \qquad \mathcal{O}_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \, \bar{\ell}\gamma_\mu\gamma_5 \ell$$

$$\mathcal{O}_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b \, F_{\mu\nu}$$

Charmonium resonance effects arise from 4-quark operators:

$$\mathcal{O}_1 = \bar{c}^\alpha \gamma^\mu P_L b^\beta \, \bar{s}^\beta \gamma^\mu P_L c^\alpha \qquad \mathcal{O}_2 = \bar{c}^\alpha \gamma^\mu P_L b^\alpha \, \bar{s}^\beta \gamma^\mu P_L c^\beta$$

$$\mathcal{O}_{3..6} = \bar{s}^\alpha \gamma^\mu P_L b^{\alpha,\beta} \sum_q \bar{q}^\beta \gamma^\mu P_{L,R} q^{\beta,\alpha}$$

Form factors

$B \to K$ form factors

$$\langle K(k)|\bar{s}\gamma^\mu b|B(p)\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_K^2}{q^2}\, q^\mu\right] f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2}\, q^\mu f_0(q^2)$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu} q_\nu b|B(p)\rangle = \frac{i f_T(q^2)}{m_B + m_K}\left[q^2(p+k)^\mu - (m_B^2 - m_K^2) q^\mu\right]$$

- ✤ “Gold-plated” matrix elements: QCD-stable $|i\rangle$ and $|f\rangle$ states
- ✤ Observables: differential branching fraction $d\Gamma/dq^2$, forward/backward asymmetry A_{FB} (zero in SM), and “flat term” F_H

$B \to V$ form factors

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma$$

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}^5 b|B(p)\rangle = 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right)$$
$$-A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+k)^\mu - \frac{m_B^2-m_V^2}{q^2}q^\mu\right)$$

$$q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau k^\sigma$$

$$-q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle = iT_2(q^2)[\varepsilon^*_\mu(m_B^2-m_V^2) - (\varepsilon^*\cdot q)(p+k)_\mu]$$
$$+iT_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]$$

$$A_{12}(q^2) = \frac{(m_B+m_V)^2(m_B^2-m_V^2-q^2)A_1(q^2) - \lambda A_2(q^2)}{16m_B m_V^2(m_B+m_V)}$$

$$T_{23}(q^2) = \frac{m_B+m_V}{8m_B m_V^2}\left[(m_B^2+3m_V^2-q^2)\,T_2(q^2) - \frac{\lambda T_3(q^2)}{m_B^2-m_V^2}\right]$$

with $\lambda = (t_+ - t)(t_- - t) \qquad t = q^2 \qquad t_\pm = (m_B \pm m_V)^2$

$\Lambda_b \to \Lambda$ form factors

In general

$$\langle \Lambda | \bar{s}\gamma^\mu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^V \gamma^\mu - f_2^V \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^V \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s}\gamma^\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^A \gamma^\mu - f_2^A \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^A \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TV} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TV} \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TA} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TA} \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

In the $m_b \to \infty$ limit

$$\langle \Lambda(p', s') | \, \bar{s}\Gamma Q \, | \Lambda_Q(v, 0, s) \rangle = \bar{u}(p', s') [F_1(p' \cdot v) + \not{v} F_2(p' \cdot v)] \Gamma \mathcal{U}(v, s)$$

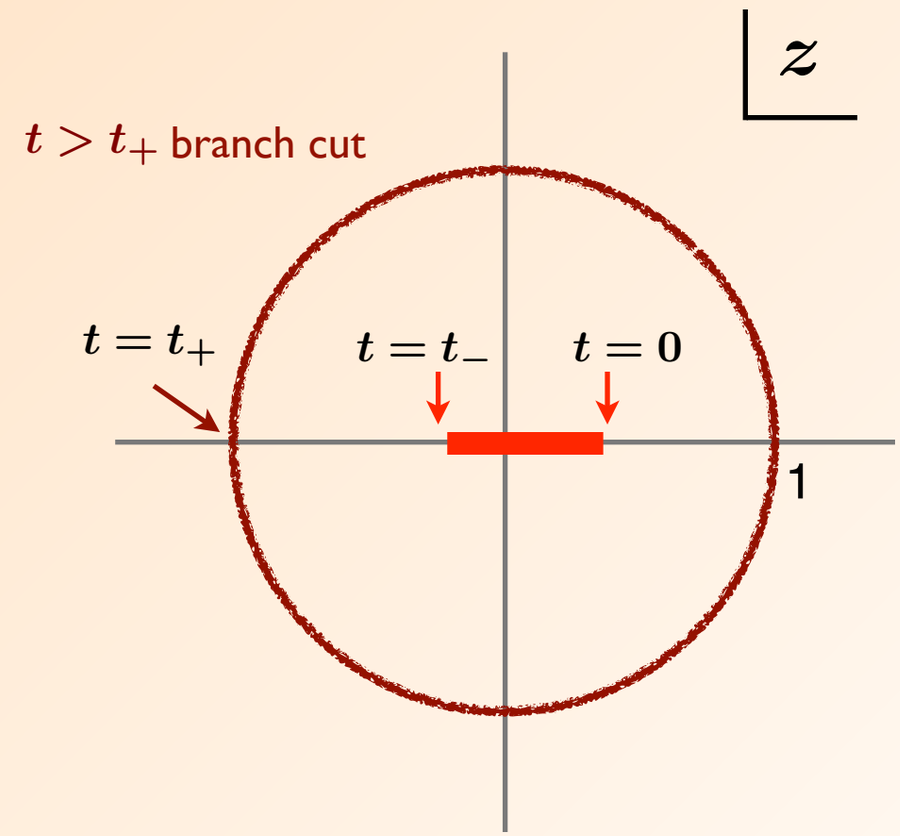
Form factor shape

Series (z) expansion

$$t = q^2 \qquad t_\pm = (m_B \pm m_F)^2$$

Choose, *e.g.* $t_0 = 12\ \mathrm{GeV}^2$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

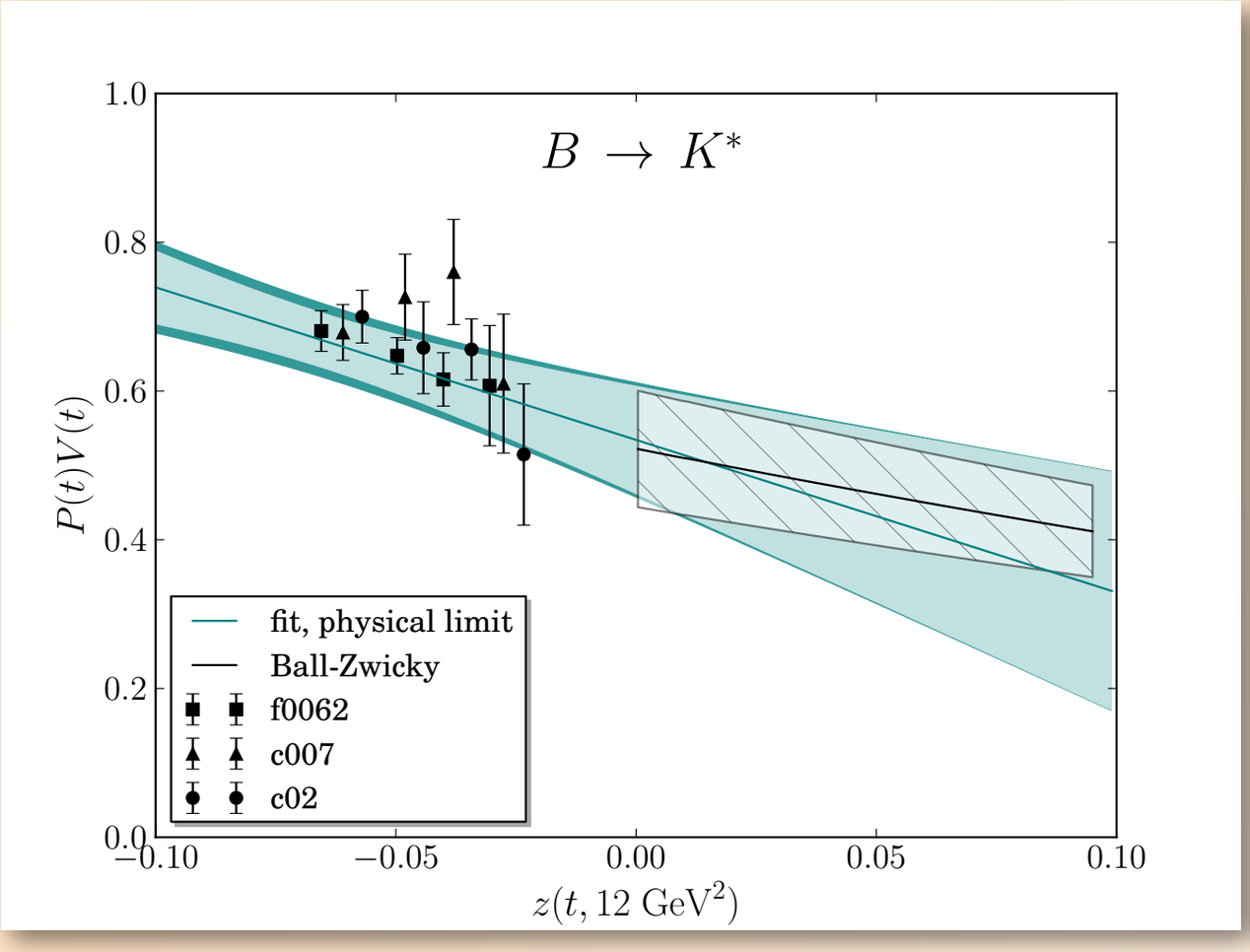


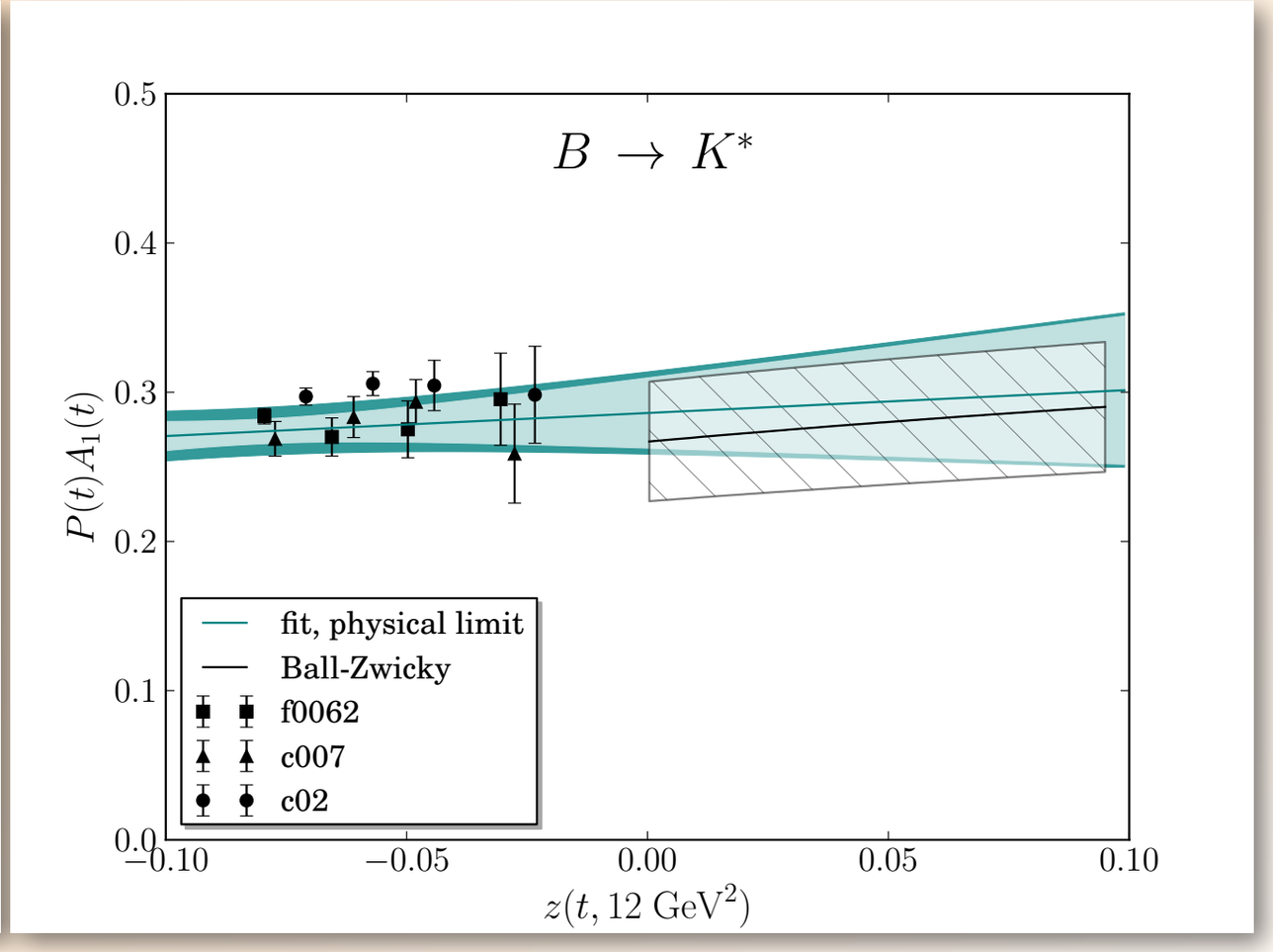
Simplified series expansion

$$F(t) = \frac{1}{1 - t/m_{\mathrm{res}}^2} \sum_n a_n z^n$$

Bourrely, Caprini, Lellouch PRD **79** (2009) following Okubo; Bourrely, Machet, de Rafael; Boyd, Grinstein, Lebed; Boyd & Savage; Arneson *et al.;* FNAL/MILC lattice collab; ...

$B \to K^*$ form factors

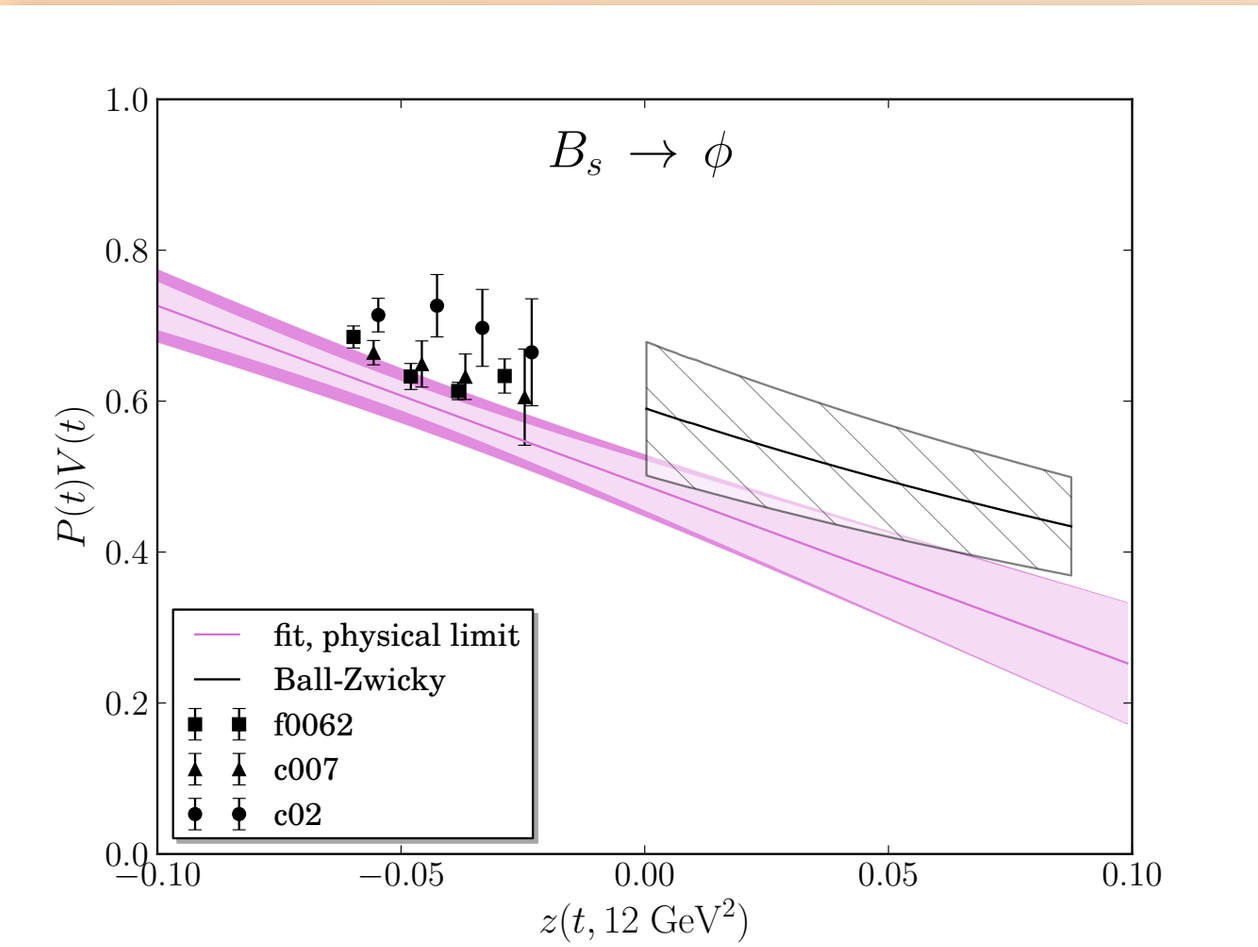


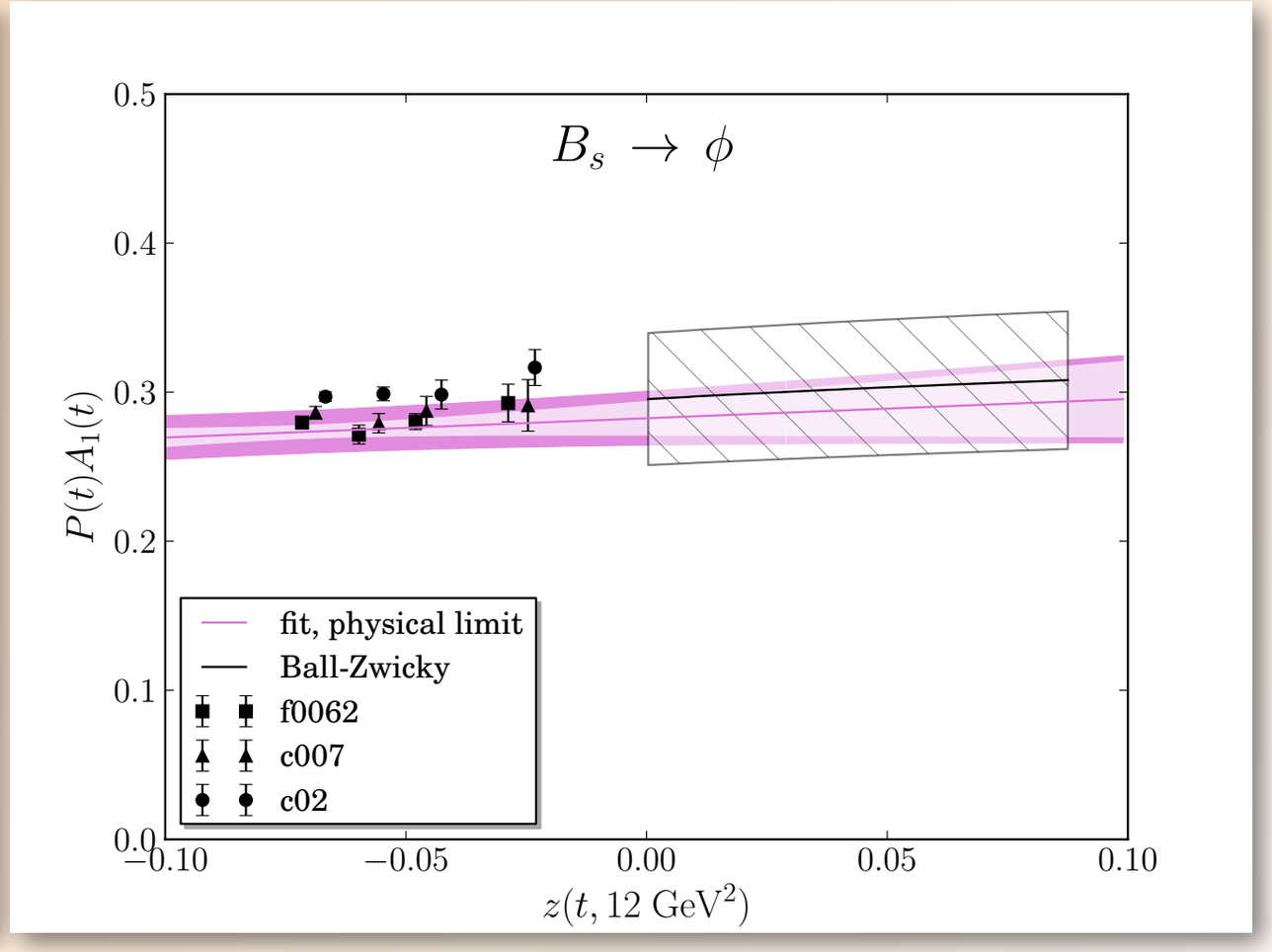


using NRQCD+asqtad valence on MILC n_f=2+1 asqtad

Horgan, Liu, Meinel, Wingate, arXiv:1310.3722

$B_s \to \varphi$ form factors





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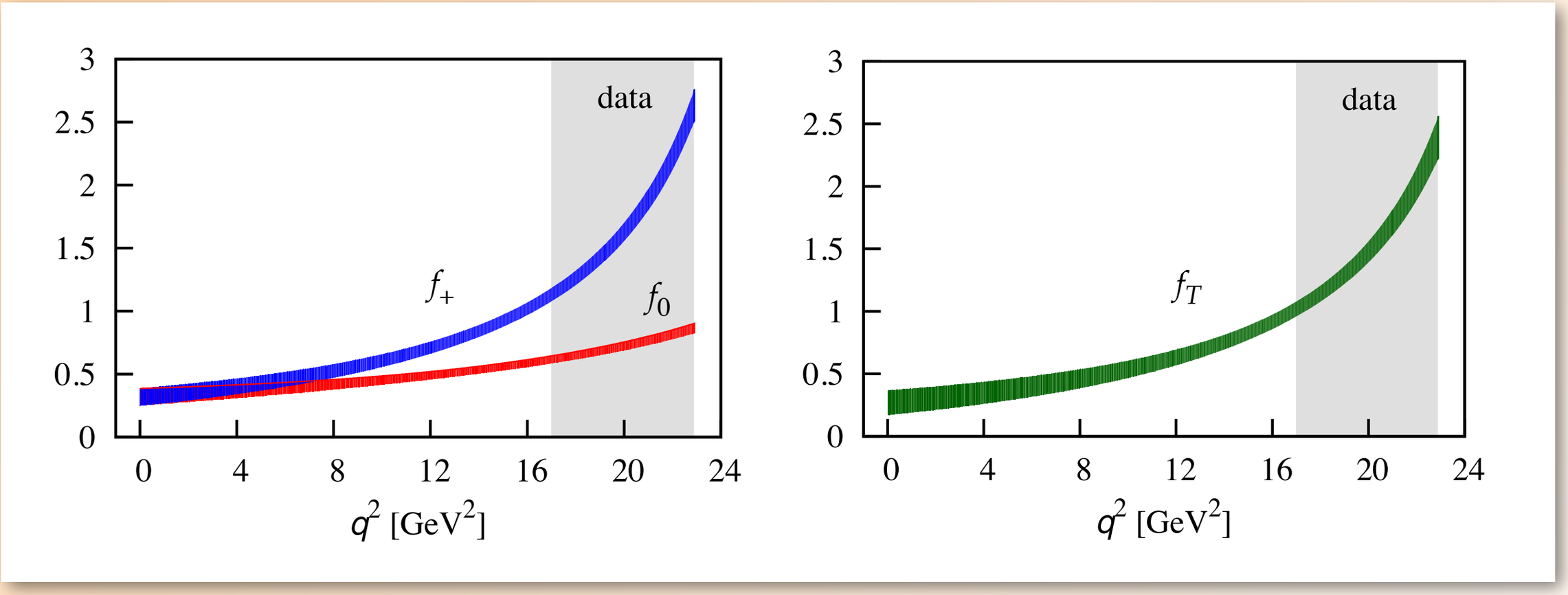
$B \to K$ form factors



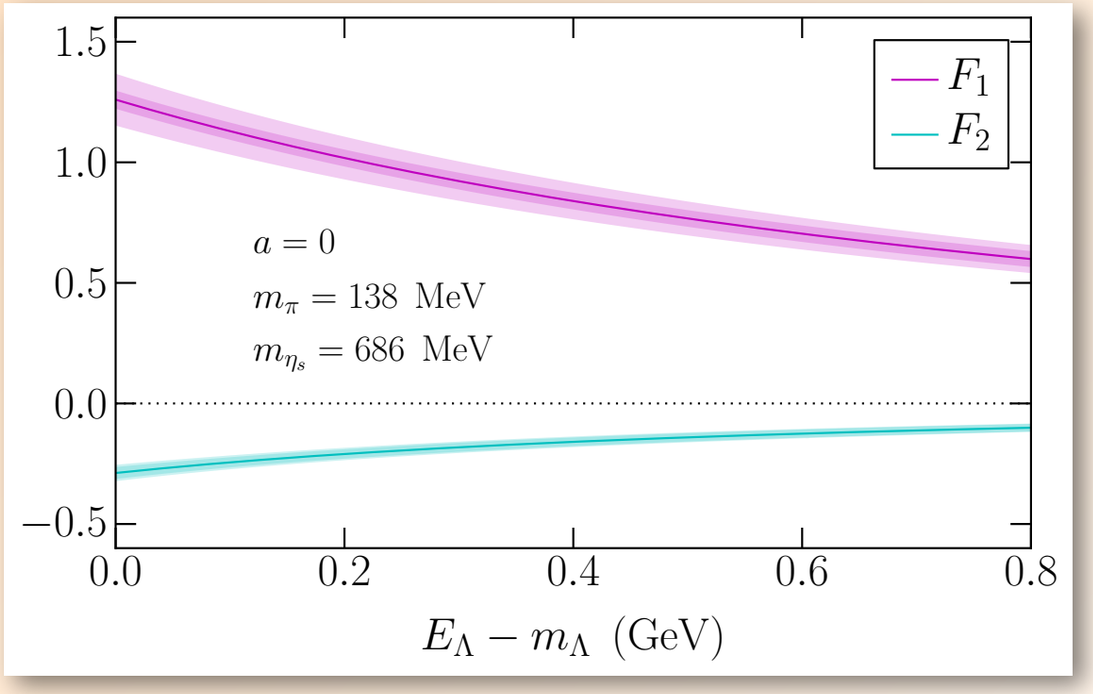
HPQCD Collaboration
(using NRQCD+HISQ valence on MILC n_f=2+1 asqtad)

C. Bouchard *et al.*, arXiv:1306.0434, arXiv:1306:2384

$\Lambda_b \to \Lambda$ form factors

Static limit, $m_b \to \infty$

(using Static+DWF on n_f=2+1 RBC-UKQCD)



In the static limit, 10 form factors reduce to 2

$$\langle \Lambda(p',s')|\,\bar{s}\Gamma Q\,|\Lambda_Q(v,0,s)\rangle \;=\; \bar{u}(p',s')\left[F_1(p'\cdot v) + \not{v}F_2(p'\cdot v)\right]\Gamma\,\mathcal{U}(v,s)$$

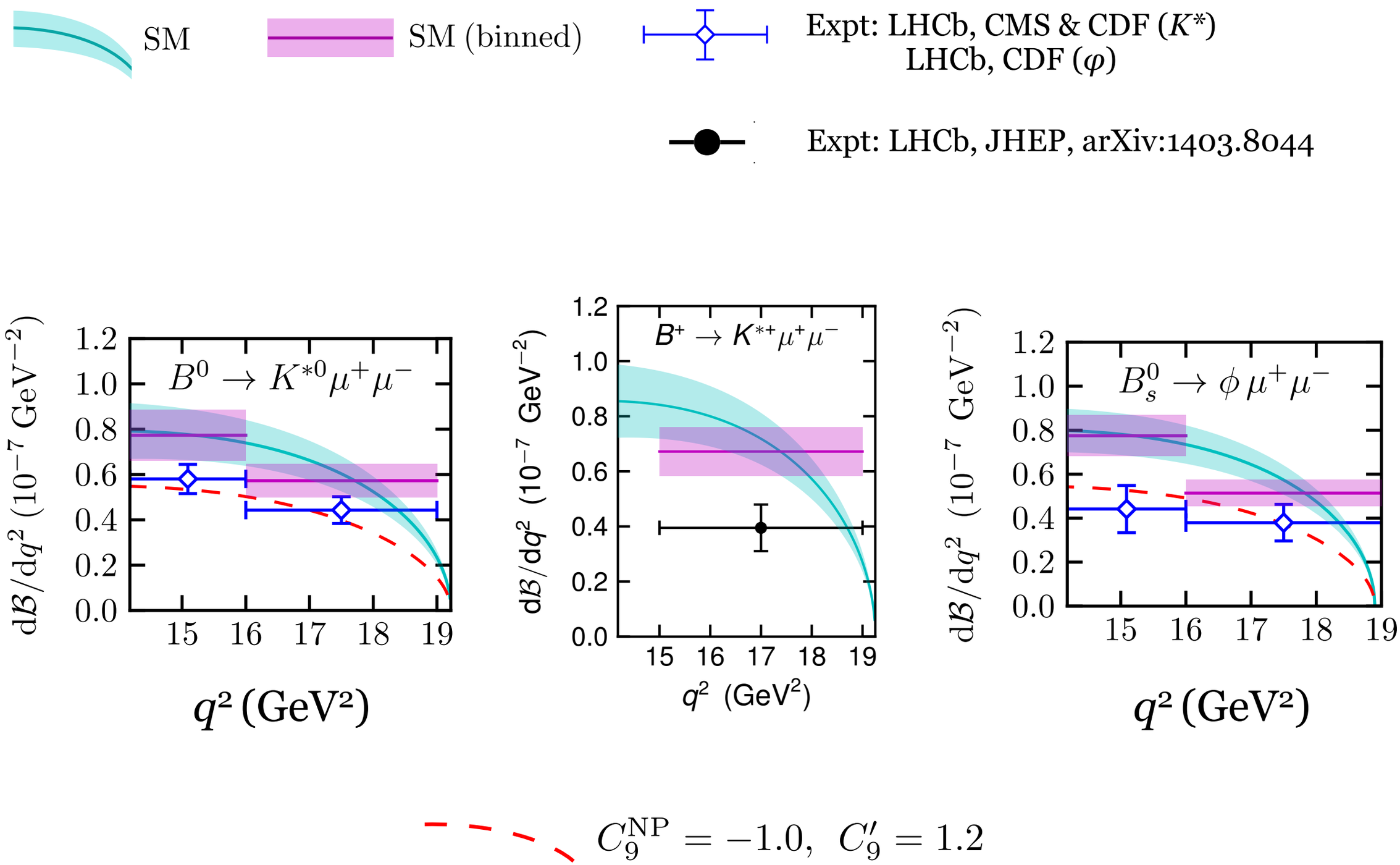
W Detmold, C-J D Lin, S Meinel, M Wingate, Phys Rev D87 (2013)

Physical m_b

Preliminary results in S Meinel, Lattice 2013, arXiv:1301.2685

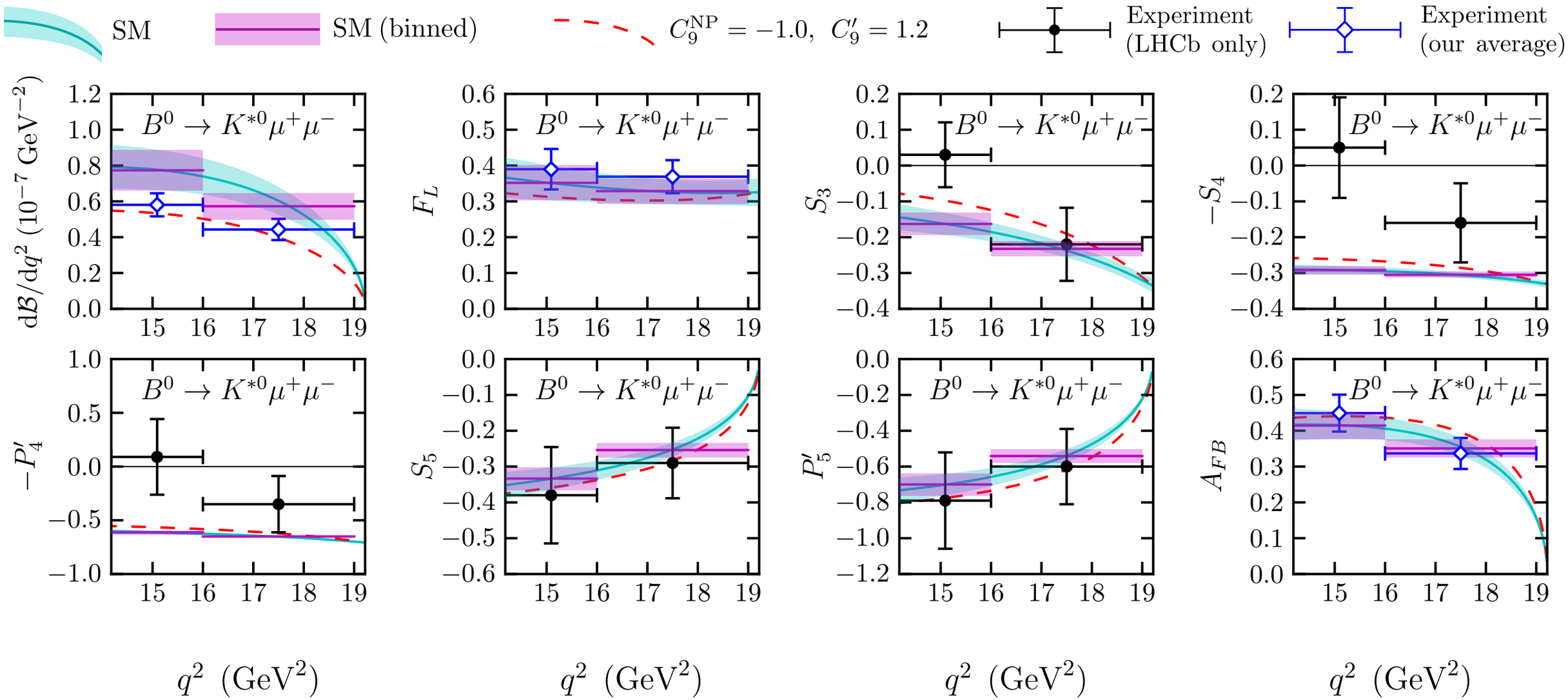
Observables

Branching fractions



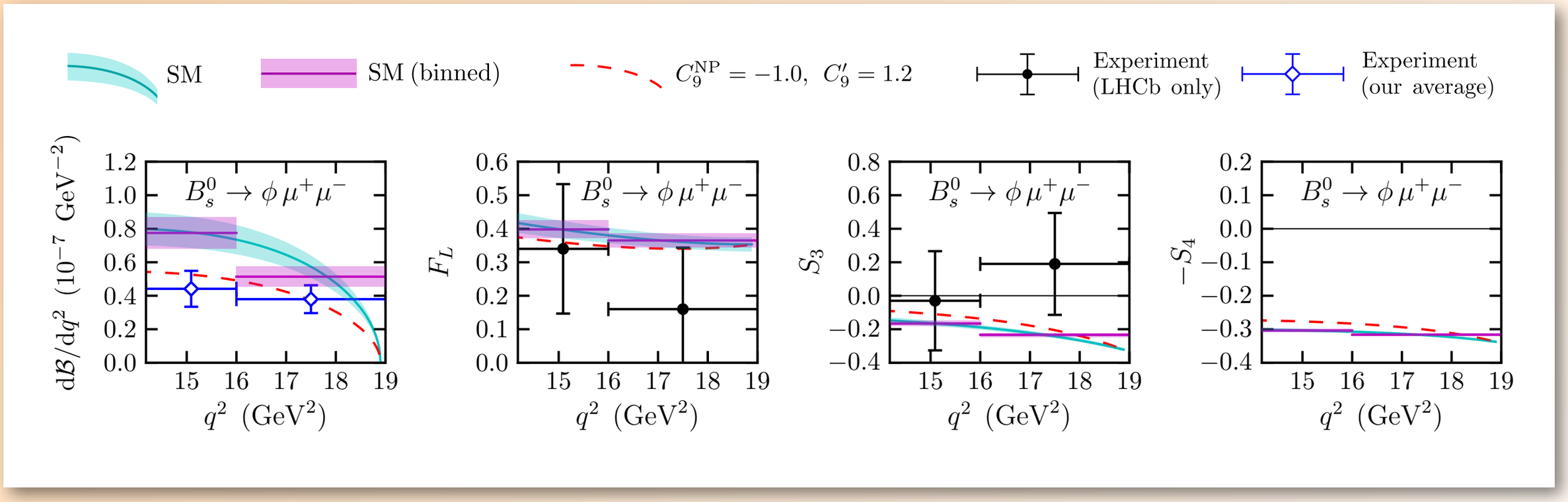
Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887; S Meinel, Paris Workshop 2014

$B \to K^* \mu^+ \mu^-$



Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887

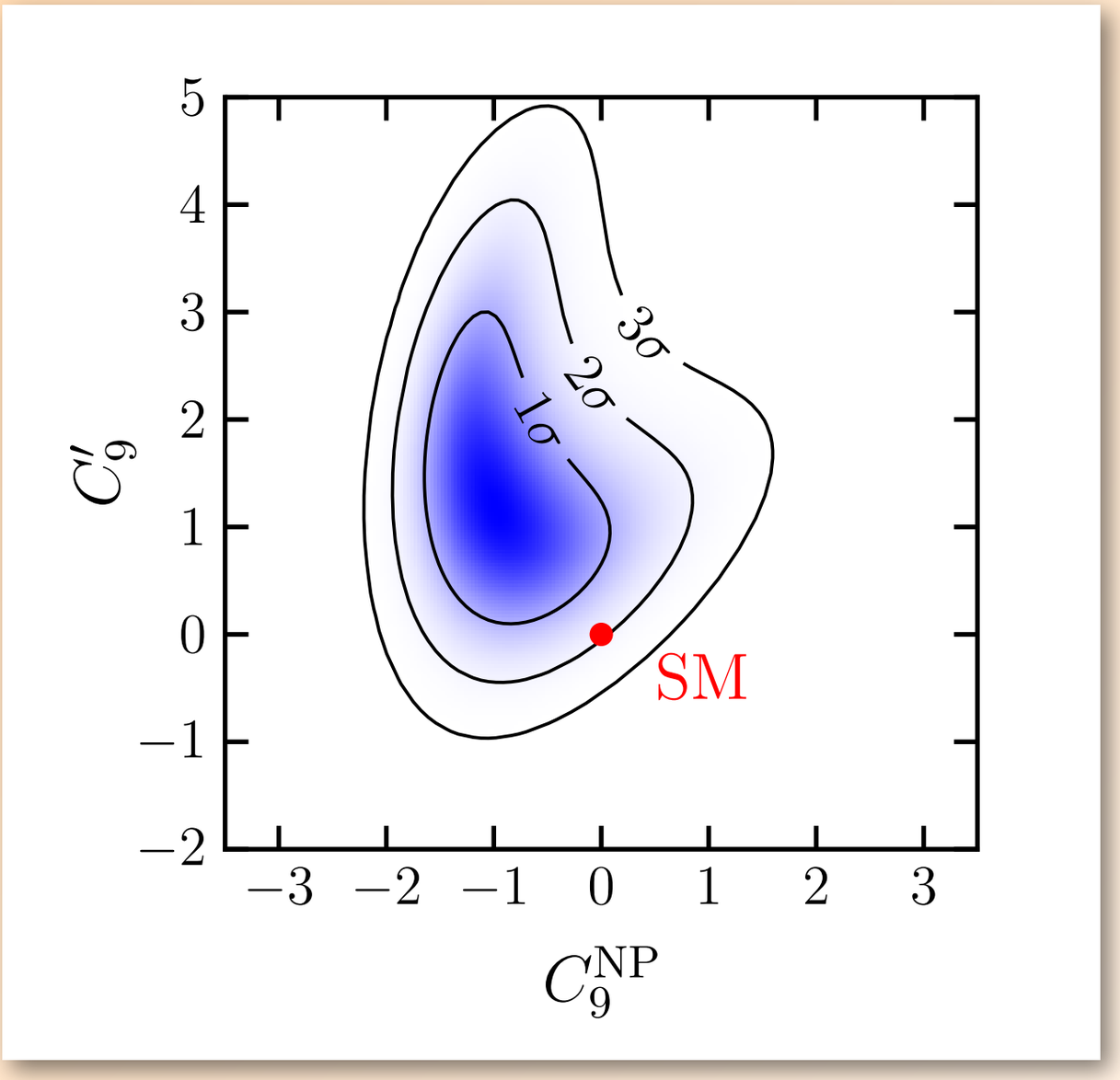
$B_s \to \varphi\, \mu^+ \mu^-$ observables



Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887

Fit to low recoil data

Best fit: $C_9^{\mathrm{NP}} = -1.0 \pm 0.6$ $C_9' = 1.2 \pm 1.0$

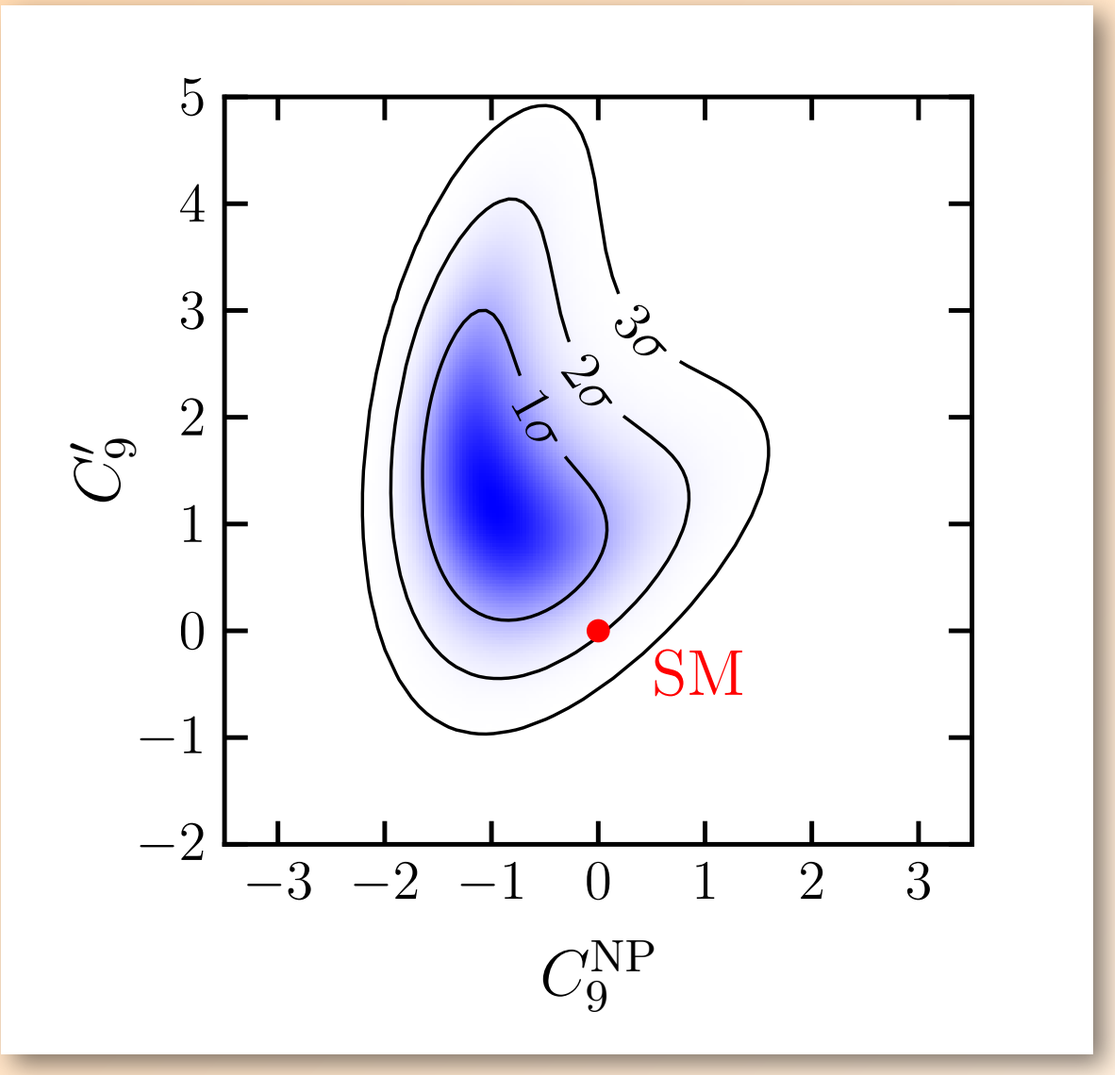


Likelihood function

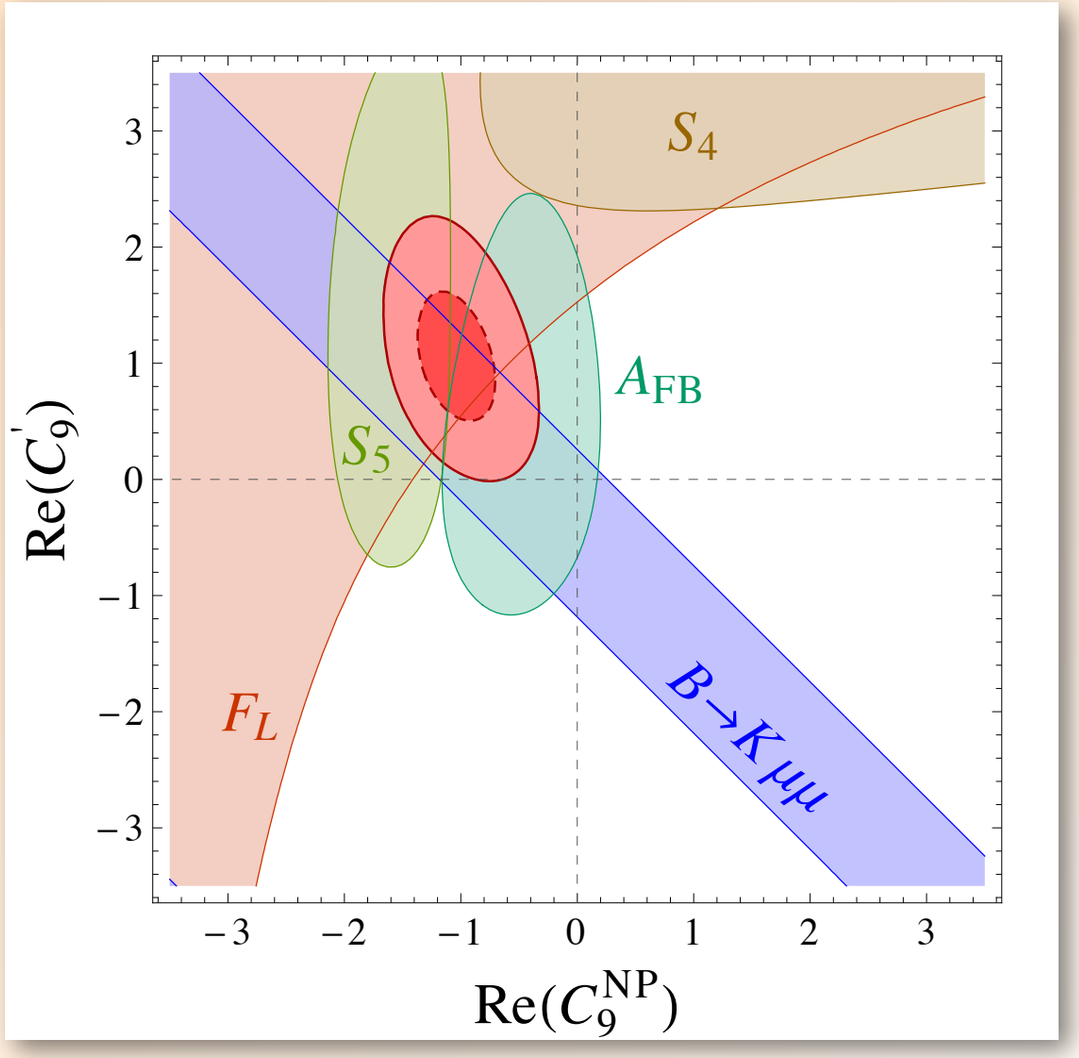
- C_9, C_9' assumed to be real
- Data in 2 highest q^2 bins
  - $B \to K^*\mu\mu$ (neutral mode): dB/dq^2, F_L, S_3, S_4, S_5, A_{FB}
  - $B_s \to \varphi\mu\mu$: dB/dq^2, F_L, S_3
- Theory correlations between observables & bins taken into account

Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887

2 complementary fits



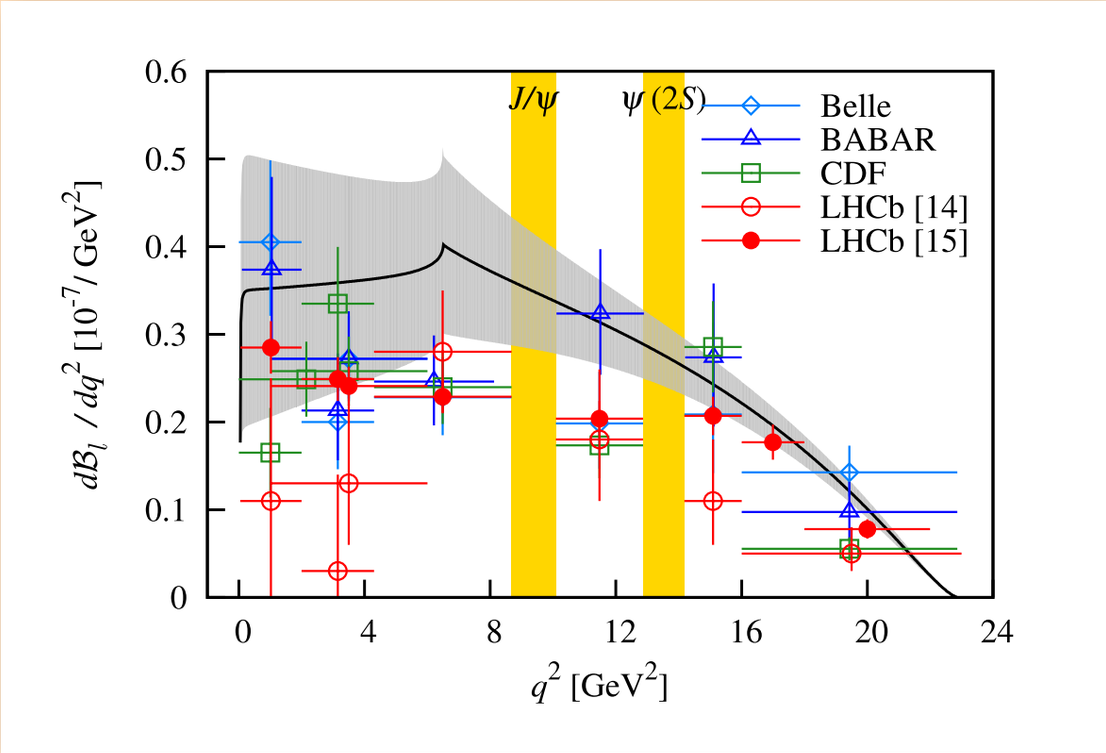
Horgan, Liu, Meinel, Wingate,
arXiv:1310.3887

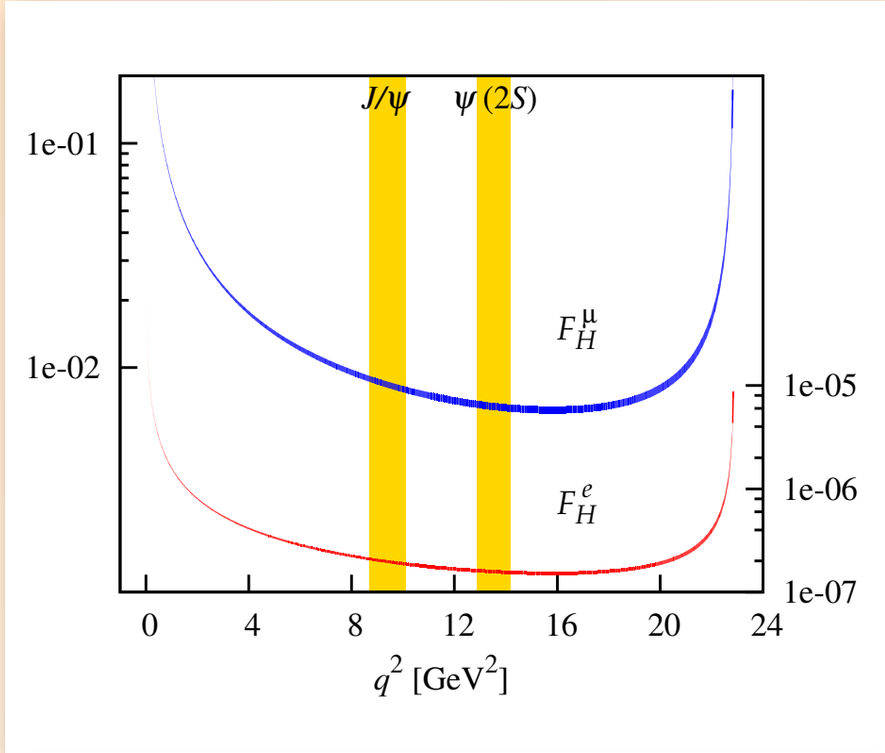


Altmannshofer & Straub,
arXiv:1308.1501

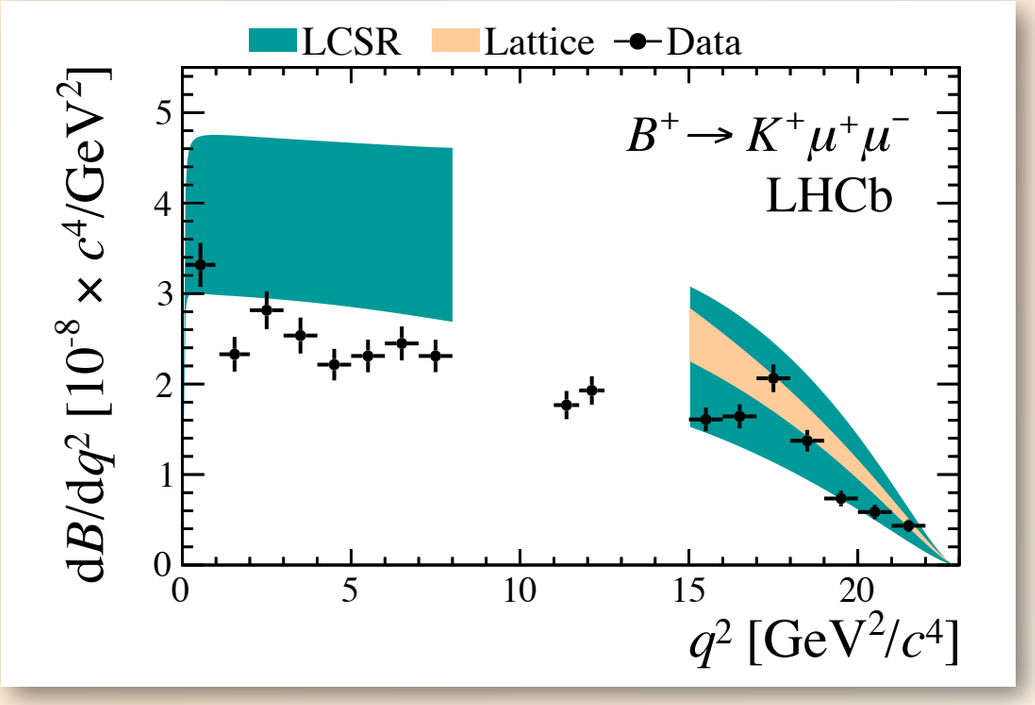
Results compatible with other groups' fits
(D-GMV arXiv:1307.5683; BBvD arXiv:1310.2478)
Except Matias' remarks at Paris workshop

$B \to K\, l^+ l^-$



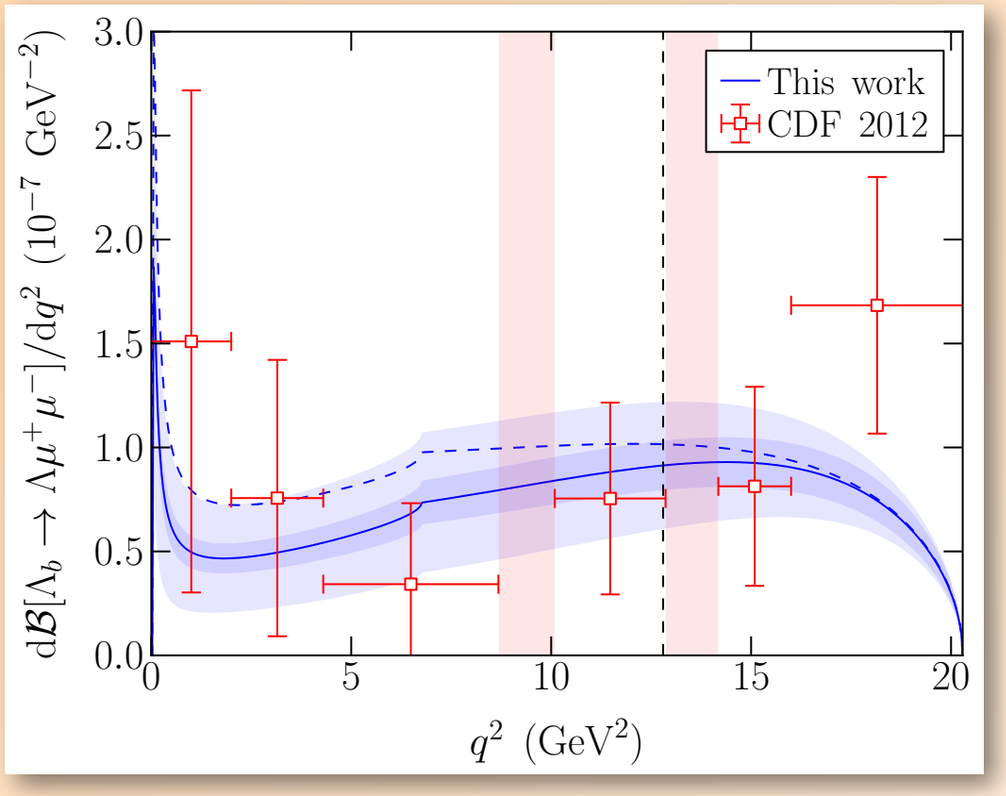


C. Bouchard *et al.*, PRL 111, (E) 112, arXiv:1306.0434v4, PRD 88, arXiv:1306:2384

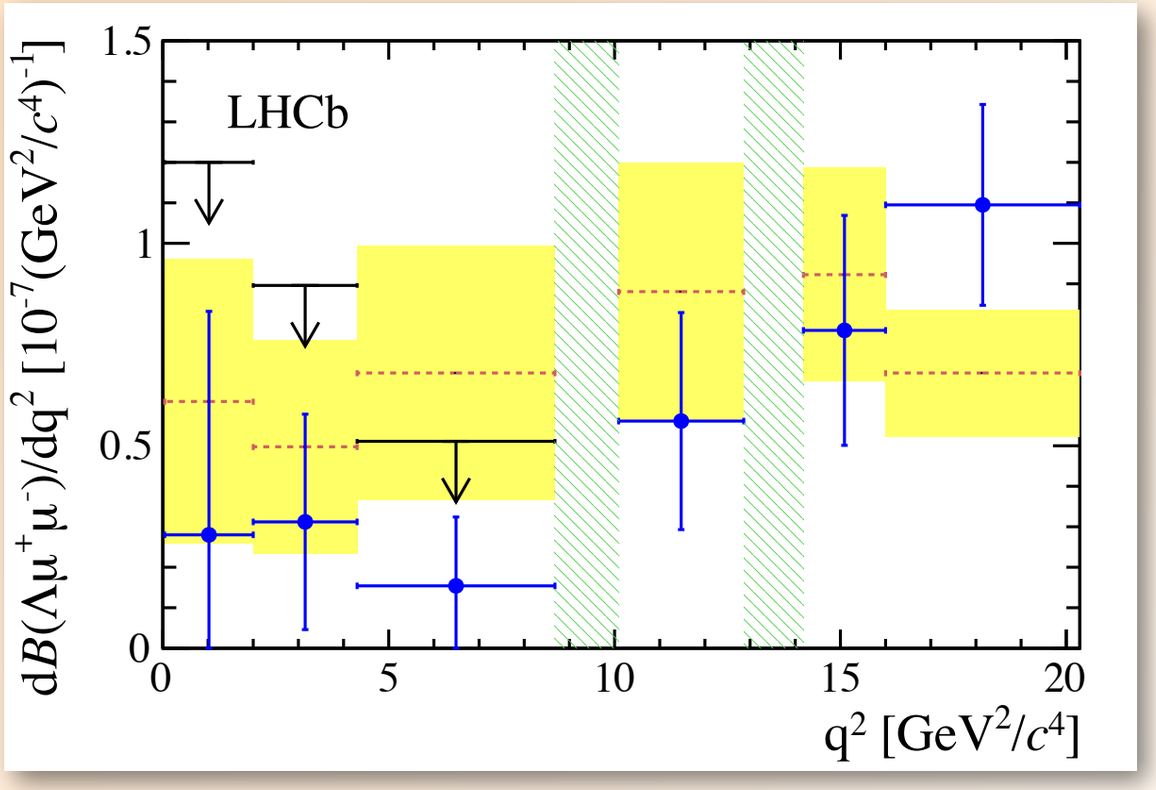


Resonant effect due to $\psi(4160)$
LHCb, PRL 111, arXiv:1307.7595
Fig. from LHCb, JHEP, arXiv:1403.8044

$\Lambda_b \to \Lambda\, l^+ l^-$



CDF: red; LQCD: blue



LHCb: blue; binned LQCD: red/yellow

Dominant theory error due to use of LO HQET (static action)
Improved calculation (w/ RHQ) underway (Meinel, Lattice 2013)

Detmold, Lin, Meinel, Wingate, Phys Rev D87 (2013)
CDF, public note 108xx, v0.1, http://www-cdf.fnal.gov/physics/new/bottom/bottom.html
LHC*b*, R Aaij, Phys. Lett. B 725 (2013) [arXiv:1306.2577]

Long distance effects?

- Grinstein & Pirjol (PRD 70 (2004)): Incorporation of nonlocal effects at large q^2 via OPE. Should hold for low recoil data binned over resonances.
- Lyon & Zwicky (arXiv:1406.0566): Investigate size nonfactorizable contributions. Conclude that under some circumstances, these could explain discrepancies between data and SM predictions

Summary

- ✤ First unquenched LQCD calculations of $b \to s$ form factors
- ✤ These reduce uncertainties in f.f., especially at large q^2
- ✤ $B_{(s)} \to K^*, \varphi$: complement sum rule calculations, many observables, caveat: $K^* \to K\pi$ threshold effects not included
  (ambitious proposal: Briceño, Hansen, Walker-Loud, arXiv:1406.5965)
  - ✦ Expt b.f. *suppressed* relative to SM
  - ✦ Best fit coincides with Altmannshofer-Straub, compatible w/ others
- ✤ $B \to K$: Precise short distance, expt. b.f. seems *suppressed* relative to SM, prominent $\psi(4160)$ resonance
- ✤ $\Lambda_b \to \Lambda$: static limit done, hint of high q^2 *enhancement* in b.f., calculation with physical m_b underway

References

$B \to K^*$
$B_s \to \varphi$
$B_s \to K^*$

R R Horgan, Z Liu, S Meinel, M Wingate
Phys. Rev. D 89, 094501 (2014) [arXiv:1310.3722]
Phys. Rev. Lett. 112, 212003 (2014) [arXiv:1310.3887]

$B \to K$

C Bouchard, G P Lepage, C Monahan, H Na, J Shigemitsu, (HPQCD)
Phys. Rev. Lett. 111, 162002 (2013); (E) PRL 112, 149902 (2014) [arXiv:1306.0434]
Phys. Rev. D 88, 054509 (2013) [arXiv:1306.2384]

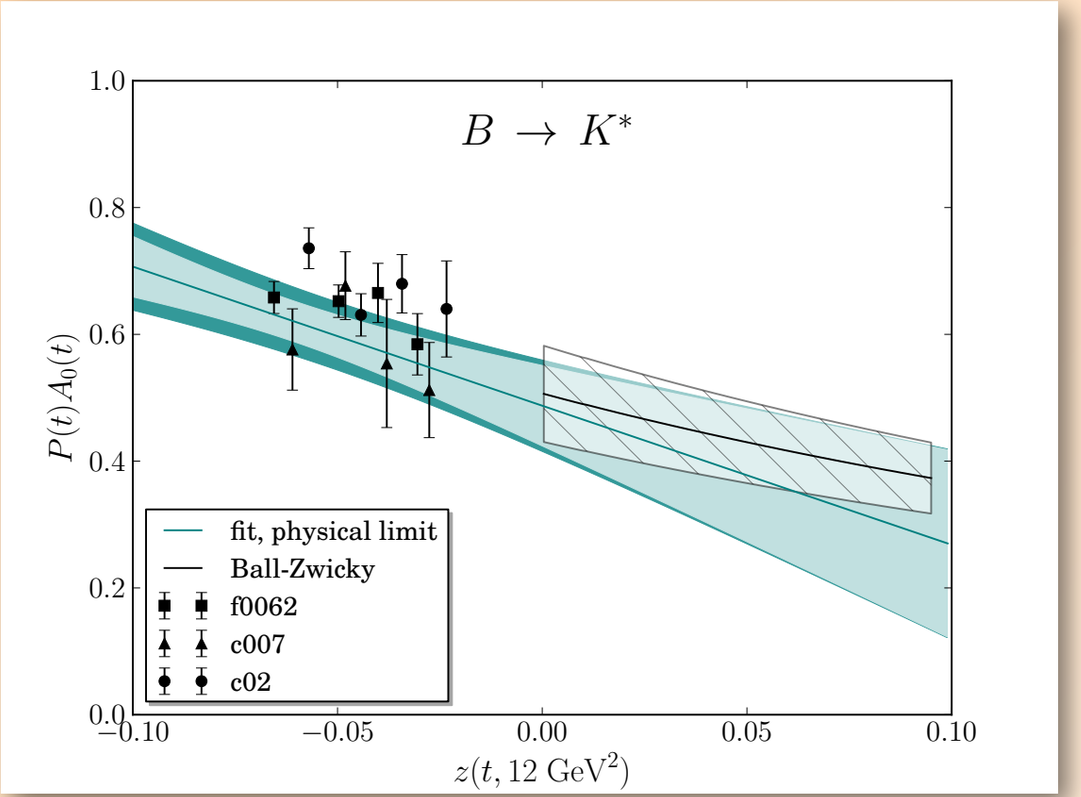
$\Lambda_b \to \Lambda$

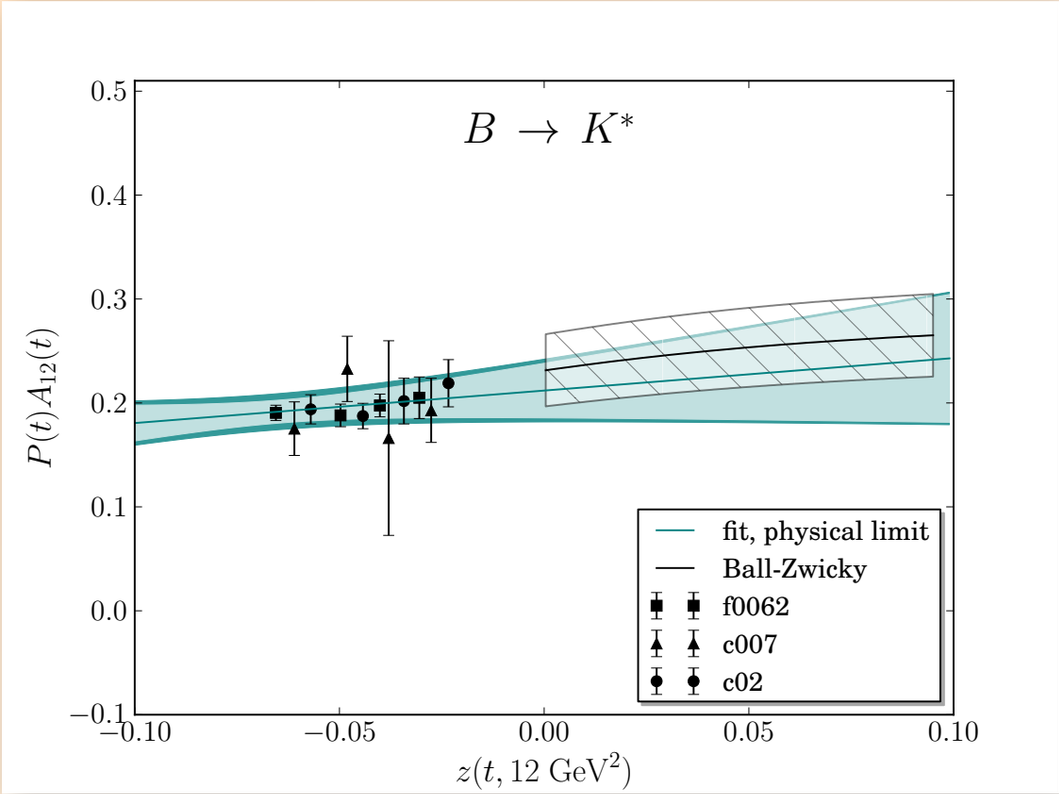
W Detmold, C-J D Lin, S Meinel, M Wingate
Phys. Rev. D 87, 074502 (2013) [arXiv:1212.4827]

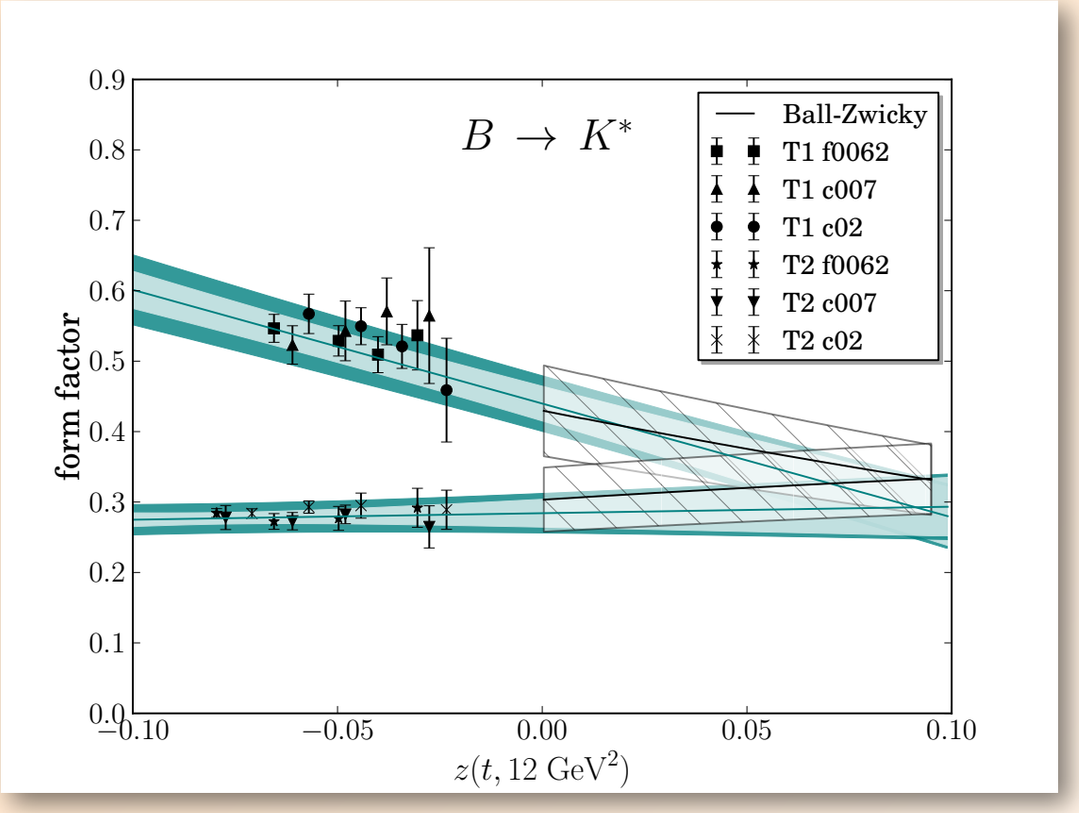
S Meinel
Lattice 2013 [arXiv:1401.2685]

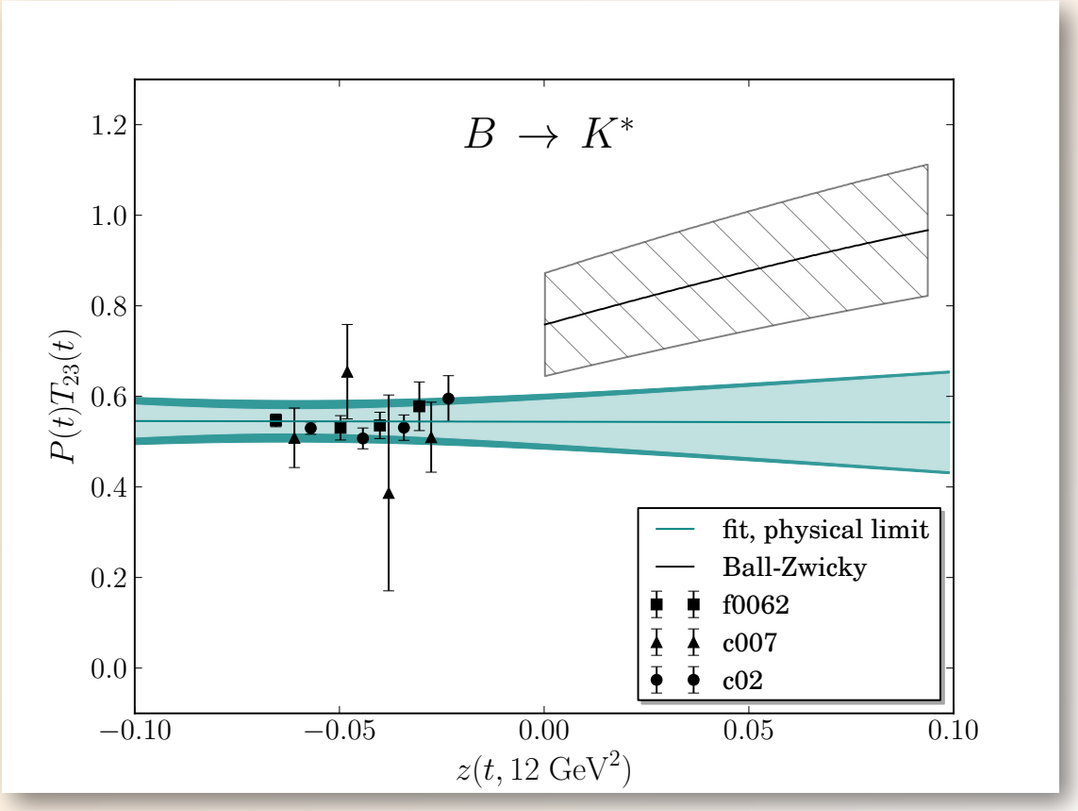
extra slides

$B \to K^*$ form factors



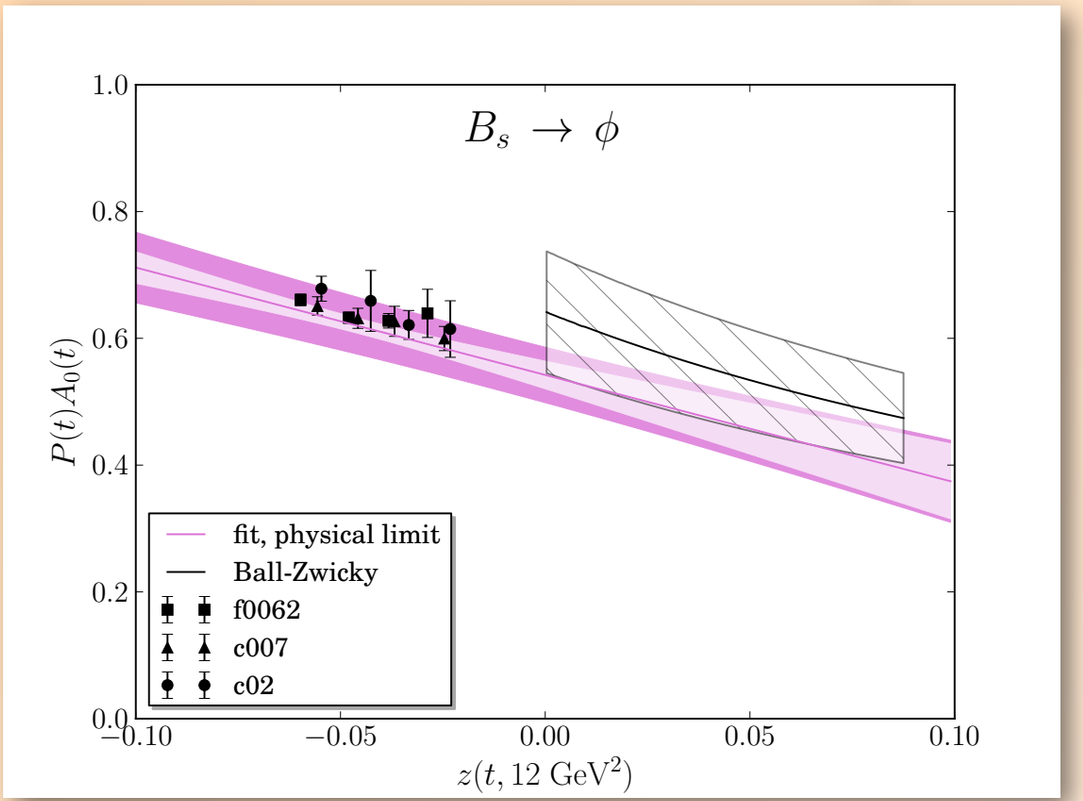


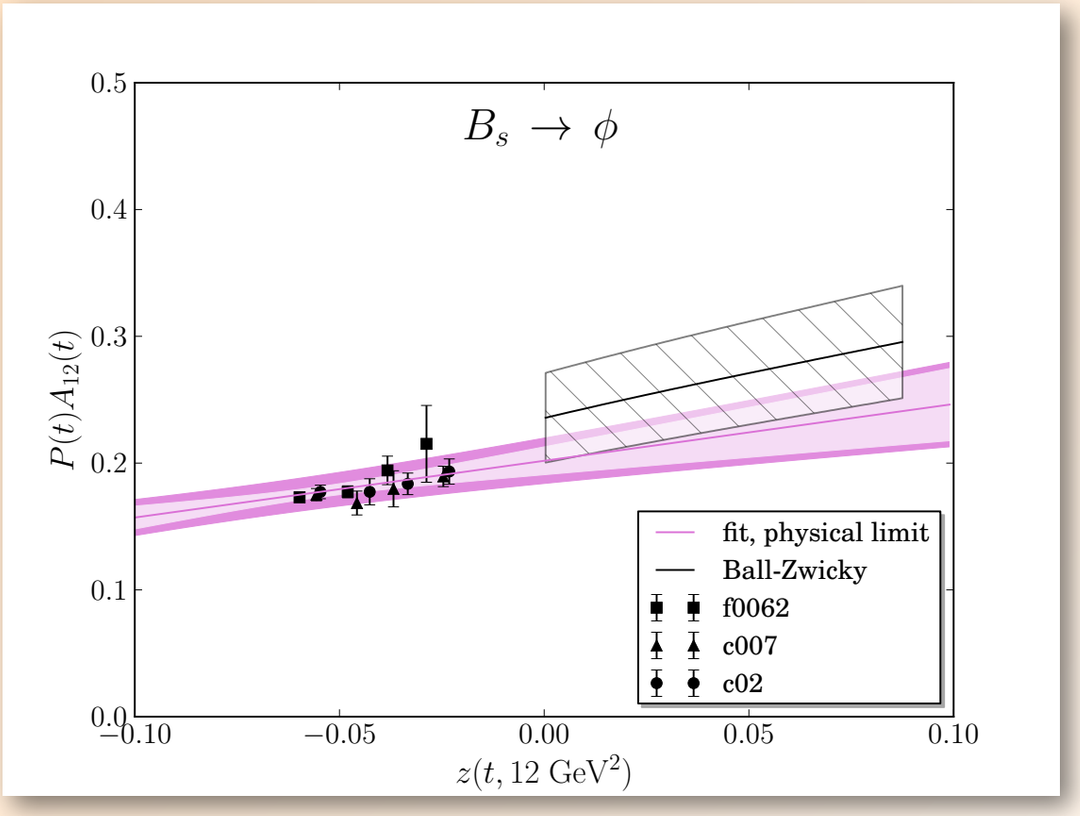


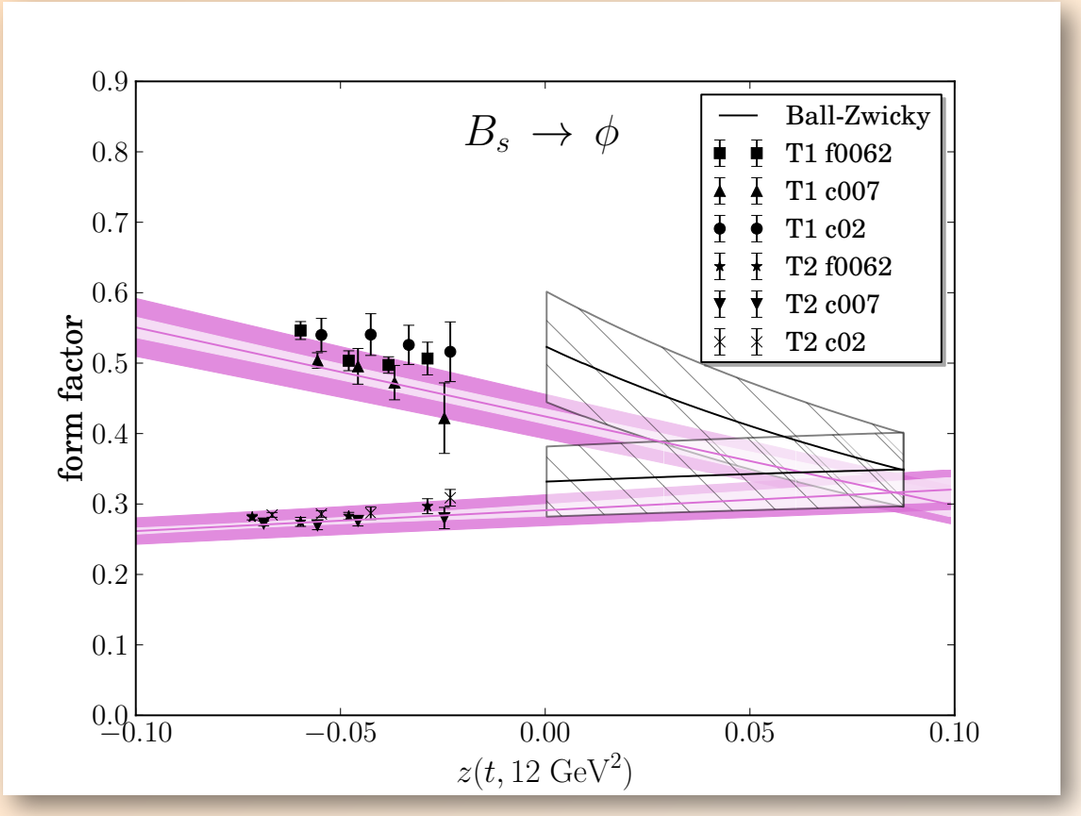


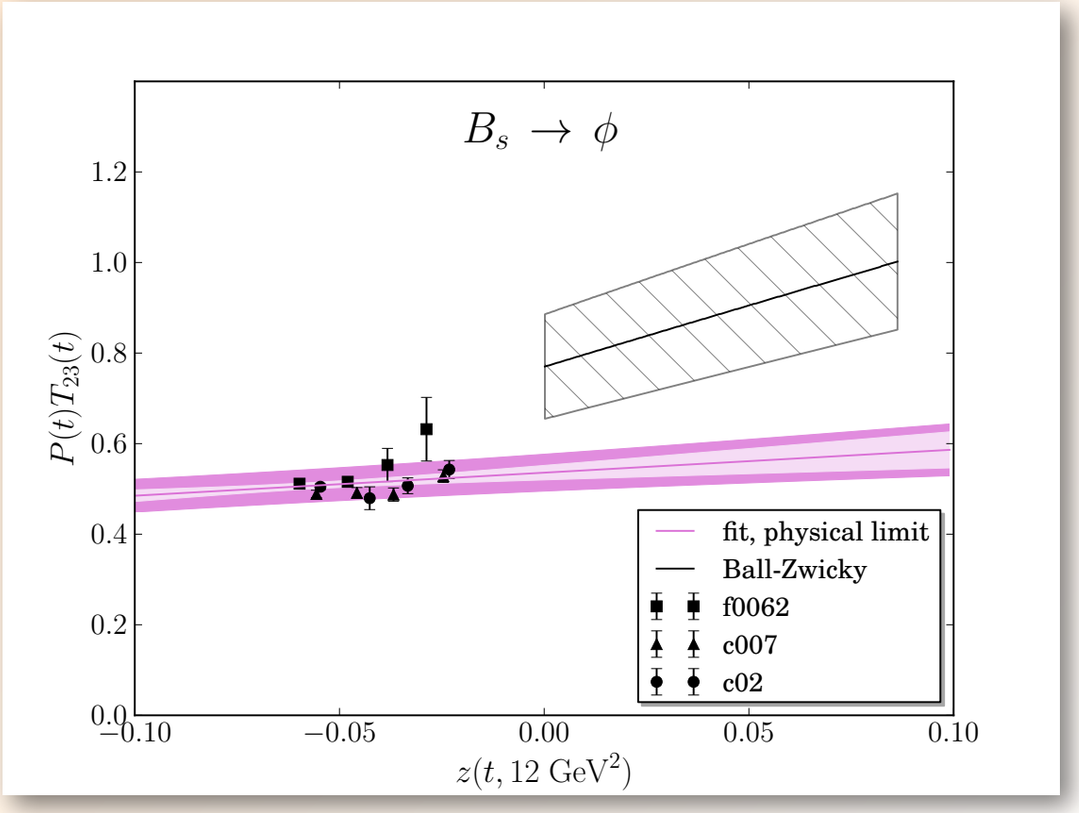
Horgan, Liu, Meinel, Wingate, arXiv:1310.3722

$B_s \to \varphi$ form factors



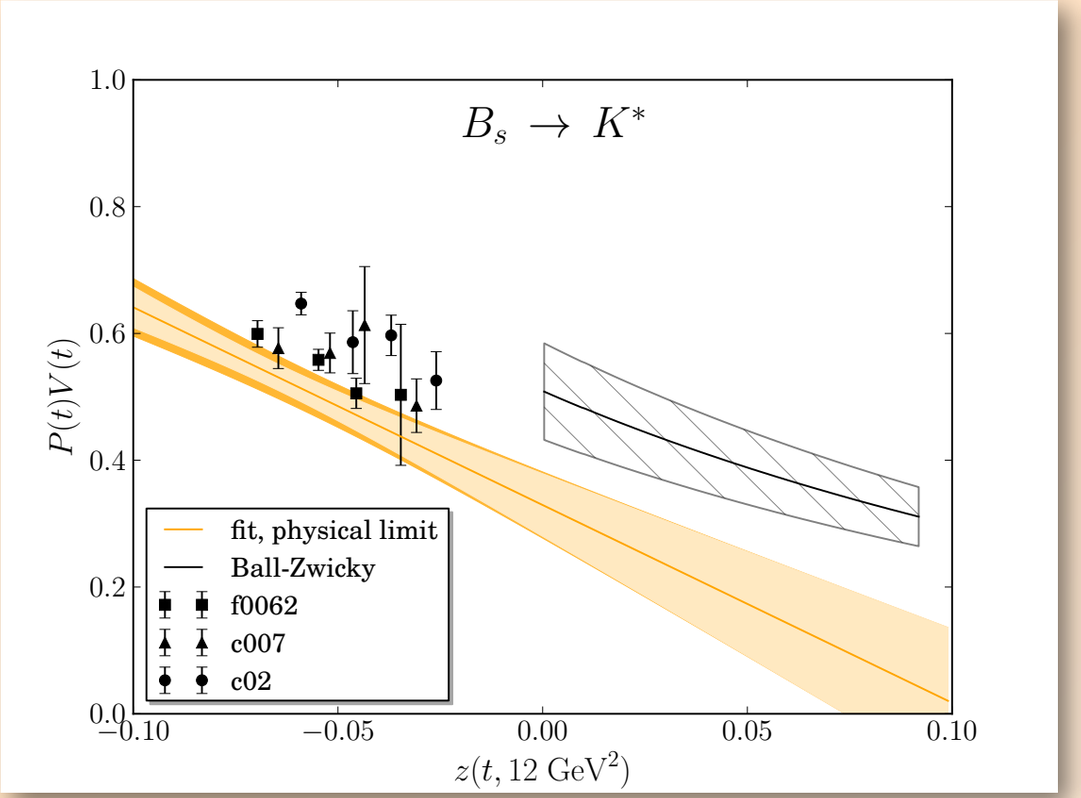


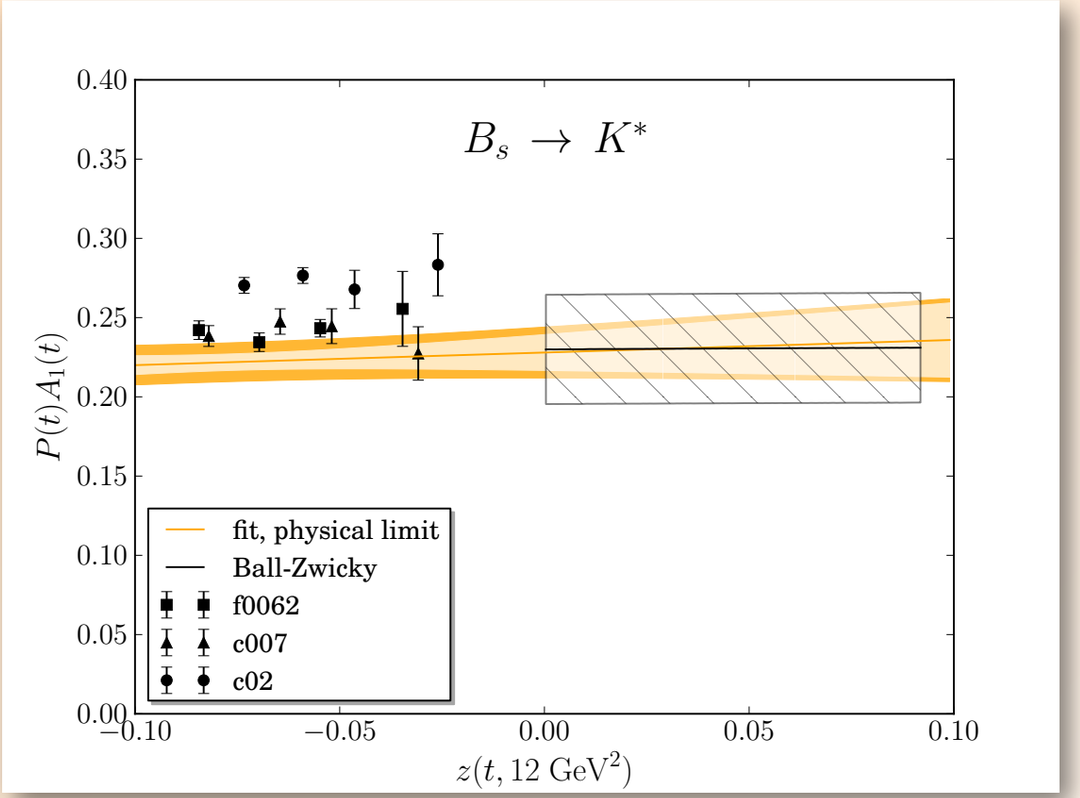


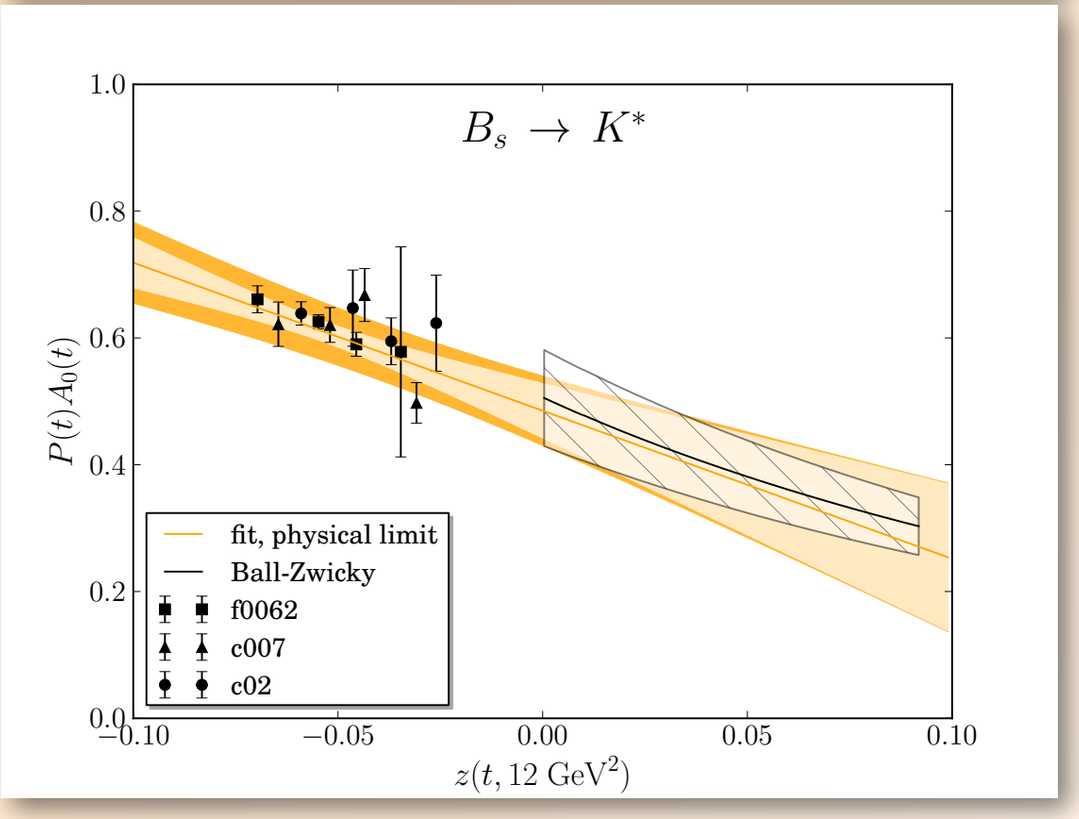


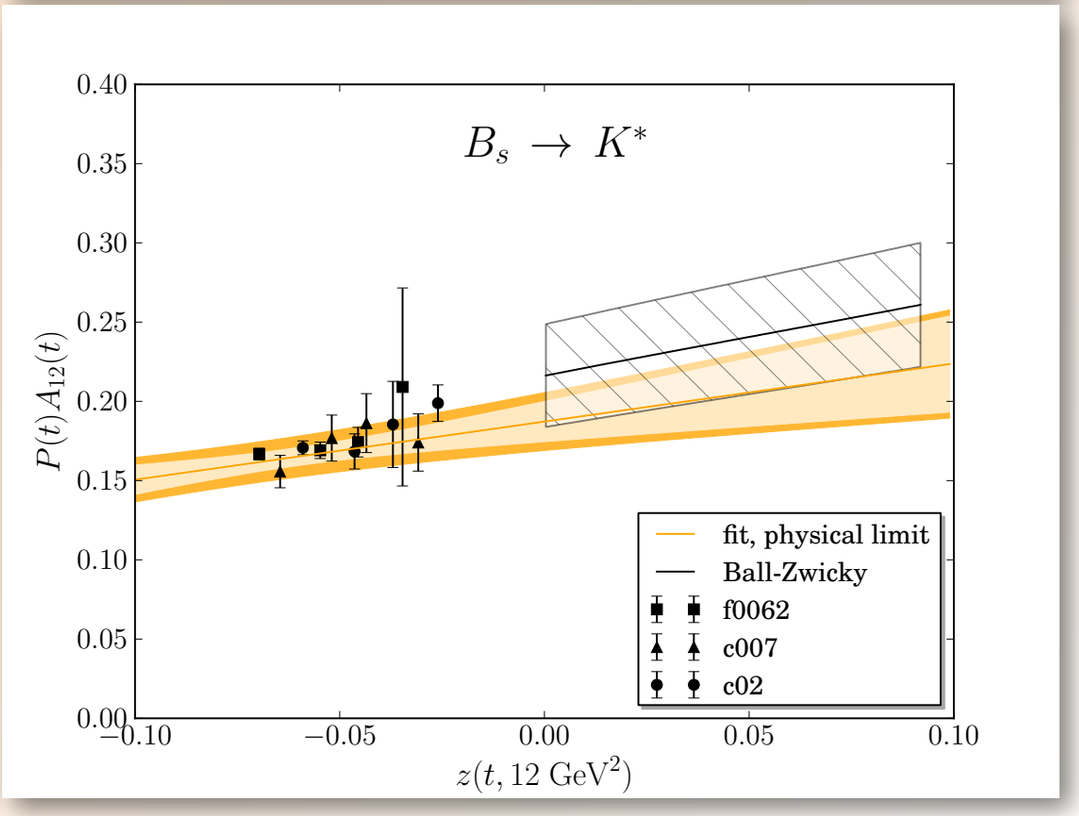
Horgan, Liu, Meinel, Wingate, arXiv:1310.3722

$B_s \to K^*$ form factors ($b \to u$ or $b \to d$)









Horgan, Liu, Meinel, Wingate, arXiv:1310.3722

Form factor error budget

source	size
Truncation of $O(\alpha_s^2)$ terms	4%
Truncation of $O(\alpha_s \Lambda_{\rm QCD}/m_b)$ terms	2%
Truncation of $O(\Lambda_{\rm QCD}^2/m_b^2)$ terms	1%
Mistuning of m_b	$<$ 1%
Mistuning of m_s	2%
Net systematic uncertainty	5%

Statistical + fitting uncertainties depend on z

Smaller than systematic uncertainty in some cases

Total uncertainty typically ≈10-20% in data range

Form factor shape

- In static limit, *z*-expansion is not applicable
- Instead, try monopole, dipole, etc. (Latter is a better fit to the data)
- Incorporate discretization and quark mass effects

$$F = \frac{N}{(X + E_\Lambda - m_\Lambda)^2}\,[1 + d(aE_\Lambda)^2]$$

$$X = X_0 + c_\ell[m_\pi^2 - (m_\pi^{\rm phys})^2] + c_s[m_{\eta_s}^2 - (m_{\eta_s}^{\rm phys})^2]$$

- In practice, *c*'s & *d*'s small, consistent with zero [except $c_{l,+}$ = 0.094(32)]