Lepton Flavor Violation in the Standard Model with general Dimension-6 Operators.

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based on JHEP 1404 (2014) 167, A. Crivellin, S. Najjari, JR

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- Lepton Flavor Violation in the SM
- SM extensions parametrization: effective higher dimension operators
- Physical observables calculation
  - radiative lepton decays $l \to l'\gamma$
  - charged lepton EDMs and $g-2$ anomaly
  - 3-body LFV charged lepton decays $l \to l'l''l'''$
  - $Z^0 \to ll'$ decays
- Numerical results and bounds
- Conclusions

1. Lepton Flavor Violation in the SM

No flavor and CP violation in the lepton sector of SM with massless neutrinos.

Neutrino mass discovery shows explicit LFV violation!

Charged lepton sector – good laboratory to search for New Physics:

SM effects negligible, GIM-suppressed by $m_\nu^2/M_W^2 \sim 10^{-25}$

LFV processes "theoretically clean", no non-perturbative QCD effects.

Two possible approaches:

1. Construct specific New Physics model; calculate LFV observables; compare with experimental bounds to constrain model parameters.

   Time and labor consuming - full calculation need to be done for each model separately.

2. Parametrize New Physics effects in terms of higher dimension operators. Express LFV observables in terms of Wilson coefficients.

   Model independent - needs to be done just once. Only Wilson coefficients need to be calculated within specific models.

2. Effective operator approach

Advantages - completeness and automatic gauge invariance.

Start from full operator basis:

$$\mathcal{L}_{\mathrm{S}M} = \mathcal{L}_{\mathrm{S}M}^{(4)} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

$\mathcal{L}_{\mathrm{S}M}^{(4)}$ - standard renormalizable dim-4 SM Lagrangian.

Full list of dimension 5- an 6-operators: Buchmiller-Wiler 1986

Reduced to minimal set: Grządkowski, Iskrzyński, Misiak, JR 2010 - 59 (64 if baryon number not conserved) independent operators.

LFV related (φ - Higgs doublet, B, W - $U(1), SU(2)$ gauge bosons):

$\ell\ell\ell\ell$		$\ell\ell X\varphi$		$\ell\ell\varphi^2 D$ and $\ell\ell\varphi^3$	
$Q_{\ell\ell}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar\ell_k\gamma^\mu\ell_l)$	Q_{eW}	$(\bar\ell_o\sigma^{\mu\nu}e_j)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi\ell}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar\ell_i\gamma^\mu\ell_j)$
Q_{ee}	$(\bar e_i\gamma_\mu e_j)(\bar e_k\gamma^\mu e_l)$	Q_{eB}	$(\bar\ell_i\sigma^{\mu\nu}e_j)\varphi B_{\mu\nu}$	$Q_{\varphi\ell}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar\ell_i\tau^I\gamma^\mu\ell_j)$
$Q_{\ell e}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar e_k\gamma^\mu e_l)$			$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar e_i\gamma^\mu e_j)$
				$Q_{e\varphi 3}$	$(\varphi^\dagger\varphi)(\bar\ell_i e_j\varphi)$
$\ell\ell qq$					
$Q_{\ell q}^{(1)}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar q_k\gamma^\mu q_l)$	$Q_{\ell d}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar d_k\gamma^\mu d_l)$	$Q_{\ell u}$	$(\bar\ell_i\gamma_\mu l_j)(\bar u_k\gamma^\mu u_l)$
$Q_{\ell q}^{(3)}$	$(\bar\ell_i\gamma_\mu\tau^I\ell_j)(\bar q_k\gamma^\mu\tau^I q_l)$	Q_{ed}	$(\bar e_i\gamma_\mu e_j)(\bar d_k\gamma^\mu d_l)$	Q_{eu}	$(\bar e_i\gamma_\mu e_j)(\bar u_k\gamma^\mu u_l)$
Q_{eq}	$(\bar e_i\gamma^\mu e_j)(\bar q_k\gamma_\mu q_l)$	$Q_{\ell edq}$	$(\bar\ell_i^a e_j)(\bar d_k q_l^a)$	$Q_{\ell equ}^{(1)}$	$(\bar\ell_i^a e_j)\varepsilon_{ab}(\bar q_k^b u_l)$
				$Q_{\ell equ}^{(3)}$	$(\bar\ell_i^a\sigma_{\mu\nu}e_a)\varepsilon_{ab}(\bar q_k^b\sigma^{\mu\nu}u_l)$

"Complete" set - other operators give LFV effects suppressed by m_ν^2/M_W^2.

19 (+1 of dim-5) Wilson coefficients - too many, small predictive power.

1. Only dim-5 term (Weinberg operator) $Q_{\nu\nu} = \varepsilon_{ab}\varepsilon_{cd}\varphi^a\varphi^c(\ell_i^b)^T C\ell_j^d$ does not contribute directly to LFV processes in the charged lepton sector.

2. $Q_{e\varphi 3} = (\varphi^\dagger\varphi)(\bar{\ell}_i e_j \varphi)$ gives off-diagonal corrections to fermion masses. Rediagonalization $\rightarrow$ LFV effect in charged lepton sector negligible.

3. Contributions of most of the 2-lepton-2 quark operators vanish or suppressed by m_l/M_W powers.

4. Goldstone/Higgs couplings to leptons - always suppressed by m_l/M_W powers $\rightarrow$ **we neglect all physical Higgs diagrams**.

9 operators remain:

2: $(\ell\ell\varphi X)$-type operators - modify $\gamma ll'$ vertex

3: $(\ell\ell)(\varphi D\varphi)$-type operators - modify $Zll', Wl'v$ vertices

3: 4-lepton contact couplings

1: 2-lepton 2-quark coupling.

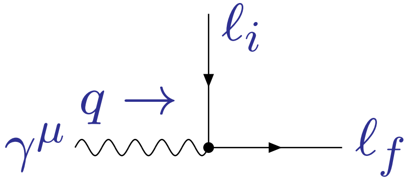
3. Effective lepton-photon coupling

Related observables: radiative lepton decays ($\mu \to e\gamma$), EDMs and AMMs of charged leptons.

The general form of the flavor violating photon-lepton vertex::

$$
\begin{aligned}
V_{\ell\ell\gamma}^{fi\,\mu} &= \frac{i}{\Lambda^2}\Big[\gamma^\mu(F_{VL}^{fi}P_L + F_{VR}^{fi}P_R) + (F_{SL}^{fi}P_L + F_{SR}^{fi}P_R)q^\mu \\
&+ (F_{TL}^{fi}i\sigma^{\mu\nu}P_L + F_{TR}^{fi}i\sigma^{\mu\nu}P_R)q_\nu\Big]
\end{aligned}
$$

Most important: " tensor" F_{TL}, F_{TR}. Tree level LFV contribution exist:



$$
i\left(e\gamma^\mu\delta^{fi} + i\sigma^{\mu\nu}\left[C_{\gamma L}^{fi}P_L + C_{\gamma R}^{fi}P_R\right]q_\nu\right)
$$

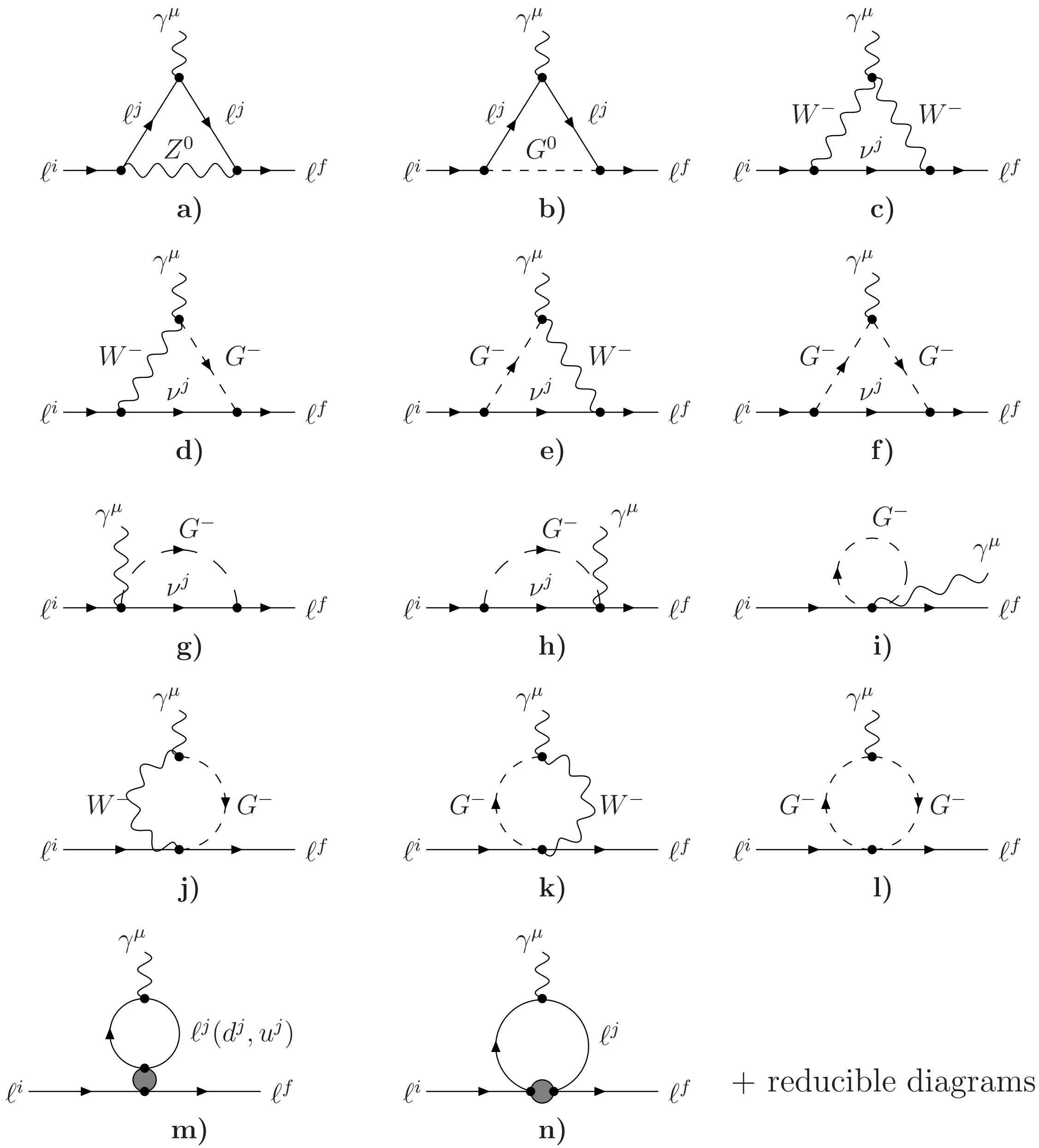
$$
C_{fi}^{\gamma R} = C_{fi}^{\gamma L\star} = \frac{v\sqrt{2}}{\Lambda^2}\left(c_W C_{eB}^{fi} - s_W C_{eW}^{fi}\right)
$$

$\bar{\ell}\sigma^{\mu\nu}\ell F_{\mu\nu}$ coupling non-renormalizable - generated radiatively, Wilson coefficients C_{eB}, C_{eW} inherently contain loop suppression factors.

Other operators can be generated at tree level, like contact 4-lepton coupling via contraction of the heavy boson propagator.

It make sense to add 1-loop terms from other operators to tree-level C_{eB}, C_{eW} terms!

1-loop calculation relatively complicated - quadruple- and quintuple-vertices appear with new Dirac structures and momentum dependence.


γ^μ
ℓ^j ℓ^j Z^0
ℓ^i ℓ^f
a)
γ^μ
ℓ^j ℓ^j G^0
ℓ^i ℓ^f
b)
γ^μ
W^- W^- ν^j
ℓ^i ℓ^f
c)
γ^μ
W^- G^- ν^j
ℓ^i ℓ^f
d)
γ^μ
G^- W^- ν^j
ℓ^i ℓ^f
e)
γ^μ
G^- G^- ν^j
ℓ^i ℓ^f
f)
γ^μ G^- ν^j
ℓ^i ℓ^f
g)
G^- γ^μ ν^j
ℓ^i ℓ^f
h)
G^- γ^μ
ℓ^i ℓ^f
i)
γ^μ
W^- G^-
ℓ^i ℓ^f
j)
γ^μ
G^- W^-
ℓ^i ℓ^f
k)
γ^μ
G^- G^-
ℓ^i ℓ^f
l)
γ^μ
$\ell^j(d^j, u^j)$
ℓ^i ℓ^f
m)
γ^μ
ℓ^j
ℓ^i ℓ^f
n)
+ reducible diagrams

Non-trivial internal test – gauge invariance, requires for $i \neq j$:

$$F_{VL} = F_{VR} = 0 \qquad \text{for} \qquad p_i^2 = m_{\ell_i}^2,\ p_f^2 = m_{\ell_f}^2,\ q^2 = 0.$$

Calculations performed in two independent approaches:

1. Passarino-Veltman reduction of tensor loop integrals in the fixed Feynman gauge (R_ξ with $\xi = 1$)

2. Asymptotic expansion of loop integrals in the general R_ξ gauge.

Both approaches agree and confirm explicit gauge invariance.

Final 1-loop results for tensor form-factors compact and simple:

$$F_{TL}^{ZWG\,fi} = \frac{4e\left[C_{\varphi\ell}^{(1)fi} m_f (1+s_W^2) - \left(C_{\varphi l}^{(3)fi} m_f + C_{\varphi e}^{fi} m_i\right)\left(\frac{3}{2}-s_W^2\right)\right]}{3(4\pi)^2}$$

$$F_{TR}^{ZWG\,fi} = \frac{4e\left[C_{\varphi\ell}^{(1)fi} m_i (1+s_W^2) - \left(C_{\varphi l}^{(3)fi} m_i + C_{\varphi e}^{fi} m_f\right)\left(\frac{3}{2}-s_W^2\right)\right]}{3(4\pi)^2}$$

$$F_{TL}^{4\ell\,fi} = \frac{2e}{(4\pi)^2} \sum_{j=1}^{3} C_{\ell e}^{fjji} m_j$$

$$F_{TR}^{4\ell\,fi} = \frac{2e}{(4\pi)^2} \sum_{j=1}^{3} C_{\ell e}^{jifj} m_j$$

$$F_{TL}^{ql\,fi} = -\frac{16e}{3(4\pi)^2} \sum_{j=1}^{3} C_{\ell equ}^{(3)fijj\star} m_{u_j} \left(\Delta - \log \frac{m_{u_j}^2}{\mu^2}\right)$$

$$F_{TR}^{ql\,fi} = -\frac{16e}{3(4\pi)^2} \sum_{j=1}^{3} C_{\ell equ}^{(3)fijj} m_{u_j} \left(\Delta - \log \frac{m_{u_j}^2}{\mu^2}\right)$$

$Q_{\ell equ}^{(3)} = (\bar{\ell}_i^a \sigma_{\mu\nu} e_a)\varepsilon_{ab}(\bar{q}_k^b \sigma^{\mu\nu} u_l)$ can only be loop generated - its (infinite) 1-loop contribution double-loop suppressed $\to$ can be neglected.

Final result for F_{TL}, F_{TR} depend on 6 Wilson coefficients:

tree level – C_{eB}, C_{eW}

one loop – $C^{(1)}_{\varphi\ell}, C^{(3)}_{\varphi l}, C_{\varphi e}, C_{\ell e}$.

Applications of effective lepton-photon coupling:

- radiative lepton decays $\ell_i \to \ell_f \gamma$ ($\mu \to e\gamma$, $\tau \to e\gamma, \mu\gamma$):

$$\mathrm{Br}\left[\ell_i \to \ell_f \gamma\right] = \frac{m^3_{\ell_i}}{16\pi\Lambda^4 \Gamma_{\ell_i}} \left(\left| F^{fi}_{TR} \right|^2 + \left| F^{fi}_{TL} \right|^2 \right)$$

- charged leptons anomalous magnetic moments:

$$a_{\ell_i} = \frac{2m_{\ell_i}}{e\Lambda^2} \mathrm{Re}\left[F^{ii}_{TR}\right]$$

- charged leptons electric dipole moments

$$d_{\ell_i} = \frac{-1}{\Lambda^2} \mathrm{Im}\left[F^{ii}_{TR}\right]$$

4. Tree-level decays: $l \to l'l''l'''$ and $Z^0 \to \ell_f^- \ell_i^+$

3-body charged lepton decays - various final state compositions:

- 3 leptons of the same flavor: $\mu^\pm \to e^\pm e^+ e^-$, $\tau^\pm \to e^\pm e^+ e^-$ and $\tau^\pm \to \mu^\pm \mu^+ \mu^-$.
- 3 distinguishable leptons: $\tau^\pm \to e^\pm \mu^+ \mu^-$ and $\tau^\pm \to \mu^\pm e^+ e^-$.
- 2 lepton of the same flavor and charge and 1 with different flavor and opposite charge: $\tau^\pm \to e^\mp \mu^\pm \mu^\pm$ and $\tau^\pm \to \mu^\mp e^\pm e^\pm$ (exotic, $\Delta L = 2$!).

Tricky phase space integral, photon propagator $1/q^2 \sim 1/m_l^2$ diverges in corners of phase space $\to$ photon contribution enhanced by logarithmic factor $\log(m_l^2/m_{l'}^2)$.

$Z^0 \to \ell_f^- \ell_i^+$ decays - interesting observation: γll and Zll decays depend on "orthogonal" combinations $\left(c_W C_{eB}^{fi} - s_W C_{eW}^{fi}\right)$, $\left(s_W C_{eB}^{fi} + c_W C_{eW}^{fi}\right)$.

5. Numerical analysis and bounds on Wilson coefficients

Assumption: no fine-tuning or large cancellations. Then:

- anomalous tree-level $\gamma ll'$ coupling best constrained by radiative lepton decays
- Zll' coupling and contact 4−lepton couplings best constrained by the three-body charged lepton decays

First approximation - constrain $\gamma ll'$ couplings from limits on Br$[\ell_i \to \ell_f \gamma]$, assuming all other Wilson coefficients negligible ($C_\gamma = c_W C_{eB}^{fi} - s_W C_{eW}^{fi}$):

$$\sqrt{|C_\gamma^{12}|^2 + |C_\gamma^{21}|^2} \leq 2.45 \times 10^{-10} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\,[\mu \to e\gamma]}{5.7 \times 10^{-13}}},$$

$$\sqrt{|C_\gamma^{13}|^2 + |C_\gamma^{31}|^2} \leq 2.35 \times 10^{-6} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\,[\tau \to e\gamma]}{3.3 \times 10^{-8}}},$$

$$\sqrt{|C_\gamma^{23}|^2 + |C_\gamma^{32}|^2} \leq 2.71 \times 10^{-6} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\,[\tau \to e\gamma]}{4.4 \times 10^{-8}}}.$$

Numbers dividing the branching ratios – the current experimental bounds.

Strong bounds on C_γ for $\Lambda \sim 1$ TeV:

10^{-10} for $\mu \to e$ transitions
10^{-6} for $\tau \to \mu, e$ transitions.

Next: assume $C_\gamma^{fi} \to 0$ and use the bounds from the $Br(\ell_i \to \ell_f \ell_f \bar{\ell}_f)$:

$$\begin{aligned} C_{\mu eee} &\leq 3.29 \times 10^{-5} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\mu \to eee\right]}{1 \times 10^{-12}}}\,, \\ C_{\tau eee} &\leq 1.28 \times 10^{-2} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\tau \to eee\right]}{2.7 \times 10^{-8}}}\,, \\ C_{\tau\mu\mu\mu} &\leq 1.13 \times 10^{-2} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\tau \to \mu\mu\mu\right]}{2.1 \times 10^{-8}}}\,, \end{aligned}$$

with $C_{\ell_i\ell_f\ell_f\ell_f}$ given by

$$\begin{aligned} C_{\ell_i\ell_f\ell_f\ell_f} &= \left|0.46 \left(C_{\varphi\ell}^{(1)fi} + C_{\varphi\ell}^{(3)fi}\right) + C_{\ell e}^{fiff}\right|^2 + 2\left|C_{\ell e}^{fiff}\right. \\ &- 0.54 \left.\left(C_{\varphi\ell}^{(1)fi} + C_{\varphi\ell}^{(3)fi}\right)\right|^2 \\ &+ \left|C_{\ell e}^{fffi} - 0.54\, C_{\varphi e}^{fi}\right|^2 + 2\left|C_{ee}^{fiff} + 0.46\, C_{\varphi e}^{fi}\right|^2 . \end{aligned}$$

Bounds on Wilson coefficient of the LFV 4-lepton and the Z^0-lepton-lepton vertices for $\Lambda \sim \mathcal{O}(1)$ TeV:

10^{-5} for $\mu \to e$ transitions

10^{-2} for $\tau \to \mu$ and $\tau \to e$ transitions.

Bounds weaker than for anomalous photon couplings, but these coefficients can be tree level generated, so potentially larger.

Last bound from $Z^0 \to \ell_f^\pm \ell_i^\mp$:

$$\sqrt{\left|C_{\varphi\ell}^{(1)12} + C_{\varphi\ell}^{(3)12}\right|^2 + \left|C_{\varphi e}^{12}\right|^2 + \left|C_Z^{12}\right|^2 + \left|C_Z^{21}\right|^2} \leq 0.06 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \mu^\pm e^\mp\right]}{1.7 \times 10^{-6}}}$$

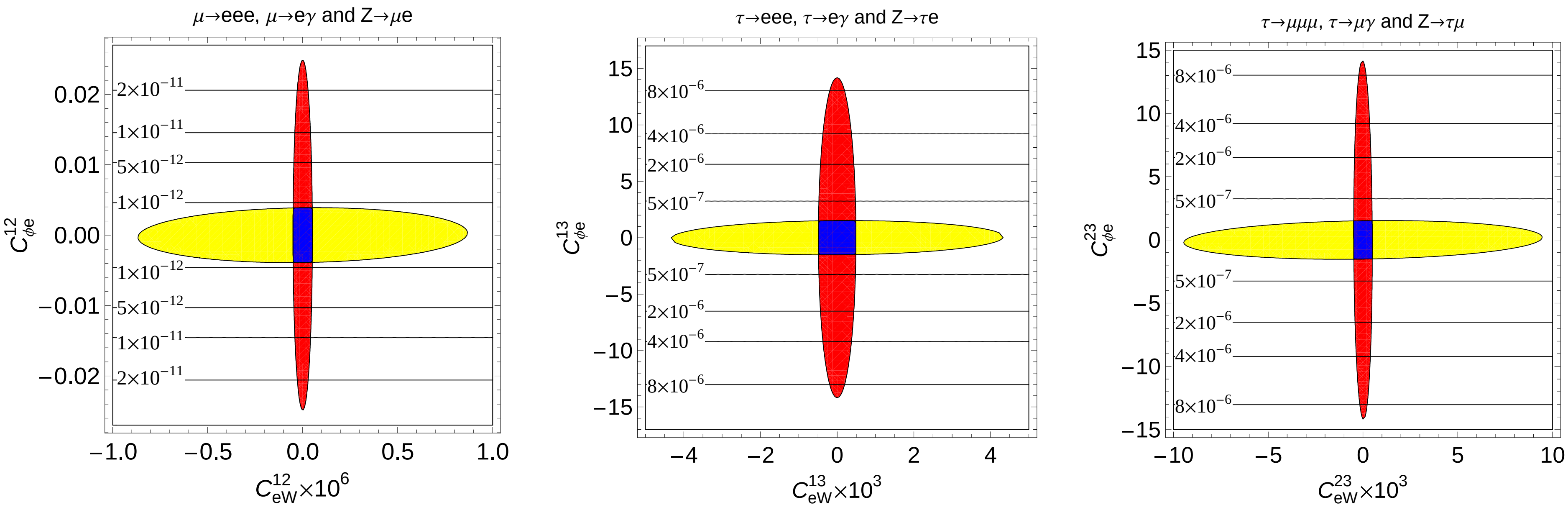
$$\sqrt{\left|C_{\varphi\ell}^{(1)13} + C_{\varphi\ell}^{(3)13}\right|^2 + \left|C_{\varphi e}^{13}\right|^2 + \left|C_Z^{13}\right|^2 + \left|C_Z^{31}\right|^2} \leq 0.14 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \tau^\pm e^\mp\right]}{9.8 \times 10^{-6}}}$$

$$\sqrt{\left|C_{\varphi\ell}^{(1)23} + C_{\varphi\ell}^{(3)23}\right|^2 + \left|C_{\varphi e}^{23}\right|^2 + \left|C_Z^{23}\right|^2 + \left|C_Z^{32}\right|^2} \leq 0.16 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \tau^\pm \mu^\mp\right]}{1.2 \times 10^{-5}}}$$

Less stringent but constrain "orthogonal" combination of C_{eB}, C_{eW}.

Correlations of Wilson coefficients.

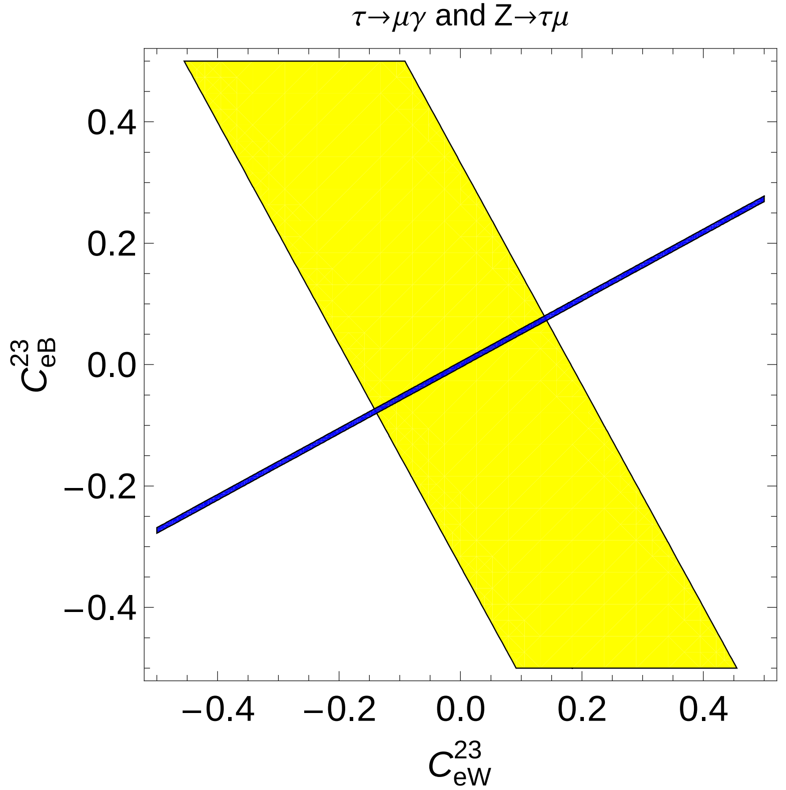
Example: correlation of $Z(W)ll$-γll couplings.



Allowed regions in the C_{eW}^{fi}–$C_{\varphi e}^{fi}$ plane for $\Lambda = 10$ TeV. Red, yellow: $\ell_i \to \ell_f \gamma$, $\ell_i \to \ell_f \ell_f \bar{\ell}_f$. The contour lines: $Br(Z^0 \to \ell_f \ell_i)$.

Photon couplings better constrained by $l' \to l\gamma$, Z, W couplings by $l \to 3l$ decays.

Constraints from $Z^0 \to \ell_f^\pm \ell_i^\mp$ decays weaker but still useful as they are complementary to bounds from $\ell_i \to \ell_f \gamma$:



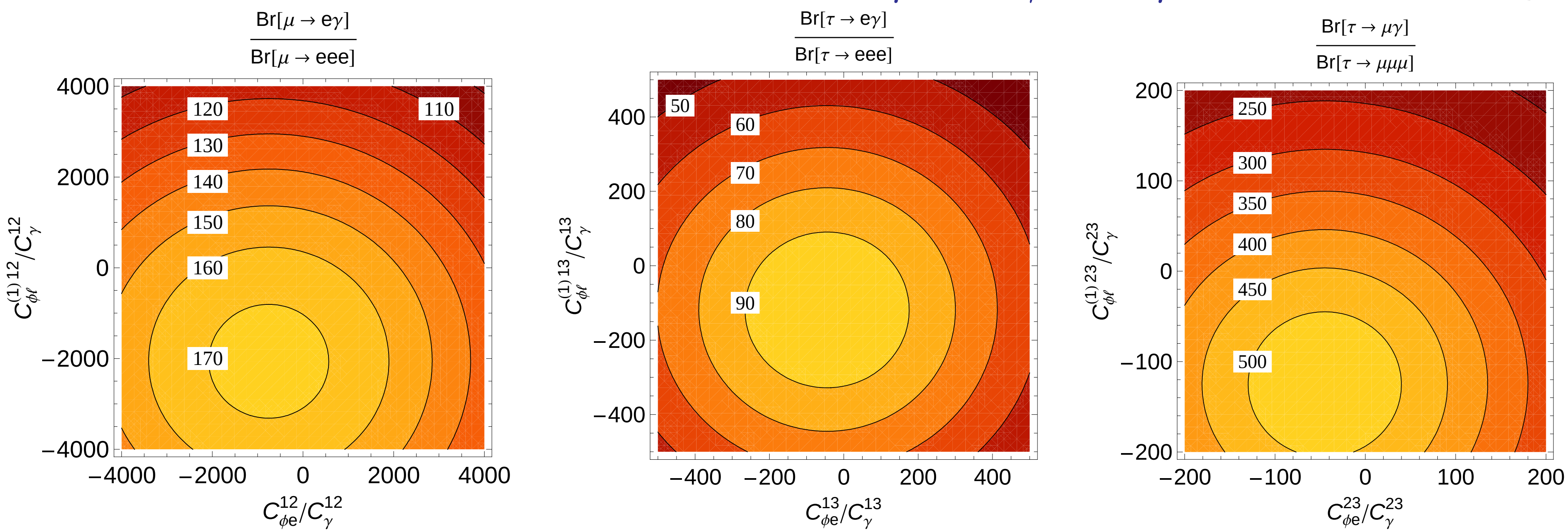
Allowed regions from Br[$Z^0 \to \tau\mu$] (yellow) and Br[$\tau \to \mu\gamma$] (blue) in the C_{eW}^{23}–C_{eB}^{23} plane for $\Lambda = 1$ TeV.

Ratios of decay rates – independent of New Physics scale Λ.

In the photon domination scenario decay rates strictly related:

$$R_{fi} \equiv \frac{Br(\ell_i \to 3\ell_f)}{Br(\ell_i \to \ell_f \gamma)} = \frac{\alpha}{3\pi}(\log \frac{m_f^2}{m_i^2} - \frac{11}{4})$$

Crucial for experiments: no need to test $\mu \to e\gamma$ and $\mu \to 3e$ separately ?



R_{fi} ratio in the $C^{fi}_{\varphi e}/C^{fi}_{\gamma} - C^{(1)\,fi}_{\varphi\ell}/C^{fi}_{\gamma}$ plane.

Lepton number conserving observables.

Anomalous magnetic moments: Example: muon $g-2$ anomaly.

$$\Delta a_\mu = 2.43 \times 10^{-4}\ \mathrm{Re}\left[2\times 10^{-5}\, C_{\ell e}^{3223} + C_\gamma^{22}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 ,$$

To be compared with measurement: $\Delta a_\mu^{exp} \approx (2.7 \pm 0.8) \times 10^{-9}$

Electric Dipole Moments (normalized to current exp. bounds):

$$d_e/d_e^{exp} = -7.9 \times 10^{10}\ \mathrm{Im}\left[2\times 10^{-5}\, C_{\ell e}^{3113} + C_\gamma^{11}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2$$

$$d_\mu/d_\mu^{exp} = -36.1\ \mathrm{Im}\left[2\times 10^{-5}\, C_{\ell e}^{3223} + C_\gamma^{22}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2$$

$$d_\tau/d_\tau^{exp} = -0.69\ \mathrm{Im}\left[C_\gamma^{33}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2$$

6. Conclusions

- Several experimentally well constrained LFV processes calculated within the SM extended with all LFV dim-6 operators.
- Predictions in terms of Wilson coefficients - all relevant contributions included, results automatically gauge-invariant.
- Approximate numerical formulae based on current exp. bounds - specific NP models can be tested just calculating Wilson coefficients.
- "Typical" bounds on LFV Wilson coefficients discussed depending on New Physics scale Λ - usually very strong for $\Lambda = \mathcal{O}(1)$ TeV.
- Examples of correlations between various Wilson coefficients shown.