The Titius-Bode Law explained as a Fossil of Global Non-Axisymmetric Instability in Discs

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and
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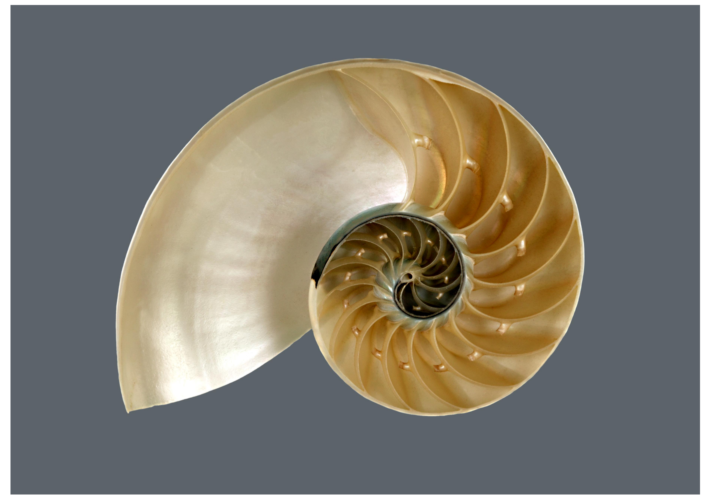
Quy Nhon, April 2014

Planetary Archaeology



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Is it time to brush up on the German language?

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Astron. Astrophys. 282, 262–268 (1994)

ASTRONOMY
AND
ASTROPHYSICS

Titius-Bode laws in the solar system

I. Scale invariance explains everything

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Abstract. According to the Titius-Bode law, the planetary distances to the sun follow a geometric progression. We review the major interpretations and explanations of the law. We show that most derivations of Titius-Bode law are implicitely based on the assumption of both rotational and scale invariance. In absence of any radial length scale, linear instabilities cause periodic perturbations in the variable $x = \ln(r/r_0)$. Since maxima equidistant in x obey a geometric progression in the variable r, Titius-Bode type of laws are natural outcome of the linear regime of systems in which both symmetries are present; we discuss possible non-linear corrections to the law. Thus, if Titius-Bode law is real, it is probably only a consequence of the scale invariance of the disk which gave rise to the planets.

Key words: planets and satellites: general – solar system: formation – hydrodynamics – instabilities

1. Introduction

1.1. Historical overview

Ever since Plato and until the recognition of the existence of chaos, the search for order and regularity in the universe has influenced many a physical theory or observation. The explanation be no order in planet orbit intervals; he critically examines data attributed to Archimedes ($\sim 286 - 212$ BC) to evidence alternating powers of 2 and 3, a platonician numerology Macrobius (400) also supports.

In modern times, the first theory within a heliocentric cosmology comes with Kepler. In his *Mysterium Cosmographicum* (1596), he assumed a construction where each planet orbit was a circle circumscribed to one of the five regular polyhedra: cube, tetra-, dodeca-, icosa- and octahedron (Crombie 1952). He then worked with Tycho Brahe to benefit from his cautious astronomical measurements, and was convinced that Mars' orbit is actually an ellipse; but even after he published his laws (Kepler 1609, 1619), he issued a late republication of the *Mysterium* sticking to the regular solids and spheres construction.

The search for an "order" in the solar system was revived in 1766 by Titius, who noticed that the known planet orbits (Mercury to Saturn) would follow a geometric progression provided a "missing" planet was inserted between Mars and Jupiter. Bode reformulated this observation in 1772 into the so called "Titius-Bode law", which Wurm expressed in 1787 under its more modern mathematical form:

$$r_n = 0.4 + 0.3 \times 2^n, \qquad (1)$$

2. Theories of Titius-Bode laws

2.1. Dynamical vs kinematical theories

Explanations to the Titius-Bode law can be divided in two categories. We refer as "dynamical" to the theories of the first type, which assume that the present law traces back to a period anterior or contemporary to planet formation; most of them describe instabilities occurring in the primordial protoplanetary disk, thus set constraints on its physical characteristics. Theories of the second category, called "kinematical", assume that the law physically originates from orbital interactions posterior to planet formation.

The pros and cons of each category have been discussed by Nieto (1972). He argues that observed deviations from the exact geometric law could be interpreted as the natural outcome of orbital evolution after planet formation. Those deviations have been quantified by Blagg (1913) and Richardson (1945) via the introduction of a periodic function in the original exponential law:

$$r_n = A(1.7)^n[B + f(\alpha + n\beta)]. \tag{3}$$

Here α and β are real constants, A and B are positive constants; f is a 2π periodic function, ranging between 0 and 1. All parameters depend on the satellite system under consideration (Solar System or giant planets). According to Nieto, f could be the result of the tendency to commensurabilities between the orbits. Such interpretation is somewhat favored by numerical simulations of Conway & Elsner (1988), which show that systems placed initially in Titius-Bode-like laws (increasing planetary distances) are very stable.

Within such a tremendous diversity, these models share a common methodology. Each of them assumes a physical phenomenon to be the origin of planet formation. Then, coming to quantitative predictions, each model needs an hypothesis on the of course unknown physical properties of the primordial system. The most natural assumption, as long as we are totally ignorant, avoids introducing unnecessary parameters. Thus the first reflex is to propose a model with no supplementary length scale, other than the radial length scale r itself. Only Prentice seems to explicitly mention his hypothesis, and apparently no author points out its relevance. Such hypotheses include: homologous collapse, constant eccentricity, disk height H or vortex size $\propto r$, r^{-2} force fields, $\nu_t \propto c_s^2 r^{3/2}$, $\sigma \propto r^{-2}$, or $s/\sigma\nu_t = cte$.

This method seems reasonable; it is very popular, but simultaneously not innocent, for a very precise reason. Indeed, it never fails to produce a geometric progression in orbit sizes, even if not desired, and whatever the underlying physical model. For a "faithful" physicist, this prediction of a Titius-Bode law evidences the validity of the model and even offers quantitative constraints. We show in the next section why we believe that, in all the models we review, Titius-Bode laws arise from a same symmetry hidden in the equations, and not from physical phenomena they tend to modelize.

4. Conclusion

We have shown the following results. (i) In any two-dimensional systems characterized by both rotational and scale invariance, the most natural Fourier basis vectors are $\exp[ik \ln(r/r_0)]$. (ii) Since gravitation respects both invariances, in a self-gravitating disk with no vertical length scale, extrema of density perturbations tend to follow a geometric progression of the form $r_n = r_0 K^n$. (iii) If the Titius-Bode laws of the solar system are more than pure numerological speculations, they may be simply interpreted as the signature of the scale and rotational invariance of the protoplanetary system. (iv) Neither symmetry considerations, nor observations of the solar system, can presently discriminate between linear and non-linear Titius-Bode laws.

We conclude that, if a model of planet or satellite formation leads to such geometric law for orbit diameters, this does not constitute a diagnostic of the model's validity, but only reflects its implicit scale and rotational invariance.

Titius-Bode laws in the solar system

II. Build your own law from disk models

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Abstract. Simply respecting both scale and rotational invariance, it is easy to construct an endless collection of theoretical models predicting a Titius-Bode law, irrespective to their physical content. Due to the numerous ways to get the law and its intrinsic arbitrariness, it is not an useful constraint on theories of solar system formation.

To illustrate the simple elegance of scale-invariant methods, we explicitly cook up one of the simplest examples, an infinitely thin cold gaseous disk rotating around a central object. In that academic case, the Titius-Bode law holds during the linear stage of the gravitational instability. The time scale of the instability is of the order of a self-gravitating time scale, $(G\rho_d)^{-1/2}$, where ρ_d is the disk density. This model links the separation between different density maxima with the ratio M_D/M_C of the masses of the disk and the central object; for instance, M_D/M_C of the order of 0.18 roughly leads to the observed separation between the planets. We discuss the boundary conditions and the limit of the WKB approximation.

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Fitting Selected Random Planetary Systems to Titius–Bode Laws

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Simple "solar systems" are generated with planetary orbital radii r distributed uniformly random in $\log r$ between 0.2 and 50 AU, with masses and order identical to our own Solar System. A conservative stability criterion is imposed by requiring that adjacent planets are separated by a minimum distance of k times the sum of their Hill radii for values of k ranging from 0 to 8. Least-squares fits of these systems to generalized Bode laws are performed and compared to the fit of our own Solar System. We find that this stability criterion and other "radius-exclusion" laws generally produce approximately geometrically spaced planets that fit a Titius–Bode law about as well as our own Solar System. We then allow the random systems the same exceptions that have historically been applied to our own Solar System. Namely, one gap may be inserted, similar to the gap between Mars and Jupiter, and up to 3 planets may be "ignored," similar to how some forms of Bode's law ignore Mercury, Neptune, and Pluto. With these particular exceptions, we find that our Solar System fits significantly better than the random ones. However, we believe that this choice of exceptions, designed specifically to give our own Solar System a better fit, gives it an unfair advantage that would be lost if other exception rules were used. We compare our results to previous work that uses a "law of increasing differences" as a basis for judging the significance of Bode's law. We note that the law of increasing differences is not physically based and is probably too stringent a constraint for judging the significance of Bode's law. We conclude that the significance of Bode's law is simply that stable planetary systems tend to be regularly spaced and conjecture that this conclusion could be strengthened by the use of more rigorous methods of rejecting unstable

Key Words: **planetary dynamics; orbits; planetary formation; celestial mechanics; computer techniques.**

1. INTRODUCTION

The Titius–Bode "law,"

$$r_i = 0.4 + 0.15 \times 2^i, \qquad i = -\infty, 1, \ldots, 8, \qquad (1)$$

roughly describes the planetary semi-major axes in astronomical units (AU), with Mercury assigned $i = -\infty$, Venus $i = 1$, Earth $i = 2$, etc. Usually the asteroid belt is counted as $i = 4$. The law fits the planets Venus through Uranus quite well, and successfully predicted the existence and locations of Uranus and the asteroids. However, (i) the law breaks down badly for Neptune and Pluto; (ii) there is no reason why Mercury should have $i = -\infty$ rather than $i = 0$, except that it fits better that way; (iii) the total mass of the asteroid belt is far smaller than the mass of any planet, so it is not clear that it should be counted as one. The question of the significance of Bode's law has taken on increased interest with discoveries of extra-solar planets and is also worth reexamination because computer speeds now permit more powerful statistical tests than were previously possible.

A partial history of the law and attempts to explain it up to the year 1971 can be found in Nieto (1972). Most modern arguments concerning the validity of Bode's law can be assigned to one of three broad classes: .

Academic Press

"For a statistician, fitting a three-parameter curve of uncertain form to ten points with three exceptions certainly brings one to the far edge of the known world."

— *Bradley Efron (1971)*

1. Attempts to elucidate the physical processes leading to Bode's law. These are based on a variety of mechanisms, including dynamical instabilities in the protoplanetary disk (Graner and Dubrulle 1994, Dubrulle and Graner 1994, Li *et al.* 1995), gravitational interactions between planetesi-

0019-1035/98 $25.00

mals (Lecar 1973), or long-term instabilities of the planetary orbits (Hills 1970, Llibre and Piñol 1987, Conway and Elsner 1988). We shall not comment on these explanations, except to say that we find none of them entirely convincing.

2. Discussions that ignore physics but try to assess whether the success of Bode's law is statistically significant:

2. METHOD

2.1. Radius-Exclusion Laws

A necessary, but not sufficient, condition for the stability of a planetary system is that its planets never get "too close to each other" (Lecar 1973). This can be formalized

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Long-range order between the planets in the Solar system

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ABSTRACT

The Solar system is investigated for positional correlations between the planets using a logarithmic distance scale. The pair correlation function for the logarithm of the semimajor axis shows a regular distribution with five to seven consecutive peaks, and the Fourier transform hereof shows reciprocal peaks of first and second order. A procedure involving random permutations for the shuffling of the inter-logarithmic distances is employed. This probes for the presence of correlations of longer range than neighbouring planets. The use of permutations is, in particular, a helpful analysis when the number of data points is small. The pair correlation function of the permutated planets lacks the sequence of equidistant peaks, and its Fourier transform has no second-order peak. This analysis demonstrates the existence of longer ranged correlations in the Solar system.

Key words: history and philosophy of astronomy – planets and satellites: general – Solar

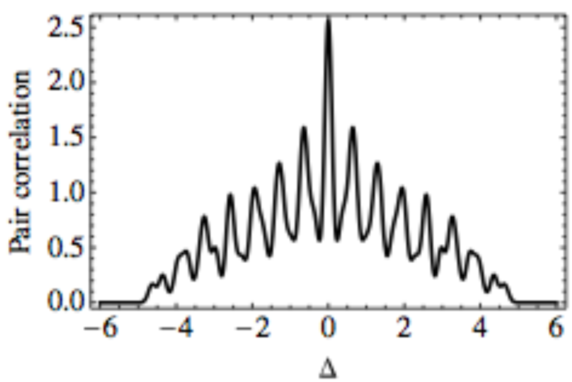

Figure 2. Pair correlation function $P(\Delta)$ in equation (1) for the Solar system. $P(\Delta)$ is a symmetric function of Δ, i.e. $P(-\Delta) = P(\Delta)$. Nearest-neighbour correlations are given by the first peak to the right of the central peak as well as by the first peak to the left of the central peak. On each side, there are five clearly visible equidistant peaks and two weaker equidistant peaks. The pair correlation function is plotted for $w_p = 0.25$. Choosing a larger w_p would further smooth out the graph of $P(\Delta)$, while the positions of the principal peaks would remain the same.

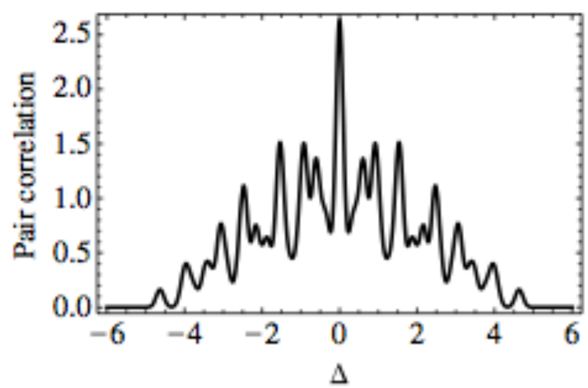

Figure 6. Pair correlation function, $P_{SS}(\Delta)$, of the shuffled Solar system obtained by a random permutation 491 526 387. We observe that the correlations tend to be destroyed compared to that of the Solar system. The six to seven major peaks on either side are not equidistant in contrast to the data for the Solar system.

On the Structural law of Exoplanetary Systems

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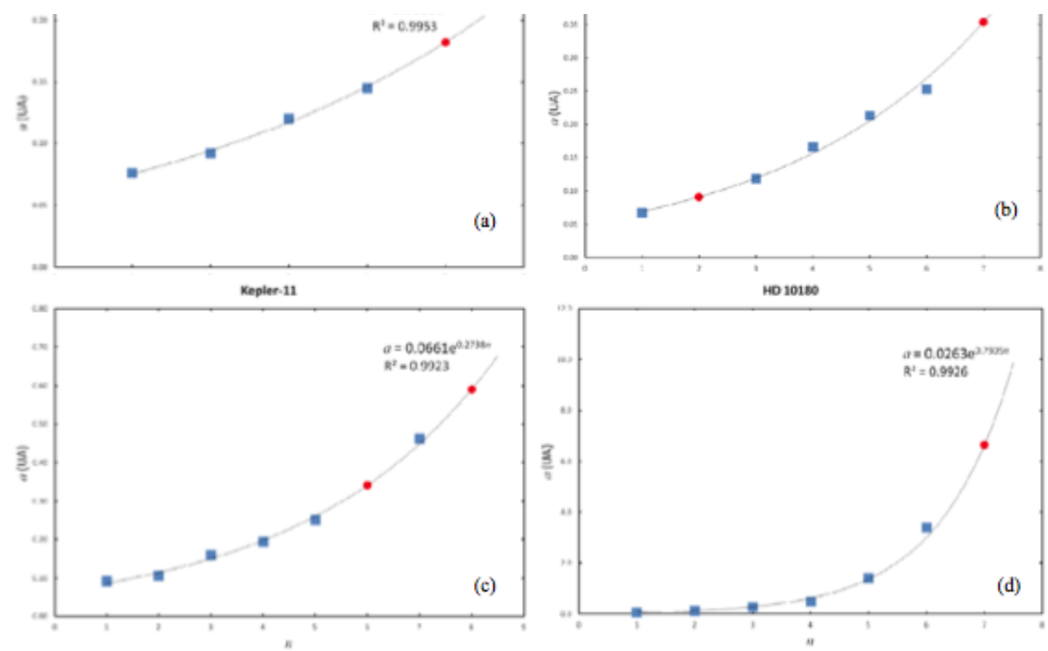
$$a_n = a_0 \times C^n = a_0 e^{bn}$$

As shown by Poveda & Lara (2008) [11-12], the planets in the Solar system can be described by such a law. The qua of the fit provided by equation 3 is poorer than that provided by the TB relation (Eq. 1) for the inner seven planets, better for Neptune.



The HARPS search for southern extra-solar planets★

XXVII. Up to seven planets orbiting HD 10180: probing the architecture of low-mass planetary systems

C. Lovis[1], D. Ségransan[1], M. Mayor[1], S. Udry[1], W. Benz[2], J.-L. Bertaux[3], F. Bouchy[4,5], A. C. M. Correia[6], J. Laskar[7], G. Lo Curto[8], C. Mordasini[9,2], F. Pepe[1], D. Queloz[1], and N. C. Santos[10,1]

A&A 528, 112 (2011)

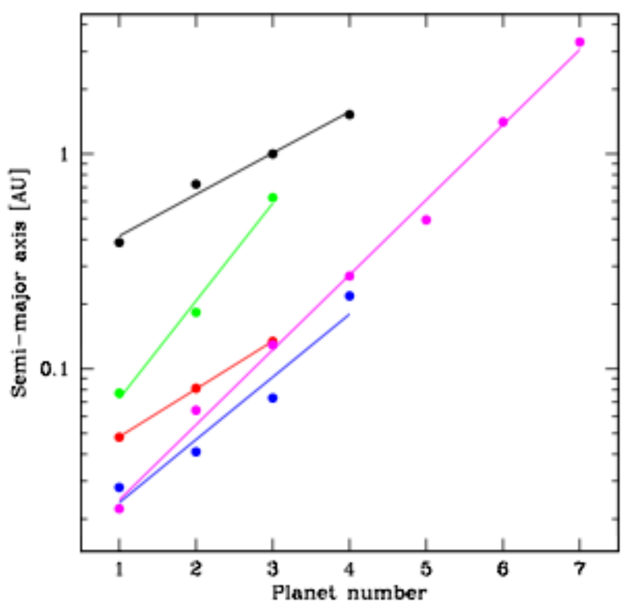


Fig. 14. Fit of exponential laws to semi-major axes as a function of planet number for the inner Solar System (black), HD 40307 (red), GJ 581 (blue), HD 69830 (green) and HD 10180 (magenta).

nential relations (e.g. GJ 581) and the physics of planet formation is so diverse and complex that we do not expect any universal rule on planet ordering to exist.

8.3. Formation and evolution

These emerging patterns, if confirmed by further discoveries of planetary systems, may provide clues on how the observed systems of close-in super-Earths and Neptunes were formed. These systems appear to be quite common, but their formation history remains a puzzle. On the one hand, it seems unlikely that they formed *in situ* given the very high inner disk densities that would be required. However, little is known about statistical properties of protoplanetary disks and their density profiles, and this possibility can probably not be completely rejected at this point. On the other hand, such systems may be the result of convergent type I migration of planetary cores formed at or beyond the ice line (e.g. Terquem & Papaloizou 2007; Kennedy & Kenyon 2008). But how can several protoplanets grow to masses in the super-Earth/Neptune range while migrating together during the disk lifetime, and end up in a configuration which is not necessarily close to mean-motion resonances? Near-commensurability of the orbits would be expected according to Terquem & Papaloizou (2007). Loss of commensurability could occur through orbital decay due to stellar tides, but this is probably efficient only for the planets closest to the star. So this scenario still has difficulties in explaining a system such as HD 10180.

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Exoplanet predictions based on the generalized Titius–Bode relation

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ABSTRACT

We evaluate the extent to which newly detected exoplanetary systems containing at least four planets adhere to a generalized Titius–Bode (TB) relation. We find that the majority of exoplanet systems in our sample adhere to the TB relation to a greater extent than the Solar system does, particularly those detected by the Kepler mission. We use a generalized TB relation to make a list of predictions for the existence of 141 additional exoplanets in 68 multiple-exoplanet systems: 73 candidates from interpolation, 68 candidates from extrapolation. We predict the existence of a low-radius ($R < 2.5R_{\oplus}$) exoplanet within the habitable zone of KOI-812 and that the average number of planets in the habitable zone of a star is 1–2. The usefulness of the TB relation and its validation as a tool for predicting planets will be partially tested by upcoming Kepler data releases.

Key words: planets and satellites: detection – planets and satellites: dynamical evolution and stability – planets and satellites: formation – planets and satellites: general – planet–disc interactions – protoplanetary discs.

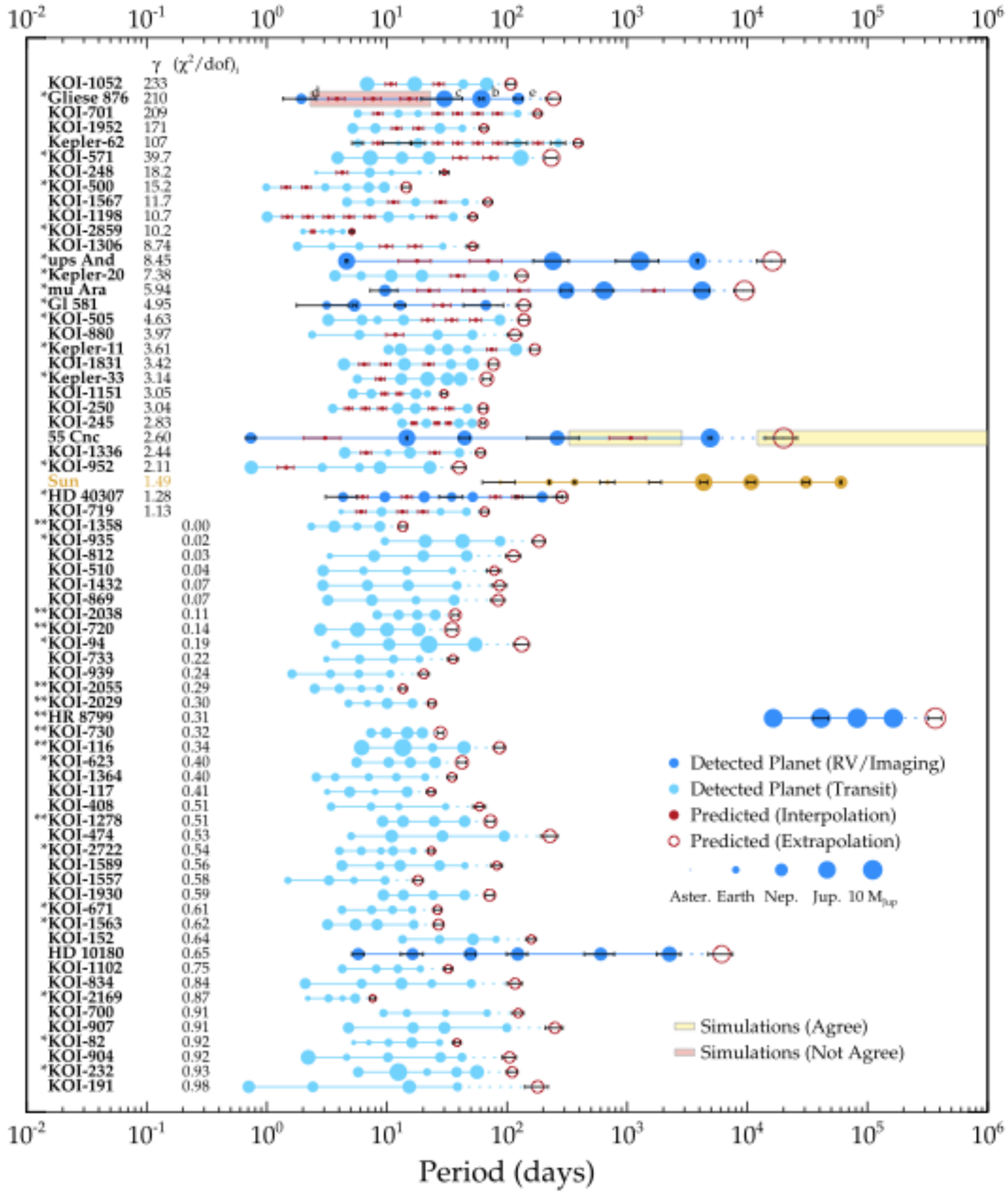


Figure 5. Orbital periods of planets in multiple planet systems containing at least four detected planets. Systems where planet insertions have been made are sorted in descending order of the highest γ found in each system (equation 5 and Table 2). Systems without planet insertions (lower 2/3 of figure) are sorted in ascending order of χ^2/d.o.f. (Equation 4). Inserted and extrapolated planets are shown with red filled circles and red open circles, respectively. The γ value for the Solar system is calculated by excluding, then including, the Asteroid Belt. '*' indicates at least two adjacent planet pairs in the system have Δ values <10 if we had inserted a planet between each pair. '**' indicates all adjacent planet pairs have Δ values <10 if we had inserted an additional planet between each

The Titius-Bode law in polar coordinates

$$a_n = a_0 \times C^n = a_0 e^{bn}$$ (

As shown by Poveda & Lara (2008) [11-12], the p … by such a law. The qua
of the fit provided by equation 3 is poorer than th … the inner seven planets,
better for Neptune.

In polar coordinates (r, θ) the logarithmic curve can be written

http://en.wikipedia.org/wiki/Logarithmic_spiral

Logarithmic spiral - Wikipedia, the free encyclopedia

as[1]

$$r = ae^{b\theta}$$

or

$$\theta = \frac{1}{b}\ln(r/a),$$

with e being the base of natural logarithms, and a and b being arbitrary positive real constants.

The spiral has the property that the angle φ between the tangent and radial line at the point (r, θ) is constant. This property can be expressed in differential geometric terms as

$$\arccos \frac{\langle \mathbf{r}(\theta), \mathbf{r}'(\theta) \rangle}{\|\mathbf{r}(\theta)\| \|\mathbf{r}'(\theta)\|} = \arctan \frac{1}{b} = \phi.$$

The derivative of $\mathbf{r}(\theta)$ is proportional to the parameter b. In other words, it controls how "tightly" and in which direction the spiral spirals. In the

A section of the Mandelbrot set following a logarithmic spiral

A low pressure area over Iceland shows

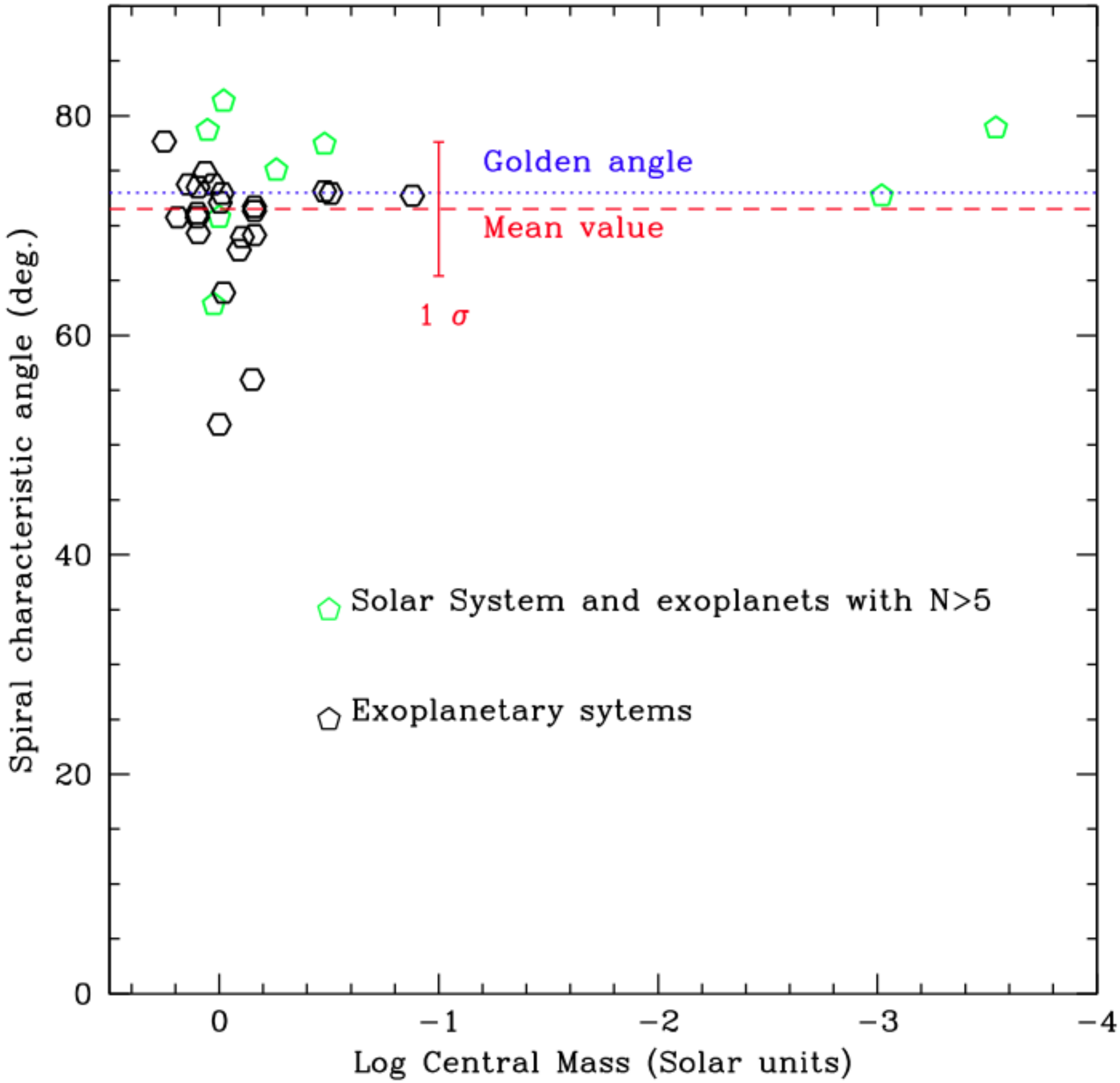
extreme case that $b = 0$ ($\phi = \frac{\pi}{2}$) the spiral becomes a circle of radius a. Conversely, in the limit that b approaches infinity ($\varphi \to 0$) the spiral tends toward a straight half-line. The complement of φ is called the *pitch*.

an approximately logarithmic spiral pattern

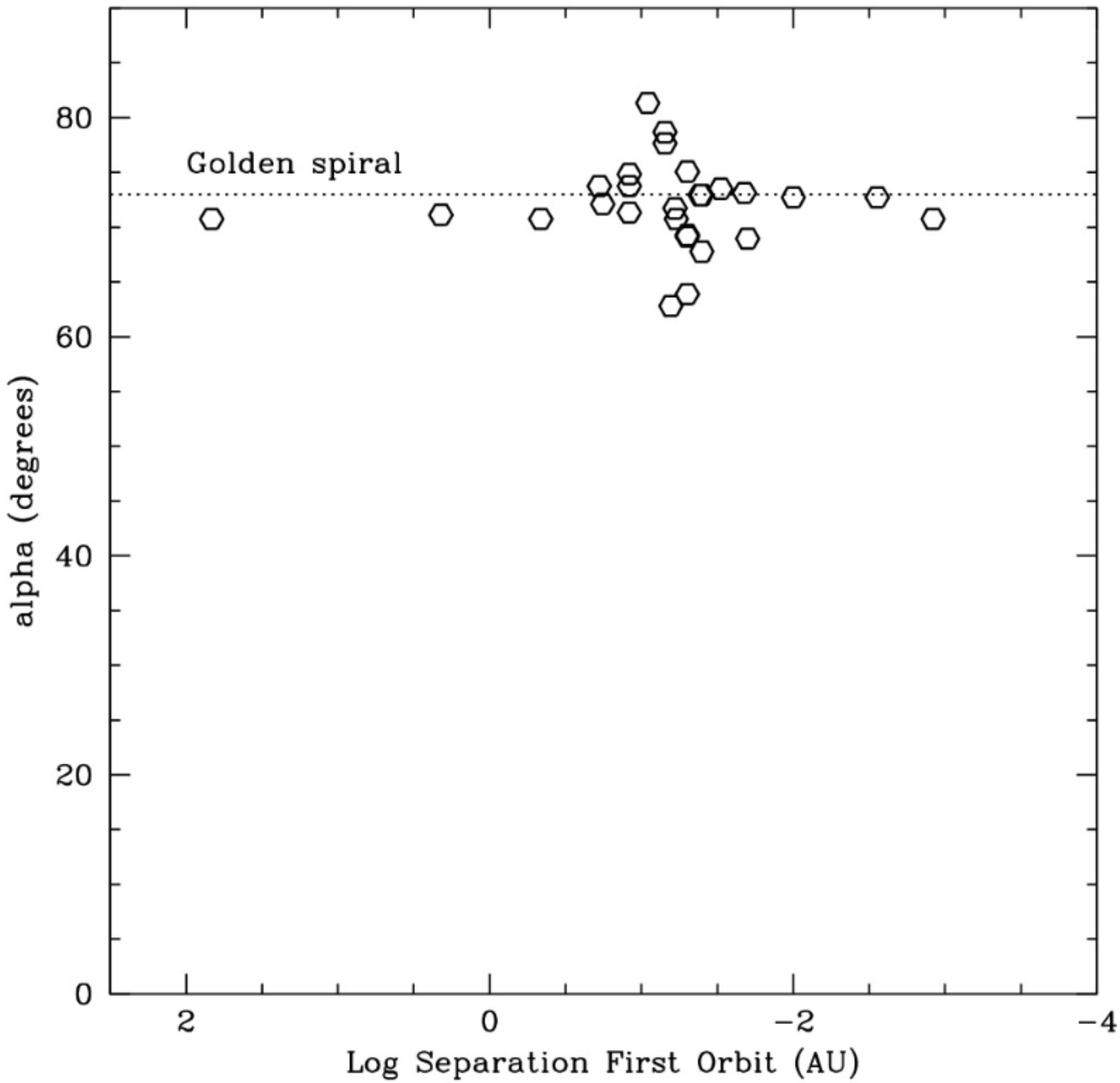
Spira mirabilis and Jacob Bernoulli

Spira mirabilis, Latin for

The arms of spiral galaxies often have the shape of a logarithmic spiral



Golden angle
Mean value
1 σ
Solar System and exoplanets with N>5
Exoplanetary sytems
Spiral characteristic angle (deg.)
Log Central Mass (Solar units)
0
20
40
60
80
0
−1
−2
−3
−4


Golden spiral
80
60
40
20
0
2
0
−2
−4
alpha (degrees)
Log Separation First Orbit (AU)

Multiple spiral patterns in the transitional disk of HD 100546*

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ABSTRACT

Context. Protoplanetary disks around young stars harbor many structures related to planetary formation. Of particular interest, spiral patterns were discovered among several of these disks and are expected to be the sign of gravitational instabilities leading to giant planets formation or gravitational perturbations caused by already existing planets. In this context, the star HD 100546 presents some specific characteristics with a complex gas and dusty disk including spirals as well as a possible planet in formation.
Aims. The objective of this study is to analyze high contrast and high angular resolution images of this emblematic system to shed light on critical steps of the planet formation.
Methods. We retrieved archival images obtained at Gemini in the near IR (Ks band) with the instrument NICI and processed the data using advanced high contrast imaging technique taking advantage of the angular differential imaging.

A&A, 560, 20 (2013)

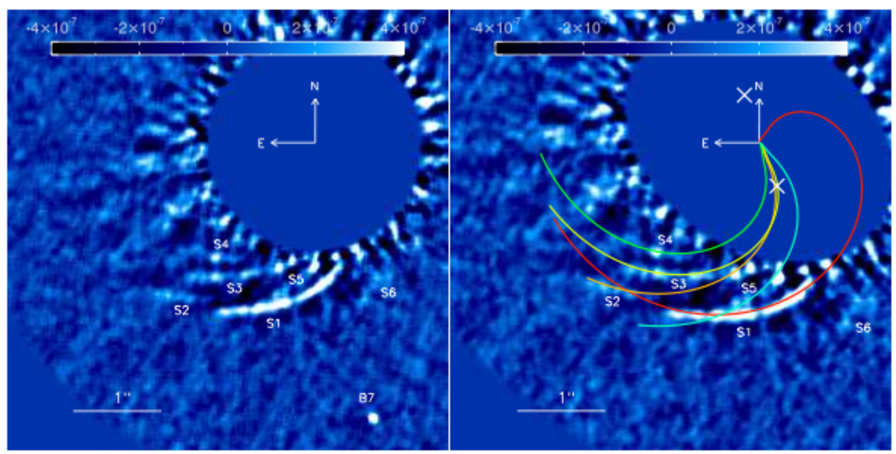
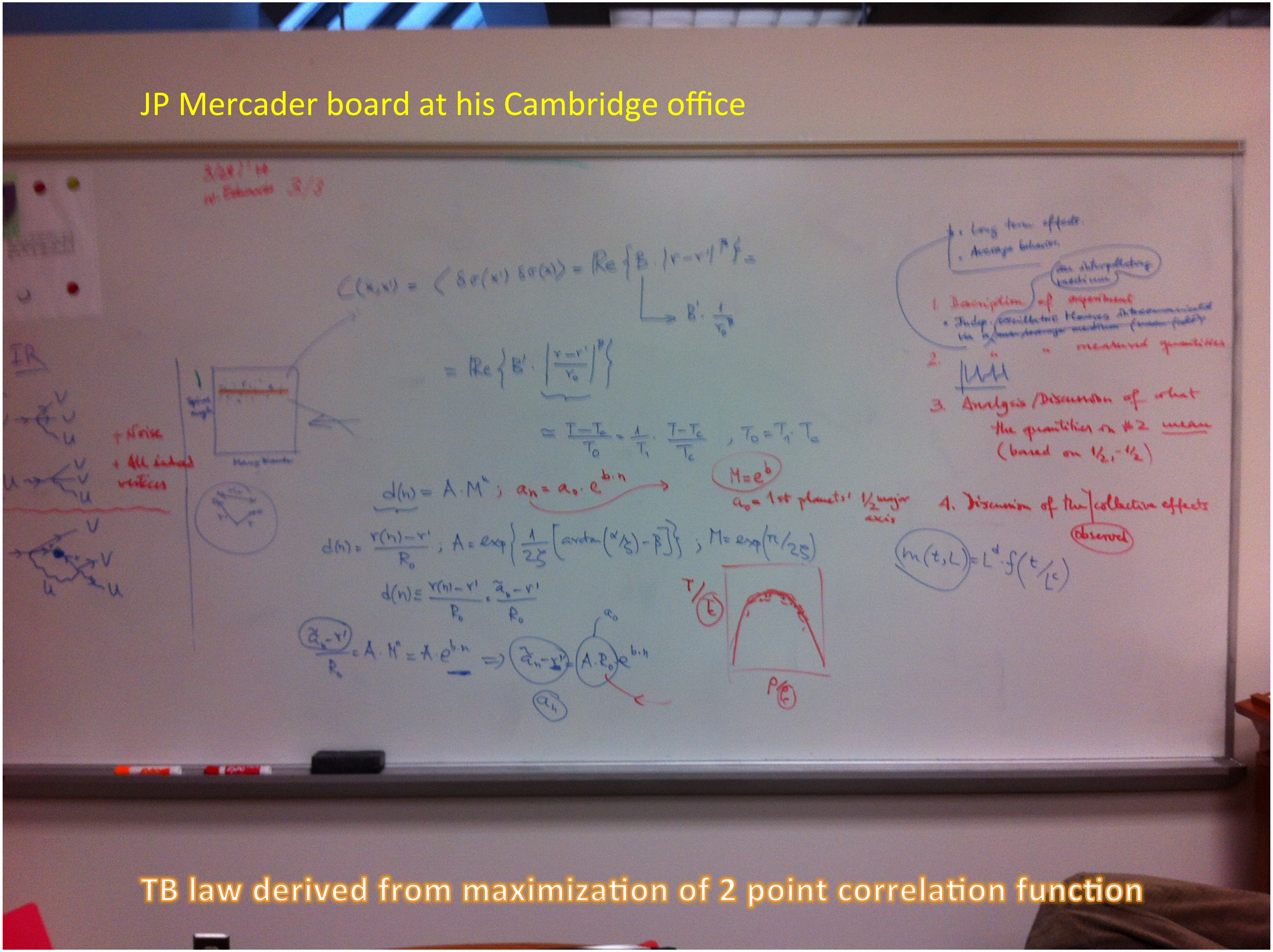


Fig. 5. The southern spiral patterns in the LOCI image, with each pieces being labeled (left), and the corresponding fits with a Muto et al. (2012) model calculated on the deprojected image (right). The positions of the Quanz et al. (2013) candidate forming planet and the southern point-source candidate identified in sec. 3 are indicated by two white crosses on the deprojected image.

Unification of Planet & Satellite Formation

1. Planetary systems around all kind of stars and satellite systems around giant planets had in common a spiral structure of which TB-like spacing in the stable orbits is a fossil remnant.
2. Ubiquitous TB law across more than 3 decades in mass and size suggest a unified hydrodynamical mechanism for all planet sytems and the satellite systems of giant planets.
3. The spiral structure originates from a rotationally driven instability in a protostar that reaches critical state.
4. a is the dilation parameter that provides information about the size of the characteristic size of the system before it becomes unstable
5. b (the exponent in the power-law) gives information about the characteristic of particular phase transitions

JP Mercader board at his Cambridge office



TB law derived from maximization of 2 point correlation function

The 2-Point Correlation Function and its Extreme

Asymptotically the 2-PCF for $r \to \infty$ scales as

$$\langle \delta\rho(r')\delta\rho(r)\rangle \propto |r - r'|^{2\chi}$$

Assume χ **complex** and $\chi = \alpha + i\zeta$.

Then

$$\langle \delta\rho(r')\ \delta\rho(r)\rangle = \mathrm{Re}\,\tilde{c}\,e^{i\beta}\left|\frac{r - r'}{r_0}\right|^{2(\alpha+i\zeta)} =$$

$$c \cdot |r - r'|^{2\alpha} \cdot \cos\left[\beta + 2\zeta \log\left(\frac{|r - r'|}{r_0}\right)\right] .$$

Extremizing 2-point correlations implies

$$|r_{(n)} - r'| = r_0 \cdot \exp\left\{\frac{1}{2\zeta}\left[\tan^{-1}(\alpha/\zeta) - \beta\right]\right\} \cdot (e^{\frac{\pi}{2\zeta}})^n$$

$$\equiv A \cdot b^n$$

with $n = 0, \pm 1, \pm 2 \cdots$.

Remarks

1. At a phase transition, all parts in the system become involved and therefore correlated. This is the origin of “opalescence” and many other phenomena
2. The above implies that the system is scale invariant
3. Scale invariance means that the system’s correlations become power-laws
4. The exponents in the power-laws are related to the critical exponents measured in the phase transition
5. Correlated density enhancements are unavoidable in a dynamical phase transition such as it takes place in the noisy hydrodynamics of a mixed, gravitationally bound fluid

What phase transitions are relevant?

1943 ACTA PHYSICOCHIMICA U.R.S.S. Vol. XVIII, No. 2–3

Letters to the Editor

On the Relation between the Liquid and the Gaseous States of Metals

By *L. Landau* and *J. Zeldovich*

A metal sharply differs from a dielectric with respect to its spectrum of electron energy levels at absolute zero temperature. The fundamental state of the

Strongly coupled H plasma??

In case 3 the rise of temperature within a certain pressure range must be expected to be accompanied by the transition of the liquid metal into a liquid non-conducting phase (on the line *BMD*), which thereafter on the line *BLG* is transformed into a gas. The loss of metallic properties takes

Fig. 1.

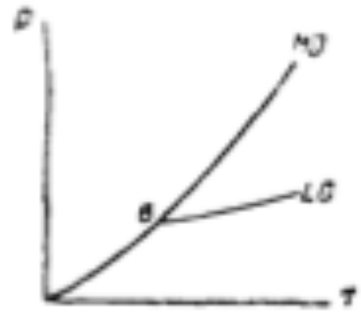


Fig. 2.

place as a phase transition metal-gas also at values of T and p much larger than those which correspond to the critical point liquid-gas. In the two latter cases a triple point T appears corresponding to the co-existence of two metallic and one dielectric phase in case 2 and one metal and two dielectric phases (liquid and gaseous) in the third case.

In the case of mercury the relatively small evaporation heat indicates that *LG* point is relatively low (1000—1500°K according to different estimates), whereas the *MD* point is probably inaccessible experimentally at the present time. There follows from our considerations that here our third case is to be expected. Our physical predictions thus are as follows: 1) there exists a non-conducting liquid phase and 2) at a temperature and pressure lying above the critical values a phase transition with a discontinuous change of the electrical conductivity, volume and other properties must take place.

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Main conclusion

- TB-like relations emerges naturally as the fossil signature of where, due to the intertwined effects of hydro- dynamics, noise and the action of gravity, the density fluctuations in the primeval protoplanetary disc had their maxima as the system underwent a fast phase transition into a coagulated system. Consequently, planet formation by gravitational clustering, coagulation and accretion of disc material occurs with higher probability at discrete distances from the dominant central object, and the final orbits are all correlated with each other.

Explanation of 3 main characteristics of planetary systems (S. Ida´s talk)

- Planetary orbits are mobile
- Planetary systems are ubiquitous
- Planetary systems are diverse

Planetary orbits are mobile

Evolution of planetary orbit likely account for some data spread around a universal T-B law

Planetary systems are ubiquitous

Common hydrodynamics working at the onset of planet forming rapid instability due to a phase change when the protostar reaches critical state

Planetary systems are diverse

Disc forming non-axysymmetric instabilities are very robust and develop under a diverse set of physical conditions such as very different spatial scales and very different gravitational potentials

New idea (S. Ida´s talk + J. Lunine contribution) for origin of Solar System

- 2 disc regions (or belts)
- Inner disc forming rocky planets later than outer disc forming giant planets

Explanation

- Two disc regions (or belts)

Two different phase transitions responsible for two different discs (e.g. 10^4--10^5 K for H, and 670 K for H2O). Boundary region between both transitions defined by asteroid belt.

- Inner disc forming rocky planets later than outer disc forming giant planets

Cool phase transition happens first when SS is large (e.g. due to reaching H2O critical point).

Hot phase transition happens later when protosun has shrunk to a smaller size (e.g. strongly coupled H plasma).