What can be learned from the dynamics of packed planetary systems?

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some Kepler gravitational molecules...

credit: Fabrycky :-)

Monday, 9 June 14
a stable pair of 10 M_j planets

period ratio = 6

e_i = e_o = 0.5
a stable pair of 10 M\textsubscript{j} planets

period ratio = 6

\(e_i = e_o = 0.5\)
an unstable pair of 10 M\textsubscript{j} planets

Period ratio = 6

e\textsubscript{i} = e\textsubscript{o} = 0.505
an unstable pair of 10 M$_j$ planets

Period ratio = 6

e$_i$ = e$_o$ = 0.505
The butterfly effect

Example: yellow: $e_o = 0.505$, aqua: $e_o = 0.505002$;
$\delta e_o / e_o = 4 \times 10^{-6}$
Why is the behaviour so different? There is no smooth transition in parameter space between these two; a system is either stable or it isn't!
1. Newtonian gravity is scale invariant
2. Angles matter
3. Dissipation acts to stabilize systems
4. Resonance protects and destroys

5. TTV teaser...
1. Newtonian gravity is scale invariant
dynamics of Newtonian point-mass systems depends only on mass ratios and length ratios (but timescales are absolute)

$P_{\text{Mercury}} = 88$ days $= P_{\text{Mercury}} = 17$ days

things just take longer to happen in the longer-period system!
dynamics of Newtonian point-mass systems depend only on mass ratios and length ratios (but timescales are absolute)

$$P_{\text{Mercury}} = 3 \text{ days, } a = 0.03 \text{ AU}$$

tides are important at this orbital period and a new length scale is introduced: the radius of the innermost planet
dynamics of Newtonian point-mass systems depend only on mass ratios and length ratios (but timescales are absolute)

\[ P_{\text{Mercury}} = 3 \text{ days}, \ a = 0.03 \text{ AU} \]

Relativistic effects are also important at this orbital period and a new velocity scale is introduced: the speed of light \( c \).
the stellar environment may also introduce a length scale
eg. a short-period system born in a star cluster is less vulnerable to a stellar flyby than a longer-period system
Period-ratio histogram for all Kepler pairs pairwise between all planets in higher order multiples, not just adjacent planets

Fabrycky et al 2012

Most periods < 100 days
The Solar System

\[
\frac{P_{\text{Pluto}}}{P_{\text{Neptune}}} = 1.5 \quad \frac{P_{\text{Neptune}}}{P_{\text{Uranus}}} = 1.96 \quad \frac{P_{\text{Saturn}}}{P_{\text{Jupiter}}} = 2.48
\]

\[
\frac{P_{\text{Jupiter}}}{P_{\text{AB}}} = 2.67 \quad \frac{P_{\text{AB}}}{P_{\text{Mars}}} = 2.34 \quad \frac{P_{\text{Uranus}}}{P_{\text{Saturn}}} = 2.85
\]

\[
\frac{P_{\text{Earth}}}{P_{\text{Venus}}} = 1.63 \quad \frac{P_{\text{Mars}}}{P_{\text{Earth}}} = 1.88 \quad \frac{P_{\text{Venus}}}{P_{\text{Mercury}}} = 2.56
\]
Period-ratio histogram for adjacent HARPS and Coralie pairs

Periods < 11 years

Most Kepler periods < 100 days

Monday, 9 June 14
2. Angles matter!!

guiding principals
Celestial mechanics for coplanar two-planet systems

\[ E_{\text{total}} = E_1 + E_2 + R \]

\[ E_1 = -\frac{G m_1 m_*}{2a_1}, \quad E_2 = -\frac{G m_2 m_*}{2a_2} \]

The ‘disturbing function’ \( R \) is responsible for moving energy and angular momentum between the orbits.
$$E_{\text{total}} = E_1 + E_2 + R$$

Three distinct frequencies:
- two orbital frequencies
- difference in apsidal motion frequencies

Thus expand disturbing function $R$ in triple Fourier series
Harmonic angle is a linear combination of all angles. It can circulate or librate. Almost all circulate.

\[ E_{\text{total}} = E_1 + E_2 + \mathcal{R} \]

\[ \mathcal{R} = \sum_{mnn'} f\left(\frac{m_1}{m_*}, \frac{m_2}{m_*}, \frac{a_2}{a_1}, e_1, e_2\right) \cos \phi_{mnn'} \]

\[ \phi_{mnn'} = nM_1 - n'M_2 + m(\omega_1 - \omega_2) \]

Here, \( M_i \) are the mean anomalies, and \( \nu_i(t - T_p) \) is a term involving time and a constant. The harmonic angle \( \phi_{mnn'} \) is a linear combination of all angles.

It can circulate or librate.
Resonance occurs when an angle librates

Resonance is Nature’s way of moving energy around.

One of the angles governing the 2:1 resonance is

\[ \phi_{212} = \lambda_1 - 2\lambda_2 + \omega_1 \]

It’s rate of change is

\[ \dot{\phi}_{212} = \nu_1 - 2\nu_2 + \dot{\omega}_1 \]

when \( \nu_1 \approx 2\nu_2 \) we have resonance.
The average value of a circulating harmonic is zero. The average value of a librating harmonic is NOT zero.

Thus resonant terms are important for the transfer of energy between orbits.

When the resonant rate of transfer of energy between orbits is greater than the migration rate of energy transfer between the disk and the orbits, RESONANCE CAPTURE occurs and the planets move together.
If a librating angle is forced to circulate by a neighbouring harmonic, chaos ensues. This normally results in the escape of one of the bodies, or perhaps collision if the finite size of the bodies is bigger than the distance of closest approach.
Secular evolution of stable coplanar systems involves the slow rotation of the orbits (apsidal motion) accompanied by periodic variations of the eccentricities.

It is governed by the angles

\[ \phi_{m00} = m(\varpi_1 - \varpi_2) \]
Secular evolution: Jupiter and Saturn

eccentricities

\[ \omega_J - \omega_S \]

\[ \omega_J (0) = 14.7^\circ, \quad \omega_S (0) = 92.4^\circ; \quad \Delta e_S \approx 0.06 \]

\[ \omega = \text{longitude of periastron} \]
Secular evolution: Jupiter and Saturn

eccentricities

\( \varpi_J - \varpi_S \)

\[ \varpi_J(0) = 0, \quad \varpi_S(0) = 0; \quad \Delta e_S \approx 0.04 \]
Secular evolution: Jupiter and Saturn

\[ e_J = 0.048, \quad e_S = 0.054 \]

eccentricities

\[ \varpi_J - \varpi_S \]

\[ \varpi_J(0) = 0, \quad \varpi_S(0) = 180; \quad \Delta e_S \simeq 0.02 \]

\[ \Delta(\varpi_J - \varpi_S) \simeq 40^\circ \]
Jupiter and Saturn are safe for all relative orientations (in the plane) and orbital phases.

But this is not true if we increase Saturn's eccentricity to, say 0.25...
All calm on the Western front...

\[ e_J = 0.048, \quad e_S = 0.25 \]

\[ \varpi_J(0) = 0, \quad \varpi_S = 180^\circ \]
All calm on the Western front...

$e_J = 0.048$, $e_S = 0.25$

$\omega_J(0) = 0$, $\omega_S = 0$
Not so calm on the Western front...

\[ e_J = 0.048, \quad e_S = 0.25 \]

\[ \varpi_J(0) = 14.7^\circ, \quad \varpi_S(0) = 92.4^\circ; \]
Not so calm on the Western front...

\[ e_J = 0.048, \ e_S = 0.25 \]
Lessons learned:

• Extent of eccentricity variation depends on initial relative orientation

• Stability study: start with $\omega_1 - \omega_2 = 0$ (or $\pi$); if this is unstable, all others will be

• Planets do not need close encounters to be unstable: Hill radius concept not always adequate
guiding principals

3. Dissipation acts to stabilize systems
Secular evolution with dissipation
   → relaxation and stabilization
Tidal dissipation: HAT-P-13

\[ P_b = 4 \text{ days} \]
\[ P_c = 429 \text{ days} \]
\[ m_b = 0.85M_J \]
\[ m_c \sin i = 15M_J \]
\[ e_c = 0.7 \]
\[ e_b^{(\text{observed})} = 0.02 \]
\[ e_b^{(\text{fixed point})} = \frac{(5/4) (a_b/a_c) e_c}{1 - \sqrt{\alpha} (m_b/m_c)} + \gamma \]

contains information about planet structure via "Love number" \( k_b \)

Putting \( e_b^{(\text{fixed point})} = e_b^{(\text{observed})} \) gives \( k_b \) !!

Mardling (2007)
Wu & Goldreich (2002)

Batygin et al (2009)
Secular evolution with dissipation
→ relaxation and stabilization

Tidal dissipation: Jupiter and Saturn
at 0.03 AU and 0.05 AU

eccentricities

\( \omega_J - \omega_S \)
\[ t \to \infty : \text{eccentricities are not zero due to planet-planet interactions} \]

\[ \text{eccentricities} \]

``forced'' eccentricities: \( \varepsilon_J \propto \frac{m_S}{m_\odot}, \varepsilon_S \propto \frac{m_J}{m_\odot} \)

``free'' eccentricities are damped to zero

the "ground state"
Another source of eccentricity damping is the protoplanetary disk at the time of formation.

Many Kepler pairs appear to be totally relaxed — they are almost perfectly coplanar and their eccentricities are their forced eccentricities.

How do we know this? Using TTVs — see: Lithwick, Xi & Wu (2012) and Wu & Lithwick (2012)

Beautiful!!

for system near 2:1 resonance:

\[ e_1 \propto \frac{m_2/m_*}{|P_2/P_1 - 2|}, \quad e_2 \propto \frac{m_1/m_*}{|P_2/P_1 - 2|} \]
4. Resonance protects and destroys
The stability boundary

$m_2 = m_3 = 0.01, \ e_i(0) = 0.5$
The stability boundary

$m_2 = m_3 = 0.01, \ e_i(0) = 0.5$

Mardling 2008, 2013
TTV teaser: a circumbinary planet...

\[ e_1 = 0.2, \quad e_2 = 0.1 \quad a_1 = 0.1 \text{ au}, \quad P_2/P_1 = 7.4 \]

\[ m_2/m_1 = 0.1, \quad m_p/(m_1 + m_2) = 0.001 \]