What can be learned from the dynamics of packed planetary systems?

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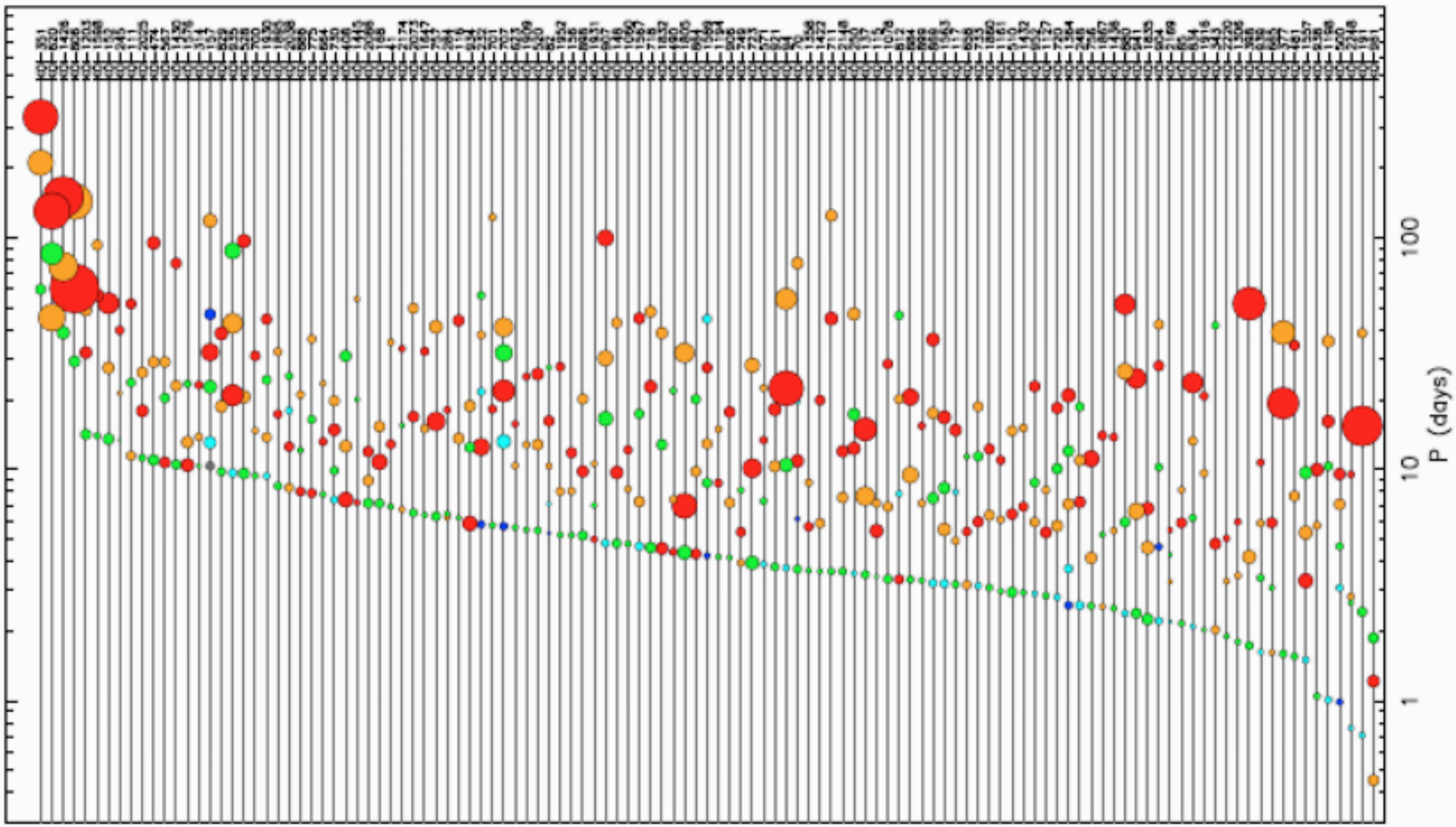
some Kepler gravitational molecules...

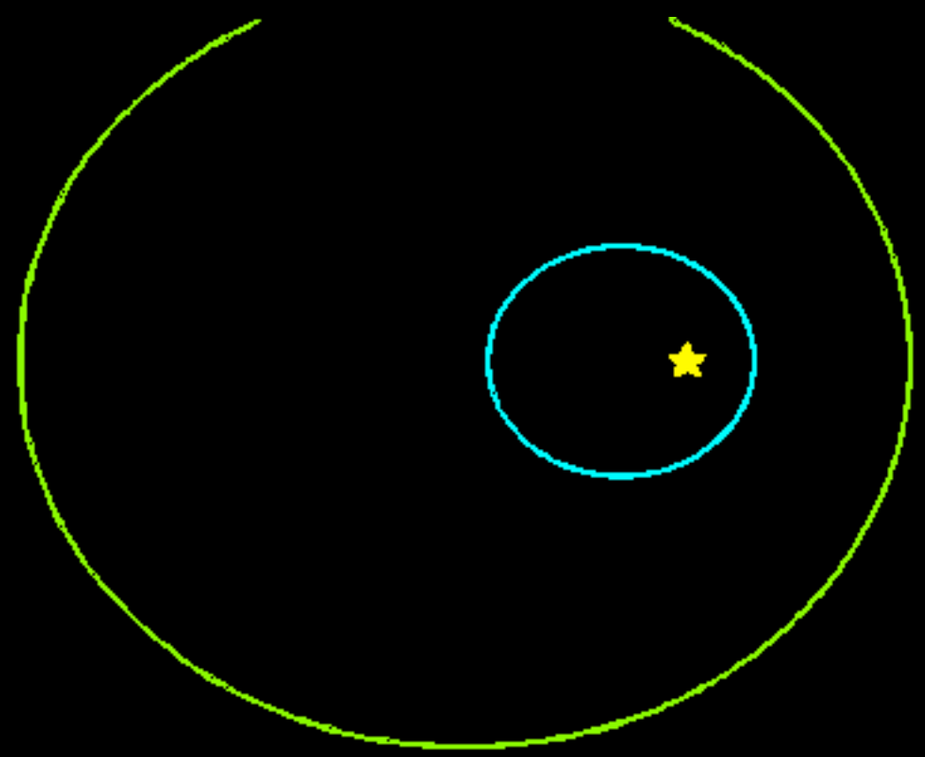


credit: Fabrycky :-)

a stable pair of 10 M_J planets

period ratio = 6

$e_i = e_o = 0.5$



a stable pair of 10 M_J planets

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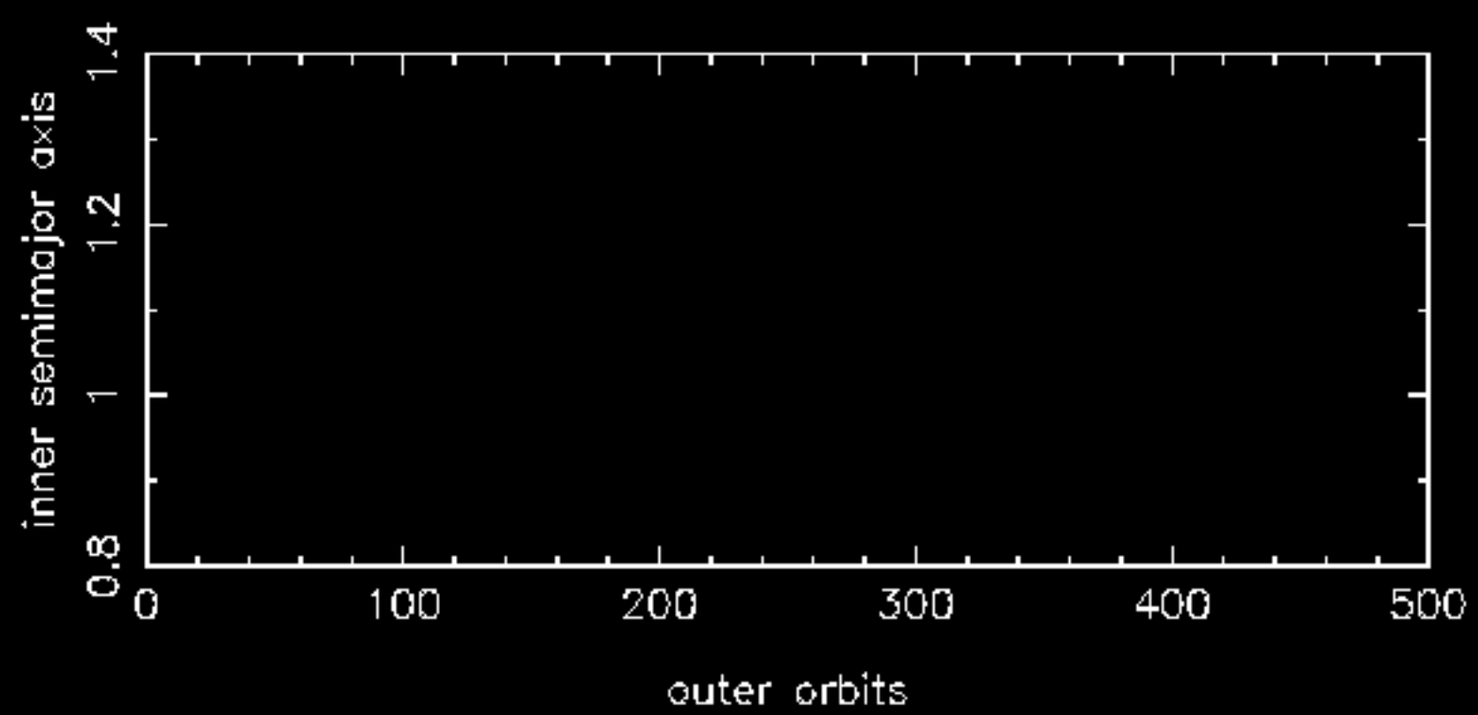
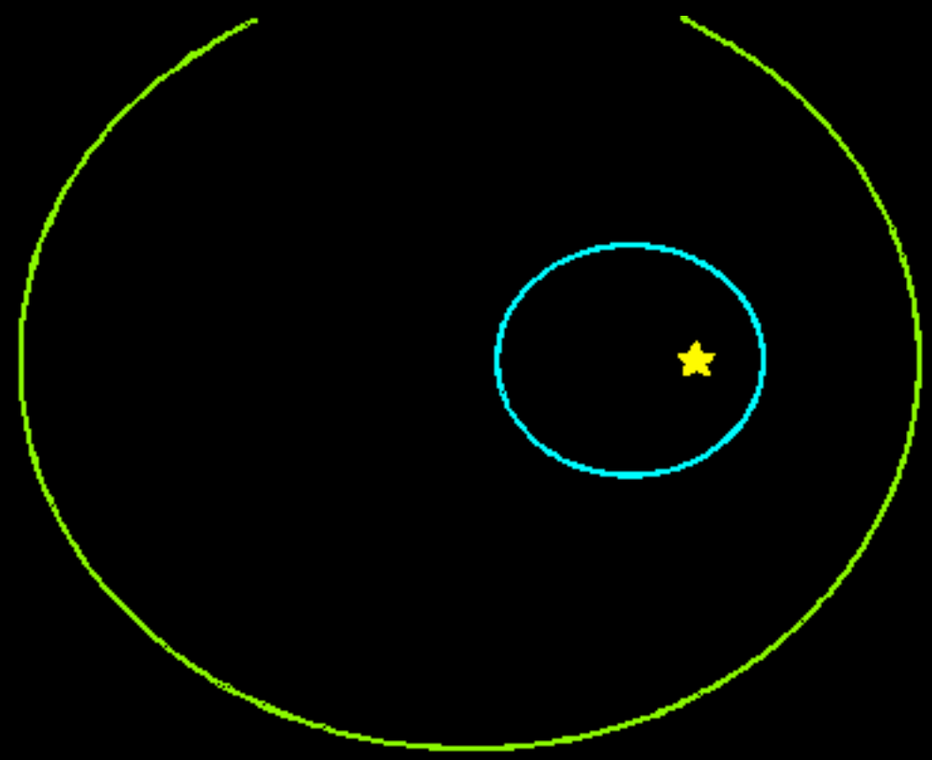


an unstable pair of 10 M_J planets

Period ratio = 6

$e_i = e_o = 0.505$



an unstable pair of 10 M_J planets

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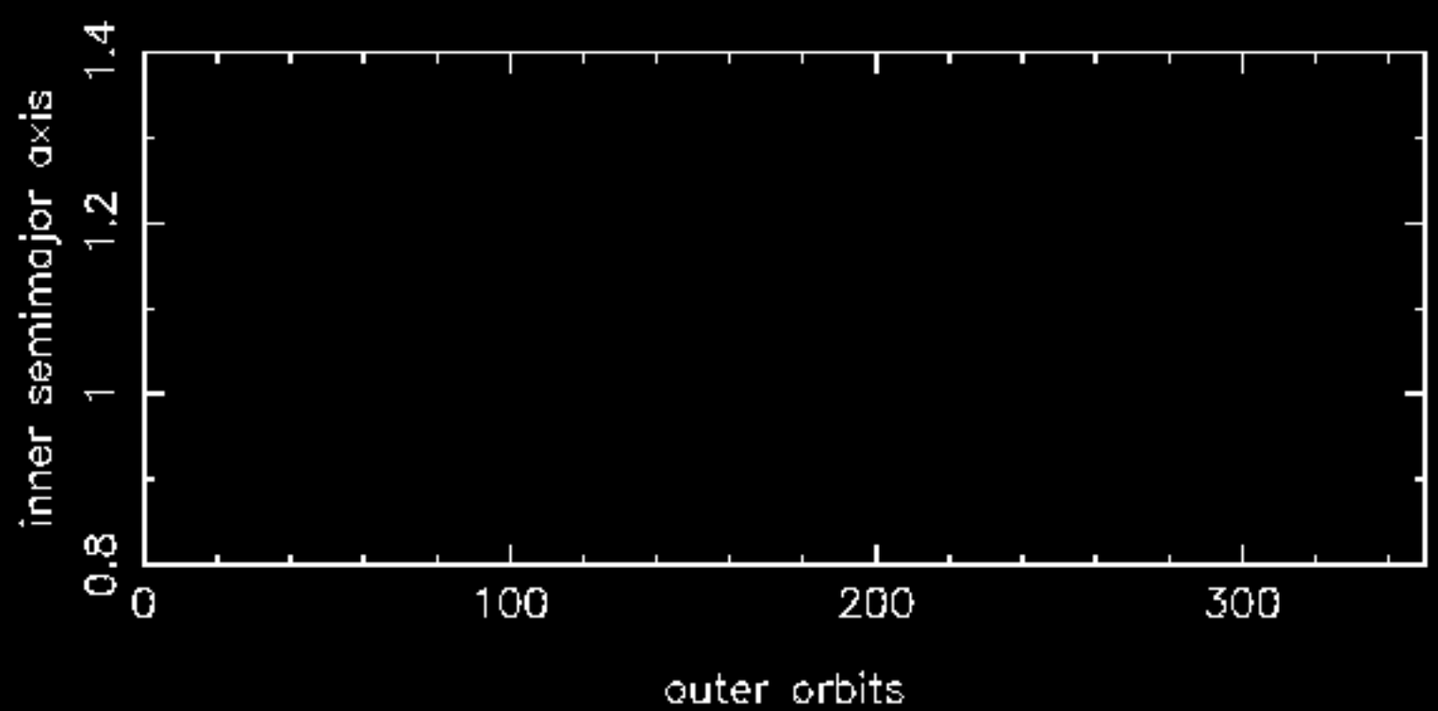
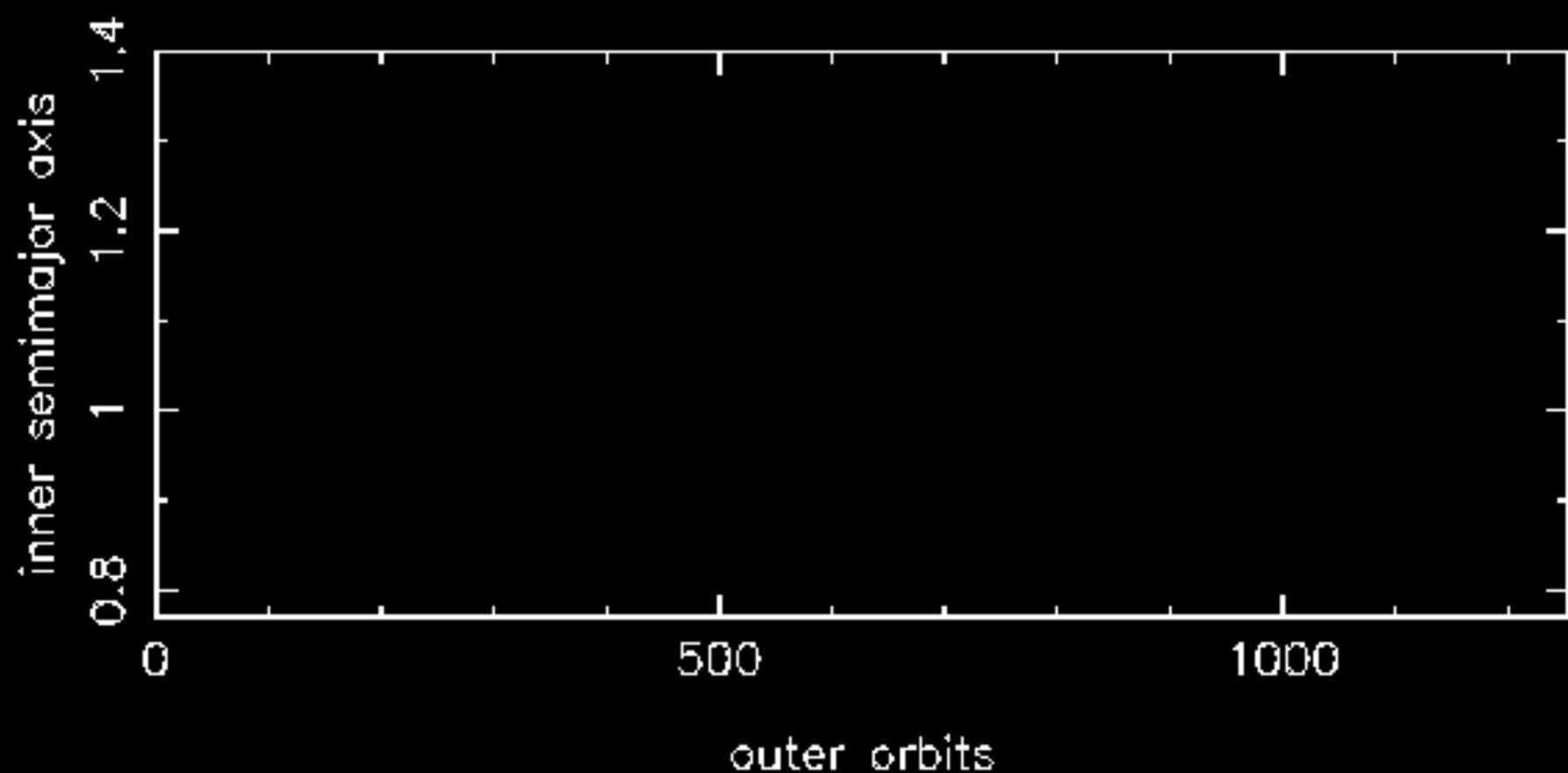
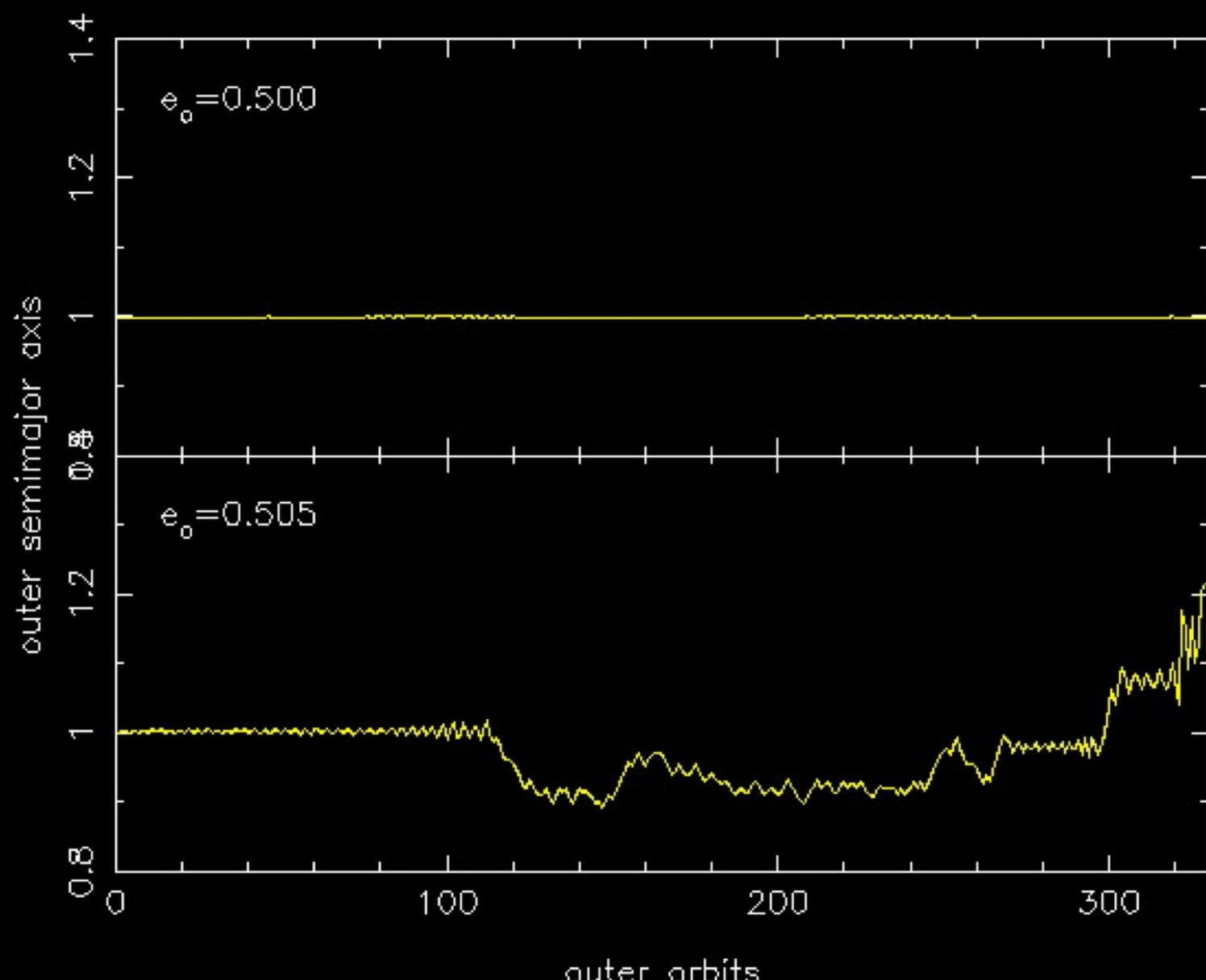


The butterfly effect

Example: yellow: $e_o = 0.505$, aqua: $e_o = 0.505002$;
$\delta e_o / e_o = 4 \times 10^{-6}$



Why is the behaviour so different? There is no smooth transition in parameter space between these two; a system is either stable or it isn't!



guiding principals

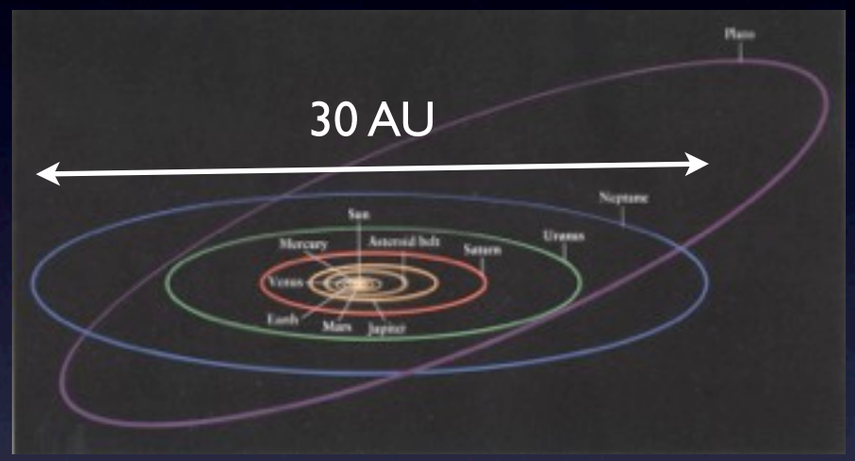
1. Newtonian gravity is scale invariant
2. Angles matter
3. Dissipation acts to stabilize systems
4. Resonance protects and destroys

5. TTV teaser...

guiding principals

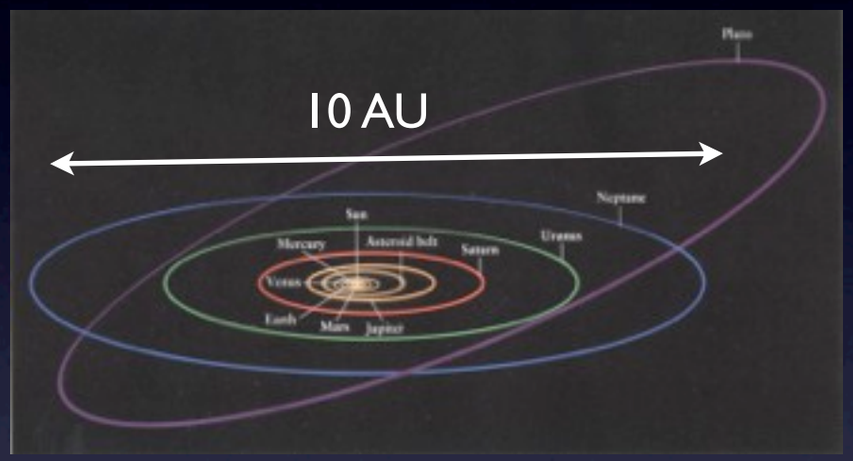
1. Newtonian gravity is scale invariant

dynamics of Newtonian point-mass systems depends only on mass ratios and length ratios (but timescales are absolute)



$P_{\mathrm{Mercury}} = 88\,\mathrm{days}$

=

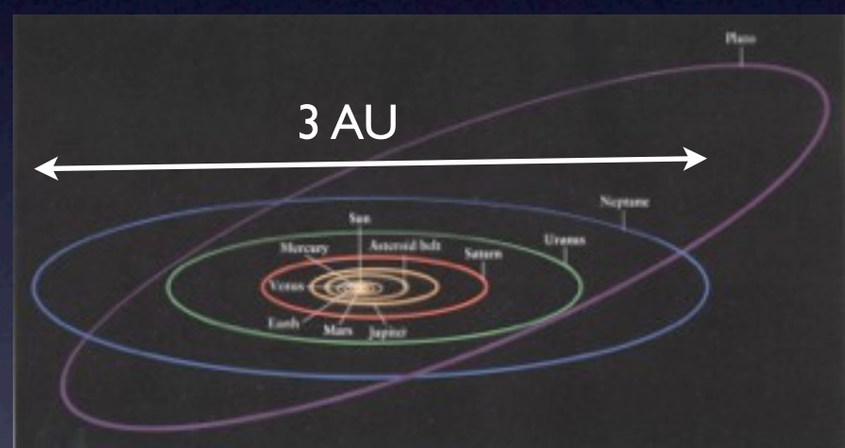


$P_{\mathrm{Mercury}} = 17\,\mathrm{days}$

things just take longer to happen in the longer-period system!

dynamics of Newtonian point-mass systems depend only on mass ratios and length ratios (but timescales are absolute)

$\neq$

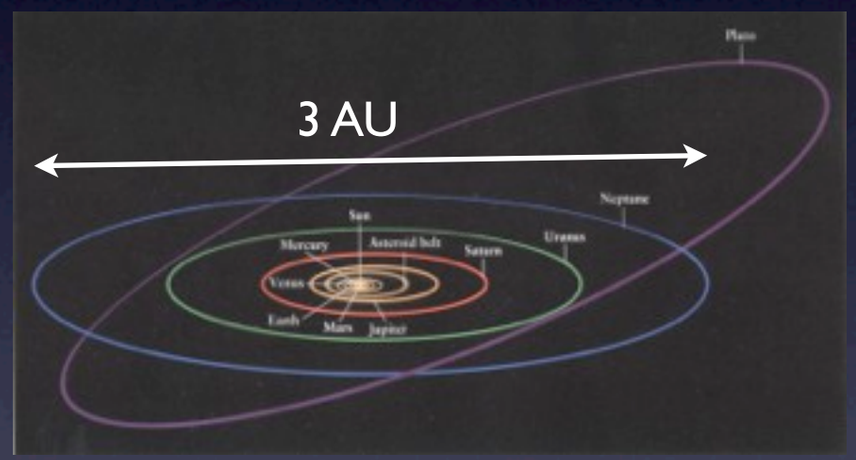


$P_{\rm Mercury} = 3\,{\rm days},\ \ a = 0.03\,{\rm AU}$

tides are important at this orbital period and a new length scale is introduced: the radius of the innermost planet

dynamics of Newtonian point-mass systems depend only on mass ratios and length ratios (but timescales are absolute)

$\neq$



$P_{\mathrm{Mercury}} = 3\,\mathrm{days},\ \ a = 0.03\,\mathrm{AU}$

Relativistic effects are also important at this orbital period and a new velocity scale is introduced: the speed of light c

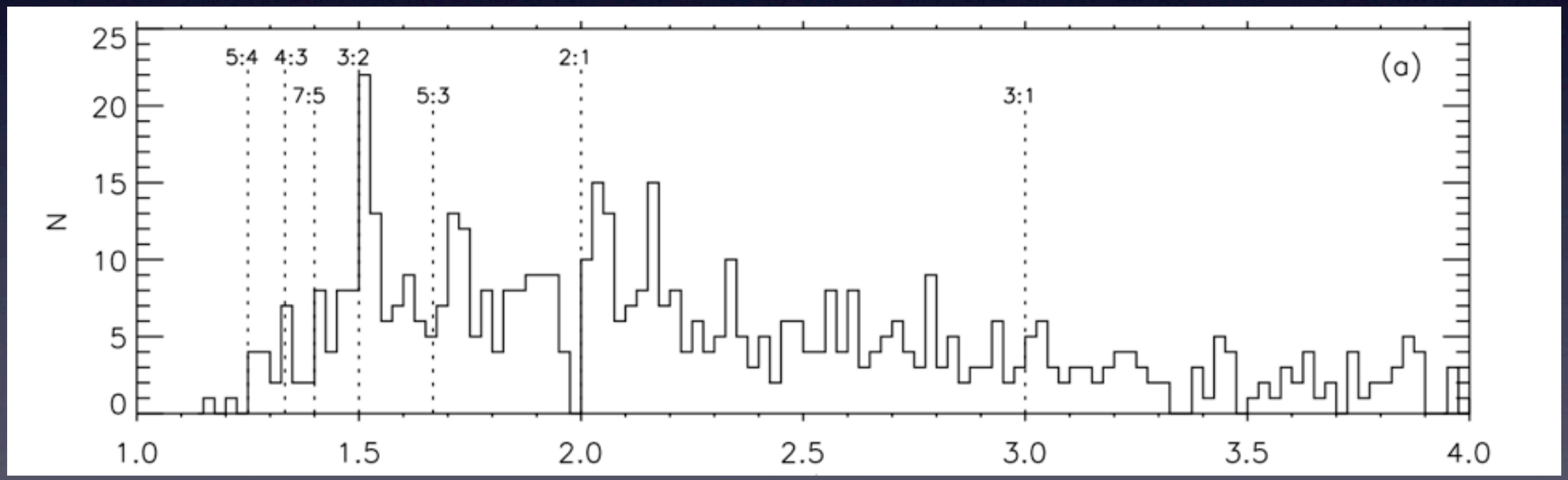
the stellar environment may also introduce a length scale

eg. a short-period system born in a star cluster is less vulnerable to a stellar flyby than a longer-period system

Period-ratio histogram for all Kepler pairs

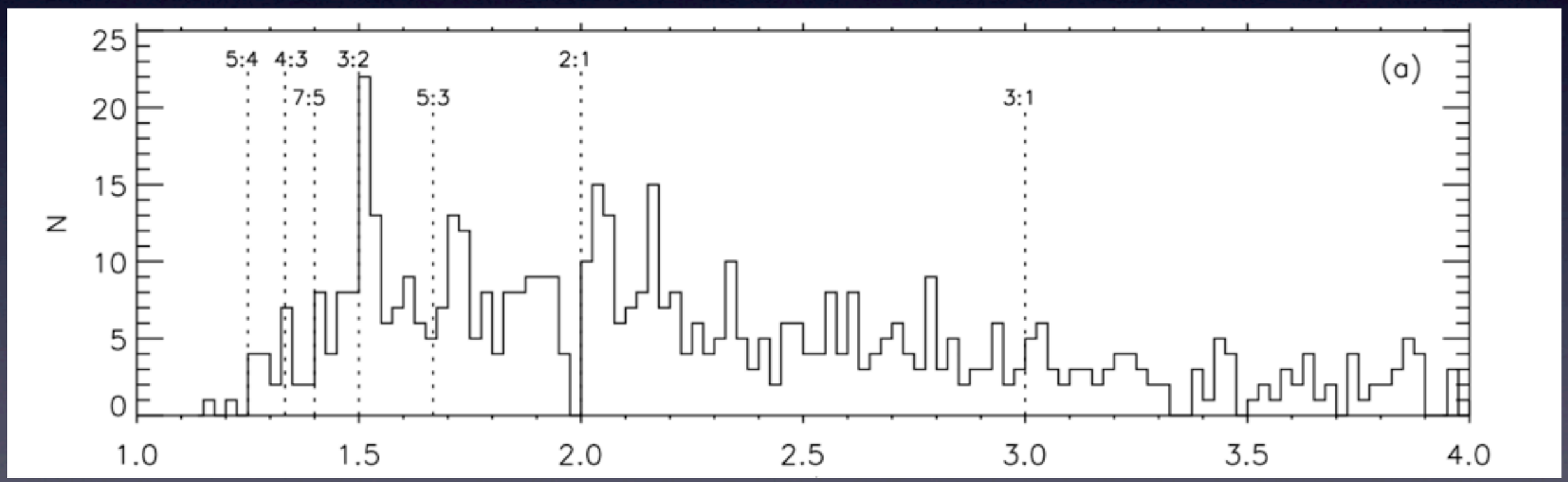
pairwise between all planets in higher order multiples, not just adjacent planets

Fabrycky et al 2012



Most periods < 100 days

The Solar System



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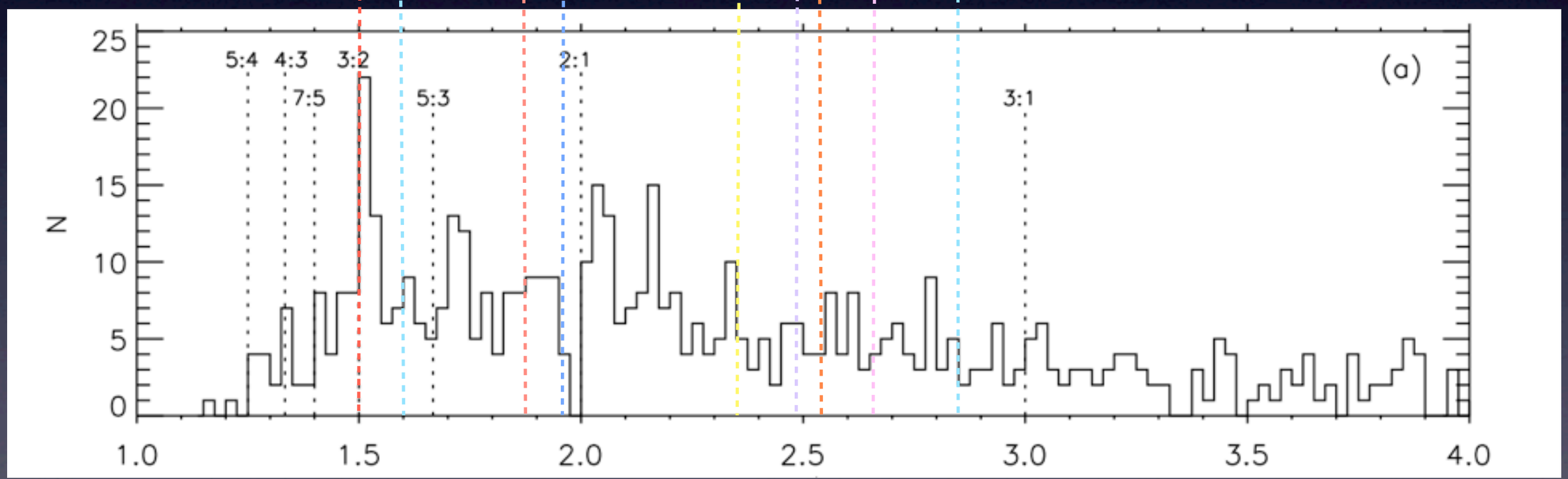
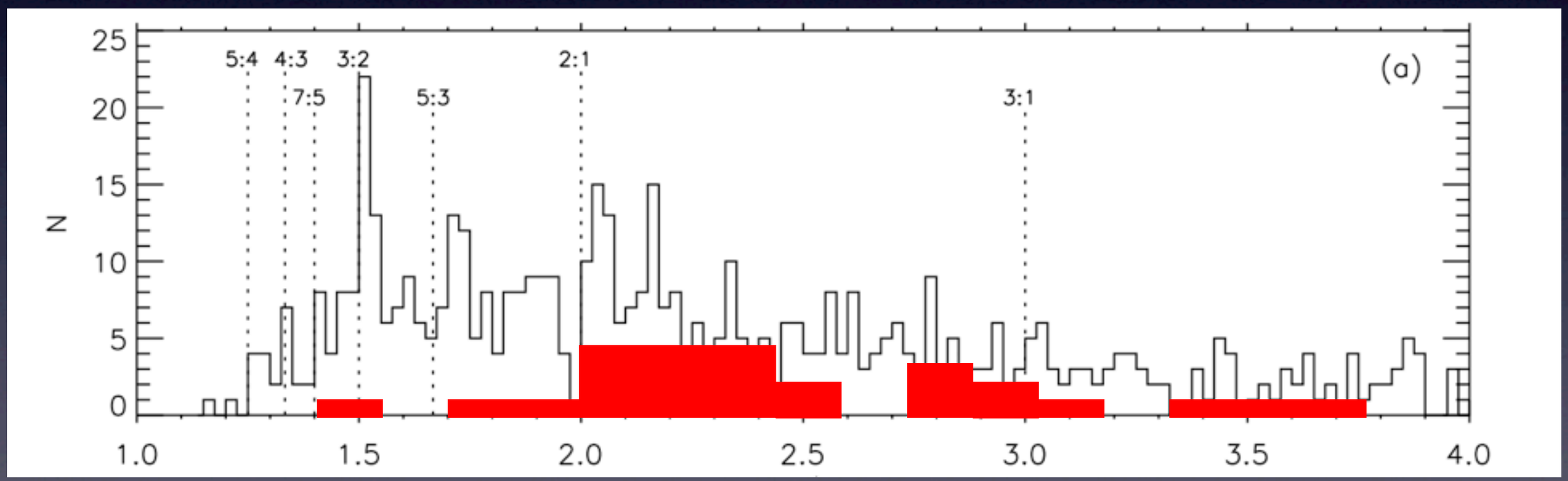
The Solar System



Fabrycky et al 2012

Period-ratio histogram for adjacent HARPS and Coralie pairs

Periods < 11 years



Most Kepler periods < 100 days

guiding principals

2. Angles matter!!

Celestial mechanics for coplanar two-planet systems

$$E_{\text{total}} = E_1 + E_2 + \mathcal{R}$$

$$E_1 = -\frac{Gm_1 m_*}{2a_1}, \quad E_2 = -\frac{Gm_2 m_*}{2a_2}$$

The 'disturbing function' $\mathcal{R}$ is responsible for moving energy and angular momentum between the orbits

$$E_{\text{total}} = E_1 + E_2 + \mathcal{R}$$

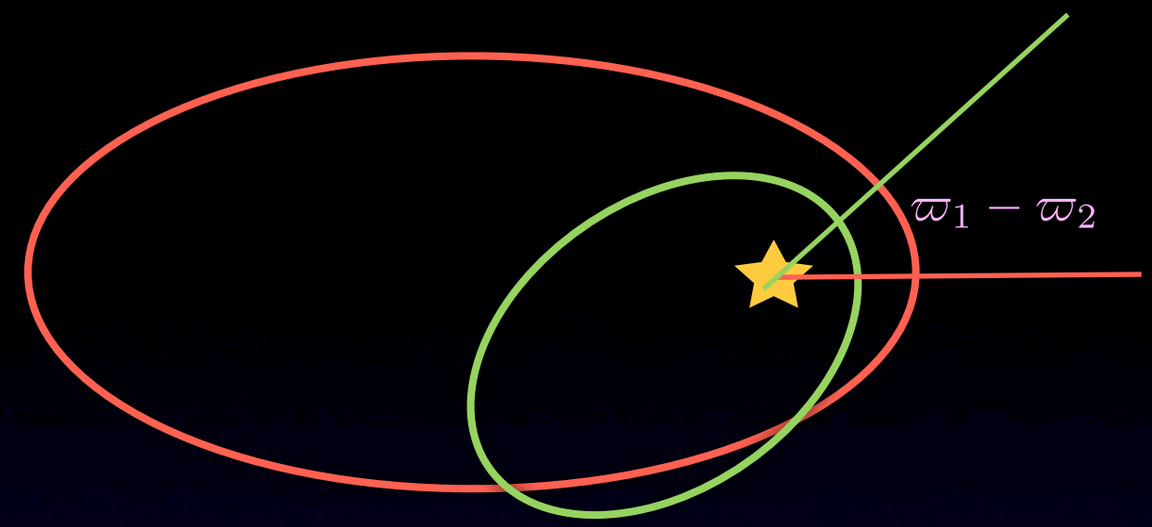
Three distinct frequencies:

two orbital frequencies

difference in apsidal motion frequencies

Thus expand disturbing function $\mathcal{R}$ in triple Fourier series

$$E_{\text{total}} = E_1 + E_2 + \mathcal{R}$$



$$\mathcal{R} = \sum_{mnn'} f(m_1/m_*, m_2/m_*, a_2/a_1, e_1, e_2) \cos \phi_{mnn'}$$

$$\phi_{mnn'} = nM_1 - n'M_2 + m(\varpi_1 - \varpi_2)$$

M_i = mean anomalies

$= \nu_i(t - T_p)$

Harmonic angle $\phi_{mnn'}$ is a linear combination of all angles.

It can circulate or librate.
Almost all circulate.

Resonance occurs when an angle librates

Resonance is Nature's way of moving energy around.

One of the angles governing the 2:1 resonance is $\phi_{212} = \lambda_1 - 2\lambda_2 + \varpi_1$

It's rate of change is $\dot{\phi}_{212} = \nu_1 - 2\nu_2 + \dot{\varpi}_1$

when $\nu_1 \simeq 2\nu_2$ we have resonance.

The average value of a circulating harmonic is zero.

The average value of a librating harmonic is NOT zero.

Thus resonant terms are important for the transfer of energy between orbits.

When the resonant rate of transfer of energy between orbits is greater than the migration rate of energy transfer between the disk and the orbits, RESONANCE CAPTURE occurs and the planets move together

If a librating angle is forced to circulate by a neighbouring harmonic, chaos ensues.

This normally results in the escape of one of the bodies, or perhaps collision if the finite size of the bodies is bigger than the distance of closest approach.

Secular evolution of stable coplanar systems involves the slow rotation of the orbits (apsidal motion) accompanied by periodic variations of the eccentricities.

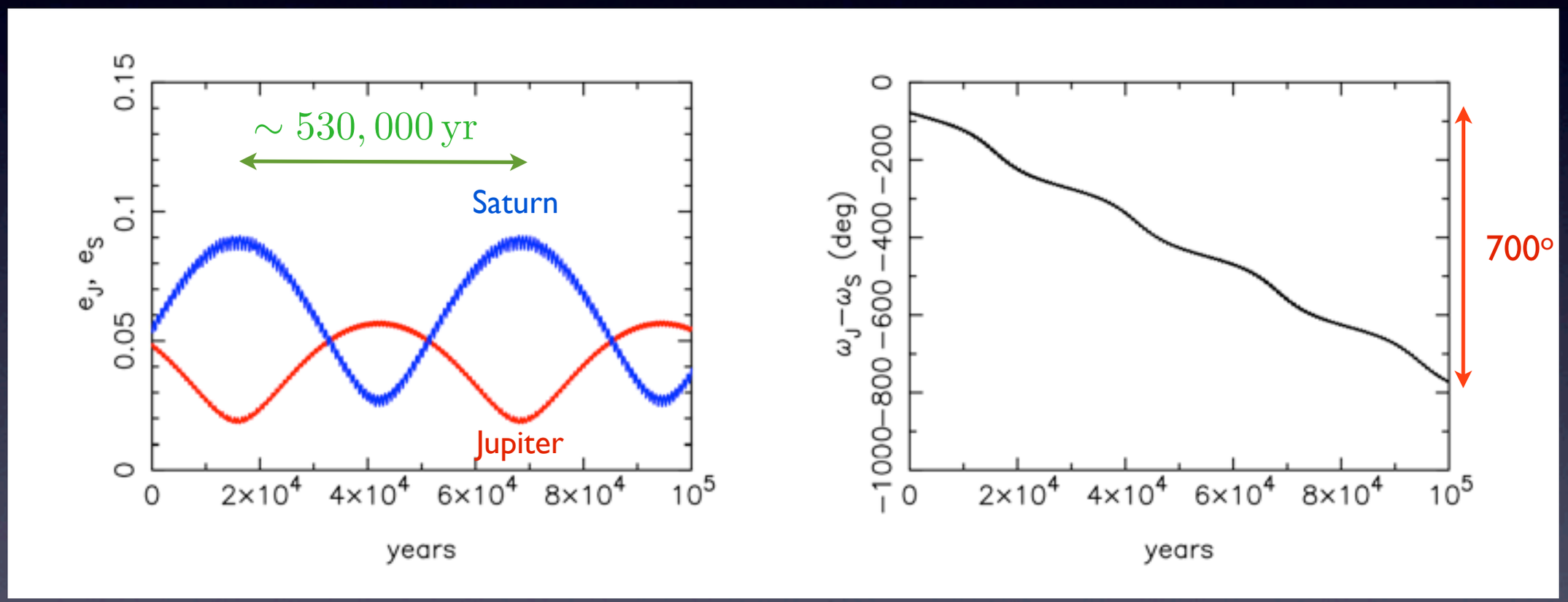
It is governed by the angles

$$\phi_{m00} = m(\varpi_1 - \varpi_2)$$

Secular evolution: Jupiter and Saturn

eccentricities

$\varpi_J - \varpi_S$

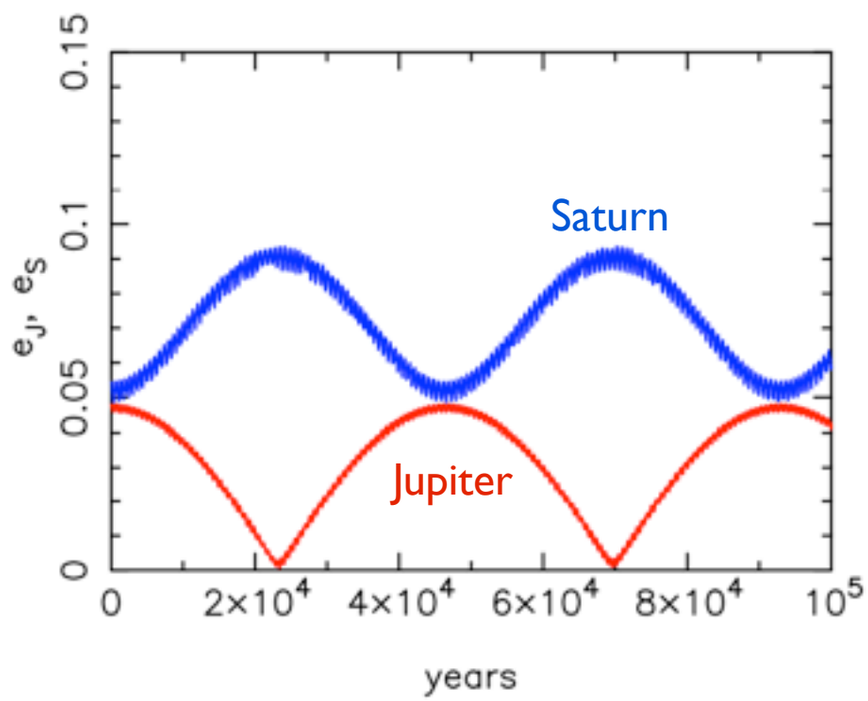


$$\varpi_J(0) = 14.7^o, \quad \varpi_S(0) = 92.4^o; \qquad \Delta e_S \simeq 0.06$$

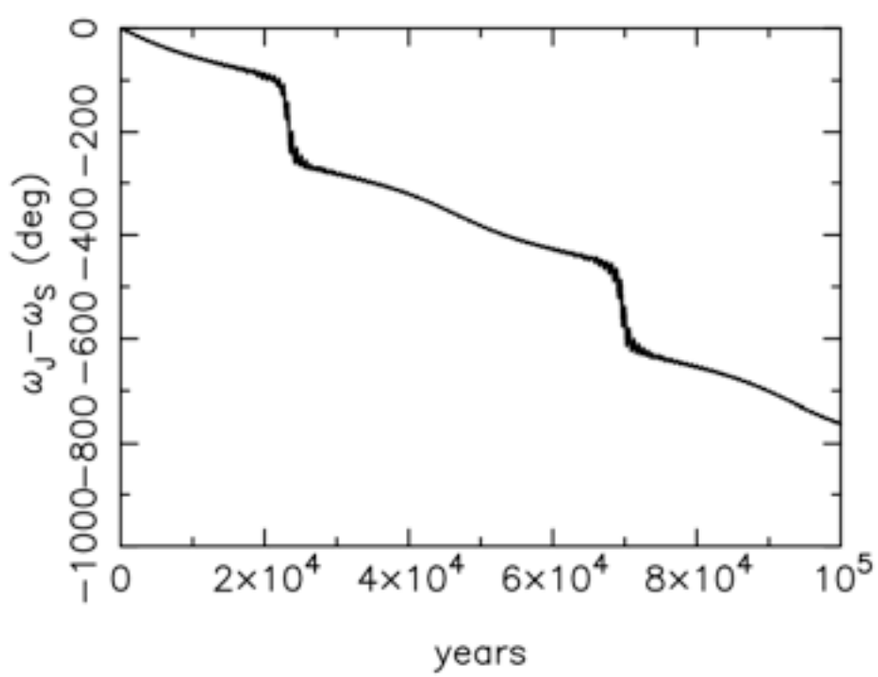
ϖ = longitude of periastron

Secular evolution: Jupiter and Saturn

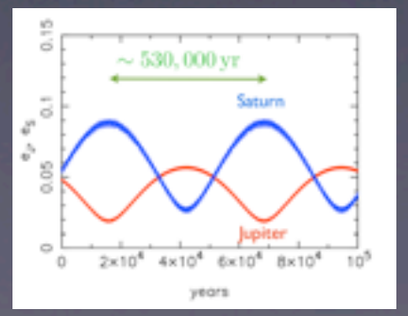
eccentricities



$\varpi_J - \varpi_S$



$$\varpi_J(0) = 0, \quad \varpi_S(0) = 0; \qquad \Delta e_S \simeq 0.04$$

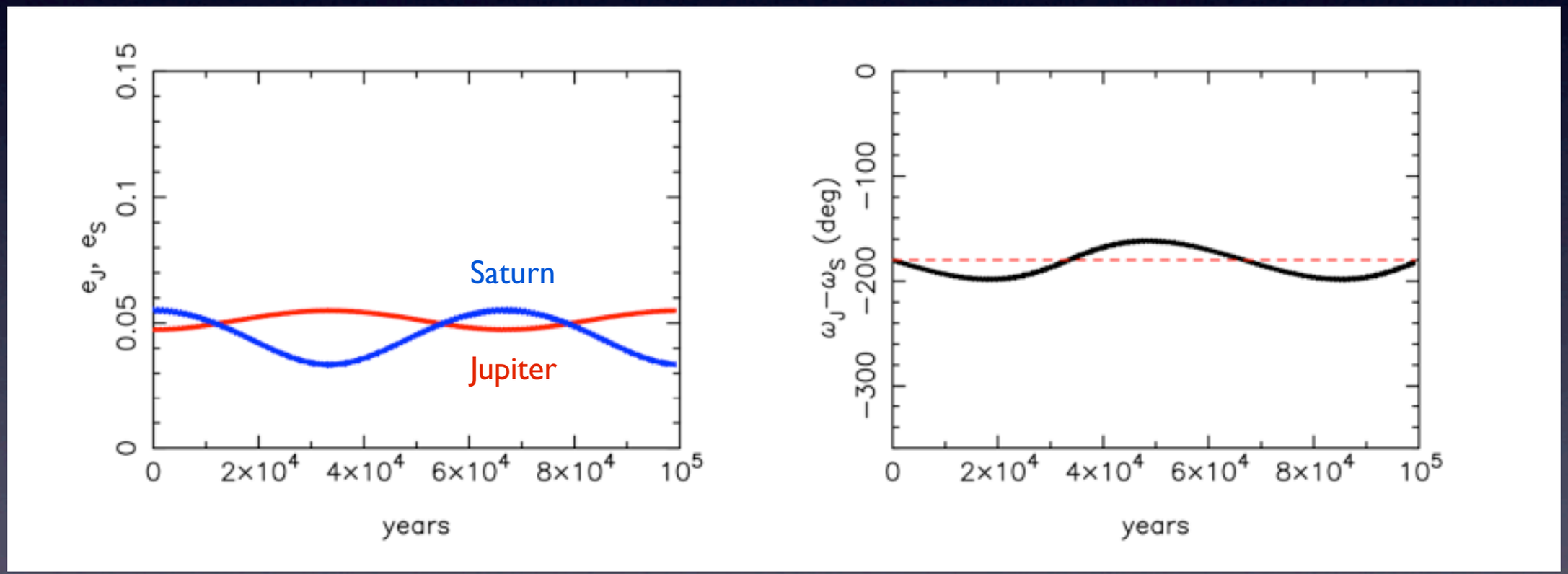


Secular evolution: Jupiter and Saturn

$e_J = 0.048,\ e_S = 0.054$

eccentricities

$\varpi_J - \varpi_S$



$\varpi_J(0) = 0,\ \ \varpi_S(0) = 180;$

$\Delta e_S \simeq 0.02$

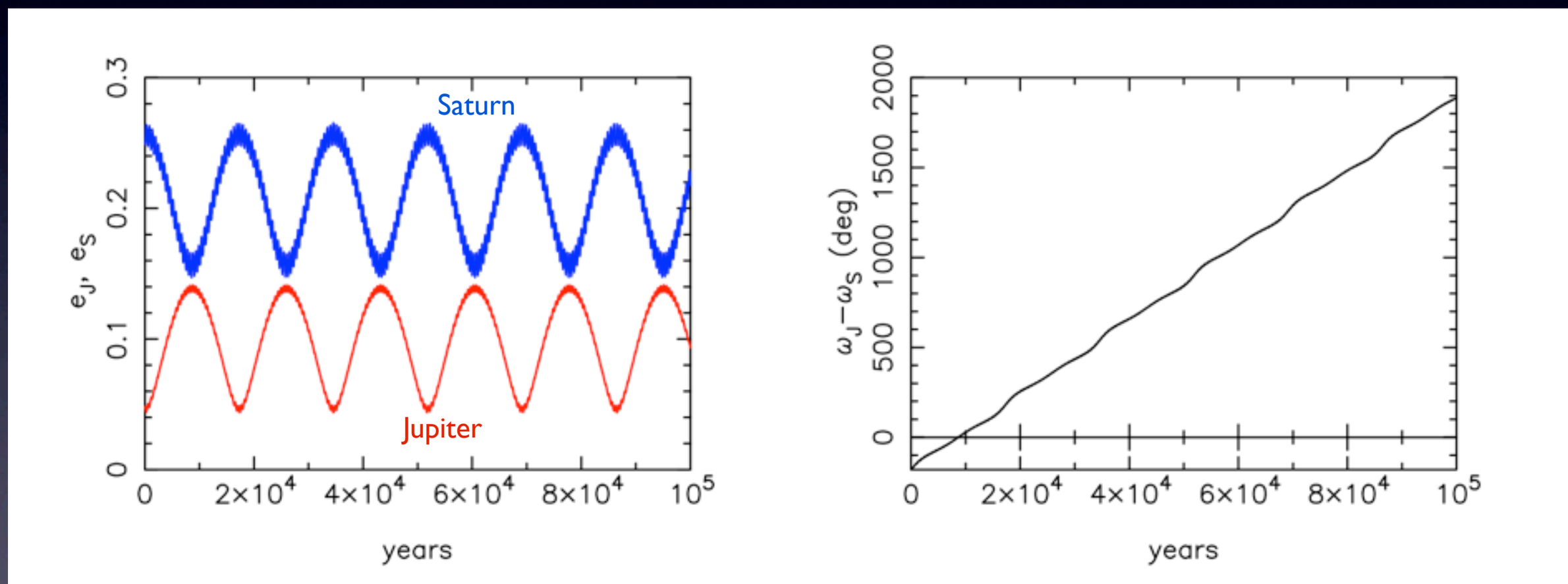
$\Delta(\varpi_J - \varpi_S) \simeq 40^o$

Jupiter and Saturn are safe for all relative orientations (in the plane) and orbital phases.

But this is not true if we increase Saturn's eccentricity to, say 0.25...

All calm on the Western front...

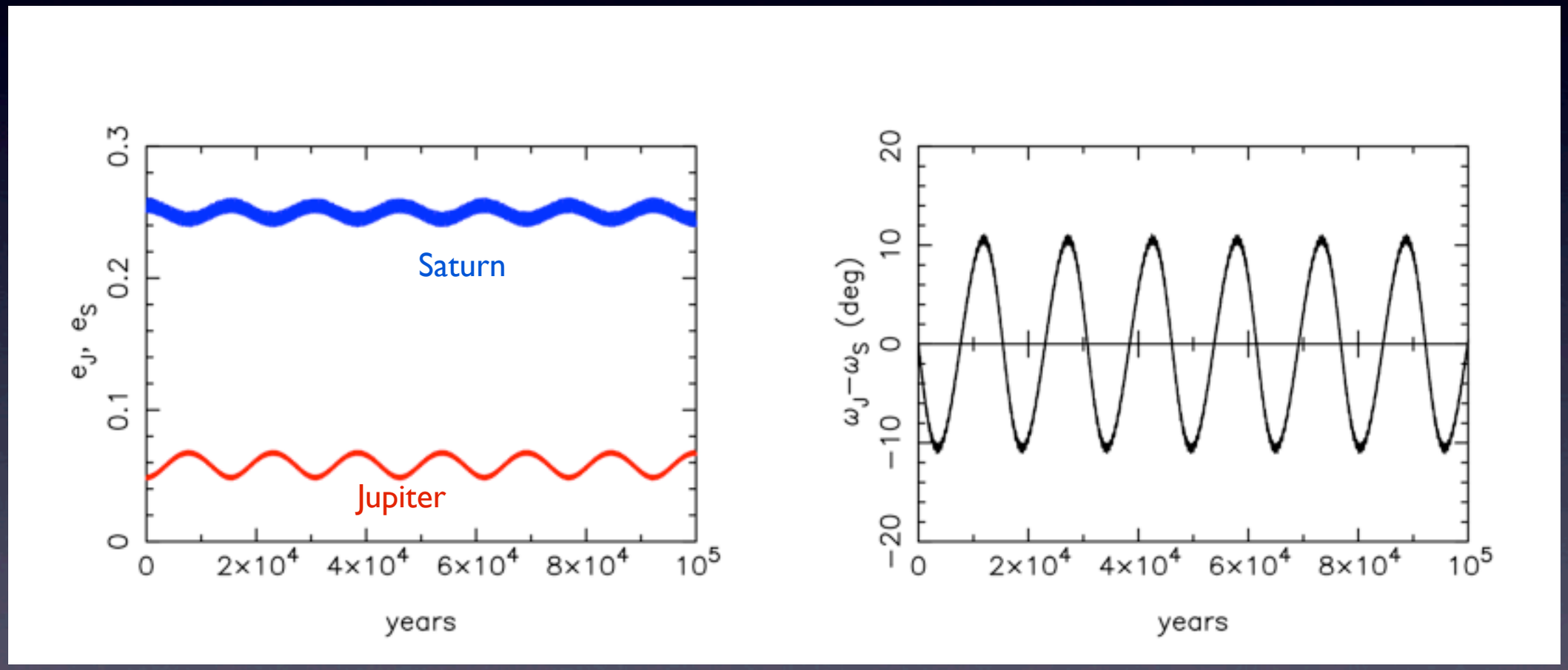
$e_J = 0.048,\ e_S = 0.25$



$$\varpi_J(0) = 0,\ \ \varpi_S = 180^o$$

All calm on the Western front...

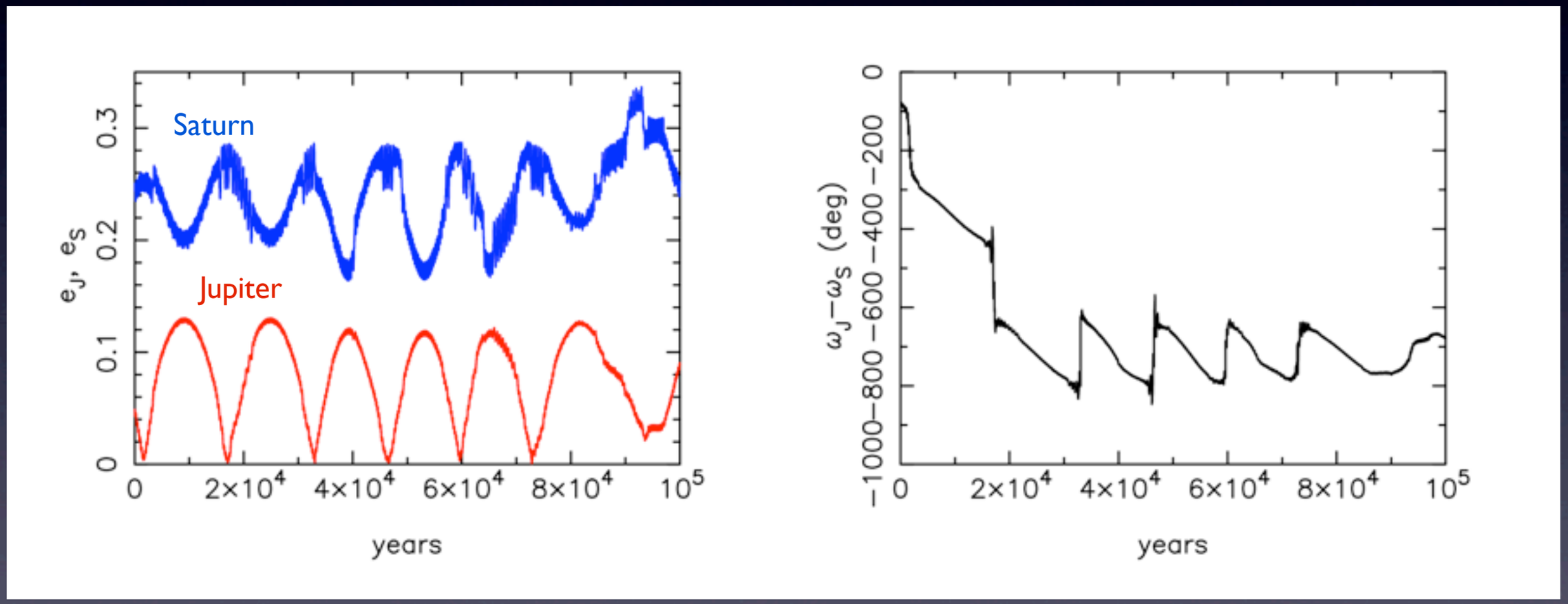
$e_J = 0.048,\ e_S = 0.25$



$$\varpi_J(0) = 0, \quad \varpi_S = 0$$

Not so calm on the Western front...

$e_J = 0.048,\ e_S = 0.25$

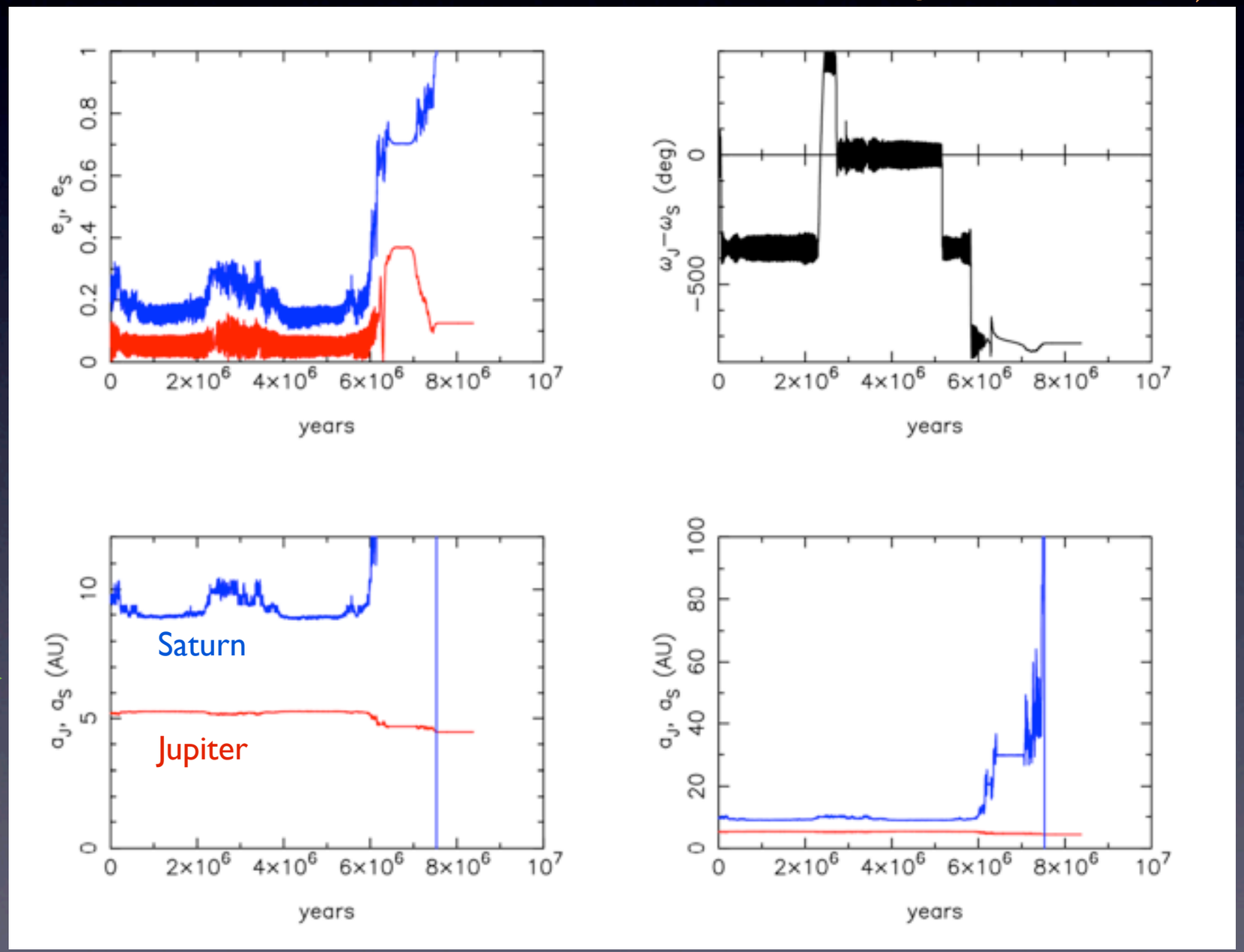


$$\varpi_J(0) = 14.7^o,\ \ \varpi_S(0) = 92.4^o;$$

Not so calm on the Western front...

$e_J = 0.048,\ e_S = 0.25$

no close encounter



Lessons learned:

- Extent of eccentricity variation depends on initial relative orientation

- Stability study: start with $\varpi_1 - \varpi_2 = 0$ (or π); if this is unstable, all others will be

- Planets do not need close encounters to be unstable: Hill radius concept not always adequate

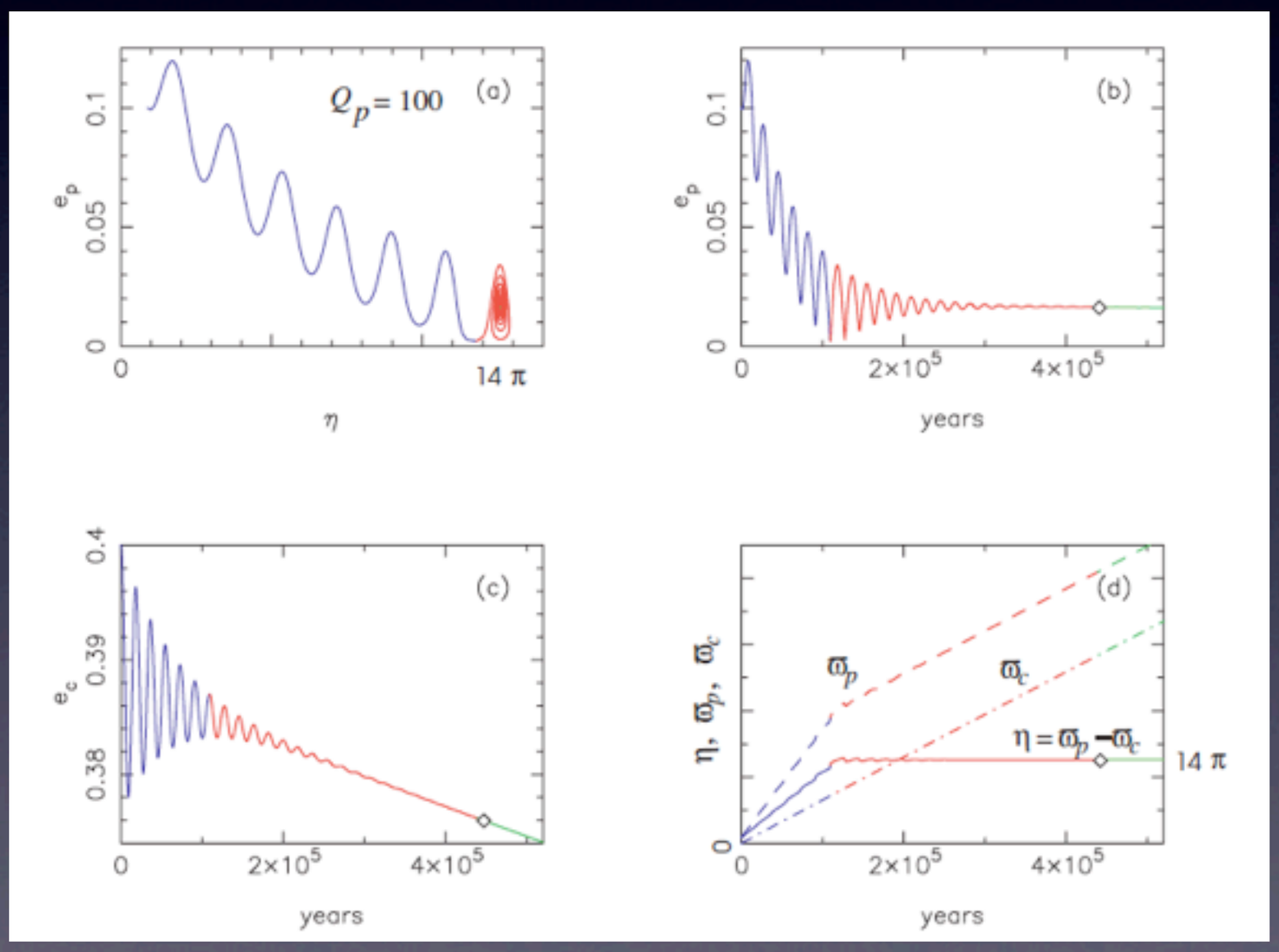
guiding principals

3. Dissipation acts to stabilize systems

Secular evolution with dissipation

$\longrightarrow$ relaxation and stabilization

Tidal dissipation: HAT-P-13



$P_b = 4$ days

$P_c = 429$ days

$m_b = 0.85 M_J$

$m_c \sin i = 15 M_J$

$e_c = 0.7$

$e_b^{(\mathrm{observed})} = 0.02$

$$e_b^{(\text{fixed point})} = \frac{(5/4)(a_b/a_c)e_c}{1-\sqrt{\alpha}(m_b/m_c)+\gamma}$$

Mardling (2007)
Wu & Goldreich (2002)

contains information
about planet structure
via ``Love number'' k_b

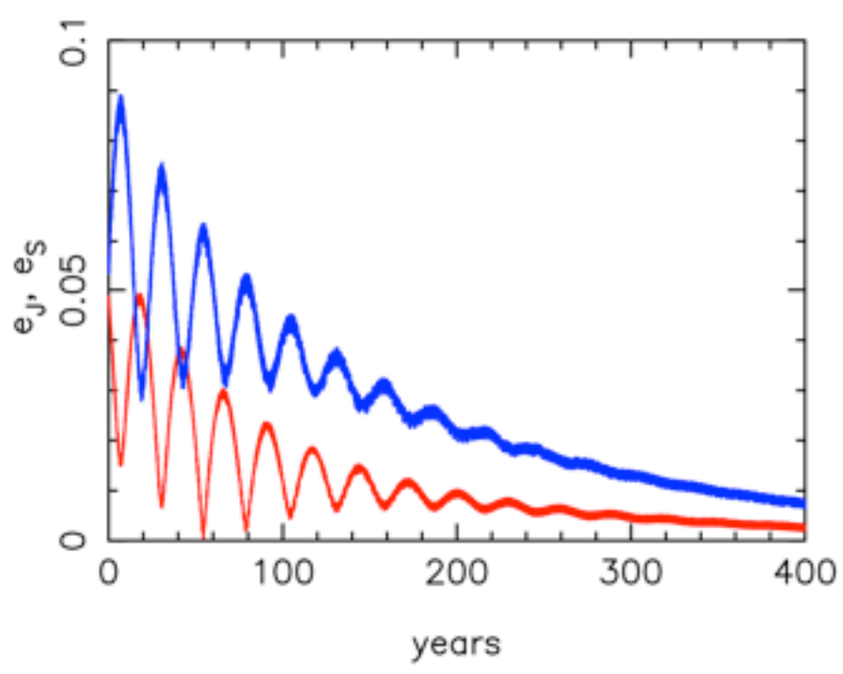
Putting $e_b^{(\text{fixed point})} = e_b^{(\text{observed})}$ gives k_b !!

Batygin et al (2009)

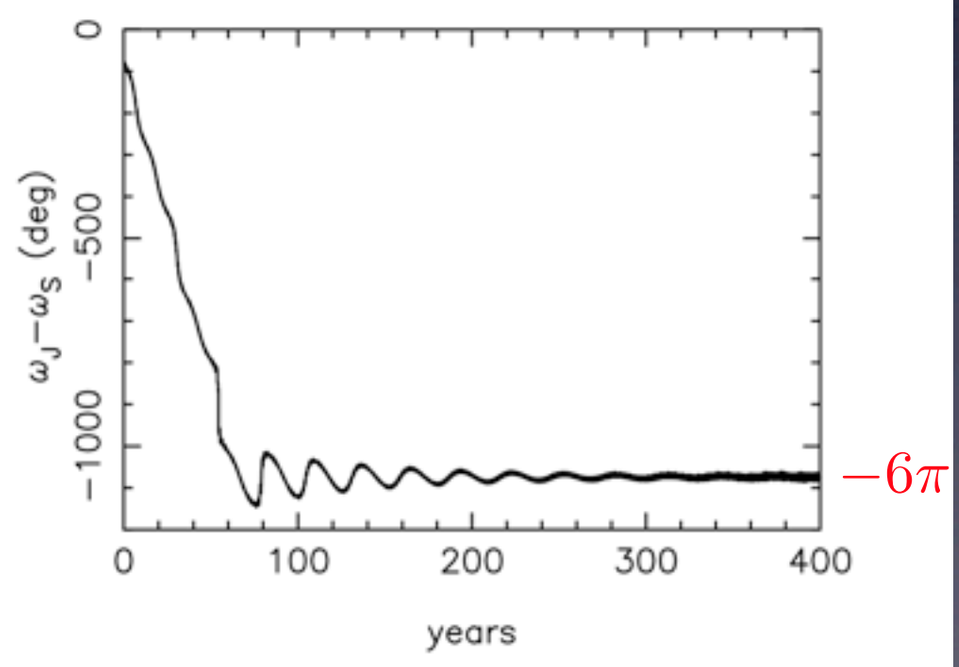
Secular evolution with dissipation
$\longrightarrow$ relaxation and stabilization

Tidal dissipation: Jupiter and Saturn at 0.03 AU and 0.05AU

eccentricities

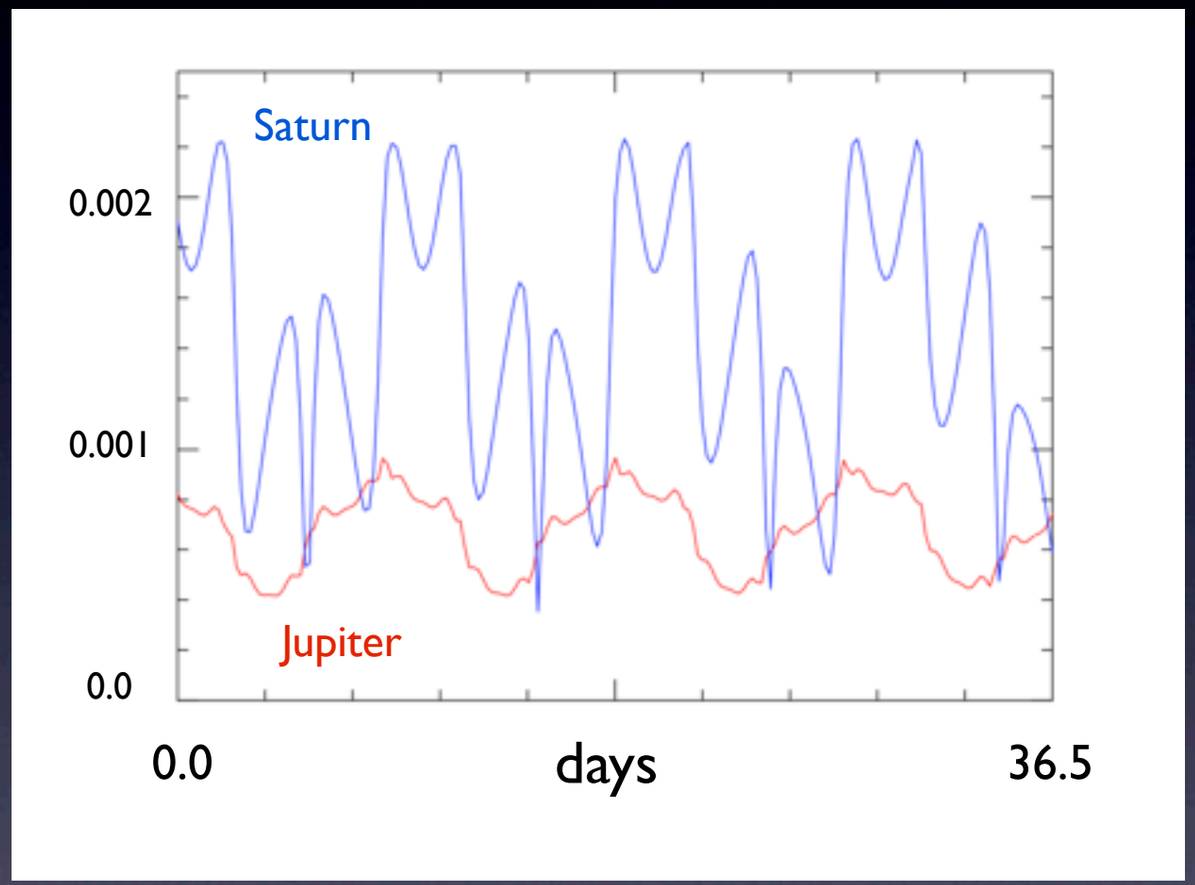


$\varpi_J - \varpi_S$



$t \longrightarrow \infty$: eccentricities are not zero due to planet-planet interactions

eccentricities



the "ground state"

"forced" eccentricities: $e_J \propto m_S/m_\odot,\ e_S \propto m_J/m_\odot$

"free" eccentricities are damped to zero

Another source of eccentricity damping is the protoplanetary disk at the time of formation

Many Kepler pairs appear to be totally relaxed – they are almost perfectly coplanar and their eccentricities are their forced eccentricities.

How do we know this? Using TTVs – see: Lithwick, Xi & Wu (2012) and Wu & Lithwick (2012)

Beautiful!!

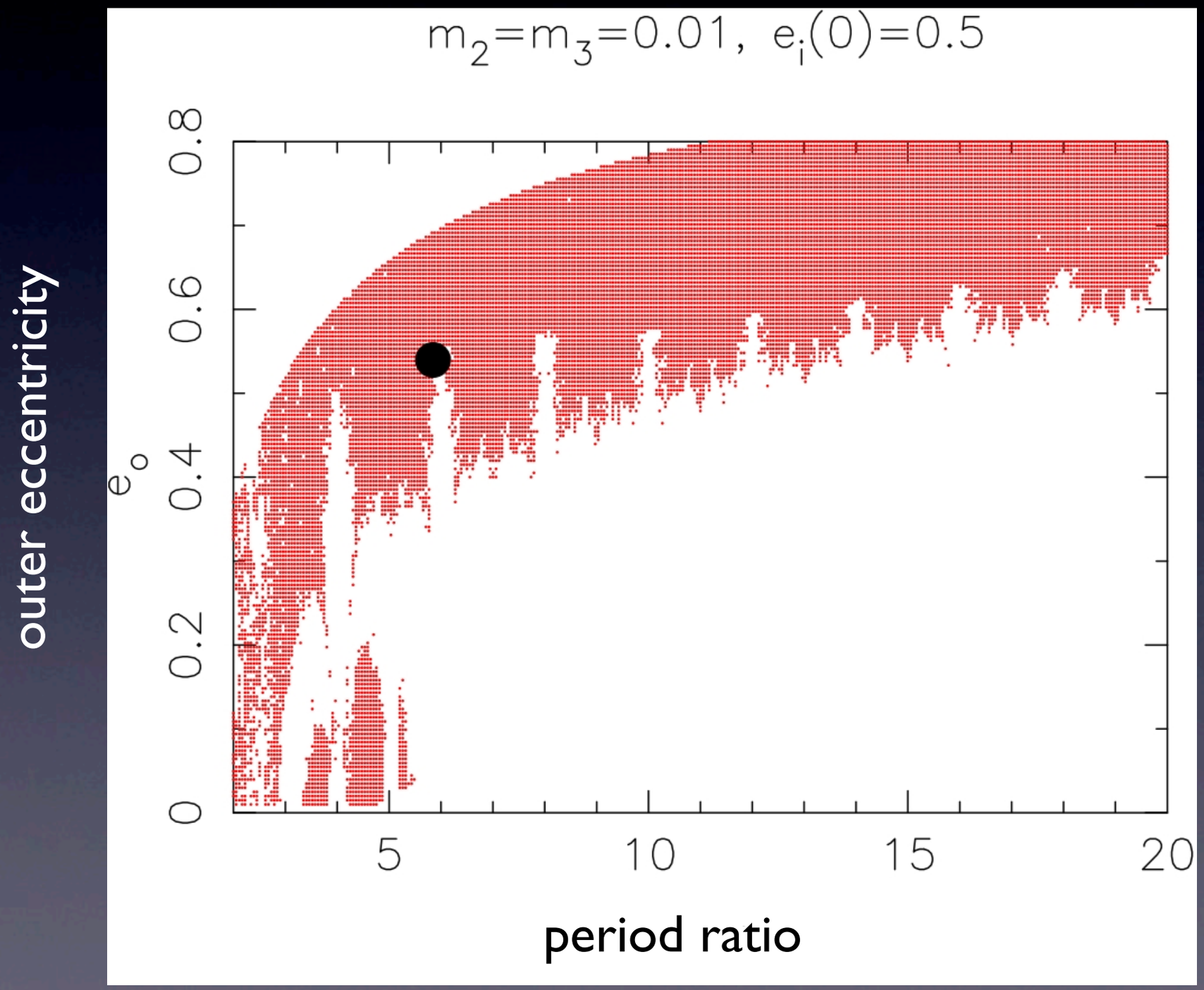
for system near 2:1 resonance:

$$e_1 \propto \frac{m_2/m_*}{|P_2/P_1 - 2|}, \quad e_2 \propto \frac{m_1/m_*}{|P_2/P_1 - 2|}$$

guiding principals

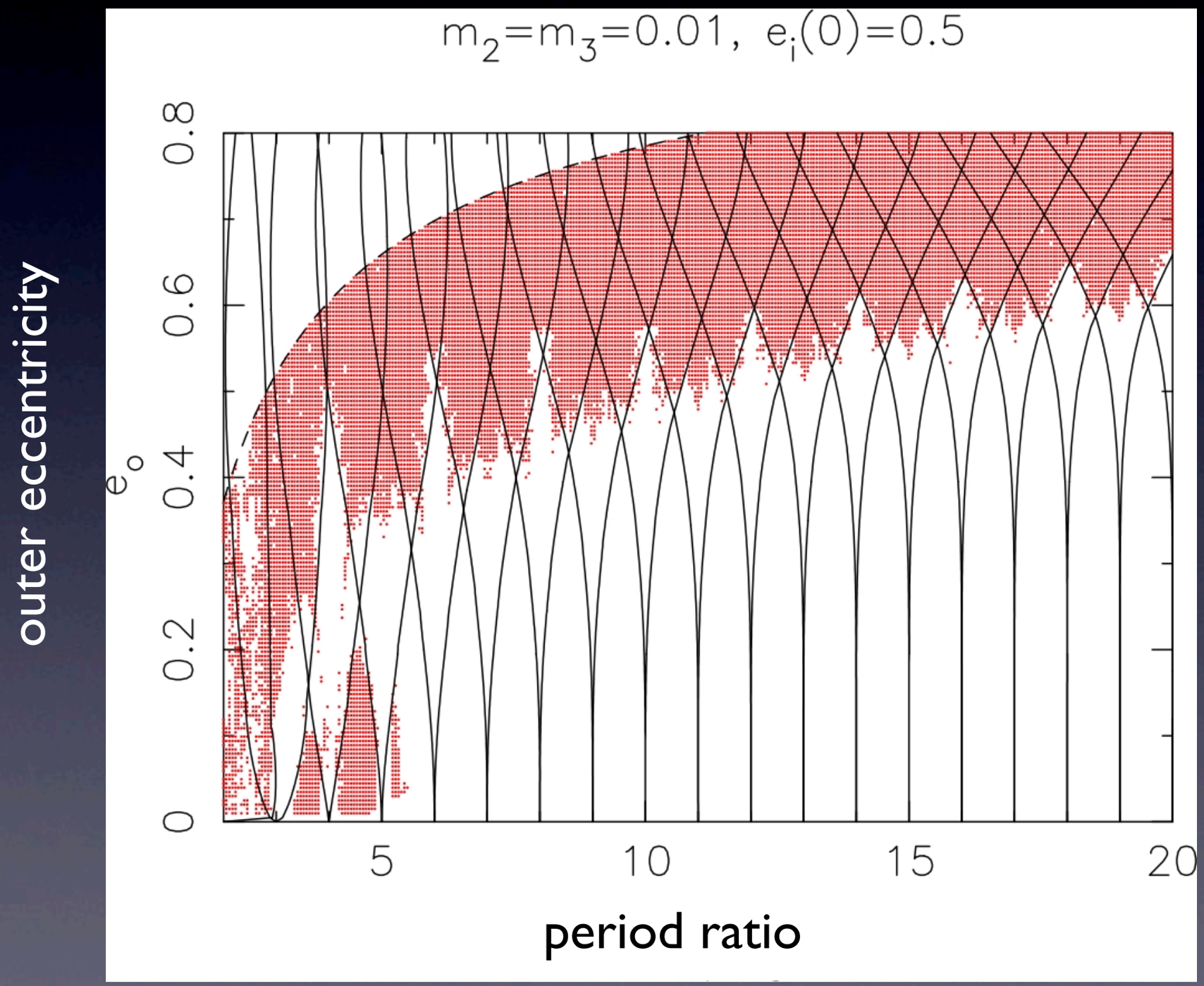
4. Resonance protects and destroys

The stability boundary

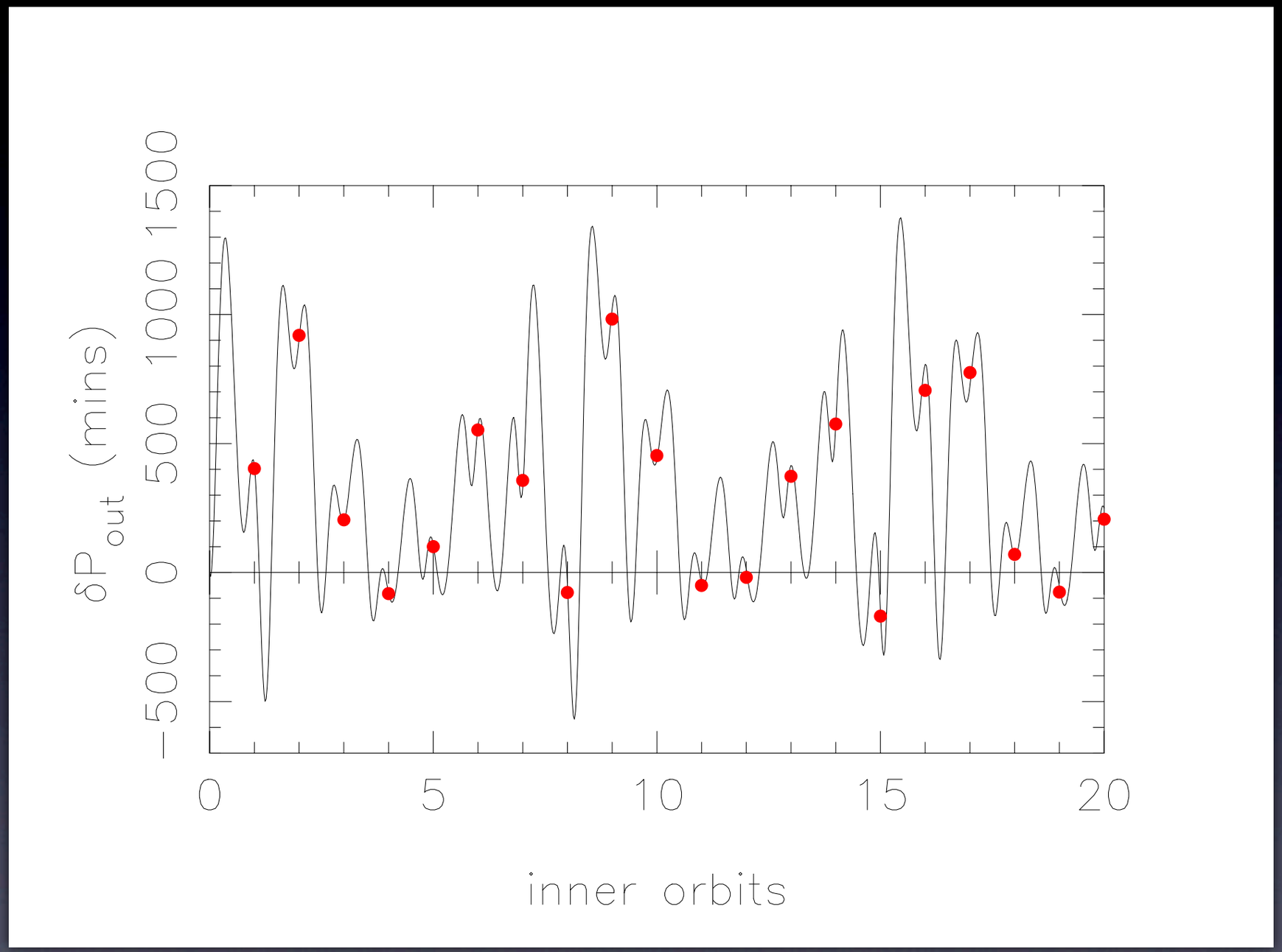


The stability boundary

Mardling 2008, 2013



TTV teaser: a circumbinary planet...



$$e_1 = 0.2,\ e_2 = 0.1\ a_1 = 0.1\,\mathrm{au},\ P_2/P_1 = 7.4$$

$$m_2/m_1 = 0.1,\ m_p/(m_1 + m_2) = 0.001$$