The full $\mathcal{O}(\alpha)$ electroweak radiative corrections $e^+e^- \to t\bar{t}\gamma, e^+e^-\gamma$ at ILC with GRACE-Loop.

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In collaboration with

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KEK
HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION

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1. **Introduction**
2. **The GRACE-Loop system**
3. **Discussion on the difficulties of 2 $\rightarrow$ 3, 4 processes's calculation**
4. **Calculation the physical processes with GRACE-Loop:** $e^+e^- \rightarrow t\bar{t}\gamma$ *as a example.*
5. **The physical results of the processes:** $e^+e^- \rightarrow t\bar{t}\gamma, e^+e^-\gamma$.
6. **Future plan and conclusions**

Introduction

1. **Thank to the achievements of the LHC: the discovery a new boson compatible with a SM Higgs [1,2].**
2. **We expect that the main goals of the ILC program are**
   - precise measurements of Higgs properties: Higgs Boson mass, Spin, CP, Higgs couplings.;
   - precise measurements of the interaction of top quarks, gauge bosons, ...;
   - searches for physics beyond the Standard Model (BSM).

   $\Longrightarrow$Electroweak radiative corrections to e^+e^- collision play important role at the high precision program of ILC.
3. **In this talk, we present the calculation of two important processes: $e^+e^- \to t\bar{t}\gamma, e^+e^-\gamma$ at ILC with GRACE-Loop.**

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[1] **Physics Letters B 716 (2012) 30-61**

[2] **Phys.Lett. B716 (2012) 1-29**

Motivation of $e^+e^- \to t\bar{t}\gamma$ calculation

1. **The experimental results of CDF and D0 observed a large top quark forward-backward asymmetry.**
2. **QCD radiative corrections to top pair production from proton-proton collisions were calculated by several authors.**
   $\Leftarrow$ However, it is affected by a huge background from QCD.
3. **In the future, we expect that the measurement will be performed at the ILC without QCD background**
   $\Longrightarrow$ The precise calculations of top pair production and top pair with photon production in e^+e^- collisions are considered.
4. **One-Loop EW corrections to $e^+e^- \to t\bar{t}$ were calculated by**
   - J. Fujimoto and Y. Shimizu et al, Mod. Phys. Lett. 3A, 581 (1988);
   - J. Fleischer, A. Leike, T. Riemann et al.Eur. Phys. J. C 31, 37 (2003).

$\Longrightarrow$ **The calculation of $e^+e^- \to t\bar{t}\gamma$ with GRACE-Loop is presented in this talk.**

Motivation of $e^+e^- \to e^+e^-\gamma$ calculation

1. **Electroweak radiative corrections to Bhabha scattering are important for the luminosity determination.**
2. **The status of electroweak radiative corrections to Bhabha scattering.**
   (a) One-loop electroweak corrections to $e^+e^- \to e^+e^-$:
      - J. Fujimoto et al: *Prog. Theor. Phys. Supplement (1990) 100.*
      - M. Böhm et al: *Nuclear Physics B304 (1988) 687-711*

   (b) Two-loop QED correction to $e^+e^- \to e^+e^-$:
      - A.A. Penin: *PhysRevLett.95.010408*.

$\Longrightarrow$ **We also present a full $\mathcal{O}(\alpha)$ electroweak radiative corrections to process $e^+e^- \to e^+e^-\gamma$ GRACE-Loop in this talk.**

GRACE-Loop system

GRACE-Loop is a generic automated program for calculating High Energy Physics processes [3].

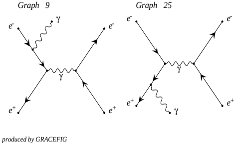
- *All Feynman diagrams for a given process at fixing order of perturbation theory.*
- *A FORM or REDUCE code.*
- *A Fortran code generated for amplitude calculations.*
- *Kinematic library.*
- *The multi-dimensional integration by BASES.*
- *Event generation by SPRING.*

The GRACE-Loop system has also been used to calculate $2 \to 3$-body processes such as $e^+e^- \to ZHH$, $e^+e^- \to t\bar{t}H$, $e^+e^- \to \nu\bar{\nu}H$ and $2 \to 4$-body process as $e^+e^- \to \nu_\mu\bar{\nu}_\mu HH$.

[3] **Phys. Rept. 430 (2006) 117**

Discussion on the difficulty of $2 \to 3, 4$ process calculations

1. **The large numerical cancellation problem.**



$$\epsilon^\mu \epsilon^\nu \to -g^{\mu\nu} + \frac{q^\mu n^\nu + q^\nu n^\mu}{n.q} - n^2 \frac{q^\mu q^\nu}{(n.q)^2}$$

Amplitude	Non-Axial Gauge	Axial Gauge
$\mathcal{M}_1^2 + \mathcal{M}_2^2$	$0.1116212357 \times 10^{+13}$	$0.3644158264 \times 10^{+02}$
$2\mathcal{M}_1^* \mathcal{M}_2$	$-0.1116212356 \times 10^{+13}$	$0.1546482734 \times 10^{+03}$
$\lvert\mathcal{M}_1 + \mathcal{M}_2\rvert^2$	$0.1910871582 \times 10^{+03}$	$0.1910898560 \times 10^{+03}$

2. **The Monte-Carlo integration step costs much in CPU time.**

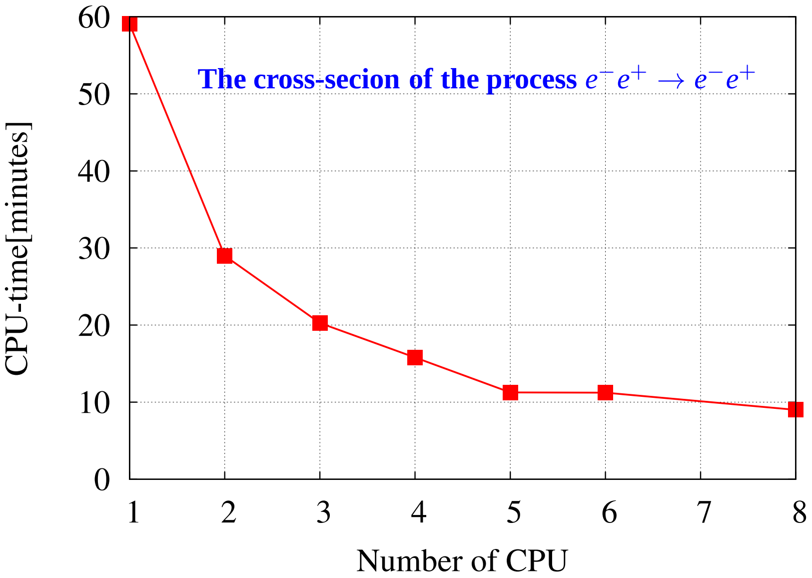
The process: $e^+e^- \to e^+e^-\gamma$

CPU	Memory	CPU time
Intel(R) Xeon(R), X5660@2.80GHz	49 GB	$\geq$3 months @$\sqrt{s}$.

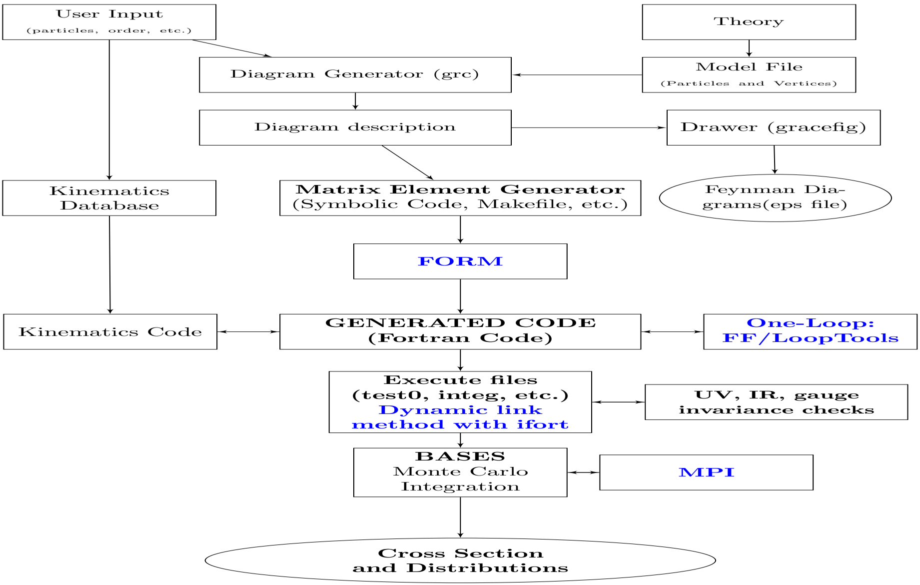
$\Longrightarrow$ **BASES with MPI[4] in quadruple precision calculation.**

[4]**The Message Passing Interface:** ***http://www.mcs.anl.gov/research/projects/mpi***

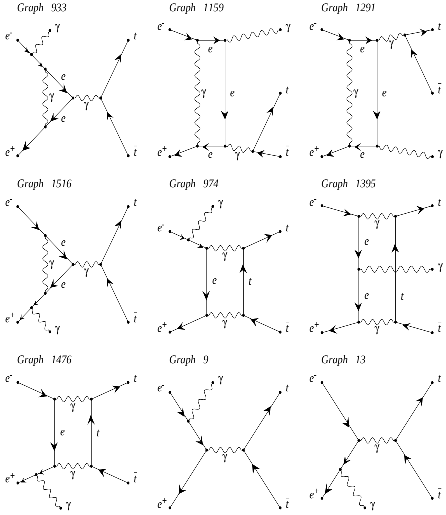
Discussion on the difficulty of $2 \to 3, 4$ process calculations



The GRACE-Loop system



Calculation the physical processes with GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.



produced by GRACEFIG

Model = ”nlg2301.mdl”;

Process;

ELWK = $\{5, 3\}$;

Initial = {electron, positron} ;

Final = {photon, t, t-bar} ;

Expand = Yes;

OPI = No;

Kinem = ”2302”;

Pend;

- 16 tree diagrams,
- 1814 one-loop diagrams.

The total cross section is

$$\begin{aligned}\sigma_{tot} &= \sigma_{Tree} + \sigma_{Loop}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda) \\ &+\sigma_{Tree}\delta_{soft}(\lambda, E_\gamma < k_c) + \sigma_{Hard}(k_c)\end{aligned}$$

The hard contribution is the process: $e^-e^+ \to t\bar{t}\gamma\gamma$.

- Soft photon with $\lambda \leq E_{soft} \leq k_c$;
- Hard photon with $E_{hard} \geq k_c$.

The non-linear gauge fixing Lagrangian condition[5]

$$\begin{aligned}\mathcal{L}_{GF} &= -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} \\ &+\xi_W\frac{g}{2}(v + \tilde{\delta}H + i\tilde{\kappa}\chi_3)\chi^+|^2 \\ &-\frac{1}{2\xi_Z}(\partial.Z + \xi_Z\frac{g}{2c_W}(v + \tilde{\varepsilon}H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial.A)^2 .\end{aligned}$$

[5] Phys. Rept. **430**, 117 (2006)

Calculation the physical processes with GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.

Test on the calculation

$$\begin{aligned}\sigma_{tot} &= \sigma_{Tree} + \sigma_{Loop}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda) \\ &\quad + \sigma_{Tree}\delta_{soft}(\lambda, E_\gamma < k_c) + \sigma_{Hard}(k_c)\end{aligned}$$

1. C_{UV} **independence of the result.**
2. **Photon mass (λ) independence of the result.**
3. **Gauge invariance of the result.**
4. k_c **independence of the result.**
5. **Cross-check with orther calculation.**

Calculation the physical processes with GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.

Test on the calculation

$$\sigma_{tot} = \sigma_{Tree} + \sigma_{Loop}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda) + \sigma_{Tree}\delta_{soft}(\lambda, E_\gamma < k_c) + \sigma_{Hard}(k_c)$$

1. C_{UV} independent of the amplitude

C_{UV}	$2\mathcal{R}(\mathcal{M}^+_{Tree}\mathcal{M}_{Loop})$
0	$-6.7575992336127728658083765531206107 \times 10^{-3}$
10^2	$-6.7575992336127728658083765531205867 \times 10^{-3}$
10^4	$-6.757599233612772865808376553118930 8 \times 10^{-3}$

The result is stable over 30 digits in quadruple precision.

Calculation the physical processes with GRACE-Loop:

$e^+e^- \to t\bar{t}\gamma$ as a example.

2. λ independence of the result.

λ [GeV]	$2\mathcal{R}(\mathcal{M}^+_{Tree}\mathcal{M}_{Loop})$+soft contribution
10^{-19}	$-1.674389236949202039765435422043876 \times 10^{-3}$
10^{-21}	$-1.674389236949202038289240208334962 \times 10^{-3}$
10^{-23}	$-1.674389236949202038274434890116147 \times 10^{-3}$

The result are stable over 18 digits.

3. Gauge invariance of the amplitude check

$(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\epsilon})$	$2\mathcal{R}(\mathcal{M}^+_{Tree}\mathcal{M}_{Loop})$
$(0,0,0,0,0) \times 10^0$	$-6.757599233612772865808376553120 6 \times 10^{-3}$
$(1,2,3,4,5) \times 10^1$	$-6.757599233612772865808383145619 3 \times 10^{-3}$
$(1,2,3,4,5) \times 10^2$	$-6.757599233612772865809055637884 2 \times 10^{-3}$

The result is stable over 21 digits in quadruple precision.

Calculation the physical processes with GRACE-Loop: $e^+e^- \to t\bar{t}\gamma$ as a example.

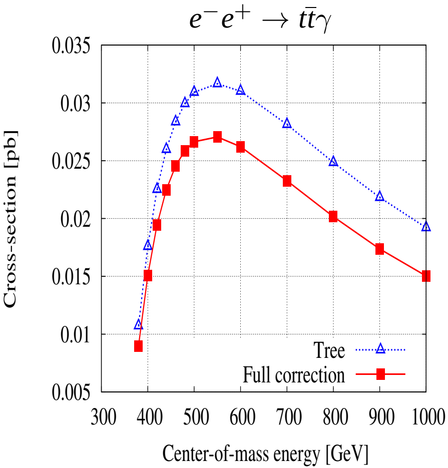
Test on the calculation

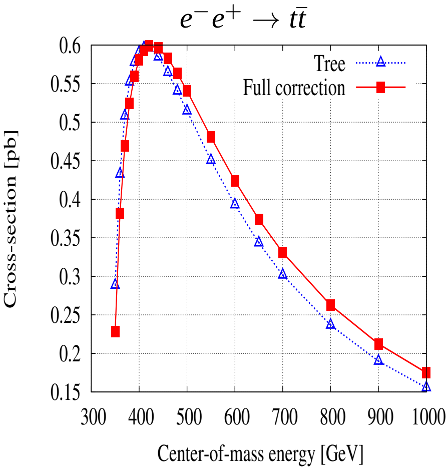
$$\sigma_{tot} = \sigma_{Tree} + \sigma_{Loop}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda) + \sigma_{Tree}\delta_{soft}(\lambda, E_\gamma < k_c) + \sigma_{Hard}(k_c)$$

4. k_c independence of the result.

k_c[GeV]	σ_H	σ_S	σ_{S+H}
10^{-5}	4.172723×10^{-02}	5.885469×10^{-02}	0.10058192
10^{-3}	2.926684×10^{-02}	7.131737×10^{-02}	0.10058421
10^{-1}	1.678994×10^{-02}	8.377319×10^{-02}	0.10056313

The physical results of the process $e^+e^- \to t\bar{t}\gamma$

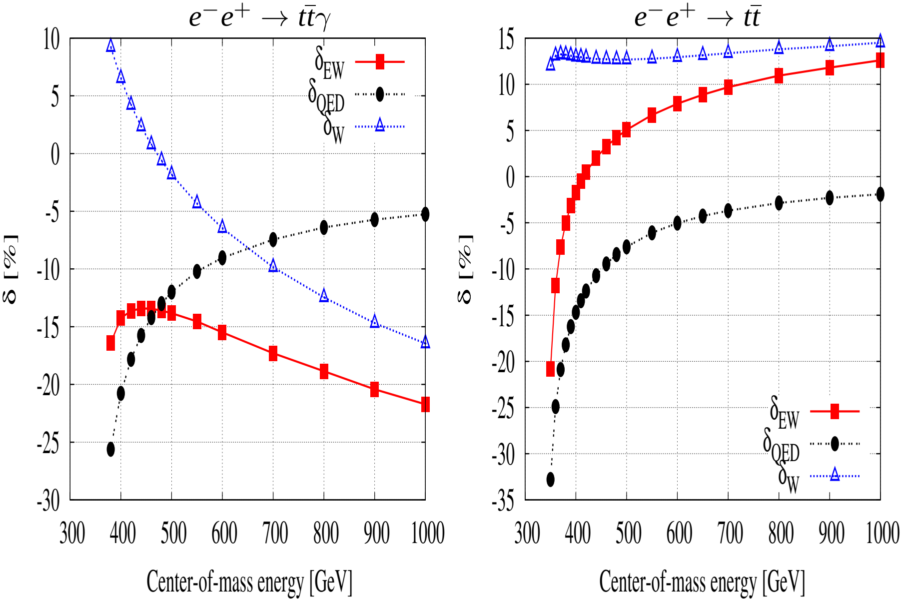




$E_\gamma \geq 10GeV, 0^0 \leq \theta_\gamma \leq 170^0.$
$P.H.Khiem, Y.Kurihara, et\ al,$
$Eur. Phys. J. C\mathbf{73}, 2400(2013)$

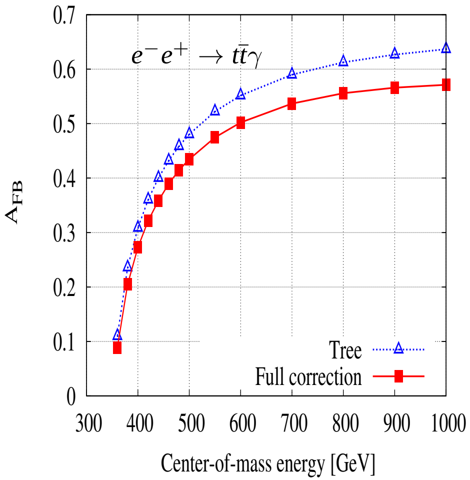
$Mod. Phys. Lett. \mathbf{3A}, 581(1988);$
$Eur. Phys. J. C\mathbf{31}, 37(2003).$

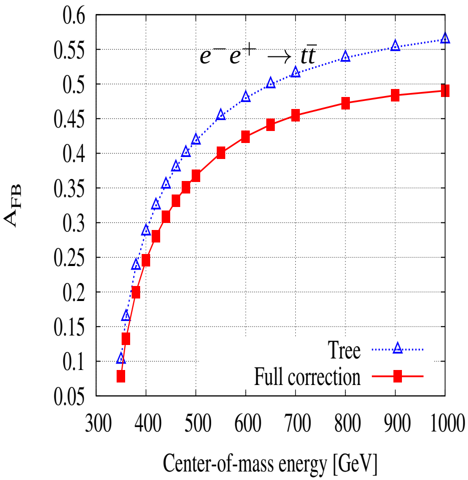
The physical results of the process $e^+e^- \to t\bar{t}\gamma$



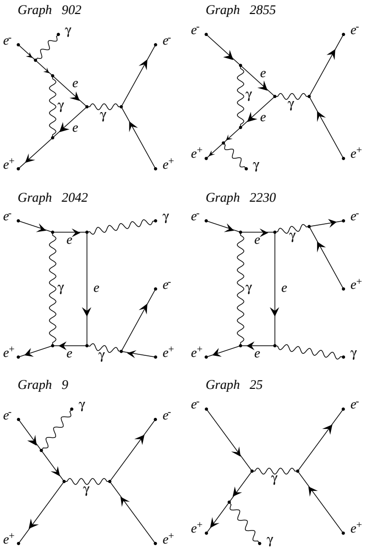
The physical results of the process $e^+e^- \to t\bar{t}\gamma$

$$A_{FB} = \frac{\sigma(0^0 \le \theta_t \le 90^0) - \sigma(90^0 \le \theta_t \le 180^0)}{\sigma(0^0 \le \theta_t \le 90^0) + \sigma(90^0 \le \theta_t \le 180^0)}$$





The process $e^+e^- \to e^+e^-\gamma$ with GRACE-Loop.



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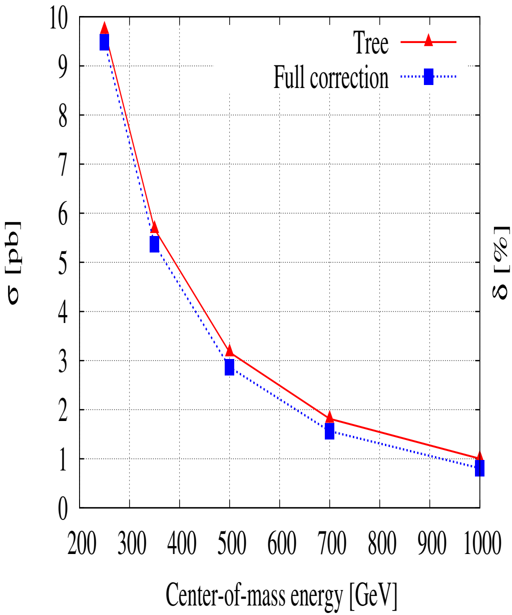
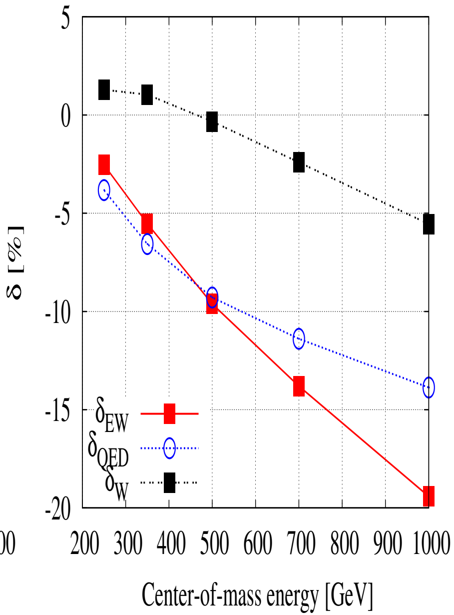
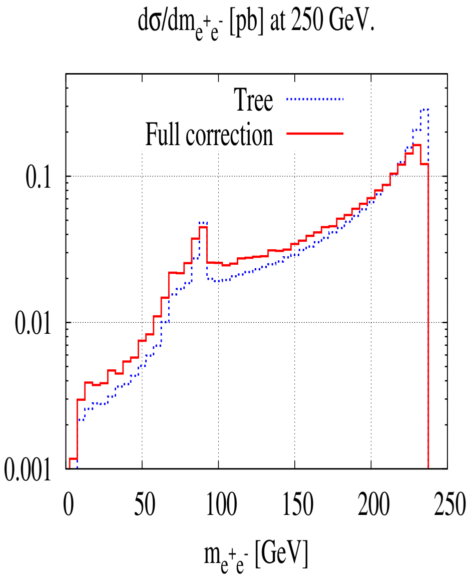
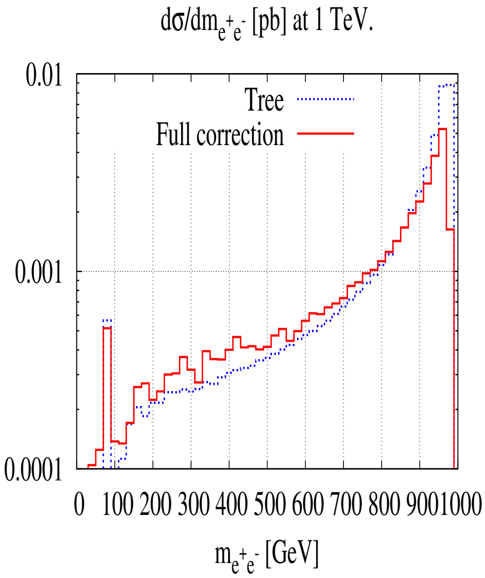
The physical results of process $e^+e^- \to e^+e^-\gamma$





The physical results of process $e^+e^- \to e^+e^-\gamma$





Conclusions

- We introduced to the GRACE-Loop system which is a generic automated program for calculating High Energy Physics processes.
- The full $\mathcal{O}(\alpha)$ electroweak radiative corrections $e^+e^- \to t\bar{t}\gamma, e^+e^-\gamma$ at ILC were calculated sucessfully with GRACE-Loop.

The physical results of the process $e^+e^- \to t\bar{t}\gamma$

- We find that the numerical value of the weak corrections varies from 10% to -16% in the range of center-of-mass energy from 360 GeV to 1TeV.
- We also obtain a large value for the top quark forward-backward asymmetry in the $t\bar{t}\gamma$ process as compared with the one in $t\bar{t}$ production.

Conclusions

- We introduced to the GRACE-Loop system which is a generic automated program for calculating High Energy Physics processes.
- The full $\mathcal{O}(\alpha)$ electroweak radiative corrections $e^+e^- \to t\bar{t}\gamma, e^+e^-\gamma$ at ILC were calculated sucessfully with GRACE-Loop.

The physical results of the process
$e^+e^- \to e^+e^-\gamma$

- We find that the numerical value of the full electroweak radiative corrections varies from -2% to -20% in the range of center-of-mass energy from 250 GeV to 1TeV.
- This contribution is sizable. The full electroweak correction to the process play important role for the determination luminosity at ILC in the future.

Our next target's calculation is
$pp \to VV + 1$ jet at LHC

1. $\gamma\gamma + 1$ jet: background for $H \to \gamma\gamma$ search.
2. $W^+W^- + 1$ jet
   - Background for $H \to W^+W^-$ search,
   - BSM signal searchs,
   - The precise measurement VVV coupling at LHC.
3. Your sugesstion are valuable for us!!!

Thank you very much for your attention!