Cosmological Constraints on Dark Energy via Bulk Viscosity from Decaying Dark Matter

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Outline

- Introduction
- Cosmological model
- Statistical analysis with the observation data
Evidence of Dark Matter

- Galaxy Rotation

![Observed vs. Predicted Keplerian](image)
Evidence of Dark Matter

- Gravitation lensing: seeing the invisible

Diagram:
1. **A Distant Source**
   - Light leaves a young, star-forming blue galaxy near the edge of the visible universe.
2. **A Lens of 'Dark Matter'**
   - Some of the light passes through a large cluster of galaxies and surrounding dark matter directly in the line of sight between Earth and the distant galaxy. The dark matter's gravity acts like a lens, bending the incoming light.
3. **Focal Points: Earth**
   - Most of this light is scattered, but some is focused and directed toward Earth. Observers see multiple, distorted images of the background galaxy.
Evidence of Dark Matter

- Clusters are filled with hot X-ray emitting intergalactic gas
  Merging cluster: optical image
Evidence of Dark Matter

- Clusters are filled with hot X-ray emitting intergalactic gas
- Merging cluster: gravitation lensing
Evidence of Dark Matter

- Clusters are filled with hot X-ray emitting intergalactic gas
  Merging cluster: x-ray

Merging clusters: Chandra x-ray image of 1E 0657-56
Evidence of Dark Matter

- Clusters are filled with hot X-ray emitting intergalactic gas
  Merging cluster: x-ray
Evidence for Dark Energy

Supernovae were dimmer in the past
\( \Omega_B = 0.04, \Omega_\gamma = 0.001, \Omega_{DM} = 0.23, \Omega_\Lambda = 0.73 \)
In Wilson, Mathews, Fuller PRD(2007), Mathews, Lan, Kolda, PRD (2008). Lan, Mathews, Com. Phys.(2009) it was proposed that a unity of dark matter and dark energy might be explained if the dark energy could be produced from a delayed decaying dark-matter particle and just consider with the supernova-redshift constraint.

We consider a simultaneous fit to the CMB, as a means to constrain this paradigm to unify dark matter and dark energy. We deduce constraints on the parameters characterizing decaying the dark matter cosmology by using the Markov Chain Monte Carlo method applied to the CMB data.
Modified Friedmann equation in which we allow for flat and the usual cosmological constant $\Lambda$.

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} \quad (1)$$

where, $\rho$ is now composed of several terms

$$\rho = \rho_{DM} + \rho_b + \rho_\gamma + \rho_h + \rho_r + \rho_{BV} \quad (2)$$
\[ \rho_h = \rho_h(t_d) a^{-3} e^{-(t-t_d)/\tau_d}, \quad (3) \]

\[ \rho_r = a^{-4} \lambda \rho_h(t_d) \int_{t_d}^{t} e^{-(t'-t_d)/\tau_d} a(t') dt', \quad (4) \]

\[ \rho_{BV} = a^{-4} 9 \int_{t_d}^{t} H^2 a(t')^4 \zeta(t') dt', \quad (5) \]

The integral term in the last equation gives the effective dissipated energy (Weinberg71) due to a cosmic bulk viscosity coefficient \( \zeta \). This term induces the cosmic acceleration once a model is formulated for the bulk viscosity coefficient \( \zeta \).
The effect of the bulk viscosity is to replace the fluid pressure with an effective pressure.

\[ p_{\text{eff}} = p - \zeta \frac{\dot{a}}{a} \]  \hspace{1cm} (6)

\[ \zeta \frac{\dot{a}}{a} = \Delta p \]  \hspace{1cm} (7)

where \( \Delta p = \tilde{p} - p \) is the difference between the constant volume equilibrium pressure and the actual fluid pressure.
\[ \Delta p \sim \left( \frac{\partial p}{\partial T} \right)_n (T_M - T) = \frac{4 \rho \gamma \tau_e}{3} \left[ 1 - \left( \frac{3 \partial p}{\partial \rho} \right) \right] \frac{\partial U^\alpha}{\partial x^\alpha} , \tag{8} \]

The timescale \( \tau_e \) to restore pressure equilibrium in an expanding cosmology from an initial pressure deficit of \( \Delta p(0) \) can be determined from,

\[ \tau_e = \int_0^\infty \frac{\Delta p(t)}{\Delta p(0)} \, dt \approx \frac{C \tau_d}{[1 + 3(\dot{a}/a)\tau_d]} . \tag{9} \]

where the coefficient \( C \geq 1 \) accounts for the possibility of higher corrections to the linearized transport equation. The final form for the bulk viscosity of the cosmic fluid is then (mathew 2010, Weinberg71),

\[ \zeta = \frac{4 \rho_h \tau_e}{3} \left[ 1 - \frac{\rho_l + \rho_\gamma}{\rho} \right]^2 . \tag{10} \]
We have modified the publicly available CosmoMC package to satisfy this decaying dark matter model as described above. Following the usual prescription we then determine the best-fit values using the maximum likelihood method. We take the total likelihood function $\chi^2 = -2\log L$ as the product of the separate likelihood functions of each data set and thus we write,

$$\chi^2 = \chi^2_{SN} + \chi^2_{CMB} + \chi^2_{LSS} + \chi^2_{BAO} + \chi^2_{CMB} \quad .$$  \quad (11)
Type supernovae data and constraint:

\[ D_L = \frac{c(1 + z)}{H_0} \left\{ \int_0^z dz' \left[ \Omega_\Lambda + \Omega_\gamma(z') + \Omega_{DM}(z') \right. \right. \\
+ \left. \left. \Omega_b(z') + \Omega_h(z') + \Omega_r(z') + \Omega_{BV}(z') \right]^{-1/2} \right\}, \]

(12)

\( \Omega_\Lambda = \Lambda / 3H_0^2, \)
\( \Omega_\gamma = (8\pi G \rho_{m0} / 3H_0^2)(1 + z)^4, \)
\( \Omega_{DM} = (8\pi G \rho_{DM} / 3H_0^2)(1 + z)^3 \)
\( \Omega_b = (8\pi G \rho_b / 3H_0^2)(1 + z)^3, \)
Type supernovae data and constraint:

This luminosity distance is related to the apparent magnitude of supernovae by the usual relation,

\[ \triangle m(z) = m(z) - M = 5 \log_{10}[D_L(z)/\text{Mpc}] + 25, \tag{13} \]

where \( \triangle m(z) \) is the distance modulus and \( M \) is the absolute magnitude which is assumed to be constant for type Ia supernovae standard candles. The \( \chi^2 \) for type Ia supernovae is given by

\[ \chi_{SN}^2 = \sum_{i,j=1}^{N} \left[ \triangle m(z_i)^{obs} - \triangle m(z_i)^{th} \right] \times \left( C_{SN}^{-1} \right)_{ij} \left[ \triangle m(z_i)^{obs} - \triangle m(z_i)^{th} \right] \tag{14} \]

Here \( C_{SN} \) is the covariance matrix with systematic errors.
Type supernovae data and constraint:

Late decaying DM with bulk viscosity can produce cosmic acceleration without Dark Energy or a Cosmological constant.
Cascading decays: Sterile neutrinos

$\nu_1 \rightarrow \nu_2 \rightarrow \nu_3 \rightarrow \nu_4 \rightarrow \nu_5 \rightarrow \nu_6 \rightarrow$ regular neutrinos or Late decays due to time varying mass or a late phase transition
The characteristic angular scale $\theta_A$ of the peaks of the angular power spectrum in CMB anisotropies is defined by Page et al 2003

$$\theta_A = \frac{r_s(z_*)}{r(z_*)} = \frac{\pi}{l_A} ,$$

(15)

$l_A$: acoustic scale  
$z_*$: the redshift at decoupling  
$r(z_*)$: the comoving distance at decoupling  
$r_s(z_*)$: the comoving sound horizon distance at decoupling.
The quantity $r_s(z_*)$ is the comoving sound horizon distance at decoupling.

$$r_s(z_*) = \int_0^{z_*} \frac{(1 + z)^2 R(z)}{H(z)} dz ,$$  \hspace{1cm} (17)
CMB constraint

the redshift at decoupling \( z_* \) proposed by Hu and Sugiyama 1996

\[
z_* = 1048[1 + 0.00124(\Omega_b h^2) - 0.738][1 + g_1(\Omega_0 h^2)g_2]
\]

where

\[
g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}},
g_2 = \frac{0.56}{1 + 21.1(\Omega_b h^2)^{1.81}}
\]

The \( \chi^2 \) of the cosmic microwave background fit is constructed as

\[
\chi^2_{CMB} = -2\ln L = \sum X^T (C^{-1})_{ij} X
\]

where

\[
X^T = (l_A - l_A^{* WMAP}, R - R_A^{* WMAP}, z_* - z_*^{* WMAP})
\]

with \( l_A^{* WMAP} = 302.09 \), \( R_A^{* WMAP} = 1.725 \), and \( z_*^{* WMAP} = 1091.3 \).
CMB constraint

Table 1 shows the inverse covariance matrix used in our analysis.

**Table: Inverse covariance matrix given by Komatsu 2011**

<table>
<thead>
<tr>
<th>case</th>
<th>$l_A$</th>
<th>$R$</th>
<th>$z_*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_A$</td>
<td>2.305</td>
<td>29.698</td>
<td>-1.333</td>
</tr>
<tr>
<td>$R$</td>
<td>29.698</td>
<td>6825.27</td>
<td>-113.18</td>
</tr>
<tr>
<td>$z_*$</td>
<td>-1.333</td>
<td>-113.18</td>
<td>3.414</td>
</tr>
</tbody>
</table>
### Table: Fitting results of the parameters with 1σ errors.

<table>
<thead>
<tr>
<th>parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_D$</td>
<td>0.112 ± 0.01</td>
</tr>
<tr>
<td>$t_d$</td>
<td>10.5 ± 2</td>
</tr>
<tr>
<td>$\Omega_b$</td>
<td>0.0225 ± 0.002</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>0.235 ± 0.01</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.0968 ± 0.001</td>
</tr>
<tr>
<td>$h$</td>
<td>0.71 ± 0.01</td>
</tr>
</tbody>
</table>
Figure: The constraints of the parameters $\Omega_m h^2$ and $\Omega_I h^2$, $h$ based upon the SN+CMB.
CMB constraint

Figure: Likelihood function for $\Omega_D h^2$ based upon the SN+CMB data.
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We have studied the evolution of the delayed decaying dark matter model with bulk viscosity by using a MCMC analysis to fit the SNIa and CMB data.
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We find that this cosmology produces an equivalent fit to that of the standard $\Lambda$CDM model, but without a cosmological constant.
Thank you very much for your attention!