Observational consequences of composite inflation models

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Outlines

1. Composite inflation model

2. Evolution equations

3. Power spectrum of perturbation and bound on the model parameters

4. Conclusions

Composite inflation model

The inflaton is a composite field made out of the bound state of particles in strongly coupled theory. The general action for composite inflation is

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} F(\phi) R - \frac{1}{2} G(\phi) \partial_\nu \phi \partial^\mu \phi - V(\phi) \right\} , \quad (1)$$

where d is the mass dimension of the composite field ϕ and

$$F(\phi) = 1 + \frac{\xi}{M_p^2} \phi^{\frac{2}{d}} \quad \text{and} \quad G(\phi) = \phi^{\frac{2-2d}{d}} .$$

Evolution equations

The Friedmann equation and the evolution equation for the background field are respecttively given by

$$3FH^2 + 3\dot{F}H = \tfrac{1}{2}G\dot{\phi}^2 + V(\phi)\,, \qquad (2)$$

$$G\ddot{\phi} + \tfrac{1}{2}G_{\phi}\dot{\phi}^2 + 3HG\dot{\phi} + V_{\phi} = 3F_{\phi}H^2\,(2-\epsilon)\,, \qquad (3)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = -\frac{F_{\phi}}{2HF} + \frac{G\dot{\Phi}^2}{2H^2M_{\rm P}^2F} + \frac{F_{\Phi\Phi}\dot{\Phi}^2}{2H^2F} + \frac{F_{\Phi}\ddot{\Phi}}{2H^2F}\,, \qquad (4)$$

During inflation,

$$G\dot{\phi}^2 \ll V(\phi)\ . \tag{5}$$

During the time at which the observable perturbations exit the horizon,

$$\epsilon \ll 1 \sim \text{constant}\,, \quad \mathcal{F}_t = \frac{\dot{F}}{2FH} \sim \text{constant}\,. \tag{6}$$

The power spectrum for the primordial curvature perturbation

We suppose that during the horizon exit $\mathcal{F}_t$ and ϵ are approximately constant, so that

$$\mathcal{P}_\zeta \simeq \frac{\left(1+\mathcal{F}_t\right)^{1/2}\left(3\mathcal{F}_t^2+G\Phi'^2/2F\right)^{1/2}}{F\left(\epsilon+\mathcal{F}_t\right)^{3/2}}\frac{H^2}{8\pi^2}\bigg|_{c_sk|\tau|=1} . \quad (7)$$

The spectrum index for this power spectrum is

$$n_s = \frac{d \ln \mathcal{P}_\zeta}{d \ln k} + 1 \simeq 1 - 2\epsilon - 2\mathcal{F}_t + \frac{\Phi'}{2} \frac{d \ln \left[G\Phi'^2 / \left(2F + 3\mathcal{F}_t^2 \right) \right]}{d\Phi} \tag{8}$$

where we have used $d/d\ln k \simeq -\phi' d/d(\ln a)$. Using $H^2 \left(1 + \mathcal{F}_t\right) \simeq V\left(\phi\right)$ during inflation,we get

$$A_s \simeq \left. \frac{\left(1 + \mathcal{F}_t\right)^{1/2} \left(3\mathcal{F}_t^2 + G\Phi'^2/2F\right)^{1/2} V}{24\pi^2 F^2 \left(\epsilon + \mathcal{F}_t\right)^{3/2} \left(1 + 2\mathcal{F}_t\right)} \right|_{c_s k|\tau|=1}, \tag{9}$$

Constraints the model parameters

Techni-Inflation:

First, we consider the case where the inflaton is the composite state of techni-quarksin minimal walking technicolor theory. – Techni-Inflation.

In this case, [a]

$$V(\varphi) = \frac{\kappa}{4}\varphi^4 . \tag{10}$$

[a] P. Channuie, J. J. Joergensen and F. Sannino, "Minimal Composite Inflation," JCAP **1105**, 007 (2011) [arXiv:1102.2898 [hep-ph]].

For this model,

$$n_s = 1 - \frac{6\xi}{1+\varphi^2\xi\left(1+6\xi\right)} + \frac{4+4\xi\left(4+\varphi^2\right)}{3\left[\varphi+4\xi\varphi+\varphi^3\xi\left(1+6\xi\right)\right]^2} - \frac{}{3\varphi^2 [} \tag{11}$$

and

$$A_s = \frac{\kappa\varphi^6\left(6\varphi^2\xi^2+\left(\varphi^2+4\right)\xi+1\right)\left(\varphi^2\xi(6\xi+1)+1\right)}{768\left(\pi\varphi^2\xi+\pi\right)^2\left(6\varphi^2\xi^2+\left(\varphi^2-4\right)\xi+1\right)}, \tag{12}$$

In the $\xi \to \infty$ limit:

$$n_s \simeq 1 - \frac{4}{2\mathcal{N}+1} + \frac{f(\mathcal{N},\phi)}{\xi^2} + \mathcal{O}\left(\frac{1}{\xi^3}\right) \,,$$

$$A_s \sim \frac{\kappa \mathcal{N}(2\mathcal{N}+1)}{144\pi^2\xi^2} + g\left(\mathcal{N},\phi\right)\mathcal{O}\left(\frac{1}{\xi^3}\right) \,.$$

[a] $$n_s = 0.960 \pm 0.007 \,, \Rightarrow\; 43 \lesssim \mathcal{N} \lesssim 62 \,,$$

$$3.04 \lesssim \ln\left(A_s \times 10^{10}\right) \lesssim 3.13 \,, \Rightarrow\; 3.9 < \frac{\kappa}{\xi^2} \times 10^{-6} < 8.9 \,.$$

[a] P. A. R. Ade *et al.* [Planck Collaboration], "Planck 2013 results. XXII. Constraints on inflation," arXiv:1303.5082 [astro-ph.CO].

In the $\xi \to 0$ limit:

$$n_s \simeq \left(1 - \frac{24}{\varphi^2}\right) + \left(\frac{96}{\varphi^2} + 8\right)\xi + \mathcal{O}\left(\xi^2\right) \,,$$

$$A_s \simeq \frac{\kappa\varphi^6}{768\pi^2} - \frac{\xi\left(\kappa\varphi^6\left(\varphi^2-8\right)\right)}{768\pi^2} + \mathcal{O}\left(\xi^2\right) \simeq \frac{2\kappa(\mathcal{N}+1)^3}{3\pi^2} + \mathcal{O}\left(\xi\right) \,.$$

$A_s \sim 10^{-9}$ requires $\kappa < 10^{-7}$. From the numerical calculation:

$$\kappa > 10^{-14} \text{ for } \mathcal{N} = 55 \,. \tag{13}$$

Glueball Inflation:

We next consider the case where the inflaton is the bound state of gluon, called glueball. The potential for the glueball inflaton is [a]

$$V(\varphi) = 2\varphi^4 \ln\left(\frac{\varphi}{\Lambda}\right) \,, \tag{14}$$

where Λ is the confining scale and we have redefined the field such that φ has a canonical dimension.

[a] F. Bezrukov, P. Channuie, J. J. Joergensen and F. Sannino, "Composite Inflation Setup and Glueball Inflation," Phys. Rev. D **86**, 063513 (2012) [arXiv:1112.4054 [hep-ph]].

In the $\xi \to \infty$limit :

$$n_s \simeq 1 - f\left(\mathcal{N}\right) + \frac{g\left(\mathcal{N}, \varphi, \Lambda\right)}{\xi} + \left(\frac{\infty}{\xi^{\in}}\right) , \tag{15}$$

$$A_s \simeq \frac{(\ln[\varphi/\Lambda])^3(6\ln[\varphi/\Lambda]+1)}{\pi^2(6\ln[\varphi/\Lambda]-1)\xi^2} + \mathcal{O}\left(1/\xi^3\right) . \tag{16}$$

The contributions from the next leading order terms can be neglected if

$\xi > 10^4$ and $\Lambda > 0.1$. For $\xi > 10^4$ and $\Lambda > 0.1$,

$$0.945 \lesssim n_s \lesssim 0.975 , \quad \Rightarrow \quad 30 \lesssim \mathcal{N} \lesssim 60$$

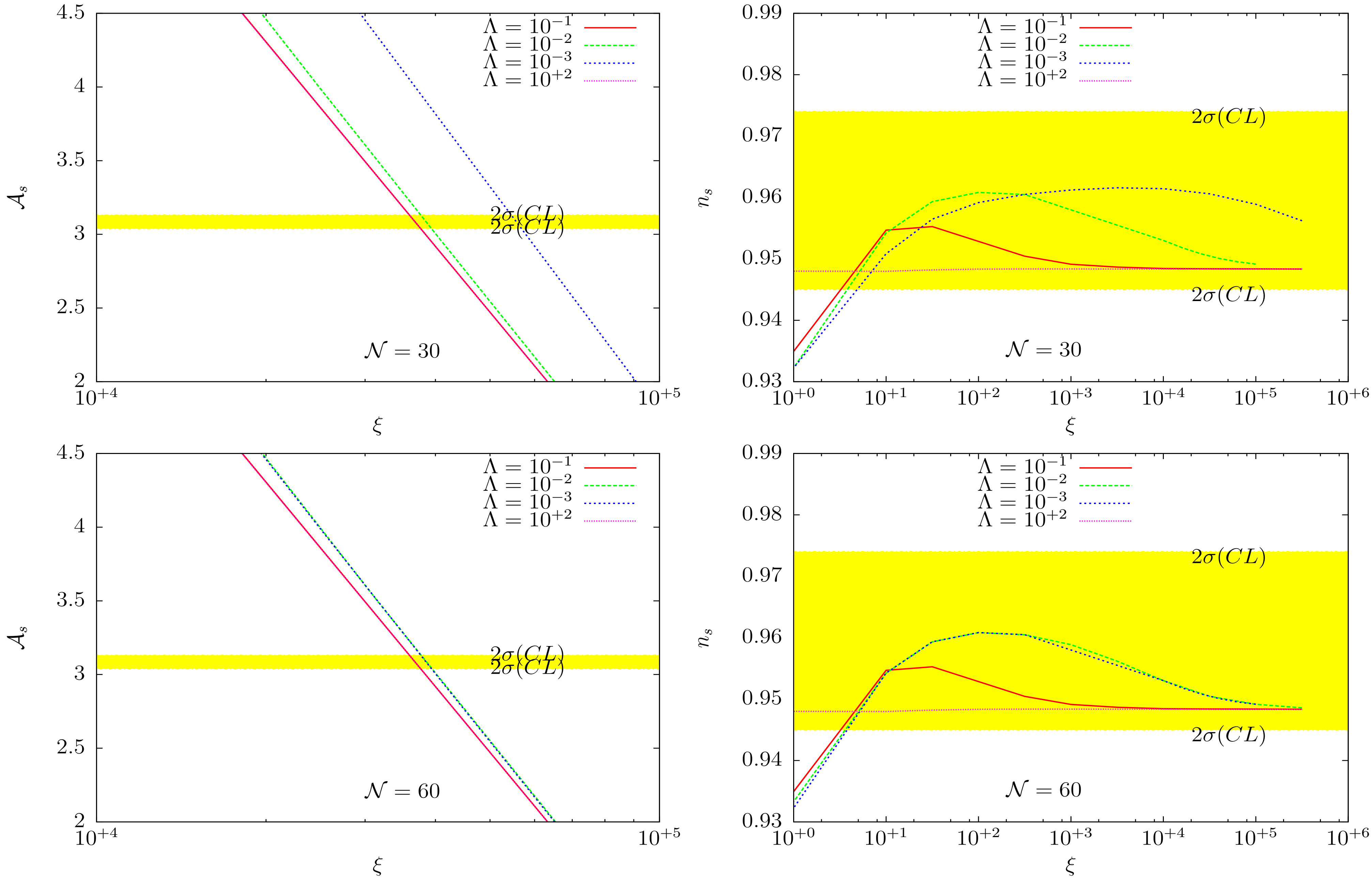
In the $\xi \to 0$ limit, we get

$$A_s \simeq \frac{16\varphi^6(\ln(\varphi/\Lambda))^3}{3\pi^2(1+4\ln(\varphi/\Lambda))^2} + \mathcal{O}(\xi) \quad (17)$$

Slow-roll evolution requires $\ln(\varphi/\Lambda) \gg 1$, so $A_s \gg 10^{-9}$.

In general, A_s increases when ξ decreases.

On the next slide, we plot how $\mathcal{A}_s \equiv \ln\left(A_s \times 10^{10}\right)$ and n_s depend on xi, Λ and the number of e-foldings $\mathcal{N}$.



$\Lambda = 10^{-1}$
$\Lambda = 10^{-2}$
$\Lambda = 10^{-3}$
$\Lambda = 10^{+2}$
$2\sigma(CL)$
$\mathcal{N} = 30$
$\mathcal{N} = 60$
$\mathcal{A}_s$
n_s
ξ
4.5
4
3.5
3
2.5
2
10^{+4}
10^{+5}
0.99
0.98
0.97
0.96
0.95
0.94
0.93
10^{+0}
10^{+1}
10^{+2}
10^{+3}
10^{+4}
10^{+5}
10^{+6}

$$\xi \gtrsim 3.5 \times 10^4 \,, \qquad \Lambda \text{ cannot be constrained .} \tag{18}$$

Super Yang-Mills Inflation:

We finally consider the case where the inflaton is the composite state of the super partner of glueball called gluino-ball in the super Yang-Mills theory.

In this case, [a]

$$V(\varphi) = 4N_c^2\varphi^4(\ln[\varphi/\Lambda])^2 , \tag{19}$$

where Λ is the confining scale and N_c is the number of colors.

[a] P. Channuie, J. J. Jorgensen and F. Sannino, "Composite Inflation from Super Yang-Mills, Orientifold and One-Flavor QCD," Phys. Rev. D **86**, 125035 (2012) [arXiv:1209.6362 [hep-ph]].

In the $\xi \to \infty$ limit :

$$n_s \simeq 1 + f(\mathcal{N}) + \frac{g(\mathcal{N}, \varphi, \Lambda)}{\xi} + \mathcal{O}\left(\frac{1}{\xi^2}\right), \quad (20)$$

$$A_s \simeq \frac{Y^4 [3Y + 1]}{2\pi^2 N_c^2 [3Y - 1] \xi^2} + \mathcal{O}(1/\xi^3), \quad (21)$$

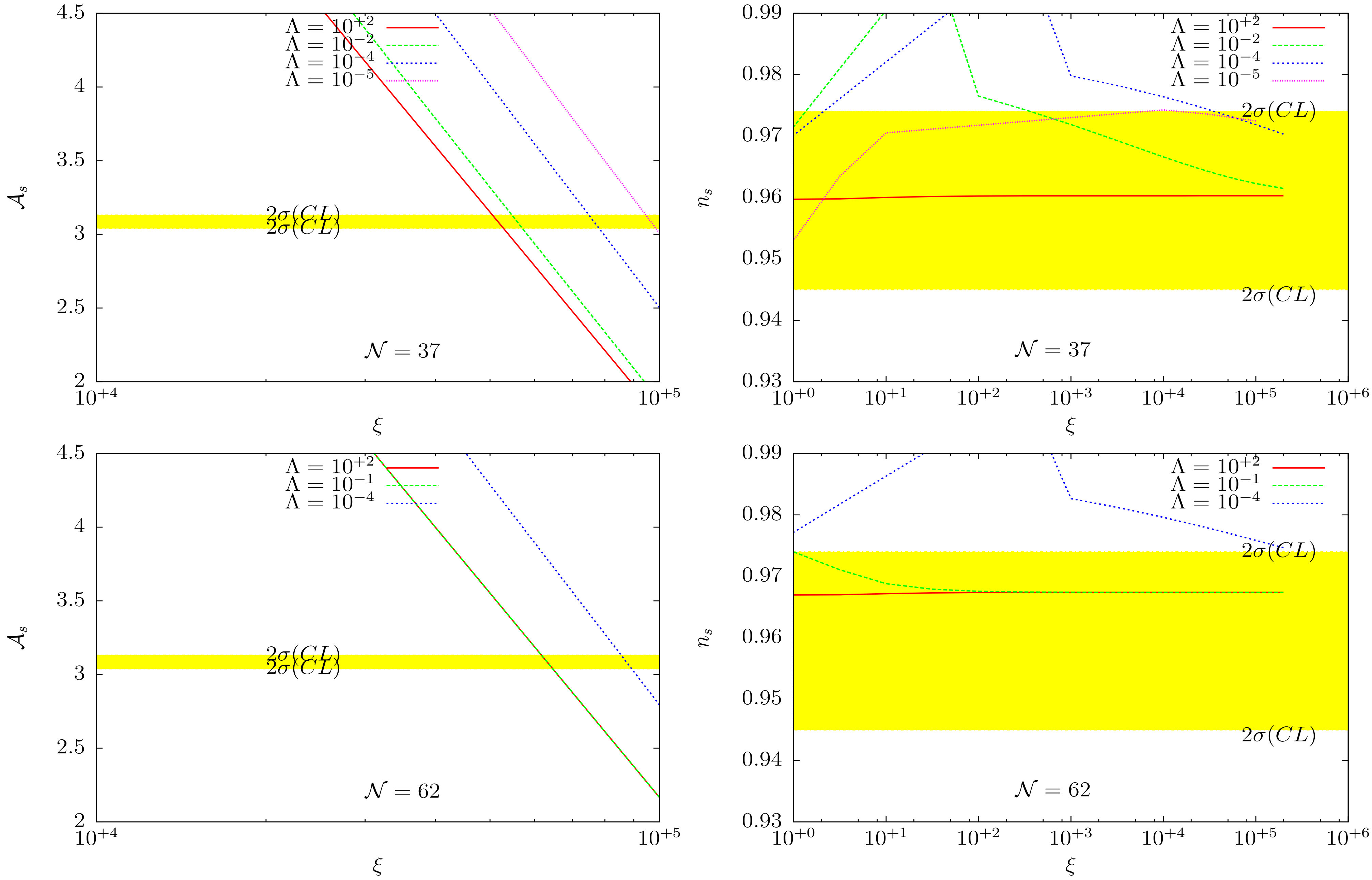
where $Y = \ln(\varphi/\Lambda)$. If $\xi > 10^4, \Lambda > 0.1$, the next leading contributions can be neglected, so that for $\xi > 10^4$ and $\Lambda > 0.1$,

$$0.945 \lesssim n_s \lesssim 0.975, \quad \Rightarrow \quad 37 \lesssim \mathcal{N} \lesssim 80$$

Similar to the glueball model, the power spectrum amplitude increases when ξ decreases, and the power spectrum amplitude cannot satisfy the observational bound if $\xi < 1$.

For this model,

$$\boxed{\xi \gtrsim 5 \times 10^4\,, \qquad \Lambda \gtrsim 10^{-5}\,.} \tag{22}$$


$\Lambda = 10^{+2}$
$\Lambda = 10^{-2}$
$\Lambda = 10^{-4}$
$\Lambda = 10^{-5}$
$2\sigma(CL)$
$2\sigma(CL)$
$\mathcal{N} = 37$
$\mathcal{A}_s$
ξ
n_s
$\Lambda = 10^{+2}$
$\Lambda = 10^{-1}$
$\Lambda = 10^{-4}$
$\mathcal{N} = 62$

Conclusions

We study how the parameters of various composite inflation model influence the spectral index n_s and the power spectrum amplitude A_s of the primordial curvature perturbations, and then constrain these parameters using the observational bound on n_s and A_s from the Planck data. We find that for the techni-inflation model, the ratio κ/ξ^2 is constrained to be $\sim 10^6$ for a large ξ. In the case of glueball-inflation, we find that the observational bound for n_s and A_s can be satisfied at 2-σ if $\xi \gtrsim 3.5 \times 10^4$. For the Super Yang-Mills Inflation, we find that ξ is constrained to be $\gtrsim 5 \times 10^4$ and $\Lambda \gtrsim 10^{-5}$