the hierarchy problem (as spelled out in the textbooks)

[w/ momentum dependent regularization (pole in D=2)]

$$\delta m_h^2 = \frac{\Lambda^2}{8\pi^2 v_W^2}\left[3m_h^2 + 3m_Z^2 + 6m_W^2 - 12m_t^2\right]$$

quadratic sensitivity to cutoff

→ cannot keep scales apart

the little hierarchy problem

"natural" scale $\Lambda_{\rm top} \simeq \dfrac{4\pi\, m_h}{\sqrt{12}} \simeq 450$ GeV

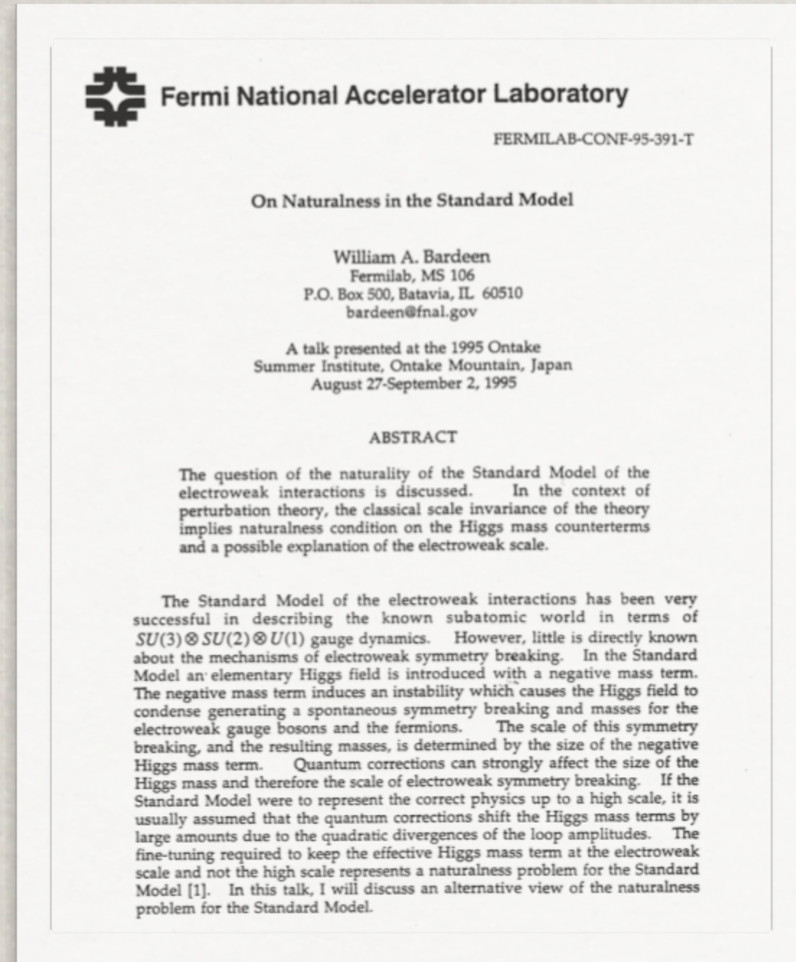
but EW radiative corrections
say no NP up to $\Lambda = 10$ TeV

troublesome points w/ cutoff regularization

- mixing of UV and IR terms
- integrating over vs. integrating out

following effective field theory

→ better use DR



Fermi National Accelerator Laboratory

FERMILAB-CONF-95-391-T

On Naturalness in the Standard Model

William A. Bardeen
Fermilab, MS 106
P.O. Box 500, Batavia, IL 60510
bardeen@fnal.gov

A talk presented at the 1995 Ontake
Summer Institute, Ontake Mountain, Japan
August 27-September 2, 1995

ABSTRACT

The question of the naturality of the Standard Model of the electroweak interactions is discussed. In the context of perturbation theory, the classical scale invariance of the theory implies naturalness condition on the Higgs mass counterterms and a possible explanation of the electroweak scale.

The Standard Model of the electroweak interactions has been very successful in describing the known subatomic world in terms of $SU(3) \otimes SU(2) \otimes U(1)$ gauge dynamics. However, little is directly known about the mechanisms of electroweak symmetry breaking. In the Standard Model an elementary Higgs field is introduced with a negative mass term. The negative mass term induces an instability which causes the Higgs field to condense generating a spontaneous symmetry breaking and masses for the electroweak gauge bosons and the fermions. The scale of this symmetry breaking, and the resulting masses, is determined by the size of the negative Higgs mass term. Quantum corrections can strongly affect the size of the Higgs mass and therefore the scale of electroweak symmetry breaking. If the Standard Model were to represent the correct physics up to a high scale, it is usually assumed that the quantum corrections shift the Higgs mass terms by large amounts due to the quadratic divergences of the loop amplitudes. The fine-tuning required to keep the effective Higgs mass term at the electroweak scale and not the high scale represents a naturalness problem for the Standard Model [1]. In this talk, I will discuss an alternative view of the naturalness problem for the Standard Model.

W.A. Bardeen, 1995

SM in the limit $m_h \to 0$

scale invariance

$$\Theta^\mu_\mu = 0$$

- trace anomaly gives only multiplicative mass corrections
- negative mass squared is a soft term

counter terms or use only DR

for the time being I am not going to worry about the quadratically divergent terms

even though they have been the motivation for SUSY in the past 40 years

W. A. Bardeen, FERMILAB-CONF-95-391-T, 1995
K.A. Meissner and H. Nicolai, 2008
M. Shaposhnikov and D. Zenhausern, 2009
F. Bazzocchi and M.F., 2012
M. Farina, D. Pappadopulo and A. Strumia, 2013
J. Lykken, 2013

no new physics beyond SM → no problem

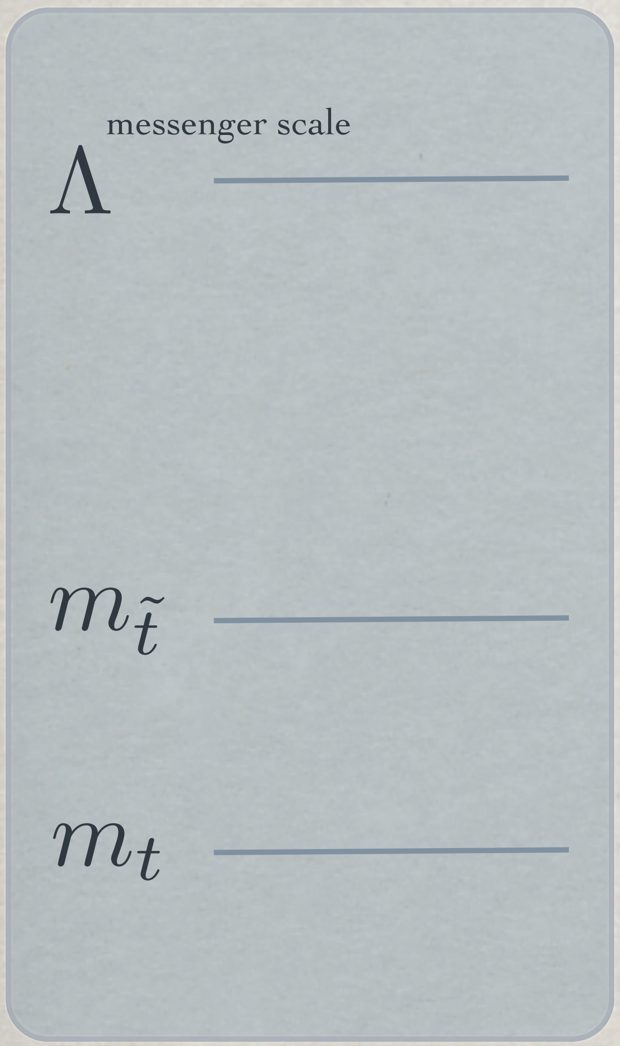
SM all the way to $M_{\rm Pl}$

new physics at scale M_χ →

$$\delta\mu_H^2(\mu) = \frac{1}{(2\pi)^2}\left[M_\chi^2 + M_\chi^2 \ln\frac{M_\chi^2}{\mu^2}\right]$$

IR sensitivity: one-loop finite terms

example: δm_h^2 in the MSSM



$$\frac{3y_t^2}{8\pi^2}\left(m_{Q_3}^2 + m_{u_3}^2 + |A_t|^2\right)\ln\frac{\Lambda}{m_{\tilde{t}}}$$

$$m_Z^2 2\cos^2 2\beta + \frac{3}{(4\pi)^2}\frac{m_t^2}{v^2}\left[\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_t^2}\left(1 - \frac{X_t^2}{12m_t^2}\right)\right]$$

L.J. Hall *et al*, 2011

$$v^2 = -\mu^2/\lambda \qquad v = O(100\,\mathrm{GeV}) \quad \lambda < 1$$

model building requires guiding principles

let new physics enter in such a way
that IR finite contributions to the Higgs boson mass
cancel

$O(m_h)$

one loop: solution to the little hierarchy problem

physics at the new scale decouples from lower scale

Low-scale seesaw mechanism and scalar dark matter

an example at work

M.F. and S. Petcov, 2013

neutrino masses and seesaw mechanism

$$\mathcal{L} = -y^{\nu}_{a\ell}\bar{N}_{aR}\tilde{H}^{\dagger}L_{\ell} - \frac{1}{2}\bar{N}^{c}_{aL}M_{Nab}N_{bR} + H.c.$$

$$\hat{y}^{\nu}_{j\ell} = M_{N_j}(RV)^{T}_{j\ell}/v_W$$

3 RH Majorana neutrinos (singlets)

couplings to LH leptons and neutrinos

$$|(RV)_{e1}|^2, |(RV)_{\mu 1}|^2, |(RV)_{\tau 1}|^2 \lesssim 10^{-3}$$

$$\left|\sum_k (RV)^{*}_{\ell' k} M_k (RV)^{\dagger}_{k\ell}\right| = |(m_v)_{\ell'\ell}| \lesssim 1 \text{ eV}$$

D.N. Dinh *et al*, 2012
Akhmedov *et al*, 2013

$$\alpha = |(RV)_{e1}|^2 + |(RV)_{\mu 1}|^2 + |(RV)_{\tau 1}|^2$$

traditional see-saw $\alpha \simeq 10^{-12}$

low-scale see-saw $\alpha \simeq 10^{-3}$

Ibarra *et al,* JHEP 1009 (2011) 013005

Higgs boson mass renormalization

$$\delta\mu_H^2(\mu) = \frac{4y^2}{(4\pi)^2} M_N^2 \left(1 - \log\frac{M_N^2}{\mu^2}\right)$$

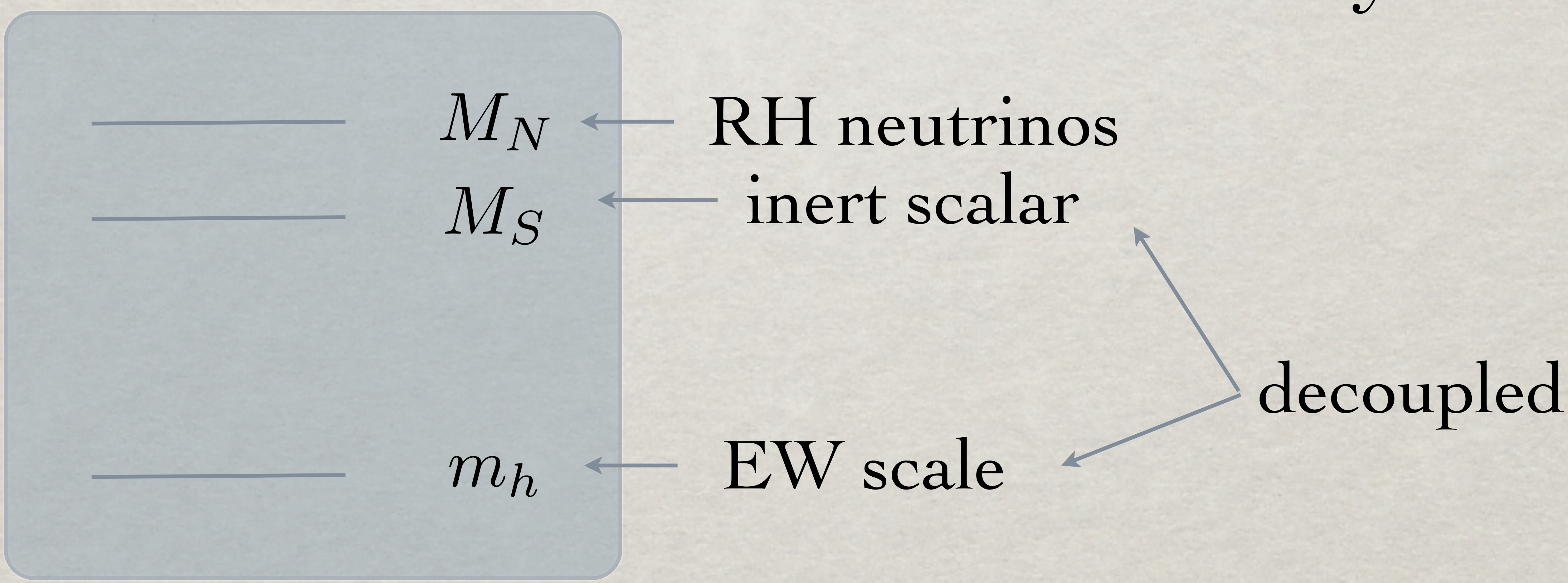
largest Yukawa coupling

$$y^2 v_W^2 = 2M_N^2 \left[|(RV)_{e1}|^2 + |(RV)_{\mu 1}|^2 + |(RV)_{\tau 1}|^2\right]$$

simplest choice: add an inert scalar

$$V(H,S) = \mu_H^2 (H^\dagger H) + \mu_S^2 S^2 + \lambda_1 (H^\dagger H)^2 + \lambda_2 S^4 + \lambda_3 (H^\dagger H) SS$$

Z_2 symmetry



one-loop renormalization

$$\delta\mu_H^2(M_S) = \frac{1}{(4\pi)^2}\left[\lambda_3 M_S^2 - 4y^2 M_N^2\left(1 - \log\frac{M_N^2}{M_S^2}\right)\right]$$

new inert scalar heavy RH neutrinos

controlling the one-loop renormalization

$$\lambda_3 = \frac{4y^2 M_N^2}{M_S^2}\left(1 - \log\frac{M_N^2}{M_S^2}\right)$$

$O(m_h)$

inert scalar as cold dark matter

V. Silvera and A. Zee, 1985
J. McDonald, 1994
C.P. Burgess *et al*, 2001
R. Dick *et al*, 2008
C.E. Yaguna, 2009
K. Cheung *et al*, 2012

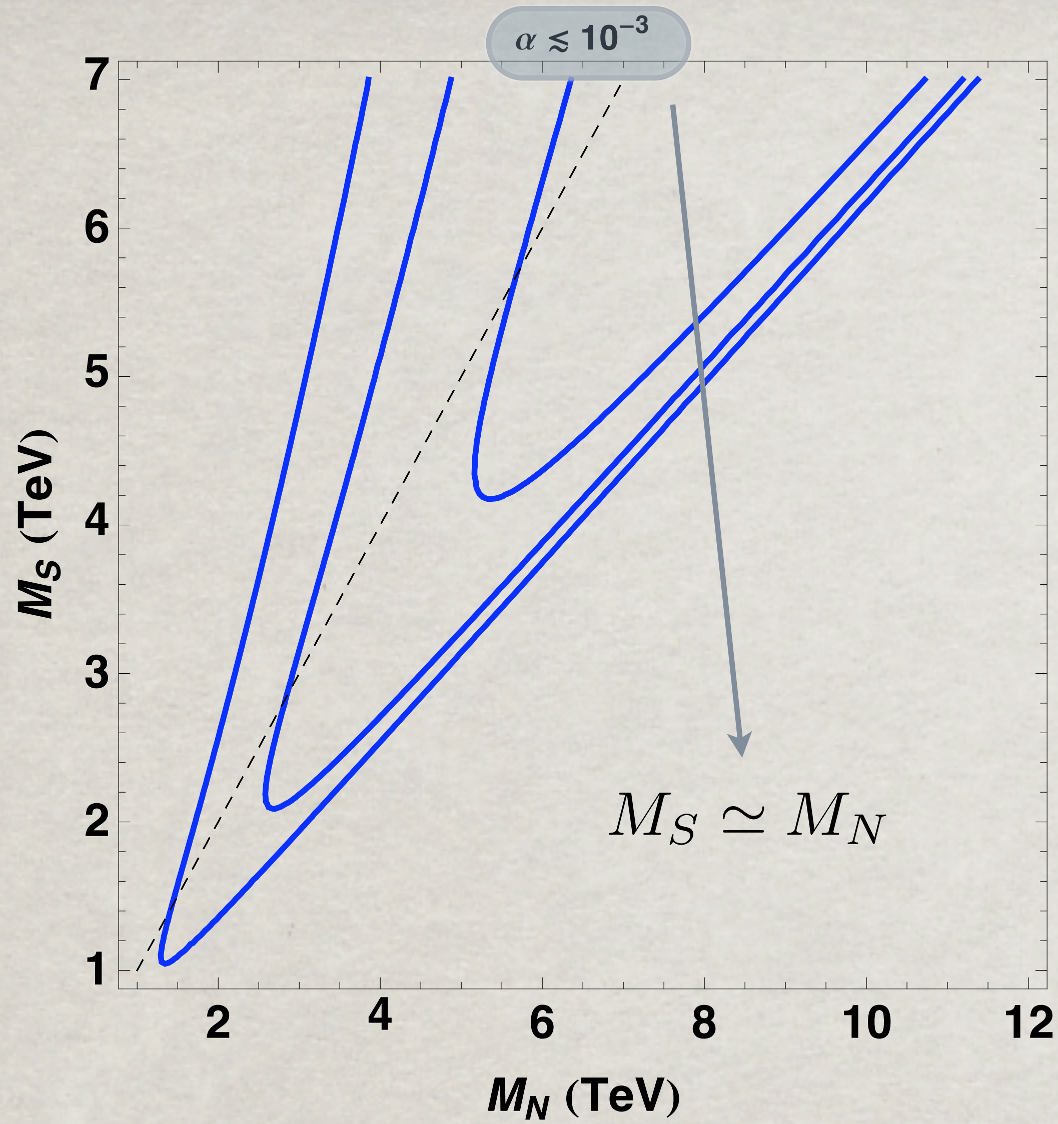
$$\Omega_S h^2 \simeq 8.41 \times 10^{-11} \frac{M_S}{T_f} \sqrt{\frac{45}{\pi g_*}} \frac{\text{GeV}^{-2}}{\langle \sigma v \rangle}$$

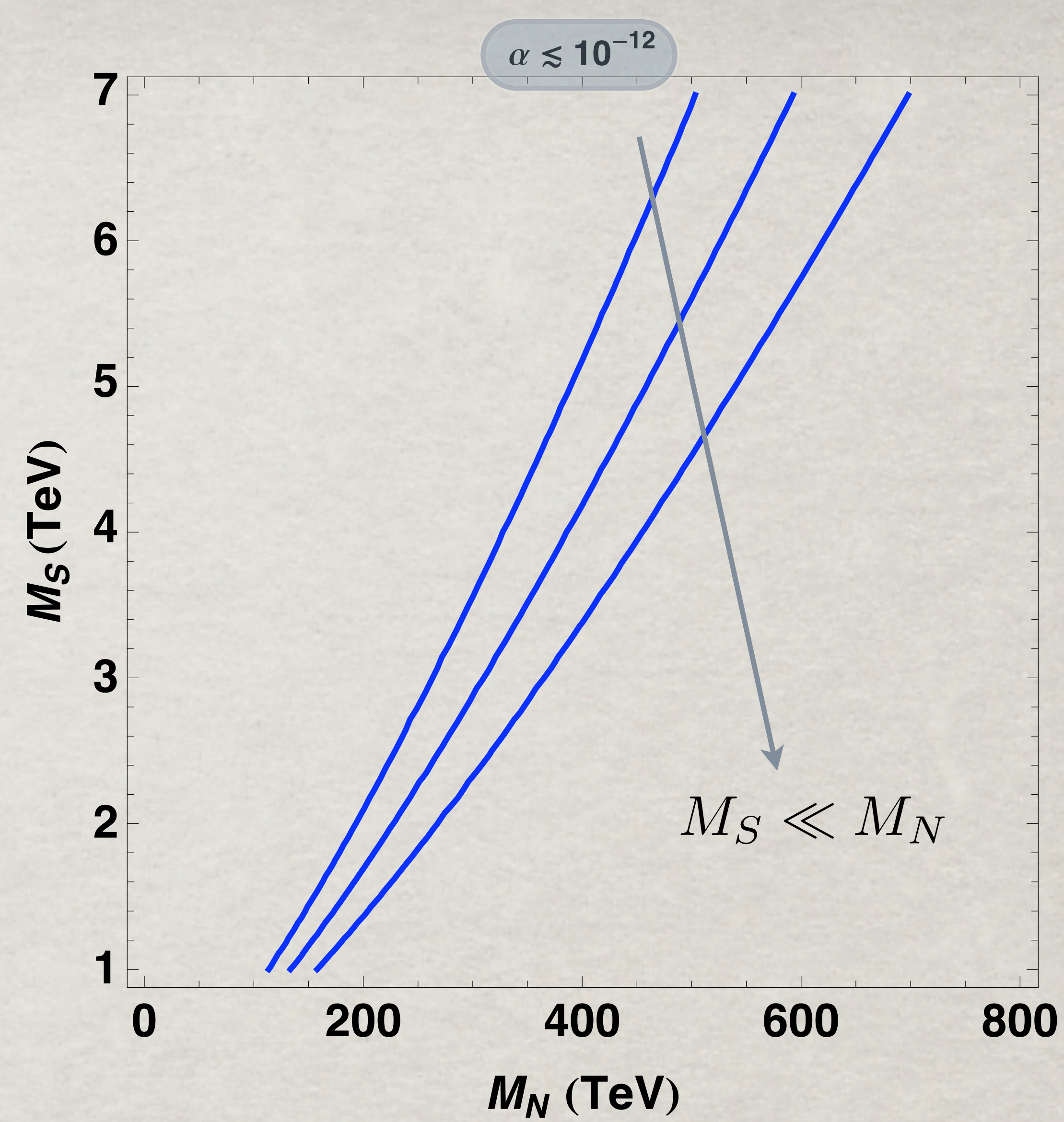
$$\langle \sigma v \rangle \simeq \frac{1}{4\pi} \frac{\lambda_3^2}{M_S^2} \sqrt{1 - \frac{m_h^2}{m_S^2}}$$

$$\Omega_{\text{DM}} h^2 = 0.1187 \pm 0.0017$$

Planck Collaboration, 2013

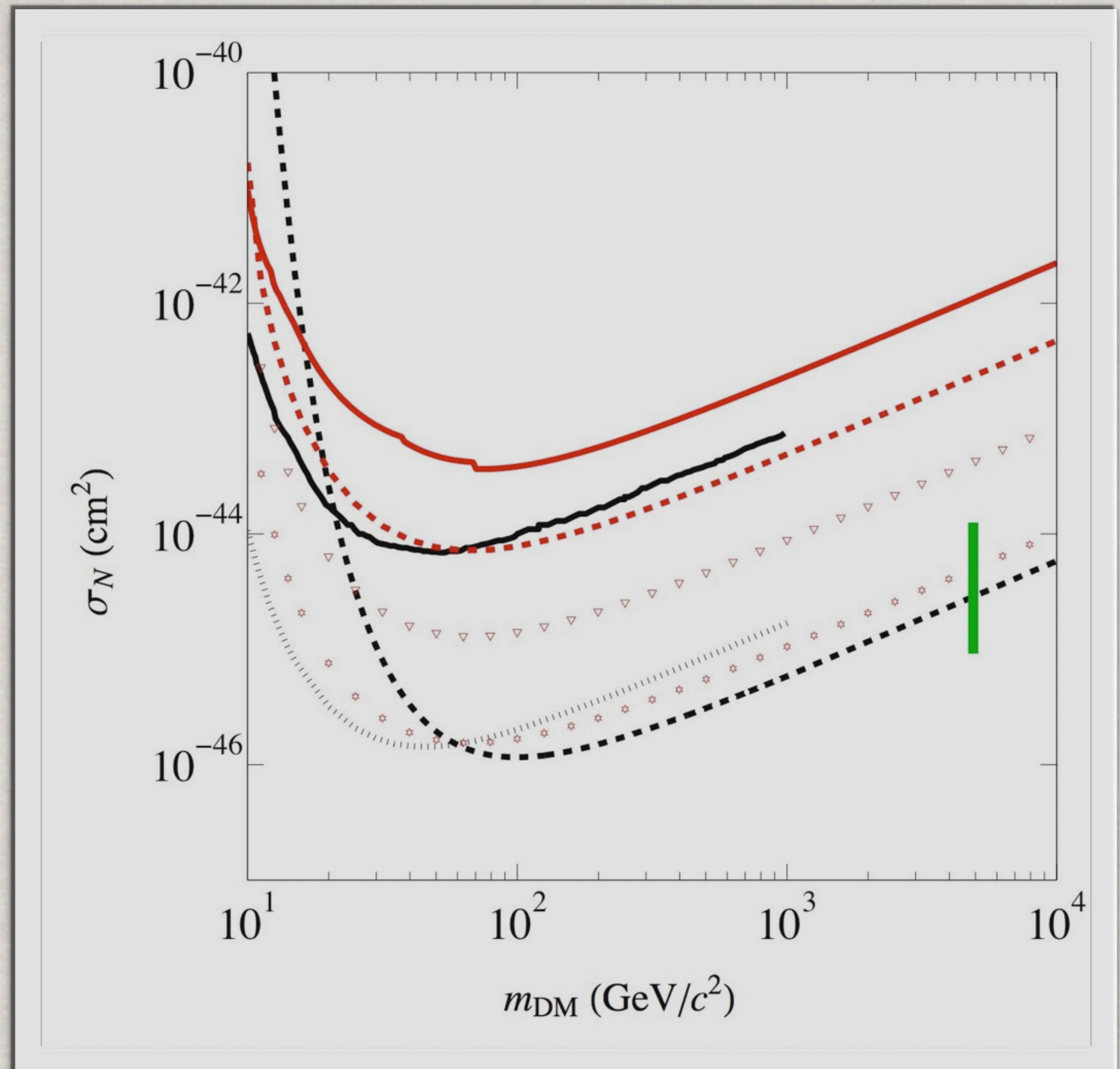
$$\Rightarrow \quad |\lambda_3| \simeq 0.15 \frac{M_S}{\text{TeV}}$$





$$0.15\, M_S^3 = 8\, \alpha\, \frac{M_N^4}{v_W^2} \left(1 - \log \frac{M_N^2}{M_S^2}\right)$$

$$\alpha = |(RV)_{e1}|^2 + |(RV)_{\mu 1}|^2 + |(RV)_{\tau 1}|^2$$



XENON100
http://dmtools.brown.edu

$$\sigma_N = f_N^2 m_N^2 \frac{\lambda_3^2}{4\pi}\left(\frac{m_r}{m_S m_h^2}\right)^2$$