NLO QCD corrections to $WZjj$ production at the LHC

Francisco Campanario, Matthias Kerner, LE Duc Ninh, Dieter Zeppenfeld | Windows on the universe, Aug 2013, Quy Nhon
Outline

- $VVjj$ production (with leptonic decays) at the LHC: motivation
- $WZjj$ @ NLO QCD: some calculational details
- Numerical results
- Summary
**VVjj production at the LHC: why?**

- **Motivation:**
  - Sensitive to $VV \rightarrow VV$ scattering, quartic quage-boson couplings.
  - Important background for new physics searches.
  - $WVjj$ with one charged lepton unobserved: background to $W^+ W^+ jj$ production.
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- **Classification at LO: 2 mechanisms**
  - EW mechanism (vector boson fusion, VBF): $\sigma_{EW} \propto \alpha^6$
  - QCD mechanism: $\sigma_{QCD} \propto \alpha_s^2 \alpha^4$

- Interference: color and kinematically suppressed.
  - $\sim$ can be neglected for a-few-percent precision measurements at the LHC.
What have been done at NLO QCD?

- EW mechanism (VBF): consider QCD corrections to each quark lines separately \( \rightsquigarrow \) pentagons at most.
  - \( W^+ W^- jj \): [Jager, Oleari, Zeppenfeld, 2006]
  - \( ZZjj \): [Jager, Oleari, Zeppenfeld, 2006]
  - \( W^\pm Zjj \): [Bozzi, Jager, Oleari, Zeppenfeld, 2007]
  - \( W^+ W^+ jj/W^- W^- jj \): [Jager, Oleari, Zeppenfeld, 2009], [Denner, Hosekova, Kallweit, 2012], good agreement!

- QCD mechanism: two quark lines are not independent \( \rightsquigarrow \) hexagons at most.
  - \( W^+ W^+ jj/W^- W^- jj \), \( W^\pm Zjj \) are included in VBFNLO program. Available soon.
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- all VBF processes and QCD $W^+ W^+ jj/W^- W^- jj$, $W^\pm Zjj$ are included in VBFNLO program. Available soon.
LHC $\sqrt{s} = 14$ TeV, $p_{Tj} \geq 20$ GeV, $|\eta_j| < 5$, $\Delta R_{jj} \geq 0.7$,
$|\eta_\gamma| < 2.5$, $\Delta R_{j\gamma} \geq 0.7$;
$\eta_j,\text{min} + 0.7 < \eta_\gamma < \eta_j,\text{max} - 0.7$, $\eta_{j1}\eta_{j2} < 0$

The two tagging jets are more separated in VBF than in QCD background!
\[ pp \rightarrow W^\pm Zjj: \text{QCD mechanism} \]
The problem

Q: What is the NLO QCD correction to this process?
A: Before answering this question, some classifications are needed.
LO: subprocesses

- 2-quark lines \([4q]\): \(q_1 + q_2 \rightarrow q_3 + q_4 + (WV)\)

- 76 subprocesses (2 generations).
- 12 crossings.

- 1-quark line \([2g]\): \(q_1 + q_2 \rightarrow g + g + (WV)\)

- 14 subprocesses (2 generations).
- 7 crossings.

- 4 QCD gauge invariant groups: \(4q(W), 4qWV, 2g(W), 2gWV\).
NLO calculation: theory

\[ d\sigma_{NLO} = d\sigma_{2\to N}^{\text{virt}} + d\sigma_{2\to N+1}^{\text{real}}. \]

- Both terms are IR divergent. The sum is finite for IR-safe observables (e.g. jet distributions)
NLO calculation: theory vs. practice

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- Real: IR divergences can be separated using Catani-Seymour dipole subtraction method.
- Virtual: 1-loop amplitude is, unfortunately, much more complicated than tree-level one. Use Feynman-diagram and tensor reduction methods. The most difficult part.
Counting diagrams: 2 generations

- LO: 4840
- NLO real emission: 79784
- NLO virtual: 116896 (up to 6-point rank 5)

This calculation can be done with:
- good classifications: effective currents $V \rightarrow l_1 l_2$, $V \rightarrow l_1 l_2 l_3 l_4$,
- building blocks (hexlines, penlines, ...),
- use crossing symmetry (to a minimum extent to reduce computing time):
- subprocesses are not completely independent (diagrams share common parts).
- two independent calculations:
  - manual implementation using VBFNLO framework
  - more automated approach using HELAS/MadGraph, FeynArts, FormCalc
- loop integrals: 2 different codes
- numerical instabilities in the virtual part: difficult $\rightarrow$ gauge tests
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Numerical instabilities: gauge test

\[ B^N = T^N_{\mu} \epsilon^\mu (k), \quad \epsilon^\mu \rightarrow k^\mu, \]

\[ \frac{1}{q + k} = \frac{1}{q} - \frac{1}{q + k}, \]

\[ T^N_{\mu} k^\mu - A^{N-1} = 0, \]

\[ Q = 1 - \frac{A^{N-1}}{T^N_{\mu} k^\mu}, \]

if \( Q < \varepsilon \) : accept the point.

if \( Q > \varepsilon \) : use quadruple precision,

\[ \sim Q < \varepsilon ? \text{ accept or discard points.} \]
Results: inclusive cuts

\[ \text{pp} \rightarrow e^{\pm} \nu_e \mu^{\pm} \mu^{\mp} jj + X \]

\[ \sqrt{s} = 14 \text{ TeV} \]

- LO
- NLO

\[ \sigma \text{ [fb]} \]

- speed: 1% statistical error in 2.5h on a normal PC (Intel i5-3470) with 1 core.

Campanario, Kerner, LDN, Zeppenfeld, arXiv: 1305.1623

- \( p_T,j > 20 \text{ GeV}, \quad |\eta_j| < 4.5, \quad R_{jj}^{anti-kt} = 0.4, \quad R_{jl} > 0.4; \)
- \( p_T,l > 20 \text{ GeV}, \quad |\eta_l| < 2.5, \quad R_{ll} > 0.4, \quad M_{l^+l^-} > 15 \text{ GeV}. \)

Dynamic scale: \( \mu_F = \mu_R = \mu_0 = \left( \sum_{\text{jet}} p_T,\text{jet} + \sqrt{p_{T,W}^2 + m_W^2} + \sqrt{p_{T,Z}^2 + m_Z^2} \right)/2, \)

\( p_T,\nu \) (m_\nu) are reconstructed from leptons, jets pass all cuts.
Distributions of leading jets

\[ pp \rightarrow e^+\nu_e\mu^+\mu^-jj+X \]

- scale uncertainty: significantly reduced
- K factor (NLO/LO): 0.6–1 in a large range
- \( \rightarrow \) regular NLO QCD corrections (as expected)
Summary

- $VVjj$ is a special class of processes: sensitive to $VV \rightarrow VV$ scatterings, quartic gauge couplings, background for new physics searches, ...
- Two mechanisms: EW (VBF), QCD, interference effects are very small.
- For $WZjj$ with leptonic decays, QCD mechanism:
  - K factor: from 0.6–1 with a dynamic scale choice for many distributions
  - scale uncertainty: from 50% at LO to 5% at NLO for inclusive cross section
- Virtual amplitude: difficult part, gauge tests are good to deal with numerical instabilities.
- The code will be available in the next release of the VBFNLO program.

Thank you!
Dipole subtraction method

\[
\int_{N+1} d\sigma_{N+1}^{\text{real}}(p) J_{N+1}^N(p) = \int_{N+1} \left[ (d\sigma_{N+1}^{\text{real}}(p) J_{N+1}^N(p) - \sum_{i,j} S_{ij}^N(\tilde{p}_{ij}) J_{ij}^N(\tilde{p}_{ij}) \right] \tag{1}
\]

\[
+ \int_{N+1} \sum_{i,j} S_{ij}^N(\tilde{p}_{ij}) J_{ij}^N(\tilde{p}_{ij}) \quad \text{PK+I}
\]

\[
\text{PK} = \int_0^1 dx \int_N \sum_{j \neq a} S_{aj}^N(x, p) J_{a}^N(x, p) + (a \leftrightarrow b) \tag{2}
\]

\[
I = \int_N \sum_{i,j} S_{ij}^N(p) J_{ij}^N(p). \tag{3}
\]
Dipole subtraction method

\[
\begin{align*}
\int_{N+1} d\sigma_{N+1}^{\text{real}}(p)J^{N+1}(p) & = \int_{N+1} \left[ (d\sigma_{N+1}^{\text{real}}(p)J^{N+1}(p) - \sum_{i,j} S_{ij}^N(\bar{p}_{ij})J_{ij}^N(\bar{p}_{ij}) \right] \\
& + \int_{N+1} \sum_{i,j} S_{ij}^N(\bar{p}_{ij})J_{ij}^N(\bar{p}_{ij}) \\
& \equiv \text{PK+I}
\end{align*}
\]

PK \quad = \quad \int_0^1 dx \int_N \sum_{j \neq a} S_{aj}^N(x,p)J_{a}^N(x,p) + (a \leftrightarrow b) \quad (2)

I \quad = \quad \int_N \sum_{i,j} S_{ij}^N(p)J^N(p). \quad (3)

The Jet function: a cut, a histogram, PDF(Q), \( \alpha_s(Q) \), ...

- IR safety: \( J^N \rightarrow J^{N+1} \) in the IR singular limits.
- An easy mistake: in Eq. (1), set \( \alpha_s^N(\bar{Q}) = \alpha_s^{N+1}(Q) \): the result is finite, BUT wrong (almost correct) because the integrated part \( \neq \) the subtraction part.
- If we do \( J^N = J^{N+1} \) in Eq. (1), then PK term will get more complicated. [arXiv: 0802.1405]
Tree-level, virtual and real matching

- Partonic level:

\[
\begin{align*}
    d\sigma_{\text{soft}}^l + d\sigma_{\text{soft}}^{\text{virt}} &= 0, \\
    d\sigma_{\text{coll}}^l + d\sigma_{\text{coll}}^{\text{virt}} &= 0, \\
    d\sigma_{\text{coll}}^{PK} &\neq 0.
\end{align*}
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  \[ d\sigma_{\text{coll}}^{PK} \neq 0. \]

- Hadronic level:
  
  \[ d\sigma^{NLO} = d\sigma^{\text{tree}} + d\sigma^{\text{virt}} + d\sigma^{\text{real}}, \]
  \[ \text{PDF}^{NLO} \otimes d\sigma^{NLO} = \int dx_1 \int dx_2 f_a(x_1, Q)f_b(x_2, Q)d\sigma^{NLO} + \delta_{\text{PDF}}(d\sigma^{\text{tree}}, 1/\epsilon), \]
  \[ d\sigma^{PK} \equiv d\sigma^{PK}(1/\epsilon) + \delta_{\text{PDF}}(d\sigma^{\text{tree}}, 1/\epsilon) : \text{finite} \]
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  d\sigma_{\text{PK}} \equiv d\sigma_{\text{PK}}(1/\epsilon) + \delta_{\text{PDF}}(d\sigma_{\text{tree}}, 1/\epsilon) : \text{finite}
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- Default matching: PDF and \( d\sigma_{\text{real}} \) are defined in the conventional dimensional-regularization scheme (CDR), AND \( \overline{\text{MS}} \) scheme.
Tree-level, virtual and real matching

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\[ d\sigma_{\text{PK}} = d\sigma_{\text{PK}}^{\text{finite}} + \delta_{\text{PDF}}(d\sigma_{\text{tree}}^{\text{finite}}) : \text{finite} \]

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**Virtual:** use ’t Hooft-Veltman (HV) scheme (external momenta in 4D)
\[ d\sigma_{\text{virt}}^{\text{HV}} = d\sigma_{\text{virt}}^{\text{CDR}}, \quad \alpha_s^{\text{HV}} = \alpha_s^{\text{CDR}}. \]

**Dimensional regularization scheme (DRS) independence** [Catani, Seymour, Trócsányi 1997]:
\[ d\sigma_{\text{tree}} + d\sigma_{\text{virt}} + d\sigma_{\text{coll}}^I : \neq \text{DRS at partonic level!} \]

**4 flavors (no b/t) or 5 flavors (b and t loop):** \( \alpha_s \), PDF, tree, virtual, real.
Fermion loops

- Include $V$ decays:

$$\epsilon^\mu(k, \lambda) \rightarrow J_{\text{eff}}^\mu/(k^2 - M_V^2 + iM_V \Gamma_V), \quad \text{Or}$$

$$g^{\mu\nu} = - \sum_{\lambda=-1,0,1} \epsilon^\mu(k, \lambda) \epsilon^{\ast \nu}(k, \lambda) + \frac{k^\mu k^\nu}{k^2}, \quad = 0$$

$$k^\mu \bar{F}_1 \gamma_\mu(a + b \gamma_5) F_2 = 0, \quad \text{since: } m_1 = m_2 = 0 \text{ (no Goldstones)}.$$
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- Fermion loop with $\gamma_5$: anomaly free (need both $b$ and $t$)

$$\rightsquigarrow 2.5 \text{ generations not OK (we cannot decouple the top quark).}$$