MONOPOLES, POLYAKOV LINE AND CONFINEMENT.

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QCD, A SECTOR OF THE STANDARD MODEL.

$SU(3)$ GAUGE THEORY

$$L = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \Sigma_f \bar{\Psi}_f (\not{D} - m_f) \Psi_f$$

- EXISTS AS A FIELD THEORY $\rightarrow$ LATTICE FORMULATION
- $\Lambda \approx 200 Mev$ A PHYSICAL SCALE
- $Q > \Lambda$ PERTURBATIVE EXPANSION WORKS.
- $x \geq \frac{1}{\Lambda}$ NO QUARKS OBSERVED (CONFINEMENT)

$\frac{n_q}{n_p} \leq 10^{-27}$ S.C.M. $\frac{n_q}{n_p} \approx 10^{-12}$

$\frac{\sigma_q}{\sigma_{Total}} \leq 10^{-15}$ PERT. TH. $\frac{\sigma_q}{\sigma_{Total}} \approx 1$

10^{-15} ! $\longrightarrow$ A SYMMETRY.

DECONFINEMENT.

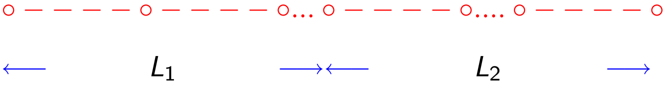
- N. CABIBBO, G. PARISI ('75) T_{Hag}: DECONFINEMENT OF COLOR. SEEN ON LATTICE, MAYBE IN H.I.C.
- FINITE TEMPERATURE FIELD THEORY
  $Z \equiv Tr\{\exp(-\beta H)\} = \int_{P.B.C.} [d\phi] \exp(\int d^3x \int_0^\beta dx_4 S(\phi))$
- QCD (GAUGE $\partial_4 A_4 = 0$)
  $\int_{P.B.C.} dA_\mu \exp(\int d^3x \int_0^\beta dx_4 S(A_\mu)) = \int dA_i(\vec{x}) dA_4(\vec{x})$
  $< A_i(\vec{x})| \exp -\beta \int d^3x [\frac{\vec{E}^2 + \vec{B}^2}{2} - i\vec{E}\vec{D}A_4(\vec{x})]|A_i(\vec{x}) >$
- $\vec{E}\vec{D}A_4 = \partial_i(E_i A_4]) - A_4 D_i E_i$
  $\int d^3x \vec{E}\vec{D}A_4 = A_4(\infty)Q - \int d^3x A_4 D_i E_i$ HOLONOMY
  ONLY IF $A_4(\infty) = 0$ AND/OR $Q = 0$
  $\int_{P.B.C.} dA_\mu \exp(\int d^3x \int_0^\beta dx_4 S(A_\mu))$
  $= \int dA_i(\vec{x}) < A_i(\vec{x})| \exp(-\beta H) 2\pi\delta(D_i E_i)|A_i(\vec{x}) >$
- LATTICE FORMULATION $N_t \times N_s^3$ $N_t \ll N_s$ $T = \frac{1}{a(g^2)N_t}$
  $T \propto \frac{1}{N_t} \exp(\frac{K}{g^2})$ [RENORMALIZATION GROUP].

POLYAKOV LOOP

- POLYAKOV LOOP : $L(\vec{x}) \equiv P\exp(i\int_t^{t+\frac{1}{T}} dt' igA_4(\vec{x},t'))$
  PARALLEL TRANSPORT ALONG TIME, PATH CLOSED BY P.B.C. ON LATTICE $L(\vec{x}) = \Pi_{i=0,N_t} U_0(\vec{x}, ia)$.
  DEF : $\langle L(\vec{x})\rangle = Tr\{L(\vec{x})\}$ GAUGE INVARIANT
  DEF : $\langle L\rangle = \frac{1}{V}\int d^3x \langle L(\vec{x})\rangle$
- GAUGE $\partial_4 A_4 = 0$ $\langle L(\vec{x})\rangle = Tr\{\exp(ig\frac{A_4(\vec{x})}{T})\}$
- $\langle L\rangle = \exp(-\frac{F_q}{T})$ $\langle L\rangle = 0 \rightarrow F_q = \infty$ (CONFINEMENT)
- $\langle L(\vec{x})L(0)\rangle \approx_{x\rightarrow\infty} C\exp(-\sigma\frac{x}{T}) + \langle L\rangle^2$. IF $\langle L\rangle = 0$
  $V(x) = -T\ln\langle L(\vec{x})L(0)\rangle \approx \sigma x$
- $T_c \approx \Lambda$ OBSERVED ON LATTICE : $T \leq T_c$, $\langle L\rangle = 0$,$(A_4 \neq 0)$;$T > T_c$, $|\langle L\rangle| = 1$,$(A_4 = 0)$
  2nd ORDER TRANSITION FOR $SU(2)$ $N_f = 0$
  1st ORDER TRANSITION FOR $SU(3)$ $N_f = 0$

POLYAKOV LOOP-2

- $T \to 0 \qquad \frac{1}{T} = aN_t \gg \frac{1}{\Lambda}$



$$L = L_1 L_2 \qquad \langle L\rangle \approx \langle L_1\rangle\langle L_2\rangle \;\; \to \langle L\rangle = \langle L\rangle^2 \;\; \langle L\rangle = 0$$

$$\langle L\rangle = 0 \qquad aN_t \geq \frac{1}{\Lambda}$$

- GAUGE GROUP $SU(2)$
  $\partial_4 A_4 = 0, A_4$ DIAGONAL (POLYAKOV GAUGE)

$$A_4 = A_4^3 \frac{\sigma_3}{2} \qquad \langle L(\vec{x})\rangle = \cos\frac{gA_4^3(\vec{x})}{2\Lambda} \quad T \leq T_c \approx \Lambda$$

$$\langle L\rangle = \frac{1}{V}\int_V d^3x' \cos\frac{gA_4^3(\vec{x}')}{2\Lambda} = 0$$

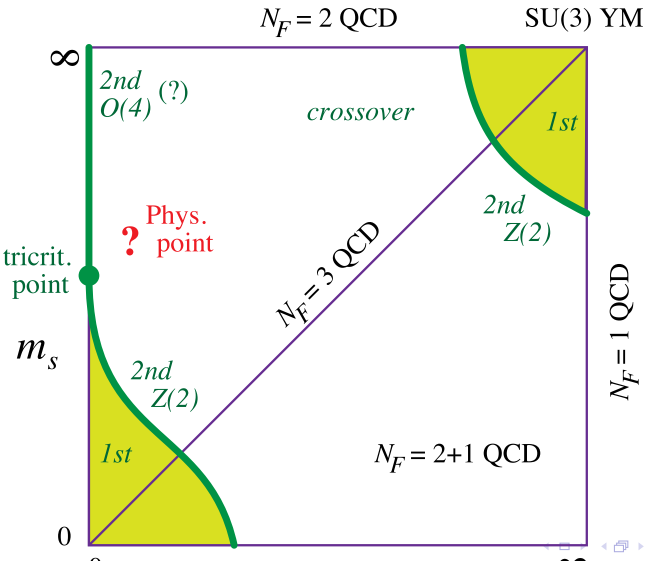
SYMMETRY

EXTRA SYMMETRY OF QCD, COMMUTING WITH GAUGE SYMMETRY:

1) FOR PURE GAUGE CENTRE OF THE GROUP $\{C\}$.
$C|0>=|0>$, $\langle CL\rangle = \langle L\rangle = \exp(2i\pi \frac{n}{N})\langle L\rangle = 0$;
IF $\langle L\rangle \neq 0$ SYMMETRY BROKEN . $\langle L\rangle$ THE ORDER PARAMETER.
QUARKS BREAK SYMMETRY. CANNOT EXPLAIN CONFINEMENT .

2) DEGREES OF FREEDOM ON THE BOUNDARY (DUAL VARIABLES) [KADANOFF 71]. 3 SPATIAL DIMENSION $\Pi_2 \rightarrow$ MONOPOLES .
MONOPOLES $SU(2)$ OBJECTS.
DUAL SUPERCONDUCTIVITY [MANDELSTAM, 'T HOOFT 75]

IS DECONFINEMENT A CHANGE OF SYMMETRY?



MONOPOLES

HIGGS MODEL: $L = -\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + \frac{1}{2}(D_\mu\vec{\Phi})^\dagger(D_\mu\vec{\Phi}) - \frac{\lambda}{4}[\vec{\Phi}\vec{\Phi} - \mu^2]$

STATIC SOLUTIONS

$$D_j G_{ji} = g\vec{\Phi} \wedge D_i\vec{\Phi} - g\vec{A}_0 \wedge D_i\vec{A}_0$$
$$D_i D_i \vec{A}_0 = g\vec{\Phi} \wedge D_0\vec{\Phi}$$
$$D_0 D_0\vec{\Phi} - D_i D_i\vec{\Phi} + \lambda(\vec{\Phi}^2 - \mu^2)\vec{\Phi} = 0$$

$A_0 = 0$ ['T HOOFT, POLYAKOV 74]

$$D_j G_{ji} = g\vec{\Phi} \wedge D_i\vec{\Phi}$$
$$-D_i D_i\vec{\Phi} + \lambda(\vec{\Phi}^2 - \mu^2)\vec{\Phi} = 0$$

NO HIGGS FIELD (QCD)

$$D_j G_{ji} = -g\vec{A}_0 \wedge D_i\vec{A}_0$$
$$D_i D_i\vec{A}_0 = 0$$

QCD MONOPOLES : $\lambda = 0$ (BPS) ; $iA_0 \equiv A_4 \leftrightarrow \Phi$

$$A_i^a = \epsilon_{ain}\frac{\hat{r}^n}{gr}(1 - K(\xi))$$
$$A_4^a = \frac{\hat{r}^a}{gr}J(\xi),\ \xi = g\mu r$$
$$B_i^a \approx_{r\to\infty} \hat{r}^a\frac{\hat{r}^i}{gr^2}$$

$$E_i^a \approx_{r\to\infty} i\hat{r}^a\frac{\hat{r}^i}{gr^2}$$
$$K(\xi) = \frac{\xi}{\sinh\xi}$$
$$J(\xi) = \xi\coth\xi - 1$$

$A_4^a(r = \infty) = \mu\hat{r}^a$

UNITARY GAUGE :

$\hat{r}^a \to \hat{n}^3$

LATTICE MONOPOLES

- MONOPOLES SEEN ON LATTICE VIA MAGNETIC FLUX. GAUGE DEPENDENT ! $\Rightarrow$ POLYAKOV GAUGE.
- MONOPOLE DOMINANCE [KANAZAWA 90, ITEP 95] . MAXIMAL ABELIAN GAUGE.
- DISORDER PARAMETER [PISA 95] $\langle\mu\rangle \neq 0$ $T \leq T_c$, $\langle\mu\rangle = 0$ $T > T_c$. μ A MAGNETICALLY CHARGED OPERATOR.
- MAX. ABELIAN GAUGE IS THE UNITARY GAUGE FOR STATIC MONOPOLES [PISA 10] . $\Rightarrow$ MONOPOLES $\approx \frac{1}{\Lambda}$ COINCIDE IN MAX. ABELIAN AND POLYAKOV GAUGE.
- CALORONS [LEE , VAN BAAL 98] FINITE T INSTANTONS $\{$WITH $A_4(\infty) \neq 0\}$. MADE OF MONOPOLES. DESCRIBE FINITE T QCD AS A GAS OF CALORONS. INTERACTIONS OF MONOPOLES SPOIL THE PICTURE. [ST PETERSBURG 05].

A SIMPLE MODEL

- A PLASMA OF BPS MONOPOLES AND ANTI-MONOPOLES, WITH EQUAL $\mu = A_4(\infty)$, i.e. EQUAL SIZE $s = \frac{1}{g\mu}$.
  $\langle L(\vec{x})\rangle = \cos[\frac{g\mu}{2\Lambda}(\coth\xi - \frac{1}{\xi})] \qquad \xi = g\mu|\vec{x}|$
  $\coth\xi - \frac{1}{\xi} \quad [0 \to 1]$ AS ξ $[0 \to \infty] \Rightarrow \frac{g\mu}{2\Lambda} = (2k+1)\pi$
- $\langle L\rangle = \int dVP(V)\frac{1}{V}\int dV' \cos[\pi(\coth\xi' - \frac{1}{\xi'})]$
  $P(V) = 2\rho\exp(-2\rho V) \qquad \rho =$ DENSITY OF MONOPOLES
  $\langle L\rangle = 3A\int_0^\infty d\xi\xi^2\ (-)Ei(-A\xi^3)\cos[\pi(\coth\xi - \frac{1}{\xi})]$
  $A = \frac{8\pi}{3}\frac{1}{(g\mu)^3}\rho = \frac{\rho}{3\pi^2\Lambda^3}$
- DISTRIBUTION IN $d\cos(\theta) \qquad \theta = \pi[\coth\xi - \frac{1}{\xi}]$
  $\bar{P}(\cos\theta) = \frac{3A\xi^2 Ei(-A\xi^3)}{\pi(\frac{1}{\xi^2} - \frac{1}{\sinh^2\xi})\sin[\pi(\coth\xi - \frac{1}{\xi})]}$

  NO SYMMETRY $\cos(\theta) \to \cos(\pi - \theta) \qquad \xi \to \frac{3}{\xi}$
  COMPARE TO LATTICE $\qquad A \approx .04$

MONOPOLE CONDENSATION

- $\phi = \frac{1}{\sqrt{(2MV)}}[a_{\vec{0}} + b^{\dagger}_{\vec{0}}] \qquad \frac{M^2}{\bar{\lambda}} = \langle g| : \phi^{\dagger}\phi : |g\rangle = \frac{|\alpha|^2}{MV}$
  $|g\rangle = \exp(-|\alpha|^2)\exp(\alpha a^{\dagger}_{\vec{0}})\exp(\alpha b^{\dagger}_{\vec{0}})|0,0\rangle \;\; |\alpha|^2 = N_m = N_{\bar{m}}$
  $\rho \equiv \frac{N_m}{V} = \frac{M^3}{\bar{\lambda}}$
- $M = \frac{4\pi\mu}{g} = \frac{8\pi^2}{g^2}\Lambda$ BPS MONOPOLE $\mu = A_4(\infty)$
- $.04 \approx A = \frac{\rho}{3\pi^2\Lambda^3} = (\frac{M}{\Lambda})^3\frac{1}{\bar{\lambda}3\pi^2} = \frac{8\pi}{3\bar{\lambda}}(\frac{4\pi}{g^2})^3$ INDICATES THAT
  $g^2\bar{\lambda} \geq 2$ TYPE II SUPERCONDUCTOR.

CONCLUSIONS

- MONOPOLE CONDENSATION A NATURAL MECHANISM FOR CONFINEMENT (A SYMMETRY).
- NON-TRIVIAL HOLONOMY $(A_4(\infty) \neq 0)$ NECESSARY TO HAVE MONOPOLES. $\langle L \rangle$ ORDER PARAMETER ?
- MONOPOLES CATALIZE INSTANTONS [RUBAKOV 82], i.e. CHIRAL SYMMETRY BREAKING. $[\rho^{(3)}_{mon}\Lambda = \rho^{(4)}_{inst}, T \leq T_c]$
- BETTER KNOWLEDGE OF MULTIMONOPOLE CLASSICAL BACKGROUND CONFIGURATIONS NEEDED. A SIMPLE MODEL INDICATES THAT THE AVERAGE DISTANCE BETWEEN MONOPOLES IS LARGER THAN THEIR SIZE.
- THE MODEL INDICATES THAT THE DUAL SUPERCONDUCTOR IS TYPE II.