

# Gauge/gravity duality and applications

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Inauguration conference, ICISE, Quy Nhon, Vietnam

# Plan

- QCD, large  $N_c$  limit
- Gauge/gravity duality
- Quark/gluon plasma, viscosity
- Applications to condensed matter physics

# QCD: nonperturbative part of the Standard Model

$$SU(3) \times \underbrace{SU(2) \times U(1)}$$

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electroweak interactions  
Glashow-Weinberg-Salam

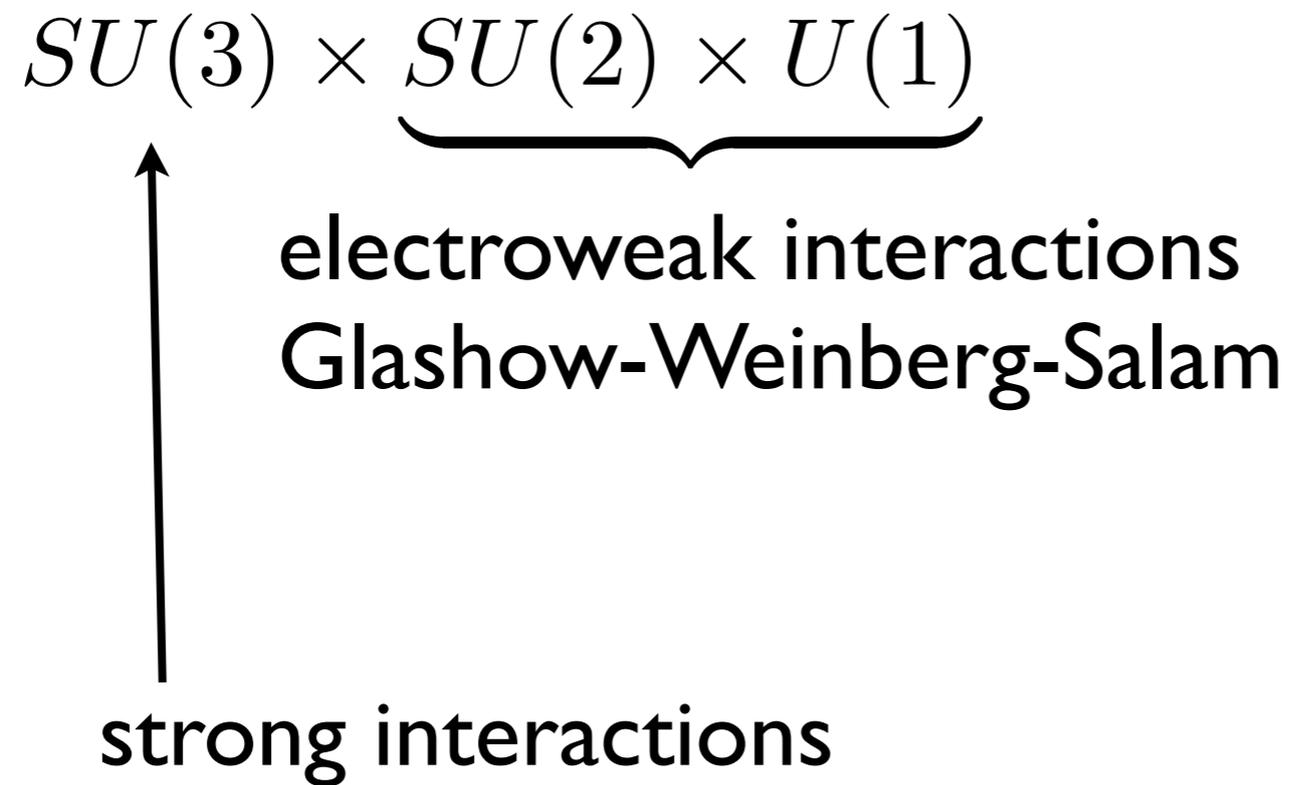
# QCD: nonperturbative part of the Standard Model

$$SU(3) \times \underbrace{SU(2) \times U(1)}$$

electroweak interactions  
Glashow-Weinberg-Salam

↑  
strong interactions

# QCD: nonperturbative part of the Standard Model



Asymptotic freedom **Gross Politzer Wilczek 1972**

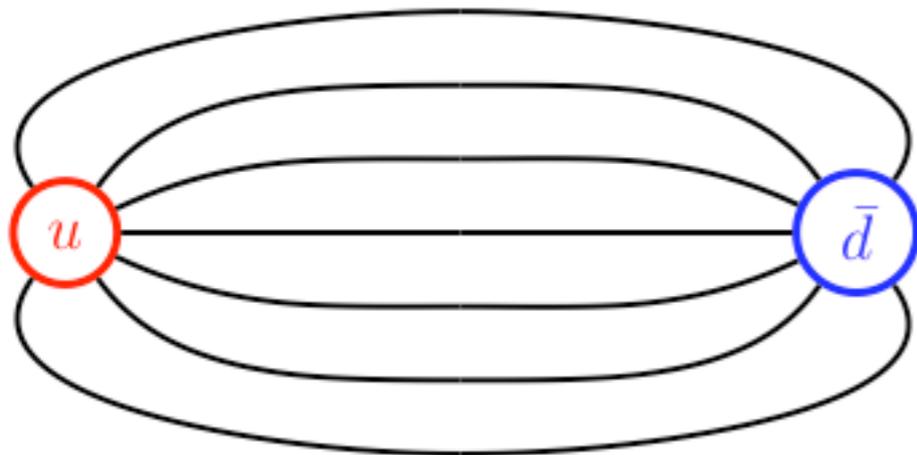
Asymptotic states are hadrons, not quarks and gluons

# 't Hooft large $N_c$ limit

$$N_c \rightarrow \infty, \quad g \rightarrow 0, \quad g^2 N_c = \text{fixed}$$

Only planar diagrams survive  
look like string worldsheets

Effective string theory description of large  $N_c$  QCD?



fat flux tube

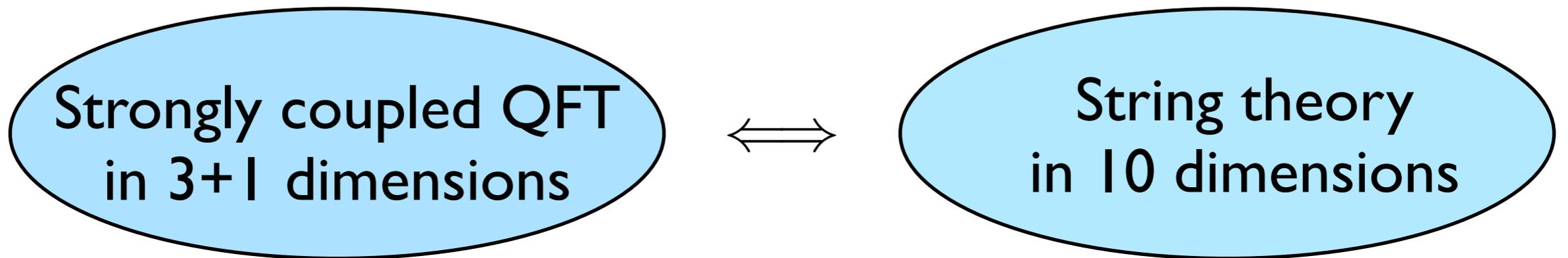
vs



elementary string

# Gauge gravity duality

- Maldacena 1997: equivalence

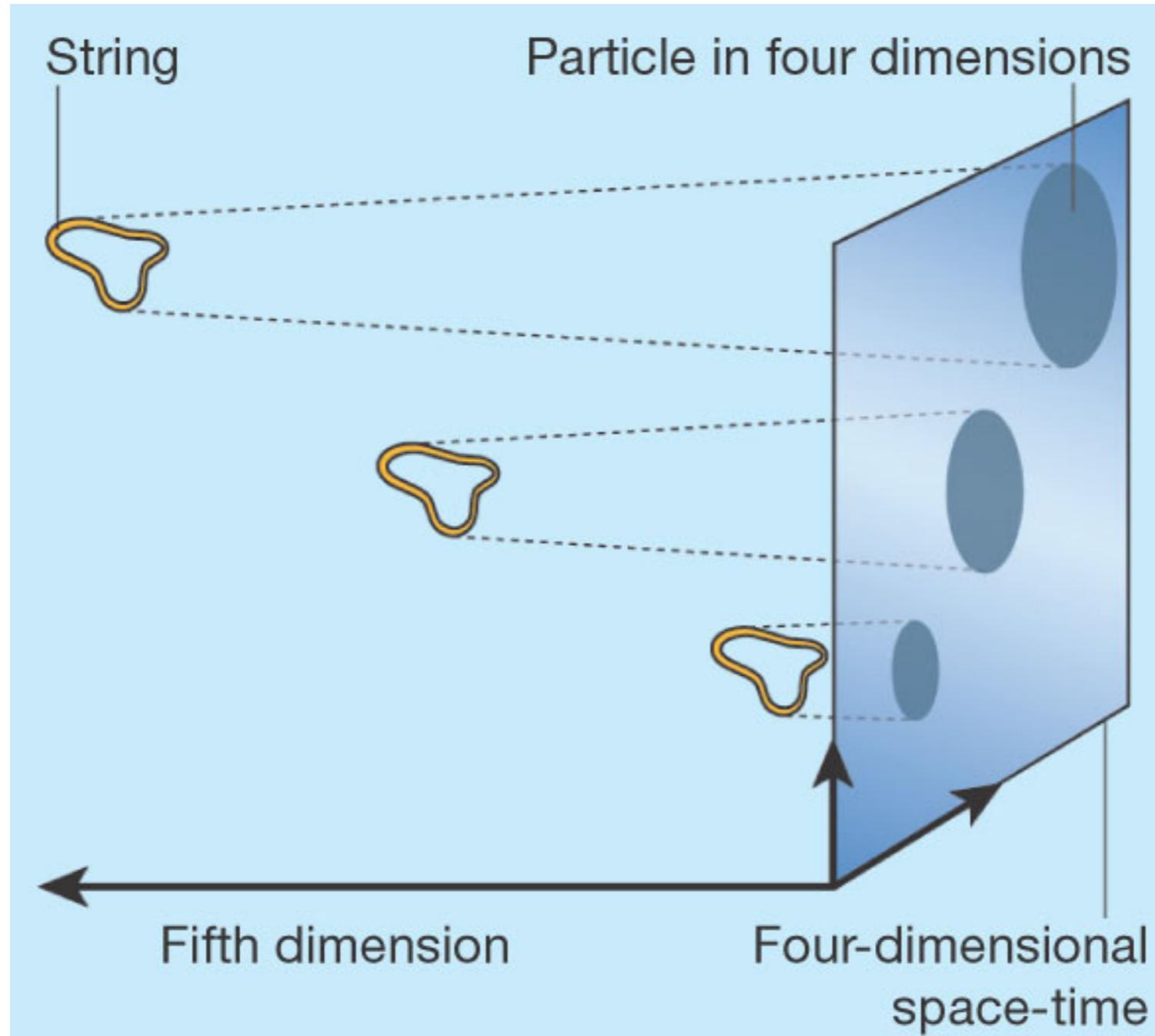


Complete surprise:

- Different numbers of dimensions
- Role of gravity

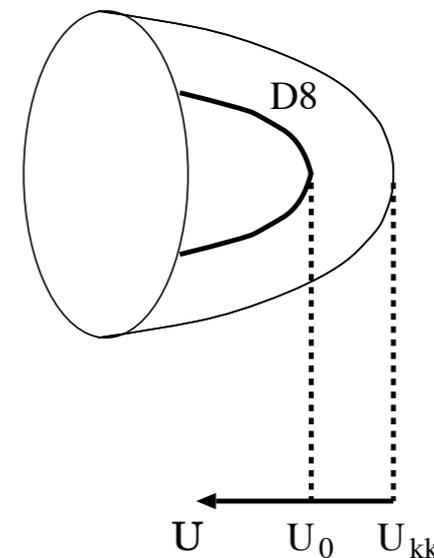
$$g^2 N_c \gg 1 \iff \text{Einstein's gravity}$$

# “Holography”



# Approaching QCD

- “Bottom-up” approach: modify the gravitational background to have confinement and chiral symmetry breaking
  - Erlich, Katz, Son, Stephanov “QCD and a holographic model of hadrons” 2005
  - Da Rold, Pomarol “ $\chi$ SB from 5D spaces” 2005
- “Top-down” approach: find string theory solutions with properties of QCD
  - Sakai-Sugimoto model



# Viscosity of the quark gluon plasma

- Very high-temperatures: hadrons melt to quarks and gluons: quark-gluon plasma
- Goal of RHIC and LHC heavy ion experiments
- Surprise of RHIC and LHC: QGP behaves like an almost ideal fluid
- Holography: QGP  $\sim$  black hole in AdS space

# “Prediction” of gauge-gravity duality

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

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viscosity  
fluid mechanics

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entropy per unit volume  
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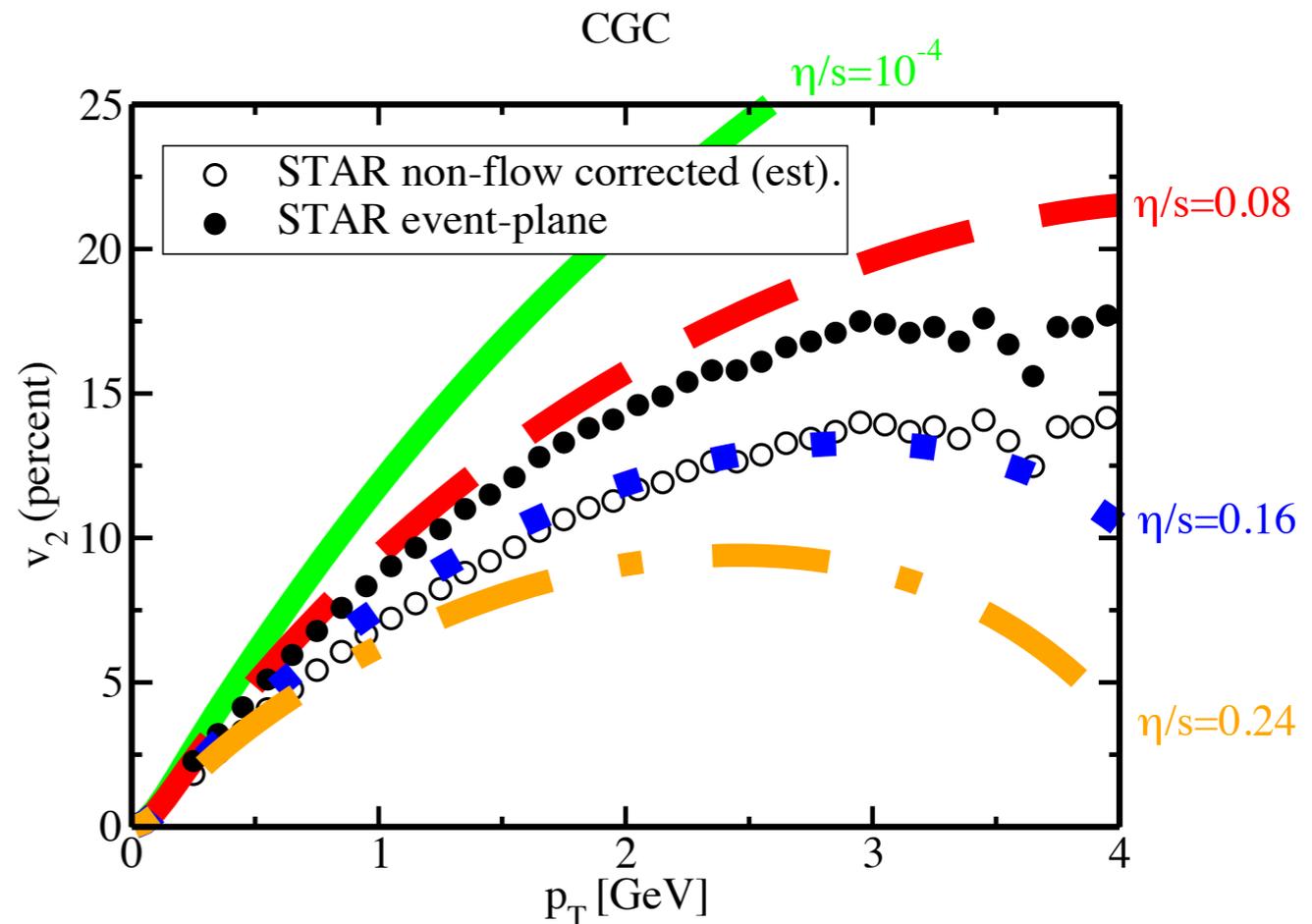
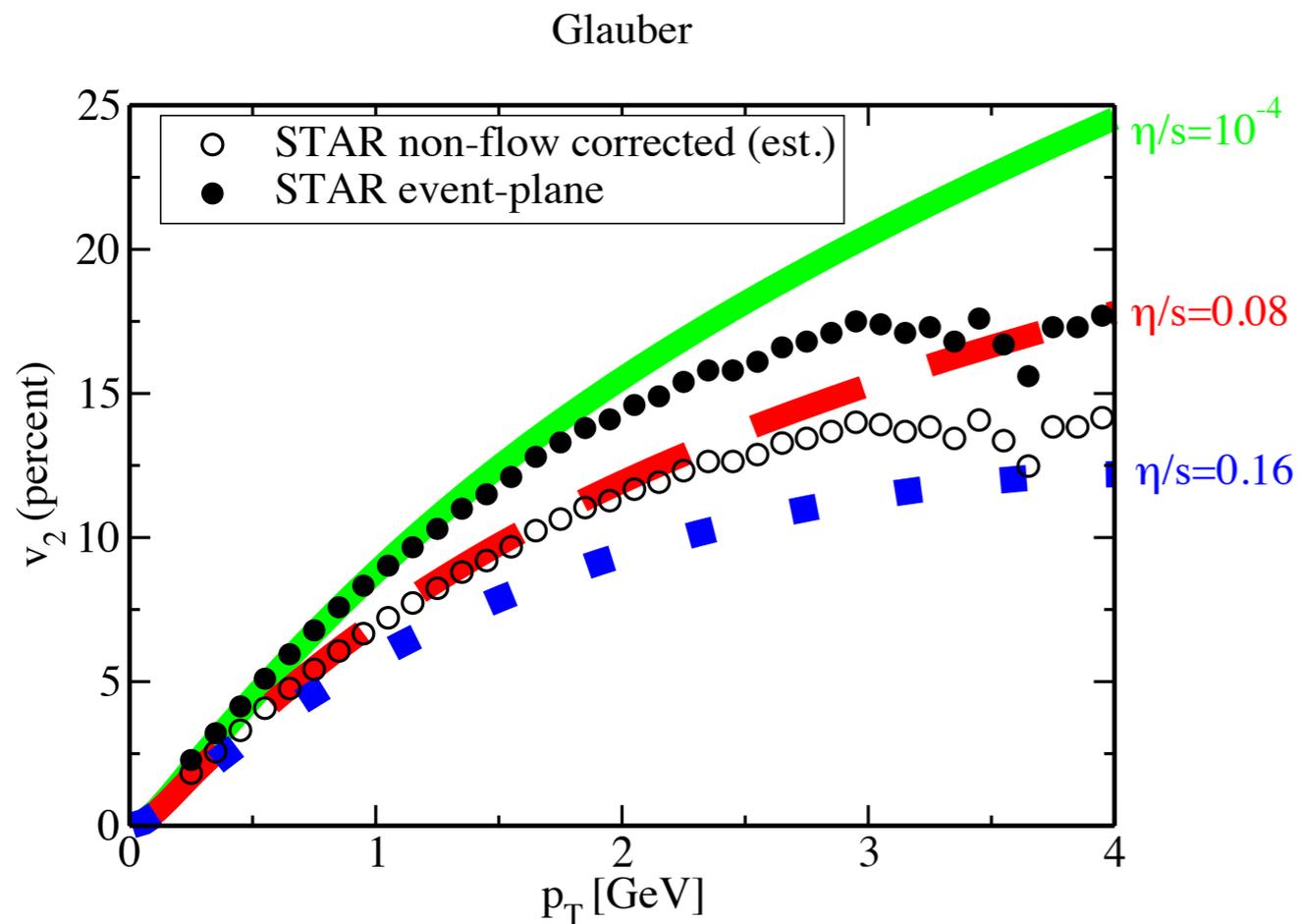
viscosity  
fluid mechanics

quantum mechanics

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

entropy per unit volume  
thermodynamics

# Viscosity of the QGP



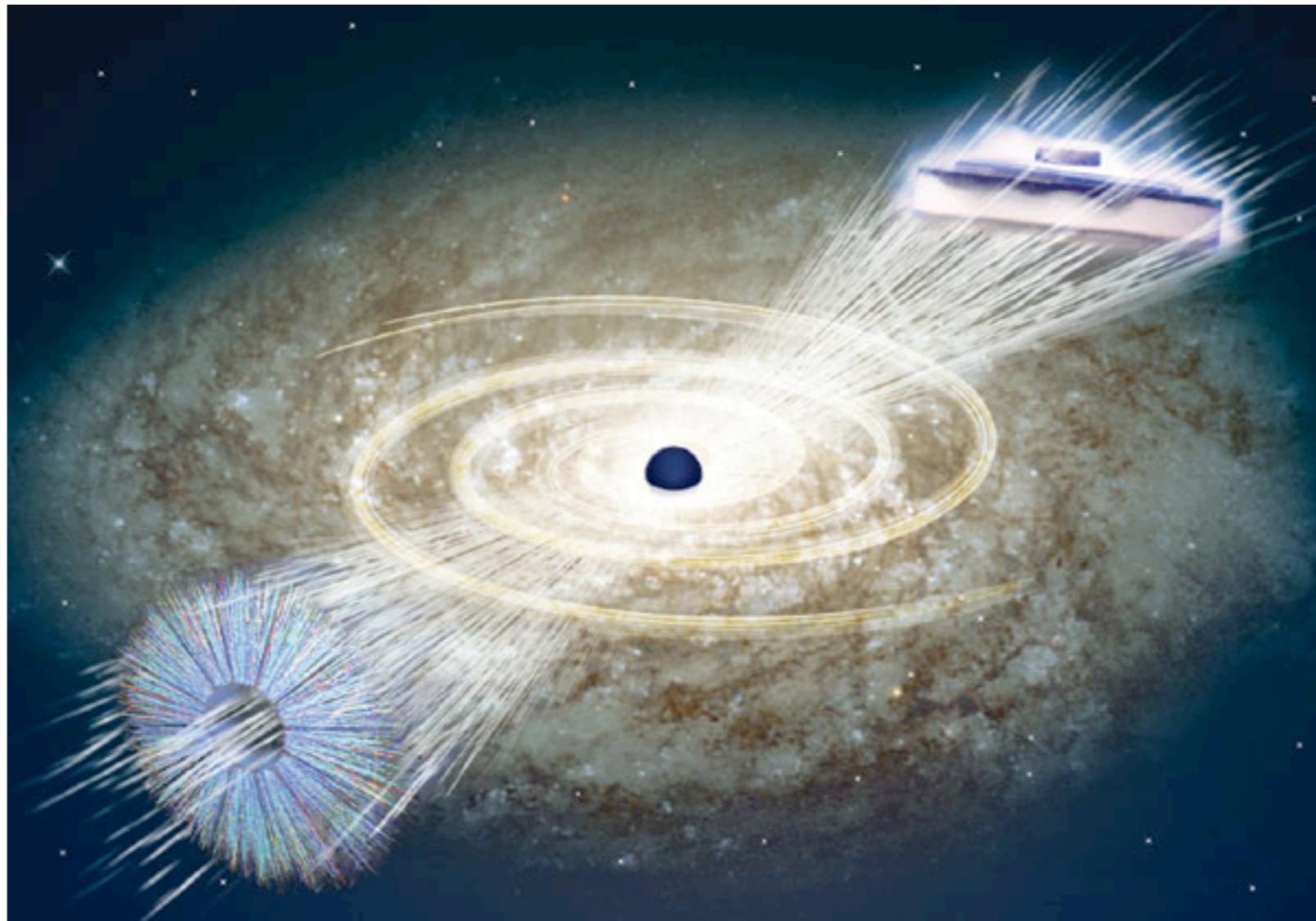
Romatschke and Luzum

$$\frac{\eta}{s} = 0.1 \pm 0.1(\text{th}) \pm 0.08(\text{exp})$$

Not too far away from  $1/4\pi$

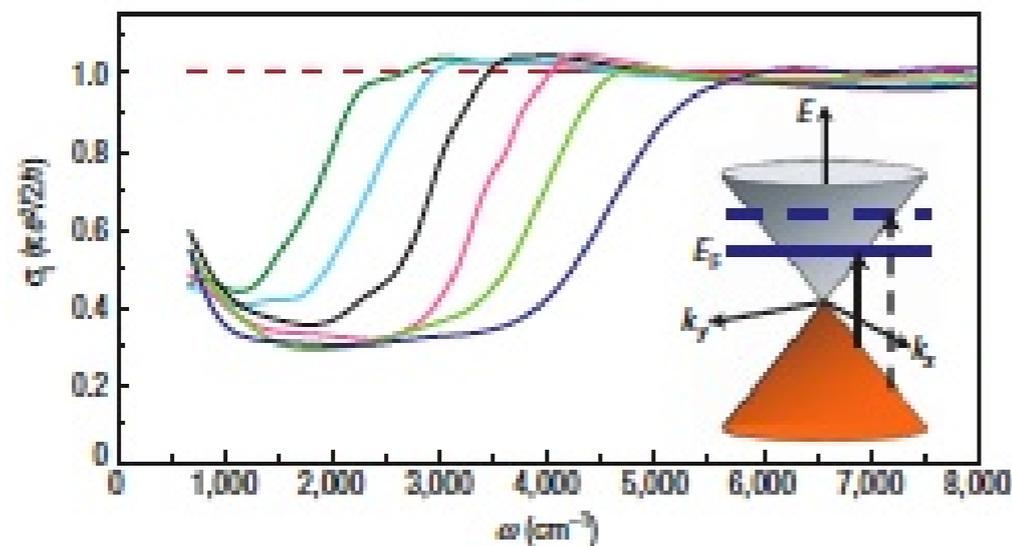
QGP is strongly coupled (sQGP)

# Applications to condensed-matter physics?

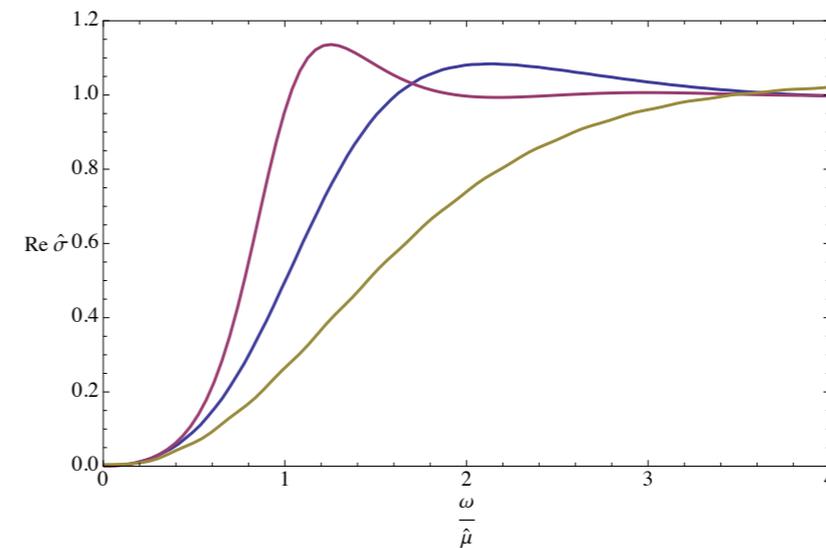


# Strongly coupled superconductors

- Motivation: high- $T_c$  superconductivity
- Normal phase with nontrivial properties (“strange metal”)
- Gauge/gravity duality: enables studies of some models of strongly coupled superconductors

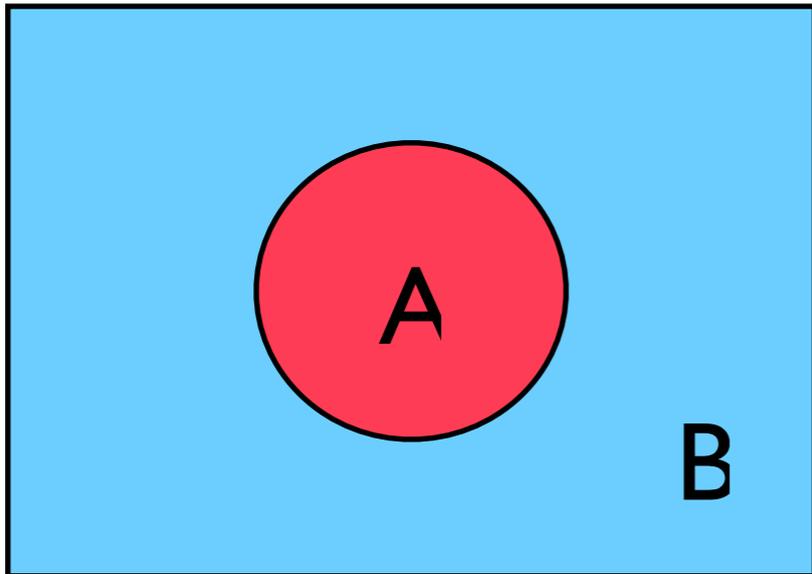


[Li, Henriksen, Jiang, Hao, Martin, Kim, Stormer, Basov '04]



Herzog, Harnoll, Horowitz

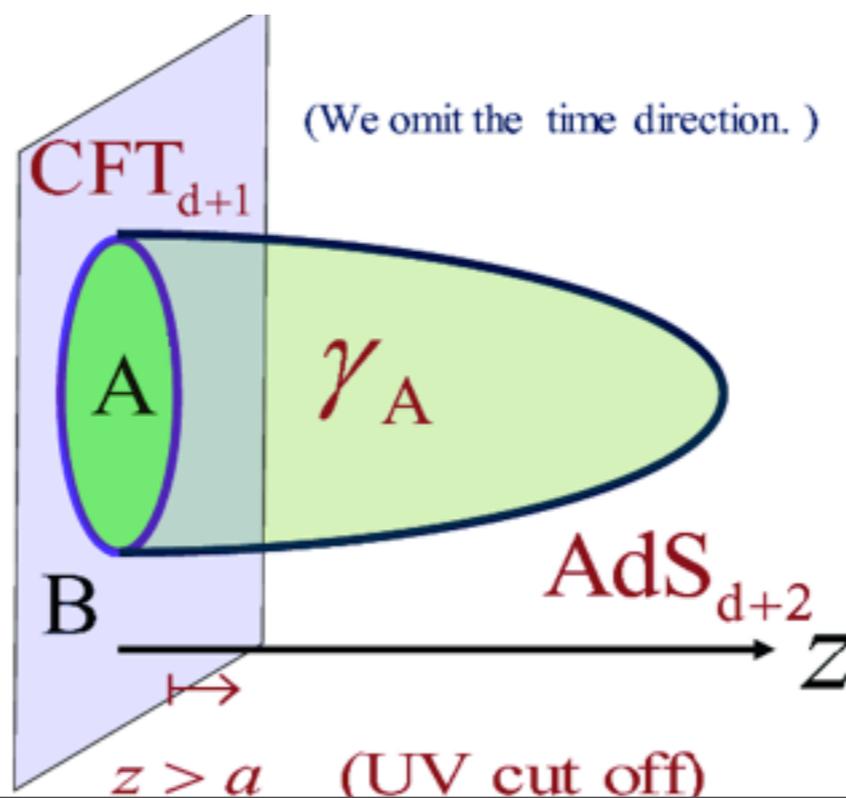
# Entanglement entropy



$$\rho_A = \text{tr}_B \rho_{\text{tot}}$$

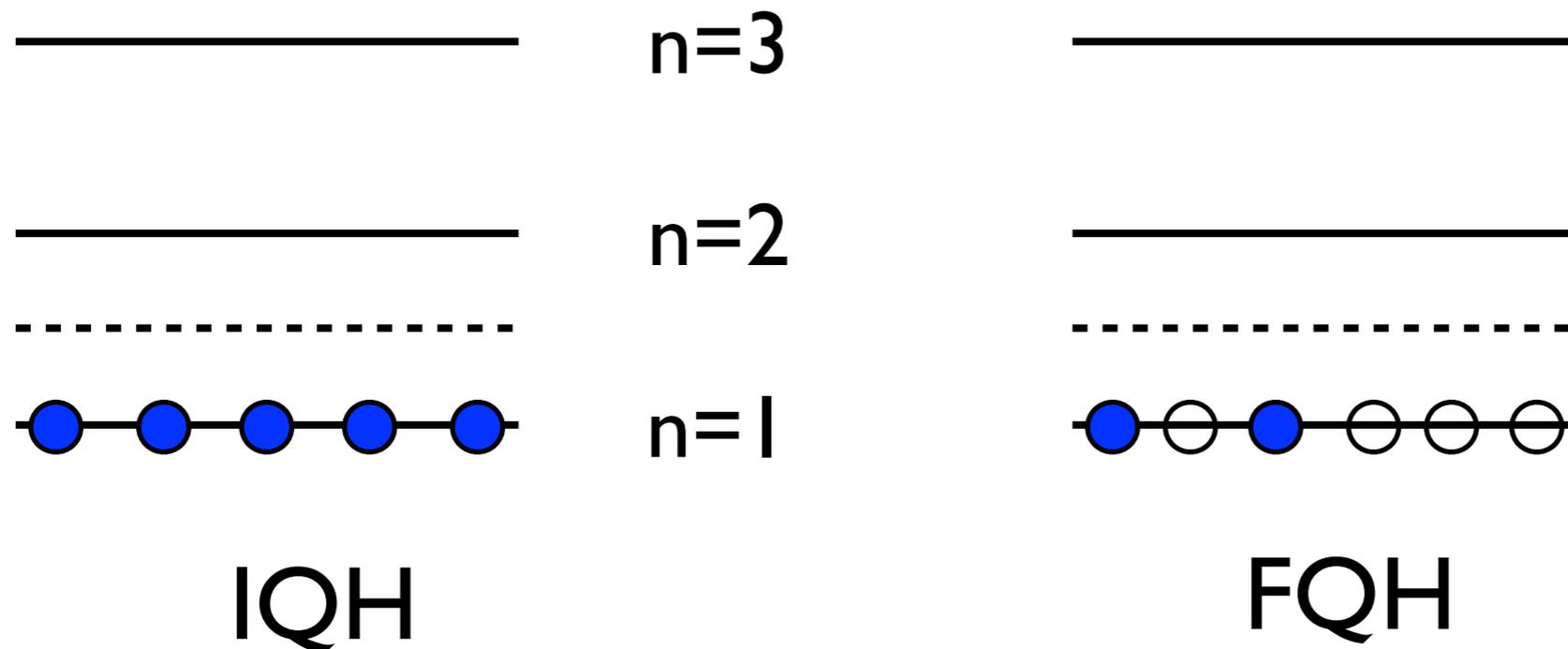
$$S_A = -\text{tr} \rho_A \ln \rho_A$$

Entanglement entropy from holography (Ryu-Takanayagi)



# Gravity in quantum Hall effect?

- 2 types of quantum Hall effects
  - integer QHE (von Klitzing 1980)
  - fractional QHE (Tsui, Stormer 1982)



# Excitations in FQH systems

- Ground state degeneracy lifted by interactions
- Excitations
  - fractionally charged quasiparticles
  - neutral magneto-rotons

Physics is strongly nonperturbative (cf. QCD)

Useful gravitational dual?

# Gravity for QHE

- The theory of QHE can be formulated to be frame-independent, like General Relativity
- A convenient framework: Newton-Cartan's geometry

# Cartan 1923-1924

Reformulation of Newton's theory of gravity

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Reformulation of Newton's theory of gravity



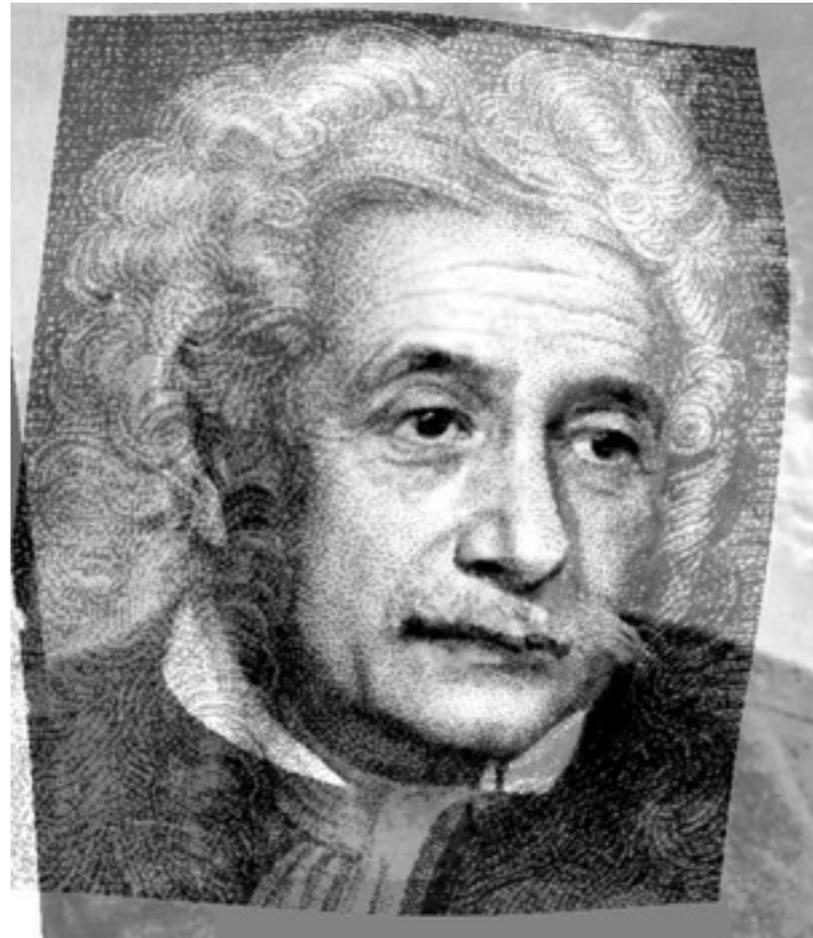
# Cartan 1923-1924

Reformulation of Newton's theory of gravity



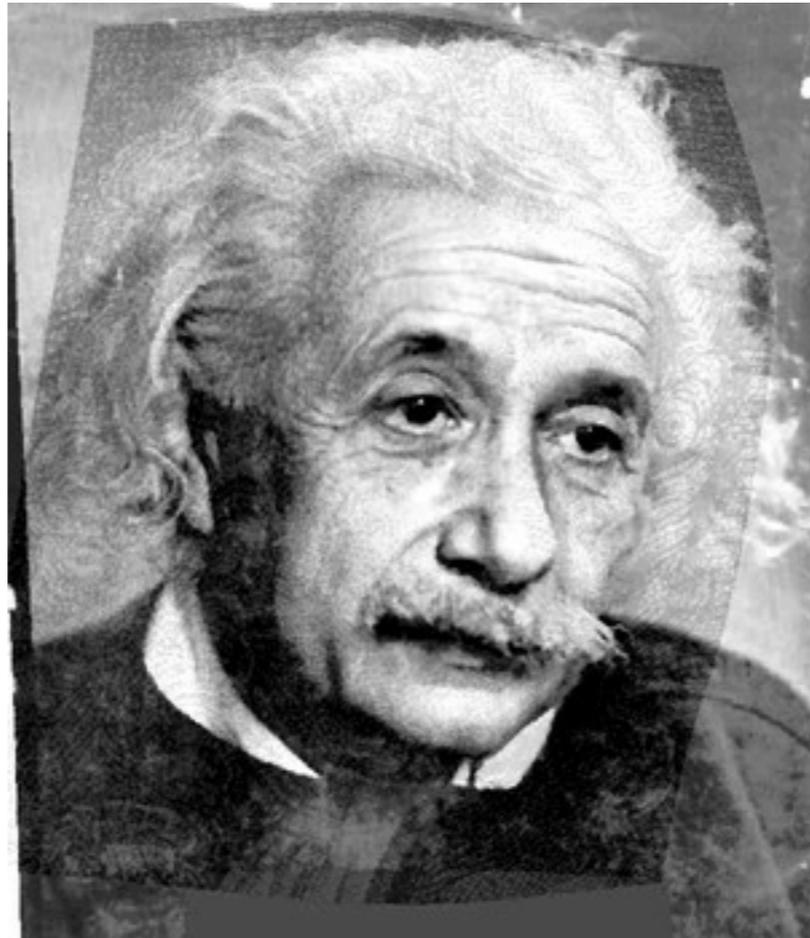
# Cartan 1923-1924

Reformulation of Newton's theory of gravity



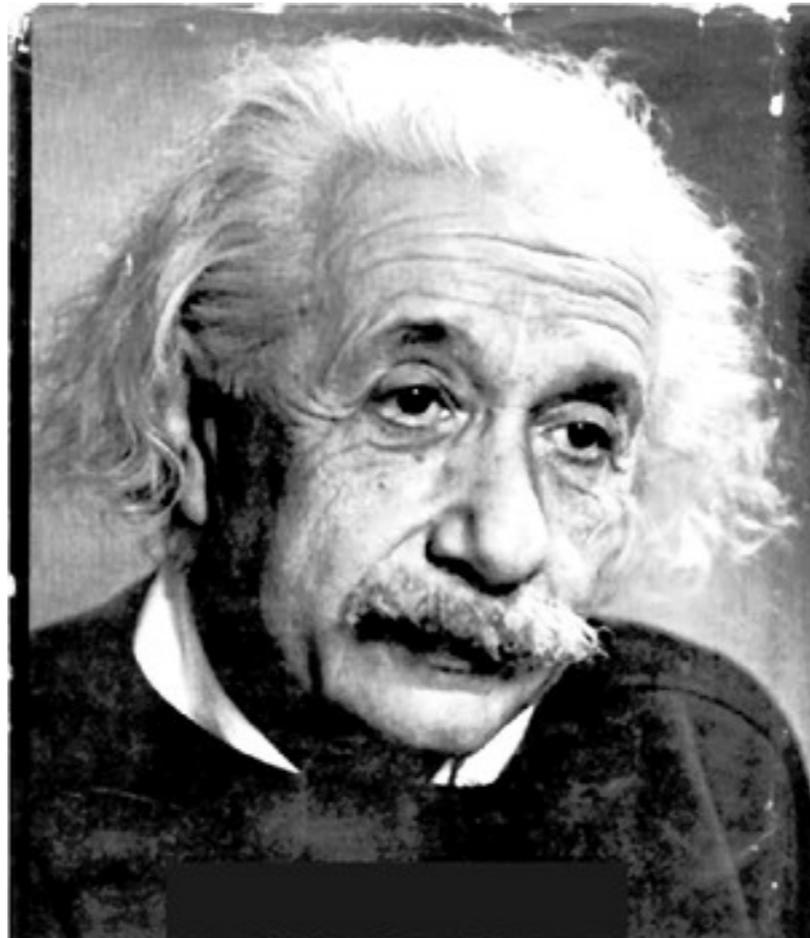
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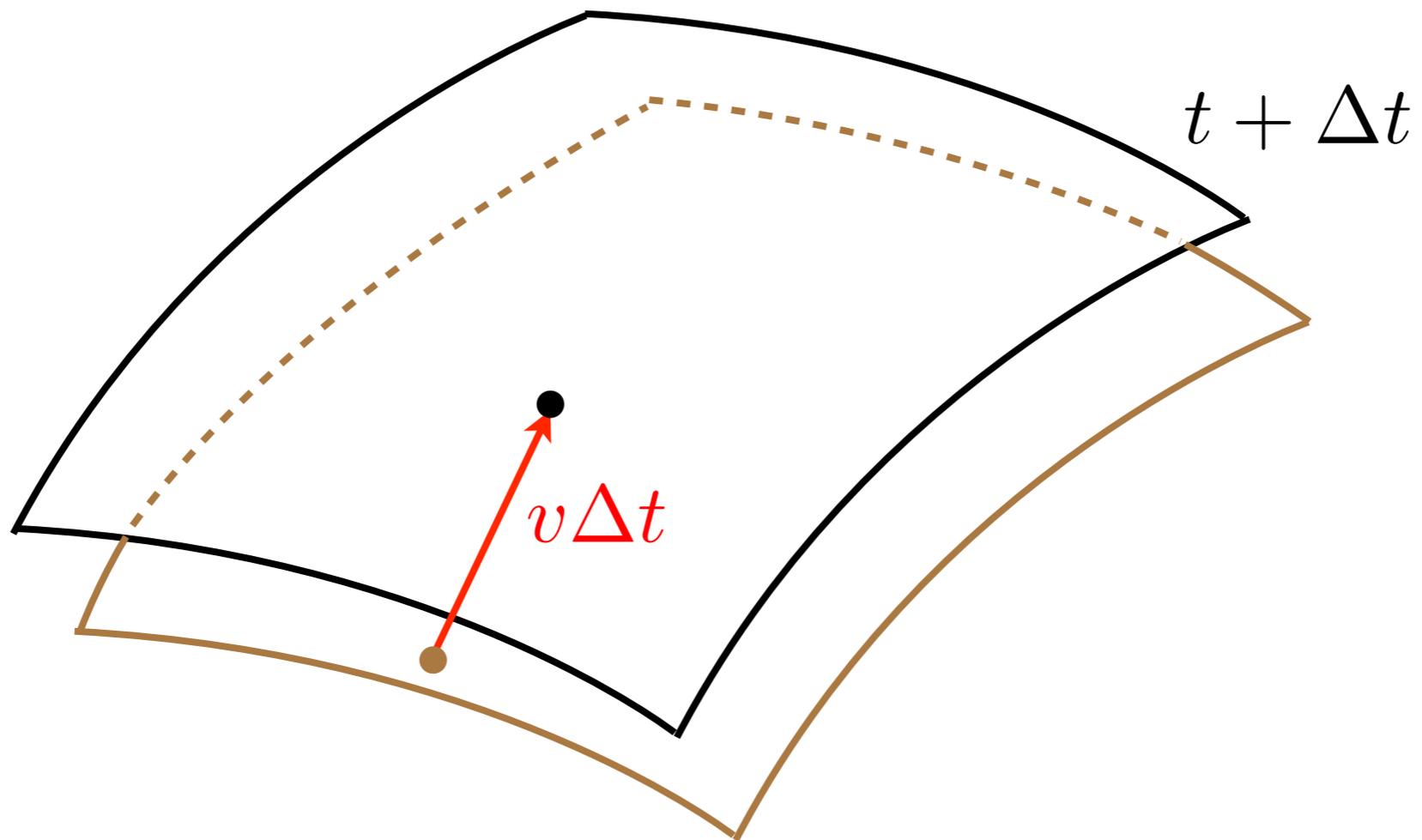
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Reformulation of Newton's theory of gravity

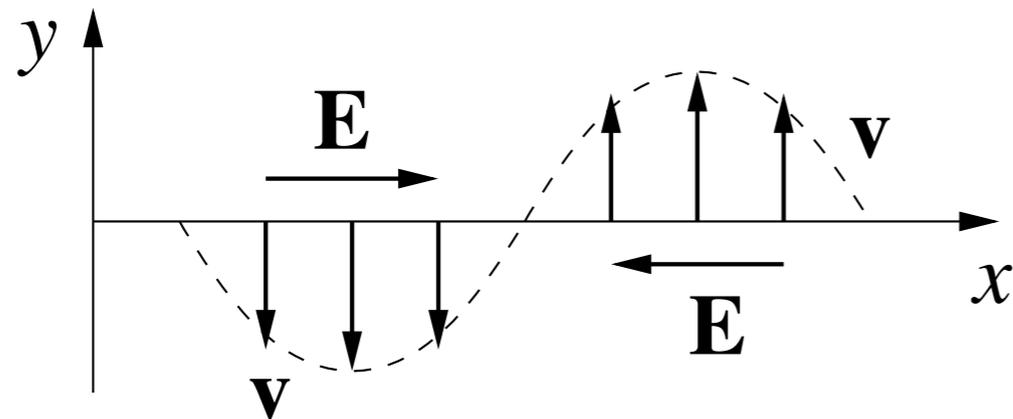




$g_{ij}, v^i$

# A prediction of Newton-Cartan formalism

Hall conductivity at nonzero wavenumber



$$E_x = E e^{iqx}$$

$$j_y = \sigma_{xy}(q) E_x$$

$$\frac{\sigma_{xy}(q)}{\sigma_{xy}(0)} = 1 + C_2 (q\ell)^2 + \mathcal{O}(q^4 \ell^4)$$

Carlos Hoyos, DTS 2011  
DTS 2013

universal coeff.

$$C_2 = \frac{1}{4\nu} - 1, \quad \nu = \frac{1}{2p+1}$$

# Magneton as emergent graviton?

- Haldane: there is a hidden metric in FQH systems
- A graviton, made massive by a gravitational Higgs mechanism *S.Golkar, Nguyễn Xuân Dũng, DTS, to appear*

$$(\partial_i u_j + \partial_j u_i + h_{ij})^2$$

fluid displacement = NG boson (eaten)

magneton = graviton

magneton at  $q=0$  has spin polarization

# Conclusion

- Gauge/gravity duality: connections between seemingly unrelated systems
- Black holes in AdS space as rough model of the strongly coupled QGP
- Maybe useful for condensed matter physics
- Fractional quantum Hall systems: a system with emergent massive graviton?