Pulsar Tests of Modified Gravity

COSMOLOGY IN THE PLANCK ERA OUT NHOU

Claudia de Rham July 30th 2013



GR has been a successful theory from mm length scales to **Cosmological scales** 10-6 10-10 10-14 10⁻¹⁸ -5 10⁻¹ period shift (s) Excluded region The Gravity Probe B 10⁰ 10-2 -10 Experiment 10-10⁻³ ... testing Einstein's Universe hc×10⁴⁰ 10-2 -1510-4 Frame-dragging Effect 10⁻³ 10-5 -39 milliarcseconds/year 000011 d-Then why Modify Gravity?

Why Modify Gravity in the IR?













In addition modified gravity (and especially massive gravity) comes hand in hand with a mechanism that can screen the effect of a cosmological field on short distances

Massive Gravity

$$S = \int \sqrt{-g} \frac{M_{\rm Pl}^2}{2} \left(R - \text{Mass Term} \right)$$

- The notion of mass requires a *reference* !
- Having the flat Metric as a Reference breaks
 Covariance !!! (Coordinate transformation invariance)
- The loss in symmetry generates new dof

$$\begin{array}{c} 2 + 4 = 6 \\ \text{GR} \leftarrow \text{Joss of 4 sym} \end{array}$$

Gravitational Waves

GR: 2 polarizations





Gravitational Waves

GR: 2 polarizations

In principle GW could have 4 other polarizations



Potential `new degrees of freedom'

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- The loss in symmetry generates new dof

2 + 4 = 6 = 5

Boulware & Deser, PRD6, 3368 (1972)

Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\rm FP} = h_{\mu\nu}^2 - h^2$$

Mass term for the fluctuations around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

• Transforms under a change of coordinate

$$x^{\mu} \rightarrow x^{\mu} + \partial^{\mu} \xi$$

 $h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\partial_{\mu}\partial_{\nu}\xi + \partial_{\mu}\partial_{\alpha}\xi\partial_{\nu}\partial^{\alpha}\xi$

Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\rm FP} = H_{\mu\nu}^2 - H^2$$

Mass term for the 'covariant fluctuations'

$$H_{\mu\nu} = h_{\mu\nu} + 2\partial_{\mu}\partial_{\nu}\pi - \partial_{\mu}\partial_{\alpha}\pi\partial_{\nu}\partial^{\alpha}\pi$$

Does not transform under that change of coordinate

$$x^{\mu} \rightarrow x^{\mu} + \partial^{\mu} \xi$$

 $h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\partial_{\mu}\partial_{\nu}\xi + \partial_{\mu}\partial_{\alpha}\xi\partial_{\nu}\partial^{\alpha}\xi$

 $\pi \rightarrow \pi - \xi$

Fierz-Pauli Massive Gravity

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Mass term for the 'covariant fluctuations'

 $H_{\mu\nu} = h_{\mu\nu} + 2\partial_{\mu}\partial_{\nu}\pi - \partial_{\mu}\partial_{\alpha}\pi\partial_{\nu}\partial^{\alpha}\pi$

• The potential has higher derivatives...

$$\mathcal{U}_{\rm FP} = \underbrace{\left(\partial_{\mu}\partial_{\nu}\pi\right)^{2} - \left(\Box\pi\right)^{2}}_{\text{Total derivative}} + \underbrace{\left(\partial^{2}\pi\right)^{3} + \cdots}_{\begin{array}{c}\text{Ghost reappears at}\\\text{the non-linear level}\end{array}}$$

Deffayet & Rombouts, gr-qc/0505134

Ghost-free Massive Gravity

$$\mathcal{U}_{\rm GF} = \mathcal{K}_{\mu\nu}^2 - \mathcal{K}^2$$

• With the tensor

 $\mathcal{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \sqrt{\delta^{\mu}_{\nu} - H^{\mu}_{\nu}} = \partial^{\mu}\partial_{\nu}\pi + h\partial\partial\pi + h^{2}\partial\partial\pi + \cdots$

• The potential has **NO** higher derivatives...

$$\mathcal{U}_{\rm GF} = (\partial_{\mu}\partial_{\nu}\pi)^2 - (\Box\pi)^2 + (\partial^2\pi)^3 + \cdots$$

Ghost DOES NOT reappear

CdR, Gabadadze, Tolley, 1011.1232

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- With the tensor
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 - The potential has NO higher derivatives...

 $\mathcal{U}_{\rm GF} = h\partial\partial\pi + h(\partial\partial\pi)^2 + \cdot$

Galileon in disguise after diagonalization

CdR, Gabadadze, 1007.0443

Galileons

• Finite Class of Interactions

$$\mathcal{L}_2 = (\partial \pi)^2$$

$$\mathcal{L}_3 = (\partial \pi)^2 \Box \pi$$

$$\mathcal{L}_4 = (\partial \pi)^2 ((\Box \pi)^2 - (\partial_\mu \partial_\nu \pi)^2)$$

$$\mathcal{L}_5 = (\partial \pi)^2 ((\Box \pi)^3 + \cdots)$$

• That enjoy a shift and Galileon symmetry

 $\pi \to \pi + v_\mu x^\mu + c$

- Have no ghost (2nd order eom)
- Enjoy a non-renormalization theorem

Nicolis, Rattazzi, Trincherini, 0811.2197



Ghost-free Massive Gravity

One can construct a consistent theory of massive gravity around any reference metric which

- propagates **5** dof in the graviton (free of the BD ghost)

- one of which is a **helicity-o** mode which behaves as a scalar field (a Galileon) couples to matter $\pi~T$

- "hides" itself via a Vainshtein mechanism

Vainshtein, PLB**39**, 393 (1972)



Origin of the vDVZ discontinuity

van Dam & Veltman, Nucl.Phys.B 22, 397 (1970) Zakharov, JETP Lett.12 (1970) 312



The non-linearities are essential in screening the helicity-o mode

Babichev & Deffayet, 1304.7240

Vainshtein, PLB**39**, 393 (1972)



The interactions for the helicity-o mode are important at a very low energy scale, $\Lambda \ll M_{\rm Pl}$

• Close to a source the interactions are important

$$\Box \pi + \frac{1}{\Lambda^3} \left((\Box \pi)^2 + \cdots \right) = -T$$

• Perturbations end up being weakly coupled to matter, $\pi = \pi_0 + \delta \pi$

$$\left(1 + \frac{\Box \pi_0}{\Lambda^3} + \cdots\right) \Box \delta \pi = -\frac{1}{M_{\rm Pl}} \delta T$$
$$Z\left(\partial^2 \pi_0\right) \gg 1$$

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$$\Box \widehat{\delta \pi} = -\frac{1}{ZM_{\rm Pl}} \delta T$$

 $Z = 1 + \frac{\partial^2 \pi_0}{\sqrt{3}} + \dots \gg 1$

- Well understood for Static & Spherically Symmetric configurations e.g. $T_0 = -M_{\odot}\delta^{(3)}(r)$
- Background is analytic for $\pi'_0(r)$

$$\frac{\pi_0'}{r} + \frac{1}{\Lambda^3} \left(\frac{\pi_0'}{r}\right)^2 = \frac{M_{\odot}}{4\pi M_{\rm Pl} r^3}$$

Vainshtein radius: $r_{\star} = \frac{1}{\Lambda} \left(\frac{M_{\odot}}{M_{\rm Pl}} \right)^{1/3}$ for $r \gg r_{\star}$, $\pi'_{0}(r) \sim \frac{M_{\odot}}{M_{\rm Pl}} \frac{1}{r^{2}}$ for $r \ll r_{\star}$, $\pi'_{0}(r) \sim \frac{M_{\odot}}{M_{\rm Pl}} \frac{1}{r_{\star}^{3/2} \sqrt{r}}$

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$$\frac{F_{r \ll r_{\star}}}{F_{r \gg r_{\star}}} = \left(\frac{r}{r_{\star}}\right)^{3/2} \sim 10^{-12} \quad \text{For Sun-Earth System}$$

$$for \qquad \begin{cases} r \sim 1UA \\ M = M_{\odot} \\ \Lambda \sim (1000 \text{ km})^{-1} \text{ or } m \sim H_0 \sim 10^{-33} \text{ eV} \end{cases}$$

• Well understood for Static & Spherically Symmetric configurations *e.g.* $T_0 = -M_{\odot}\delta^{(3)}(r)$







1. Method Effective Action Approach described by Goldberger & Rothstein hep-th/0409156 & subsequent literature

1. Method Effective Action Approach

2. Monopole & Quadrupole Radiation

Vainshtein Suppression in the Monopole $~\sim$

Vainshtein Suppression in the Quadrupole

$$rac{1}{\left(\Omega_p r_\star
ight)^{3/2}} \ rac{1}{\left(\Omega_p r_\star
ight)^{3/2}} rac{1}{\Omega_p ar r}$$

1. Method Effective Action Approach

2. Monopole & Quadrupole Radiation in Simplest Galileon

3. Subtleties in General Galileon Model...

What is an EFT ?

- A Clever way to parameterize our "ignorance"
- Instead of trying to solve the *full theory*, we only include the most important contributions in the Effective description
- This makes sense when there is Hierarchy of scales.



Hierarchy of Scales

 Ω_P^{-1}

 $r_S \sim 10 \text{ km}$ $\bar{r} \sim 10^6 \text{ km}$ $\Omega_P^{-1} \sim 10^9 \text{ km}$ $r_\star \sim 10^{15} \text{ km}$

 $r_S \ ar r$

$$r_S \ll \bar{r} \ll \Omega_P^{-1} \ll r_\star \ll m^{-1}$$

 r_{\star}



Start with both objects at the center of mass and perturb around it

 $T = \underbrace{T_0}_{0} + \delta T$ Total mass at center of mass $\pi = \pi_0(r) + \underbrace{\phi}_{\text{Radiation emitted by that scalar}}$

• Start with the Cubic Galileon $S_{\text{Gal}} = \int \mathrm{d}^4 x \left(-\frac{1}{2} (\partial \pi)^2 + \frac{1}{\Lambda^3} (\partial \pi)^2 \Box \pi + \frac{1}{M_{\text{Pl}}} \pi T \right)$

 $T = (T_0) + \delta T$

 $\pi = \pi_0(r) + \phi$

Total mass at center of mass

Radiation emitted by that scalar

• Start with the Cubic Galileon $S_{\text{Gal}} = \int \mathrm{d}^4 x \left(-\frac{1}{2} (\partial \pi)^2 + \frac{1}{\Lambda^3} (\partial \pi)^2 \Box \pi + \frac{1}{M_{\text{Pl}}} \pi T \right)$

 $T = (T_0) + \delta T$

 $\pi = \pi_0(r) + \phi$

Total mass at center of mass

Radiation emitted by that scalar

• Start with the Cubic Galileon

$$S_{\text{Gal}} = \int \mathrm{d}^4 x \left(-\frac{1}{2} Z(\pi_0) (\partial \phi)^2 + \frac{1}{M_{\text{Pl}}} \phi \delta T \right)$$

$$T = \underbrace{T_0}_{h \to T} \quad \text{Total mass at center of mass}$$
$$\pi = \pi_0(r) + \underbrace{\phi}_{\text{Radiation emitted by that scalar}}$$

• Start with the Cubic Galileon

$$S_{\text{Gal}} = \int \mathrm{d}^4 x \left(-\frac{1}{2} Z(\pi_0) (\partial \phi)^2 + \frac{1}{M_{\text{Pl}}} \phi \delta T \right)$$

Integrate out the scalar field

$$\phi(x) = \frac{i}{M_{\rm Pl}} \int \mathrm{d}^4 x' G_F(x, x') \delta T(x')$$

 $Z(\partial^2 \pi_0(r)) \,\partial_x^2 G_F(x, x') = i\delta^4(x - x')$

• Start with the Cubic Galileon

$$S_{\text{Gal}} = \int \mathrm{d}^4 x \left(-\frac{1}{2} Z(\pi_0) (\partial \phi)^2 + \frac{1}{M_{\text{Pl}}} \phi \delta T \right)$$

Integrate out the scalar field

$$S_{\rm eff} = \frac{i}{2M_{\rm Pl}^2} \int \mathrm{d}^4 x \, d^4 x' \, \delta T(x) G_F(x, x') \delta T(x')$$

 $Z(\partial^2 \pi_0(r)) \,\partial_x^2 G_F(x, x') = i\delta^4(x - x')$

- The average power emitted can then be computed out of the Im part of the effective action $/d\mathcal{E} \setminus \int_{-\infty}^{\infty} d\mathcal{E} = 2$
- $P = -\left\langle \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \right\rangle = \int_0^\infty \mathrm{d}\omega \,\omega f(\omega) \quad \text{with} \quad \frac{2}{T_P} \mathrm{Im}S_{\mathrm{eff}} = \int \mathrm{d}\omega f(\omega)$

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 - In terms of the modes spherical harmonic space $Z \partial_x^2 \left[u_\ell(r) Y_{\ell m}(\Omega) e^{-i\omega t} \right] = 0$

$$P \sim \sum_{n=0}^{\infty} \sum_{\ell,m} (n\Omega_P) \left| \frac{1}{M_{\rm Pl} T_P} \int_0^{T_P} \mathrm{d}t \mathrm{d}^3 x \, \boldsymbol{u}_{\ell}(\boldsymbol{r}) Y_{\ell,m} e^{-in\Omega_P t} \delta T \right|^2$$

Normalization

• We choose the normalization by matching to the correct modes at infinity



Normalization

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Monopole

- The leading order contribution to the monopole vanishes by conservation of energy...
- The first relativistic correction gives

$$\delta T = -\left[\sum_{i=1,2} M_i \left(1 - \frac{1}{2} (\omega r_i(t))^2 + \dots\right) \delta^{(3)}(\vec{x} - \vec{x}_i(t)) - M \delta^{(3)}(\vec{x})\right]$$

$$u_0(r) \sim \frac{1}{(\omega r_\star^3)^{1/4}} \left(1 - \frac{1}{4} (\omega r)^2 + \cdots \right)$$

With $r_i(t) = \frac{\bar{r}(1-e^2)}{1+e\cos\Omega_P t} \frac{M_{2,1}}{M}$

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Monopole

- The leading order contribution to the monopole vanishes by conservation of energy...
- The first relativistic correction gives

$$P_{\text{Monopole}} \sim \frac{\left(\Omega_P \bar{r}\right)^4}{\left(\Omega_P r_\star\right)^{3/2}} \frac{\mathcal{M}^2}{M_{\text{Pl}}^2} \Omega_P^2$$

Vainshtein Suppression in the Monopole $\sim \frac{1}{(\Omega_p r_{\star})^{3/2}}$

For the Hulse-Taylor Pulsar $\sim 10^{-10}$

Quadrupole

- A priori the quadrupole is suppressed by a few powers of velocity compared to the monopole...
- But does not need to include to relativistic corrections

$$u_2(r) \sim \frac{(\omega r)^{3/2}}{(\omega r_\star^3)^{1/4}} + \cdots$$

Quadrupole

- A priori the quadrupole is suppressed by a few powers of velocity compared to the monopole...
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$$P_{
m Quadrupole} \sim rac{\left(\Omega_P ar{r}
ight)^3}{\left(\Omega_P r_\star
ight)^{3/2}} rac{\mathcal{M}^2}{M_{
m Pl}^2} \Omega_P^2$$

Vainshtein Suppression in the Quadrupole ~ $\frac{1}{(\Omega_p r_{\star})^{3/2}} \frac{1}{\Omega_p \bar{r}}$

For the Hulse-Taylor Pulsar $\sim 10^{-8}$

Quartic Galileon

- In the Quartic Galileon, the angular direction is *not screened as much* as along the others
- The quadrupole is screened compared to GR $\frac{P_{\text{Quadrupole}}^{\text{Quadrupole}}}{P_{\text{GR}}^{\text{Quadrupole}}} \sim \frac{1}{\left(\Omega_P r_{\star}\right)^2} \sim 10^{-12} \text{ III}$
- But many multipoles contribute to the power with the same magnitude...
 - Multipole expansion breaks down

Effective metric

• In the Cubic Galileon, the Effective metric for perturbations had of the same behavior along all directions

Effective metric

• In the Quartic Galileon, the Effective metric for perturbations is *different* along different directions

$$Z_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} \sim \left(\frac{\pi_0'}{\Lambda^3 r}\right)^2 \left(-\mathrm{d}t^2 + \mathrm{d}r^2 + r_{\star}^2\mathrm{d}\Omega^2\right)$$

Effective metric

• In the Quartic Galileon, the Effective metric for perturbations is *different* along different directions

$$Z_{\mu\nu} dx^{\mu} dx^{\nu} \sim \left(\frac{\pi'_0}{\Lambda^3 r}\right)^2 \left(-dt^2 + dr^2 + r_{\star}^2 d\Omega^2\right)$$

Multipoles are no longer suppressed
by more power of velocity

Quartic Galileon

• Perturbations are under control when there is another Hierarchy of scales

eg. hierarchy of mass between the two objects Solar System

• For a generic binary pulsar system, one needs to do perturbations about a different background...

Outlook

- Massive Gravity is a specific framework to study IR modifications of Gravity
- It could play a role for
 - the late-time acceleration of the Universe
 - the cosmological constant problem
- We now have the theoretical formalism to describe a stable theory of massive gravity
- It behaves a scalar-tensor Galileon theory in some limit, hand in hand with a Vainshtein mechanism

Outlook

- The scalar field opens new channels of radiations which would have been observable in binary pulsar systems if they were not Vainshtein screened.
- The monopole is suppressed by a factor of $\frac{1}{(\Omega_p r_{\star})^{3/2}}$ • The quadrupole is suppressed by $\frac{1}{(\Omega_p r_{\star})^{3/2}} \frac{1}{\Omega_p \bar{r}}$
- While this suppression makes these effects unobservable in binary pulsar, it shows that the Vainshtein mechanism is much more subtle in time-dependent configurations.