Primordial spectra from sudden turning trajectory

MASAHIDE YAMAGUCHI

(Tokyo Institute of Technology)

08/01/13@Quy Nhon

1307.7110, Toshifumi Noumi & MY

See also JHEP06(2013)051, Toshifumi Noumi, MY, and Daisuke Yokoyama

$$c = \hbar = M_{\rm pl}^2 = 1/(8\pi G) = 1$$

Contents

- **Introduction**

  Basics of inflation, Roles of heavy fields

- **Sudden turn**

  Setup, Interaction Hamiltonian

- **Powerspectrum**

  Peak and resonance cancellation

- **Bispectrum**

  Shape and resonance

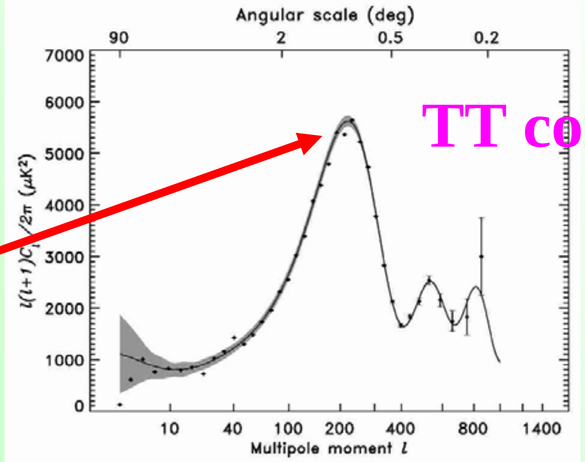
- **Discussion and conclusions**

Introduction

General predictions of inflation

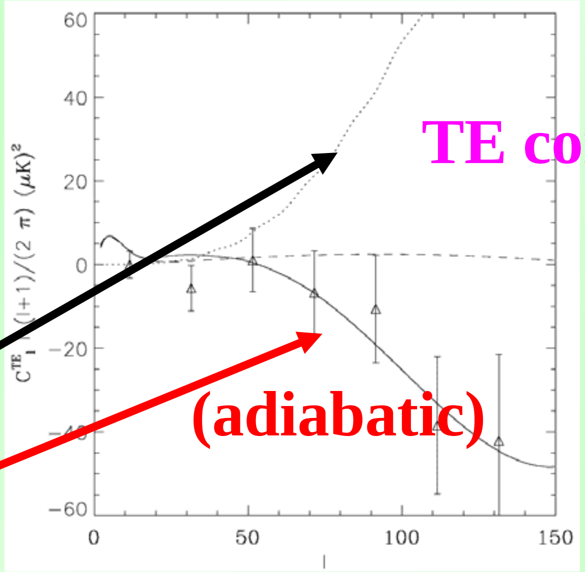
- **Spatially flat universe**

$$\Omega_{\rm total} \simeq 1.0$$



TT correlation

- **Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations**



TE correlation

Causal seed models

(adiabatic)

Inflation models

WMAP1

There are still rooms for $|f_{NL}| = O(1)$ and $O(0.1)\%$ isocurvature perturbations.

Roles of heavy fields during inflation

- **Single field inflation :**

curvature perturbation ζ ⇐==⇒ One light scalar field ϕ
(adiabatic mode) **($m_\phi \ll H$)**

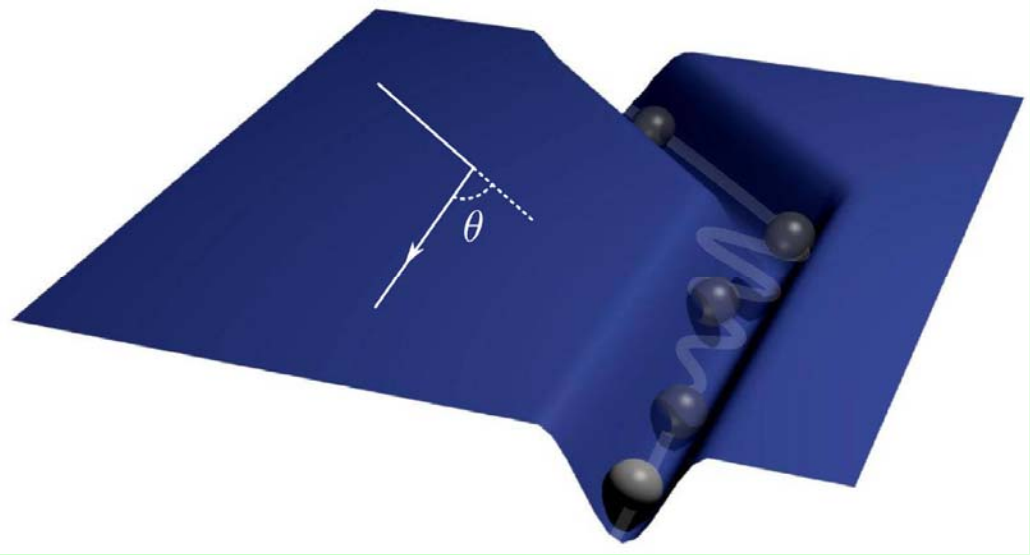
In supergravity and/or superstring theory, there are a lot of scalar fields whose masses are larger than the Hubble parameter.

Though effects of these heavy fields are usually assumed to be negligible, is this kind of neglect justified ?

If heavy fields can leave significant imprints on primordial spectra, such signatures become powerful tools to probe high energy physics.

Sudden turn

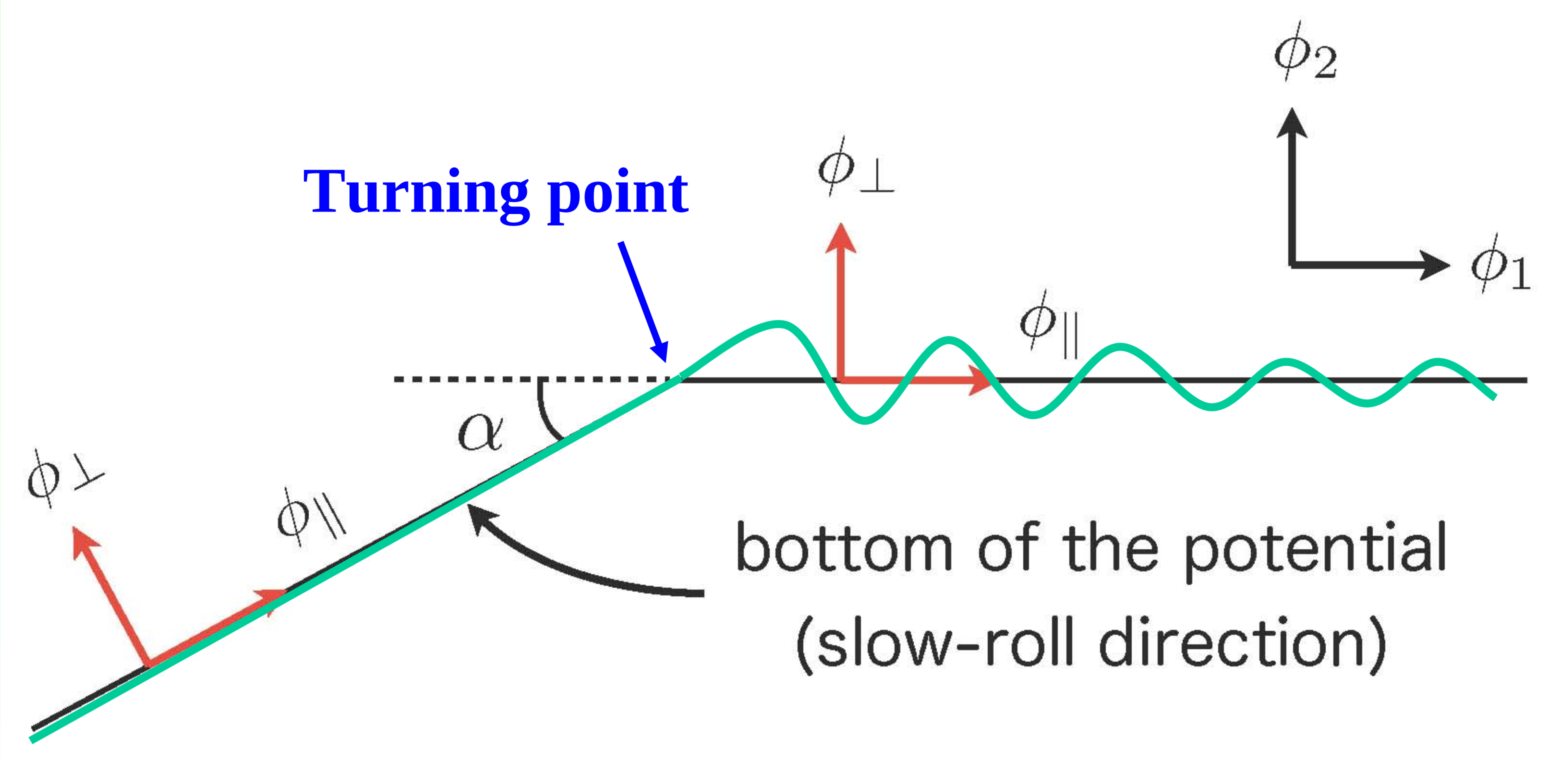
Straight & curved trajectory



(Credit: Wang, 1303.1523)

If the trajectory is straight, there is no excitation of heavy fields.
But, if the trajectory is suddenly curved, this is not the case.
For example, such a case often happens in string landscape.

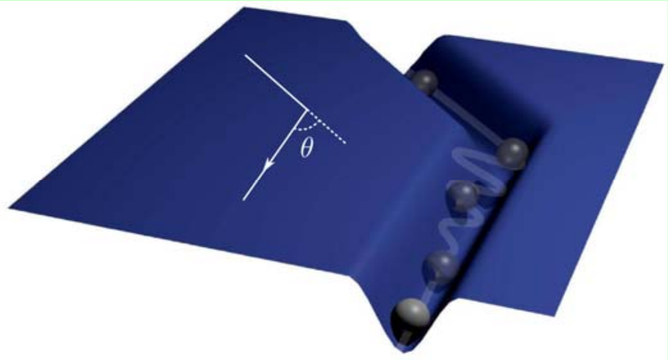
Setup



$\phi_\parallel$: **the slow-roll direction**

$\phi_\perp$: **the heavy field direction**

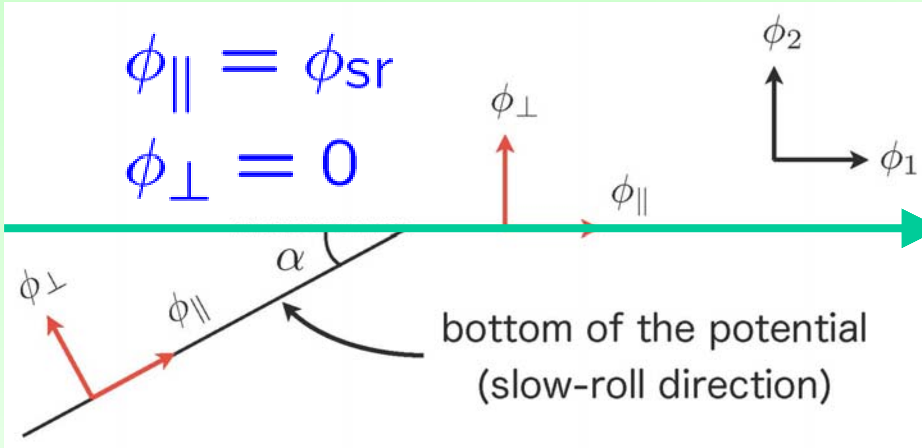
$$V_\perp(\phi_\perp) = \frac{m^2}{2}\phi_\perp^2 + \frac{\lambda}{4!}\phi_\perp^4$$

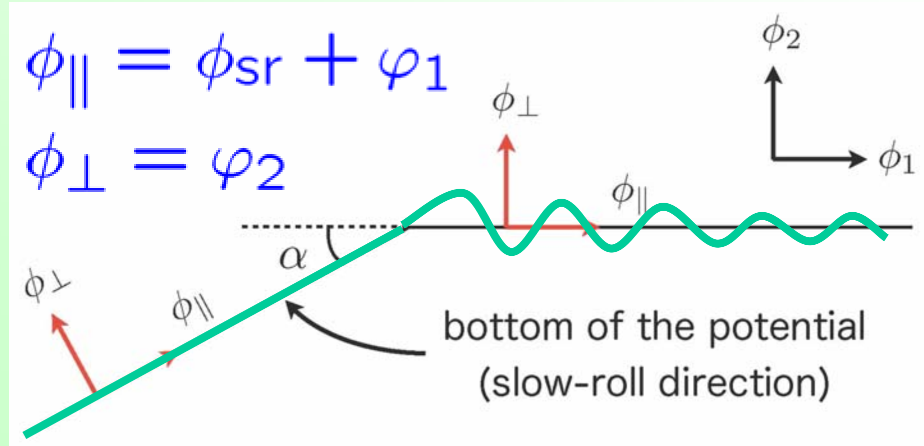


Effects of oscillating trajectory

- **Deformations of the Hubble parameter**
- **Conversion effects between adiabatic and isocurvature (heavy field) modes**

Deformations of the Hubble parameter





φ_1, φ_2 **: deviation from the slow-roll**

$$-2M_{\rm pl}^2\dot{H} = \left(\dot{\phi}_{\rm sr} + \dot{\varphi}_1\right)^2 + \dot{\varphi}_2^2.$$

If there is no turn and
the background trajectory is straight,

$$\Rightarrow \quad -2M_{\rm pl}^2\dot{H}_{\rm sr} = \dot{\phi}_{\rm sr}^2.$$

Friedmann Eq.

$$\Rightarrow \quad \dot{H} = \dot{H}_{\rm sr} + \delta\dot{H}$$

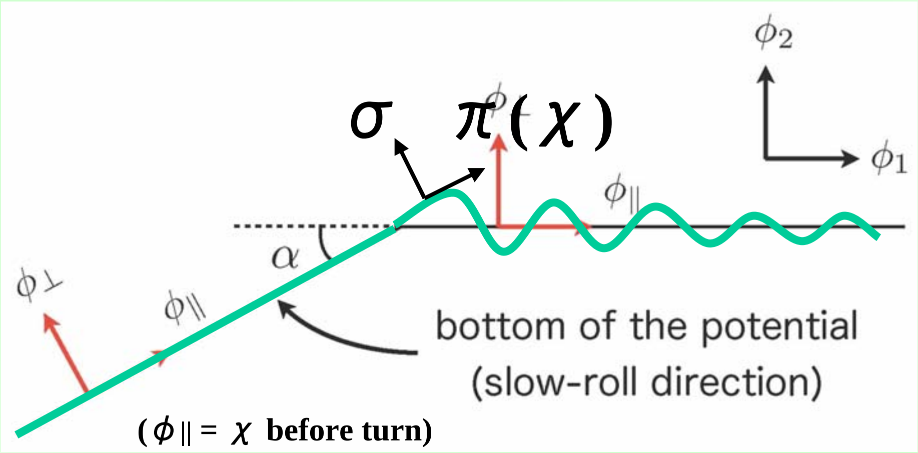
$$-2M_{\rm Pl}^2\delta\dot{H} = 2\dot{\phi}_{\rm sr}\dot{\varphi}_1 + \dot{\varphi}_2^2$$

oscillating

The slow-roll conditions are satisfied.

$$\epsilon_{\rm sr} = -\frac{\dot{H}_{\rm sr}}{H_{\rm sr}^2} \ll 1, \quad \eta_{\rm sr} = \frac{\dot{\epsilon}_{\rm sr}}{H_{\rm sr}\epsilon_{\rm sr}} \ll 1.$$

Deformations of the Hubble parameter II



$$\dot{H} = \dot{H}_{\mathrm{sr}} + \delta \dot{H}$$

$\pi = \delta\chi/\dot{\chi}$ **: adiabatic mode**

σ **: isocurvature mode**

$$\zeta = -H\pi$$

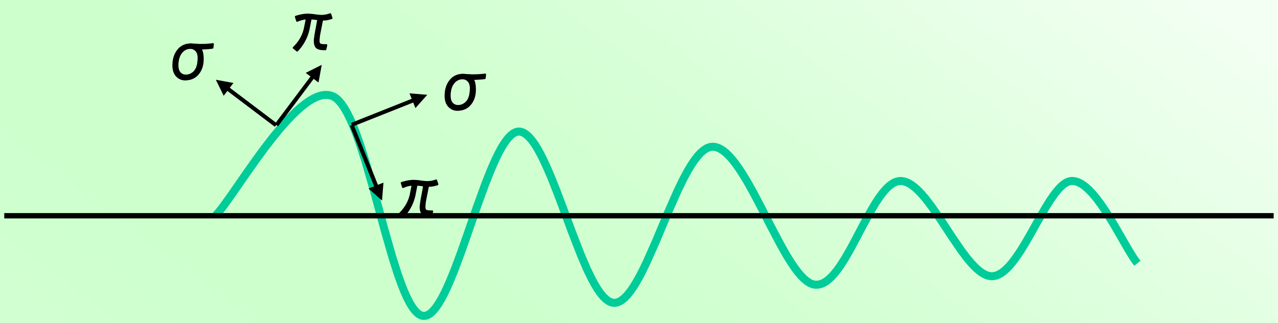
$$\left(ds^2 = -dt^2 + a(t)^2 e^{-2\zeta} d\boldsymbol{x}^2\right)$$

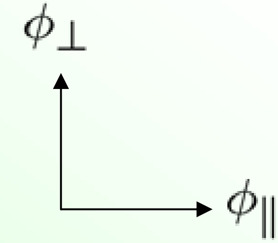
Canonical kinetic terms: $\frac{1}{2}\left(\dot{\phi}_{\parallel}^2 + \dot{\phi}_{\perp}^2\right) \ni \frac{1}{2}\dot{\delta\chi}^2 \simeq \frac{1}{2}\dot{\chi}^2\dot{\pi}^2 = -M_{\mathrm{pl}}^2 \dot{H}\dot{\pi}^2$

$$S = \int d^4x\, a^3 \left[-M_{\mathrm{Pl}}^2 \left(\dot{H}_{\mathrm{sr}} + \delta\dot{H}\right)\left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2}\right)\right]$$

kinetic term of adiabtic mode

Conversion effects



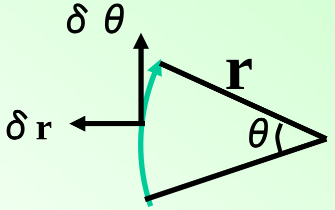


$\phi_\parallel = \phi_{sr} + \varphi_1$

$\phi_\perp = \varphi_2$

Adibatic and isocurvature directions are changing.

- **Simple example :**



$$r^2 \partial_\mu \theta \partial^\mu \theta \ni \bar{r}\dot{\bar{\theta}}\delta r \dot{\delta\theta}$$

$$(r = \bar{r} + \delta r,\ \theta = \bar{\theta} + \delta\theta) \quad = \ddot{\bar{r}}\delta r \frac{\dot{\delta\theta}}{\dot{\bar{\theta}}}$$

$\ddot{\varphi}_2 \ \sigma \ \dot{\pi}$

(Centrifugal force : $\ddot{\bar{r}} = \bar{r}\dot{\bar{\theta}}^2$ **)**

Adiabatic mode **Isocurvaton**

conversion

$$\pi = \frac{\delta\chi}{\dot{\chi}}$$

$\dot{\pi}\sigma$

σ

$$S = \int d^4x\, a^3 \beta(t) \dot{\pi}\sigma$$

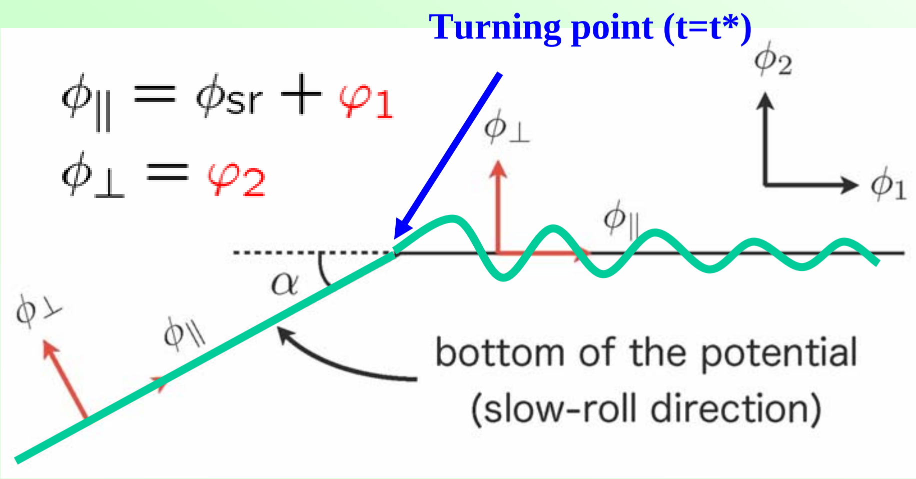
$\beta(t) = -2\ddot{\varphi}_2$ **: oscillating**

Total action

$$S_{\rm free} = \int d^4x\, a^3 \left[-M_{\rm Pl}^2 \dot{H}_{\rm sr} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{1}{2} \left(\dot{\sigma}^2 - \frac{(\partial_i \sigma)^2}{a^2} \right) - \frac{1}{2} m^2 \sigma^2 \right]$$

$$S_{\rm int}^{(2)} = \int d^4x\, a^3 \left[-M_{\rm Pl}^2 \delta\dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + \beta(t) \dot{\pi} \sigma \right]$$

$$\begin{cases} -2M_{\rm Pl}^2 \delta\dot{H} = 2\dot{\phi}_{\rm sr} \dot{\varphi}_1 + \dot{\varphi}_2^2 \\ \beta(t) = -2\ddot{\varphi}_2 \end{cases}$$



$$\begin{cases} \varphi_1(t) = \dfrac{\alpha^2 \dot{\phi}_{\rm sr}}{6H_{\rm sr}} \left(e^{-3H_{\rm sr}(t-t_*)} - 1 \right). \\ \varphi_2(t) = \dfrac{\alpha \dot{\phi}_{\rm sr}}{\mu H_{\rm sr}} e^{-\frac{3}{2}H_{\rm sr}(t-t_*)} \sin[\mu H_{\rm sr}(t-t_*)], \quad \mu = \sqrt{\dfrac{m^2}{H_{\rm sr}^2} - \dfrac{9}{4}}. \end{cases}$$

: oscillating

$$S_{\rm int}^{(3)} = \int d^4x\, a^3 \left[-M_{\rm Pl}^2 \delta\ddot{H} \pi \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + \frac{1}{2}\beta \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) \sigma + \dot{\beta} \pi \dot{\pi} \sigma - \theta(t-t_*) \frac{1}{6} \lambda \varphi_2 \sigma^3 \right]$$

Powerspectrum

Mode functions

$$
\begin{aligned}
H &= H_{\text{free}} + H_{\text{int}}^{(2)} \\
&= \int d^4x\, a^3 \left[-M_{\text{Pl}}^2 \dot{H}_{\text{sr}} \left(\dot{\pi}^2 + \frac{(\partial_i \pi)^2}{a^2} \right) + \frac{1}{2} \left(\dot{\sigma}^2 + \frac{(\partial_i \sigma)^2}{a^2} + m^2 \sigma^2 \right) \right. \\
&\qquad \left. + M_{\text{pl}}^2 \delta \dot{H} \left(\dot{\pi}^2 + \frac{(\partial_i \pi)^2}{a^2} \right) - \beta \dot{\pi} \sigma \right]
\end{aligned}
$$

We treat interaction terms as perturbations.

For free parts

$$
\pi_{\mathbf{k}} = u_k\, a_{\mathbf{k}} + u_k^*\, a_{-\mathbf{k}}^\dagger\,, \quad \sigma_{\mathbf{k}} = v_k\, b_{\mathbf{k}} + v_k^*\, b_{-\mathbf{k}}^\dagger
$$

with the standard commutation relations :

$$
[a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')\,, \quad [b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')\,.
$$

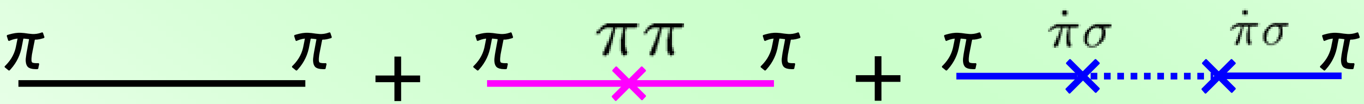
EOMs:

$$
\begin{cases}
\ddot{u}_k + 3H_{\text{sr}} \dot{u}_k + \dfrac{k^2}{a^2} u_k = 0\,, \\
\ddot{v}_k + 3H_{\text{sr}} \dot{v}_k + \left(m^2 + \dfrac{k^2}{a^2} \right) v_k = 0\,.
\end{cases}
$$

Powerspectrum of sudden turn

$$H_{\rm int}^{(2)}(t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} a^3(t) \left[M_{\rm pl}^2 \delta\dot{H} \left(\dot{\pi}_{\mathbf{k}}\, \dot{\pi}_{-\mathbf{k}} - \frac{k^2}{a^2} \pi_{\mathbf{k}}\, \pi_{-\mathbf{k}} \right) - \beta \dot{\pi}_{\mathbf{k}}\, \sigma_{-\mathbf{k}} \right](t)\,.$$

$$\begin{aligned}
\langle \pi_{\mathbf{k}}(t) \pi_{\mathbf{k}'}(t) \rangle &= \langle 0| \left[\bar{T} \exp\left(i \int_{t_0}^{t} dt' H_I(t')\right)\right] \pi_{\mathbf{k}}(t) \pi_{\mathbf{k}'}(t) \left[T \exp\left(-i \int_{t_0}^{t} dt' H_I(t')\right)\right] |0\rangle \\
&= \langle 0| \pi_{\mathbf{k}}(t) \pi_{\mathbf{k}'}(t) |0\rangle - 2\mathrm{Re}\left[i \int_{t_0}^{t} dt_1 \langle 0| \pi_{\mathbf{k}}(t) \pi_{\mathbf{k}'}(t) H_{\rm int}(t_1) |0\rangle \right] \\
&\quad + \int_{t_0}^{t} d\bar{t}_1 \int_{t_0}^{t} dt_1 \langle 0| H_{\rm int}^{(2)}(\bar{t}_1) \pi_{\mathbf{k}}(t) \pi_{\mathbf{k}'}(t) H_{\rm int}^{(2)}(t_1) |0\rangle \\
&\quad - 2\mathrm{Re}\left[\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \langle 0| \pi_{\mathbf{k}}(t) \pi_{\mathbf{k}'}(t) H_{\rm int}^{(2)}(t_1) H_{\rm int}^{(2)}(t_2) |0\rangle \right] + \dots\,,
\end{aligned}$$



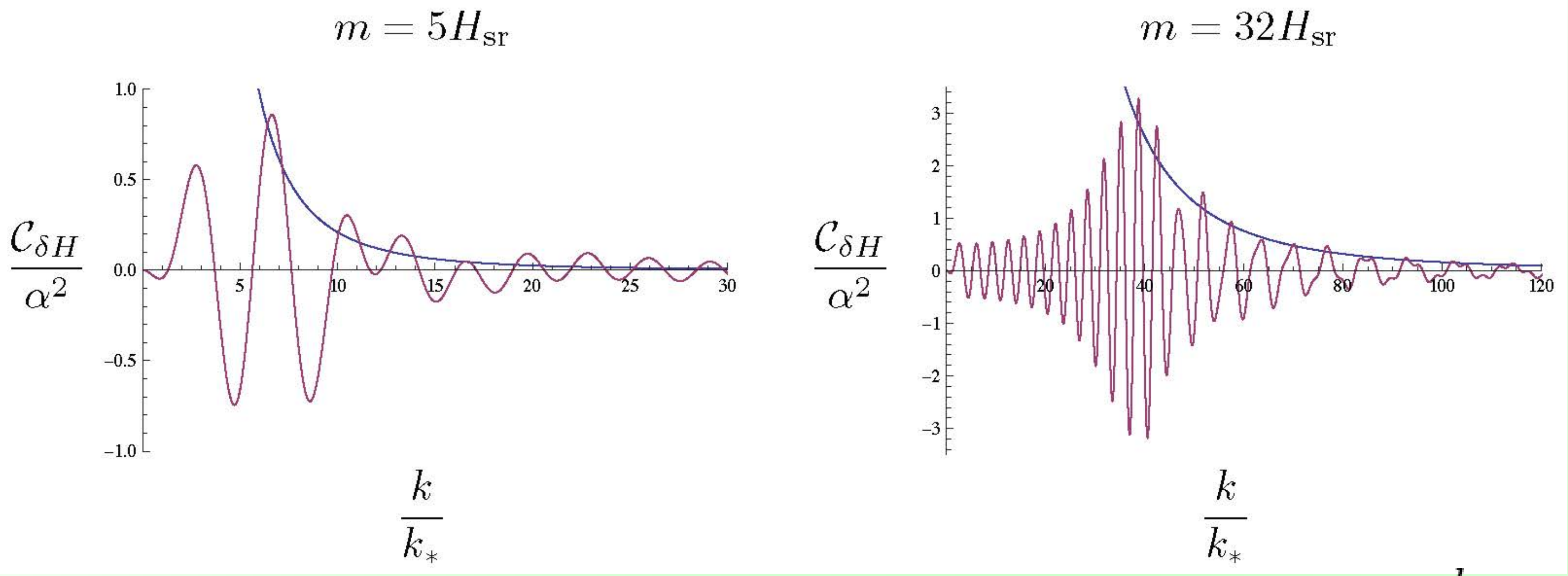
$$\langle \pi_{\mathbf{k}}(t) \pi_{\mathbf{k}'}(t) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') u_k^*(t) u_k(t) \left[1 + \mathcal{C} \right]$$

$$\mathcal{C} \;=\; C_{\delta H} + C_{\rm conv}\,.$$

represents deviation from single field inflation.

Powerspectrum from Hubble deformations

$$\mathcal{P}_\zeta(k) = \frac{H_{\rm sr}^2}{8\pi^2 M_{\rm sr}^2 \epsilon_{\rm sr}}\left(1 + \mathcal{C}_{\delta H} + \mathcal{C}_{\rm conv}\right), \qquad \mathcal{C}_{\delta H} = 2{\rm Re}\left[i\int_{-\infty}^{\infty} dt_1\, a^3 \left(2M_{\rm Pl}^2 \dot{H}_{\rm sr}\kappa(t_1)\right)\left(\dot{u}_k^2(t_1) - \frac{k^2}{a^2}u_k^2(t_1)\right)\right]$$



We observe the parametric resonance effects for $k \gtrsim \mu k_* \leftrightarrow \frac{k}{a(t_*)} \gtrsim m.$

$$-2M_{\rm Pl}^2 \delta\dot{H} = 2\dot{\phi}_{\rm sr}\dot{\varphi}_1 + \dot{\varphi}_2^2$$

Horizon exit at turn, $k_* = a(t_*)H_{\rm sr}$

The blue line represents the analytic estimate for the amplitude:

$$\mathcal{C}_{\delta H} = \frac{\sqrt{\pi}}{2}\alpha^2\mu^{1/2}\left(\frac{k}{\mu k_*}\right)^{-3}$$

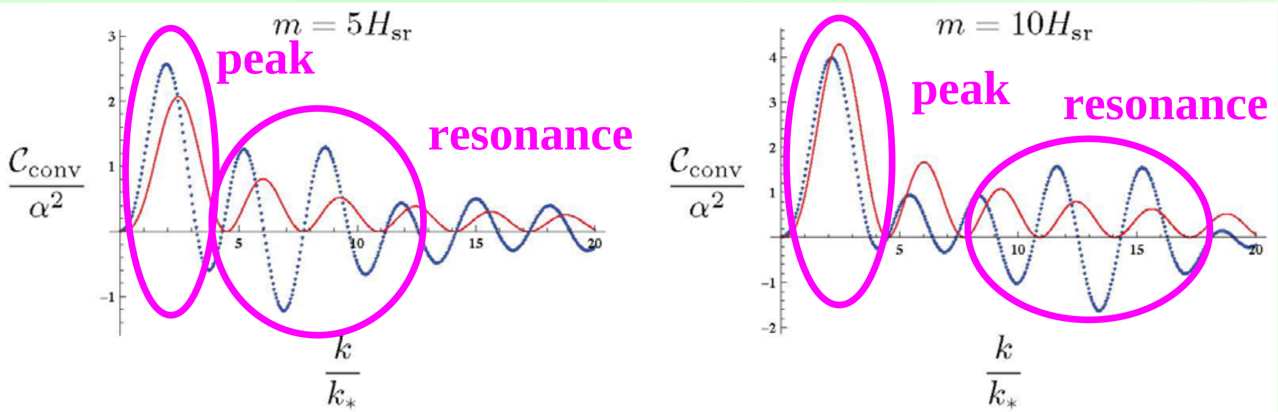
Powerspectrum from conversion effects

$$\mathcal{P}_\zeta(k) = \frac{H_{\rm sr}^2}{8\pi^2 M_{\rm sr}^2 \epsilon_{\rm sr}}(1 + \mathcal{C}_{\delta H} + \mathcal{C}_{\rm conv}),$$

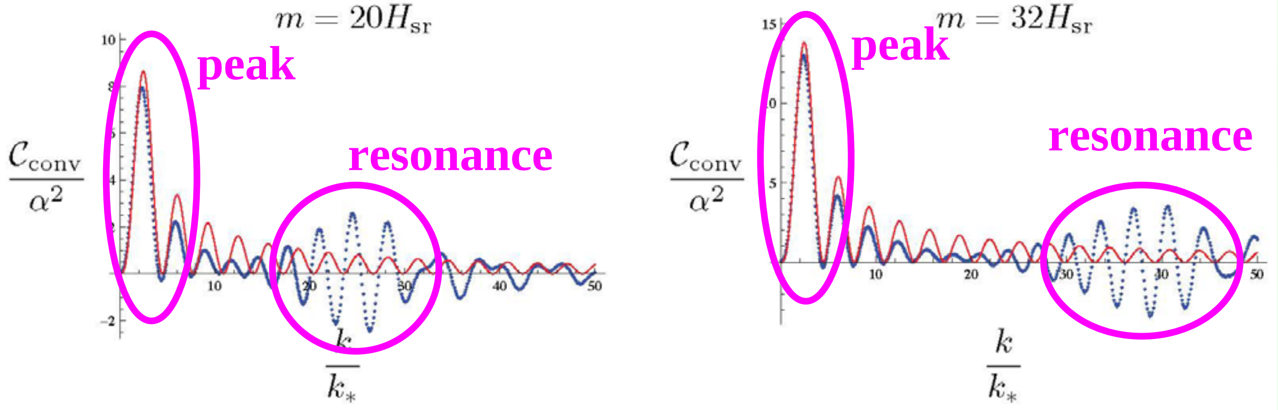
$$\mathcal{C}_{\rm conv} = \mathcal{C}_{\rm conv1} + \mathcal{C}_{\rm conv2},$$

$$\mathcal{C}_{\rm conv1} = 8\left|\int_{-\infty}^{\infty} dt_1\, a^3(t_1)\beta_1(t_1)\dot{u}_k(t_1)v_k(t_1)\right|^2,$$

$$\mathcal{C}_{\rm conv2} = -16{\rm Re}\left[\int_{-\infty}^{\infty} dt_1\, a^3(t_1)\beta_1(t_1)\dot{u}_k^*(t_1)v_k(t_1)\int_{-\infty}^{t_1} dt_2\, a^3(t_2)\beta_1(t_2)\dot{u}_k^*(t_2)v_k^*(t_2)\right].$$



Peak : Cconv ~ 0.43 $\mu\ \alpha^2$ at $x_* \sim 2.46$.



Red curve : **Analytic estimate in heavy mass limit for the peak.**

$$\mathcal{C}_{\rm conv}(k) \simeq \mu\alpha^2\frac{(\sin x_* - x_*\cos x_*)^2}{x_*^3}$$

$$x_* = k\tau_*$$

Conformal time at turn

Surprisingly, as for resonance effects,

$$\mathcal{C}_{\rm conv} = -\mathcal{C}_{\delta H}.$$

We observe the parametric resonance effects $k \gtrsim \mu k_*.$

$$\beta(t) = -2\ddot{\varphi}_2$$

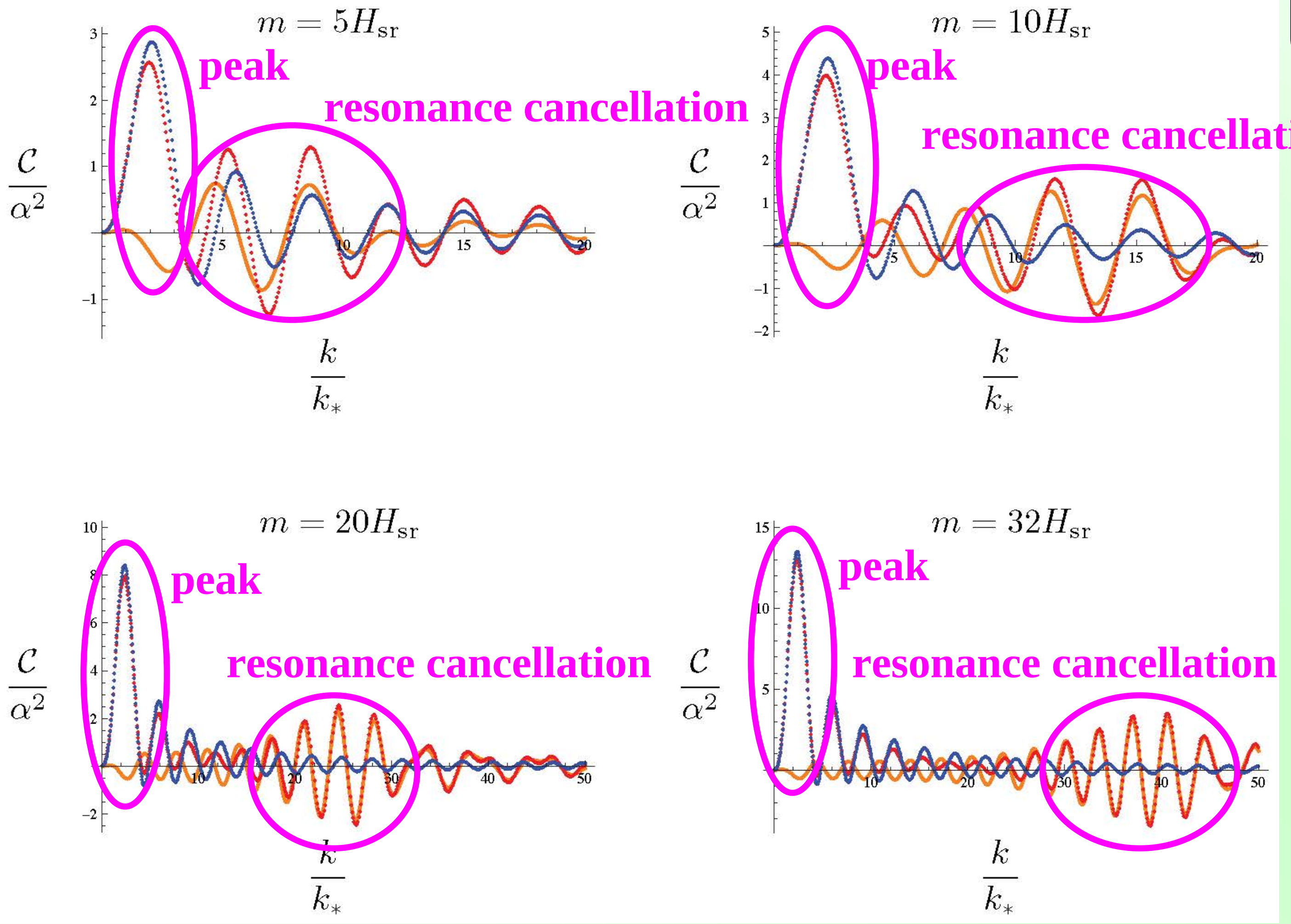
Total Powerspectrum

$$\mathcal{P}_\zeta(k) = \frac{H_{\rm sr}^2}{8\pi^2 M_{\rm sr}^2 \epsilon_{\rm sr}}(1+\mathcal{C}), \qquad \mathcal{C} = \mathcal{C}_{\delta H} + \mathcal{C}_{\rm conv}$$

Blue line : C

Red line : Cconv

Orange : - C δ H



You can observe

Cconv = - C δ H

for $k \gtrsim \mu k_*$.

Resonance effects are cancelled for the case with the canonical kinetic term

$$\mathcal{C}(k) \simeq \mu\alpha^2 \frac{(\sin x_* - x_* \cos x_*)^2}{x_*^3}, \quad x_* = k\tau_*.$$

(This result seems to coincide with the work by Gao, Mizuno, Langlois through the potential basis analysis.)

Resonance cancellation

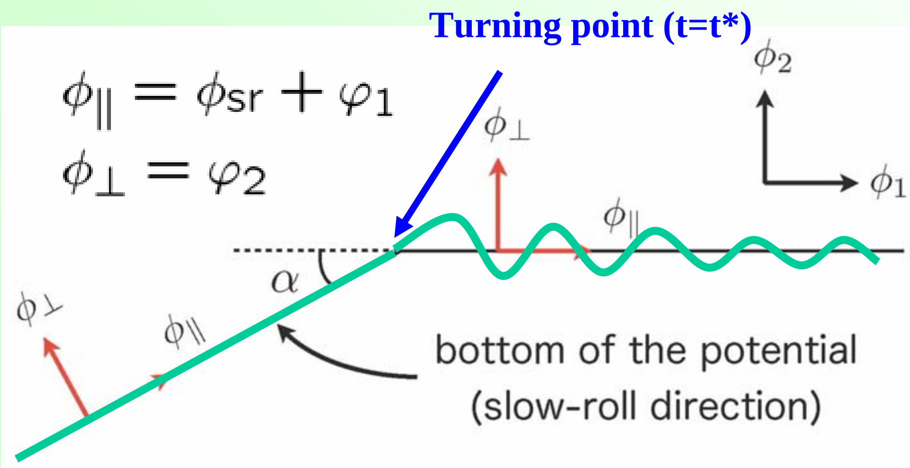
$$S^{(2)}_{\rm int} = \int d^4x\, a^3 \left[-M_{\rm Pl}^2 \delta\dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + \beta(t)\dot{\pi}\sigma \right]$$

$$\begin{cases} -2M_{\rm Pl}^2 \delta\dot{H} = 2\dot{\phi}_{\rm sr}\dot{\varphi}_1 + \dot{\varphi}_2^2 \\ \beta(t) = -2\ddot{\varphi}_2 \end{cases}$$

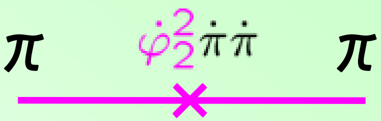
opposite phase

oscillating

$$\begin{cases} \varphi_1(t) = \dfrac{\alpha^2 \dot{\phi}_{\rm sr}}{6H_{\rm sr}} \left(e^{-3H_{\rm sr}(t-t_*)} - 1 \right). \\ \varphi_2(t) = \dfrac{\alpha \dot{\phi}_{\rm sr}}{\mu H_{\rm sr}} e^{-\frac{3}{2}H_{\rm sr}(t-t_*)} \sin[\mu H_{\rm sr}(t-t_*)]. \end{cases}$$

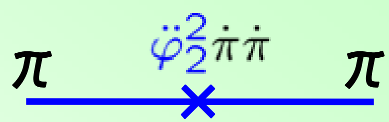


- **Hubble deform :**



- **Conversion :**

Integaration out of σ



$$\dot{\varphi}_2^2 \sim \cos^2(mt)$$

$$+$$

$$\ddot{\varphi}_2^2 \sim \sin^2(mt)$$

$$||$$

No oscillating terms in the action

No resonances are observed.

Summary

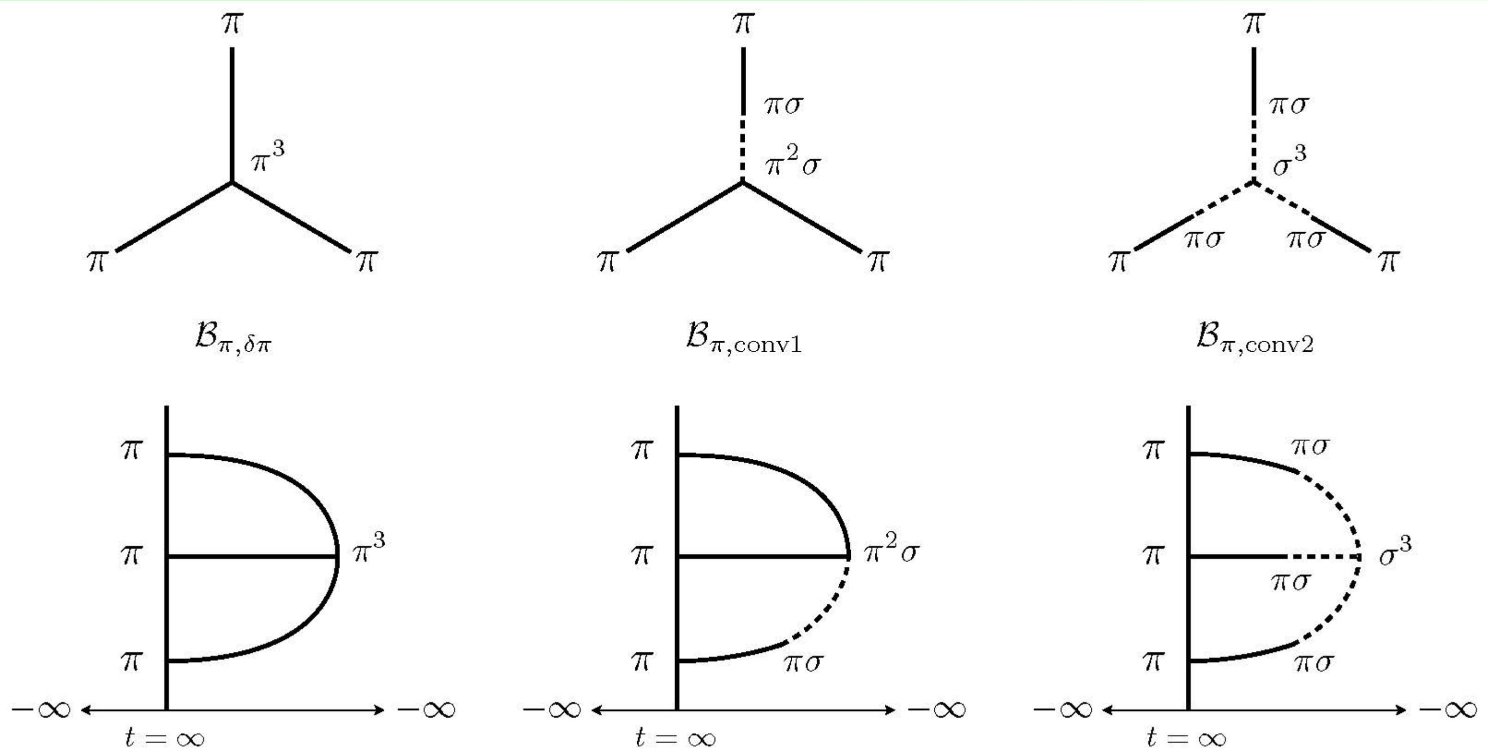
- We have investigated primordial spectra from turning trajectory, which are useful to probe the high energy physics.
- Sudden turning gives rise to the two effects:
  (1) Hubble deformation
  (2) Conversion between adiabatic and isocurvature modes
- In the powerspectrum, the peak appears around the turning scale. The above two terms cause parametric resonances, but, both effects are cancelled out in the case with the canonical kinetic term.
- In the bispectrum, such a cancellation does not happen and parametric resonance effects are observed, which could be useful to determine whether the kinetic term of such a scalar field is canonical or not.

Bispectrum

Three point interactions

Three point interactions:

$$H_{\rm int}^{(3)} = \int d^3x\, a^3 \left[\theta(t-t_*)M_{\rm Pl}^2 \dot{H}_{\rm sr}\dot{\kappa}\pi\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) - \frac{1}{2}\beta\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right)\sigma - \dot{\beta}\pi\dot{\pi}\sigma + \theta(t-t_*)\frac{\lambda\varphi_2}{6}\sigma^3\right]$$

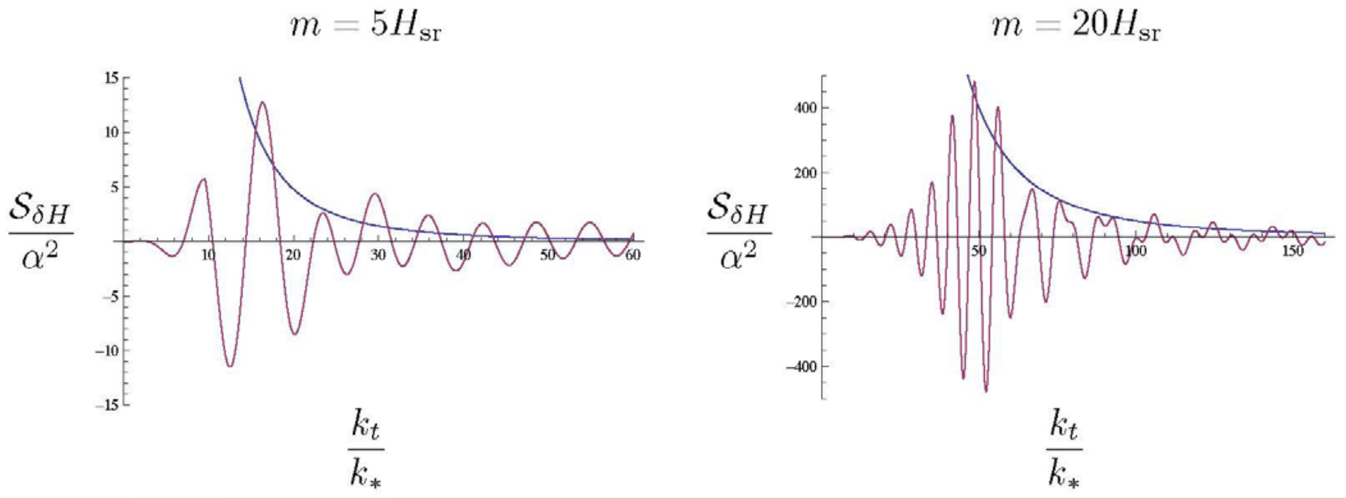


$$\mathcal{B}_\pi(k_1,k_2,k_3) = \mathcal{B}_{\pi,\delta H}(k_1,k_2,k_3) + \mathcal{B}_{\pi,\rm conv1}(k_1,k_2,k_3) + \mathcal{B}_{\pi,\rm conv2}(k_1,k_2,k_3)\,.$$

$$\langle \pi_{\mathbf{k}_1}\pi_{\mathbf{k}_2}\pi_{\mathbf{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{B}_\pi(k_1,k_2,k_3)\,.$$

Bispectrum from Hubble deformations

- **Scale dependence for the equilateral configuration:**

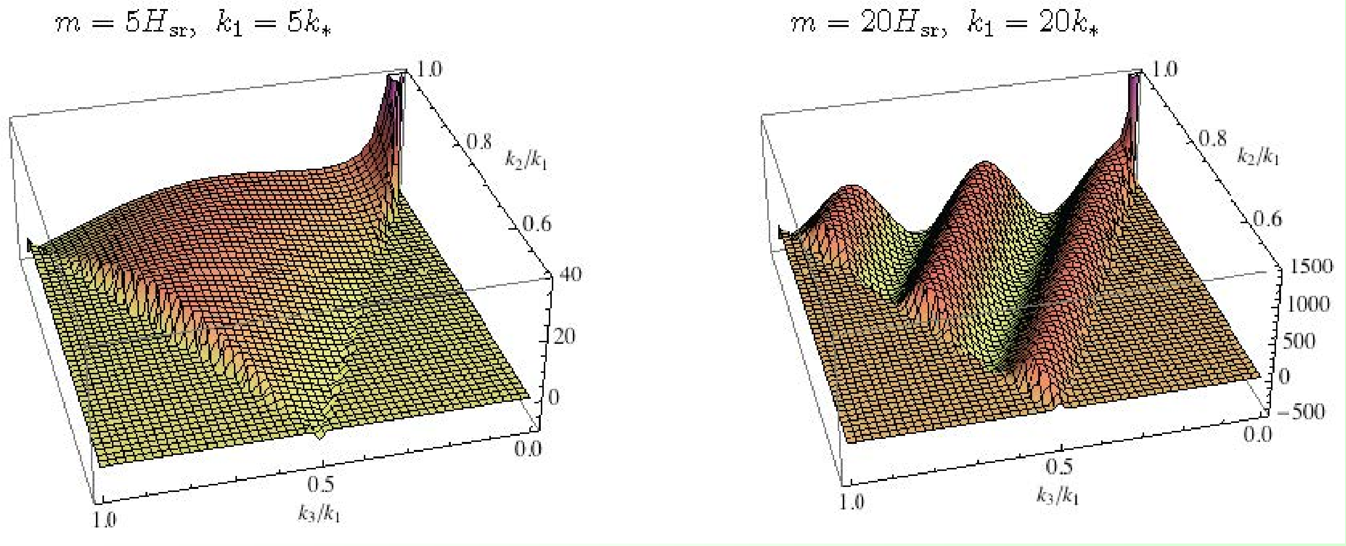


Resonance effects are observed.

$$k \gtrsim 2\mu k_*$$

$$f_{\rm NL} \sim \alpha^2 \mu^{\frac{5}{2}} \times \mathcal{O}(1).$$

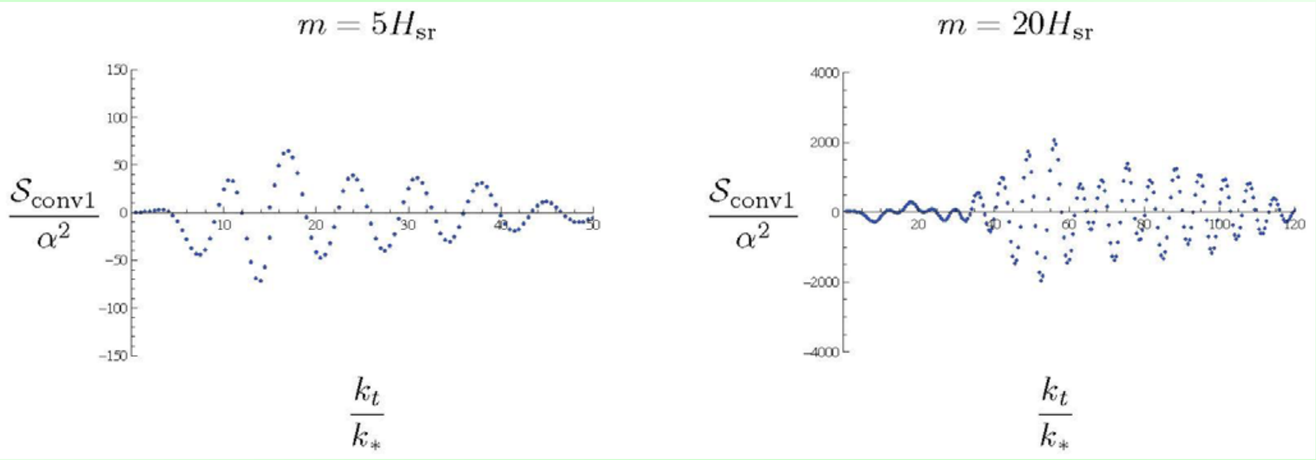
- **Shape:**



The peak appears in the squeezed limit.

Bispectrum from conversion 1

- Scale dependence for the equilateral configuration:

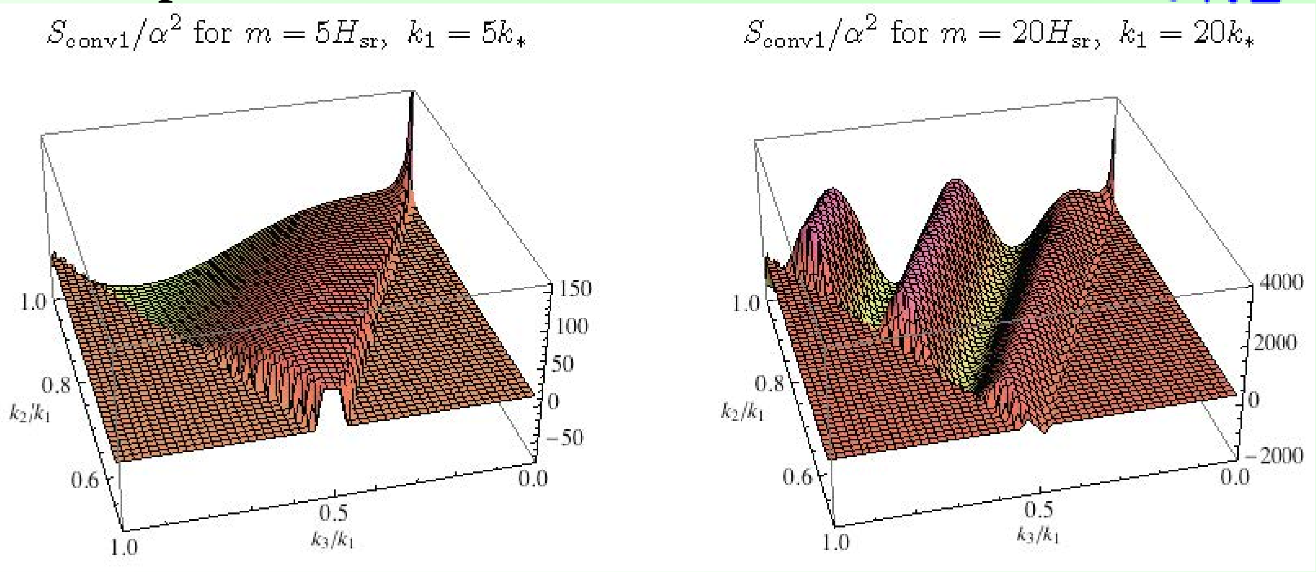


Resonance effects are observed.

$$k \gtrsim 2\mu k_*$$

- Shape:

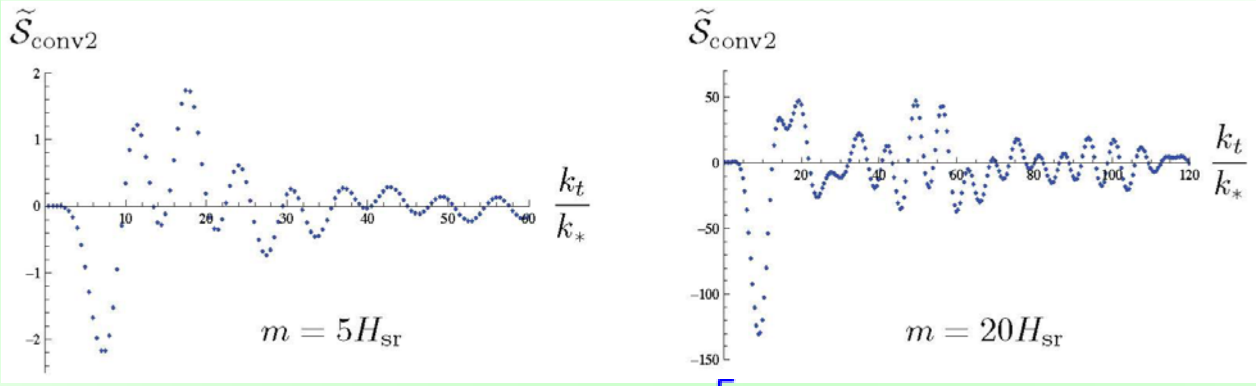
$$f_{\rm NL} \sim \alpha^2 \mu^{\frac{5}{2}} \times \mathcal{O}(1).$$



The peak appears in the squeezed limit.

Bispectrum from conversion 2

- **Scale dependence for the equilateral configuration:**



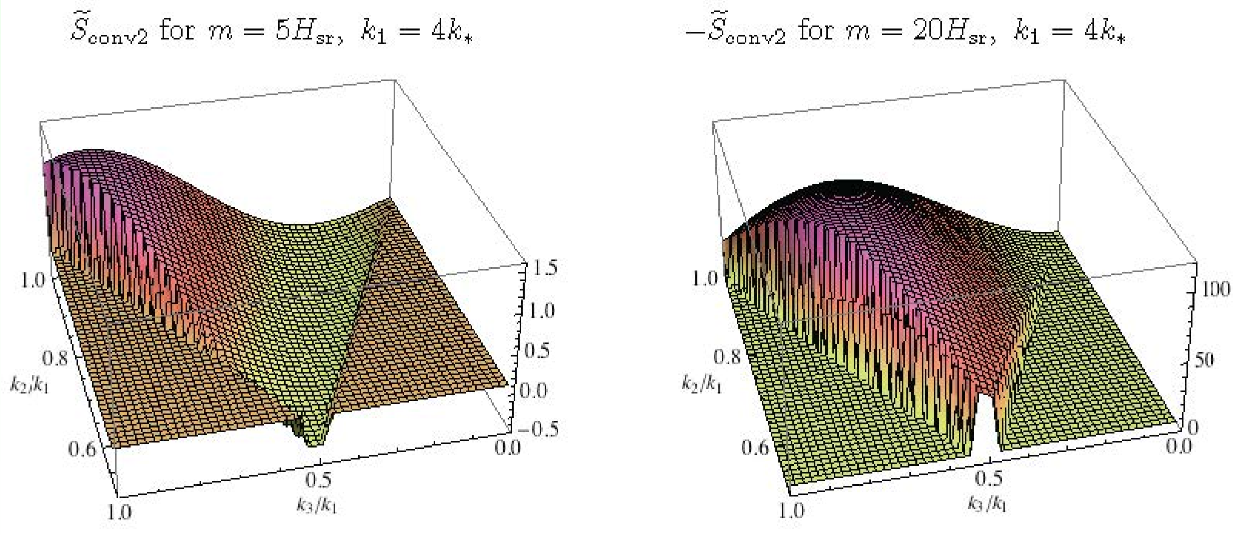
Resonance effects are observed.

The peak appears around k ~ 9 k*.

$f_{\rm NL} \lesssim \alpha^2 \mu^{\frac{5}{2}} \times \mathcal{O}(0.1)$ **around the resonance scales.**

$f_{\rm NL} \lesssim \alpha^2 \mu^{3} \times \mathcal{O}(0.1)$ **around the peak.**

- **Shape:**



The peak does not appear in the squeezed limit.

Total Bispectrum

- **O(α^2) contributions:** **Hubble deformation, Conversion effect 1**

$$f_{\rm NL} \sim \alpha^2 \mu^{\frac{5}{2}} \times \mathcal{O}(1).$$

- **O($\lambda \alpha^4$) contributions:** **Conversion effect 2**

$f_{\rm NL} \lesssim \alpha^2 \mu^{\frac{5}{2}} \times \mathcal{O}(0.1)$ **around the resonance scales.**

$f_{\rm NL} \lesssim \alpha^2 \mu^{3} \times \mathcal{O}(0.1)$ **around the peak.**

Perturbativity condition: $\dfrac{\lambda \alpha^2}{\mu^4} \lesssim 10^{-8}.$