

# *Testing the Spatial Curvature of FL Models*

*Roland Triay*

CPT - Aix Marseille University

**Hot Topics in General Relativity and Gravitation**  
**July 28th — August 3rd, 2013 • Quy Nhon, Viet Nam**

\*  $\Lambda$  effect in the cosmological expansion of voids.

*Henri-Hugues Fliche, Roland Triay - JCAP 11:022 (2010)*

\* Dynamics of Void and its Shape in Redshift Space.

*Kei-ichi Maeda, Nobuyuki Sakai, Roland Triay - JCAP 08:026 (2011)*

# Voids as Gravitational Structures

- **Voids are non linear structures** — e.g., Traveling Wavelets approach to the Gravitational Instability theory. – One dimensional Wavelets . – N. Benhamidouche, B. Torresani, R. Triay. -MNRAS 302,807(1999)
- « vacuum gravitational repulsion » ?  
e.g., Schwarzschild solution of Einstein Eq. with a Cosmological Constant

$$\vec{g} = \left( -G \frac{m}{r^3} + \frac{\Lambda}{3} \right) \vec{r}$$

No effect in the Solar neighborhood  
Small effect in the outerpart of the Galaxy  
Intervenies at the edge of LSC  
Homogeneity scale

$$r_o = \sqrt[3]{3mG/\Lambda}$$

$$\begin{aligned} r_o &\sim 10^2 h^{-2/3} \text{ yr} \\ r_o &\sim 5 \cdot 10^5 h^{-2/3} \text{ yr} \\ r_o &\sim 4 \cdot 10^8 h^{-2/3} \text{ yr} \\ &(\sim 100 \text{ Mpc}) \end{aligned}$$

- Modeling a “**single void**” embedded in a **Friedmann Lemaître Universe** as the first step to a more global investigation (foam like structure)
  - Newton - Friedmann model (FT)
  - GR “FL<sub>in</sub> - FL<sub>out</sub>” Model (MST)

# Friedmann - Newton Model



*Covariant Global solution to Euler- Poisson Eqs. System*

Reference Coordinates

$$(t, \quad \vec{x} = \frac{\vec{r}}{a}), \quad a > 0$$

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3} - \frac{K_o}{a^2} + \frac{8\pi G \rho_o}{3 a^3}}$$

$$K_o = \frac{8\pi G}{3} \rho_o + \frac{\Lambda}{3} - H_o^2 \leq \sqrt[3]{(4\pi G \rho_o)^2 \Lambda}, \quad a_o = 1$$

$$\vec{v}_c = \alpha \vec{x}, \quad \vec{g}_c = \beta \vec{x}$$

$$\frac{d\alpha}{dt} + 4\alpha^2 + 2H\alpha - \frac{2\pi G \rho_c}{3 a^3} = 0$$

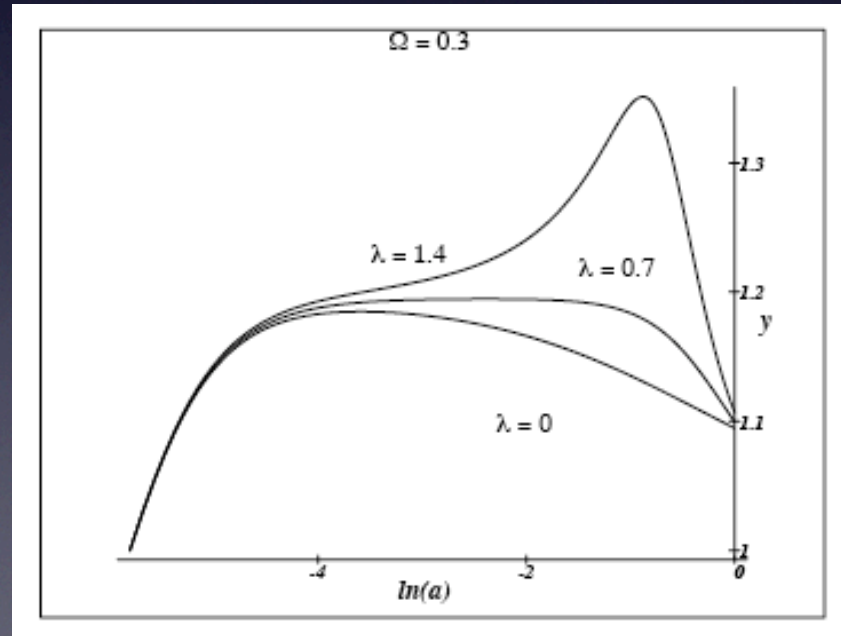
$$\beta = \frac{4\pi G}{a^3} \left( \frac{\rho_c}{3} - \frac{(\rho S)_c}{2x} \right)$$

# Magnification & Expansion Rate

$$\vec{v}_c = \alpha \vec{x}, \quad \vec{g}_c = \beta \vec{x}$$

$$X = \frac{x}{x_i}, \quad Y = \frac{\alpha}{H_0}$$

$y$  : Corrective factor to Hubble expansion



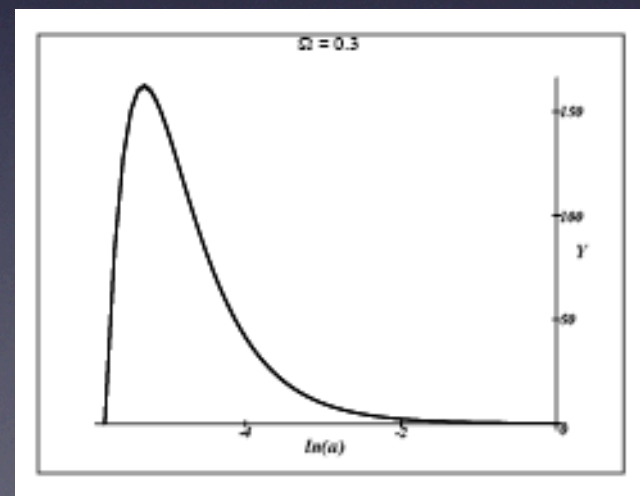
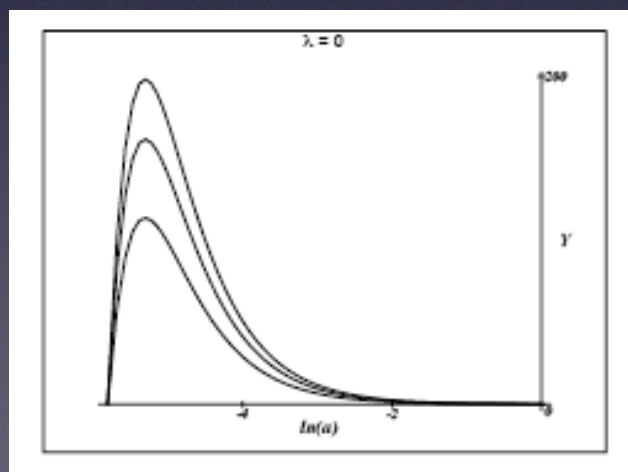
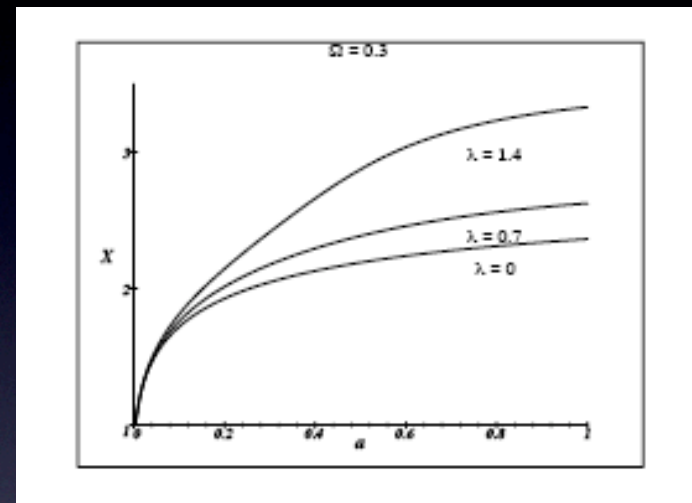
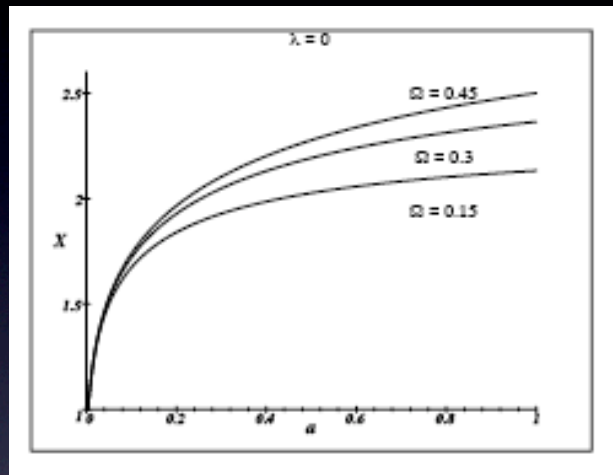
$$\vec{v} = yH\vec{r}, \quad y = 1 + \frac{Y}{h}, \quad h = \frac{H}{H_0}$$

$$\lambda_0 = \frac{\Lambda}{3H_0^2}, \quad \Omega_0 = \frac{8\pi G\rho_0}{3H_0^2}, \quad k_0 = \frac{K_0}{H_0^2}$$

**Observational Signature of  $K_0$**

$$H_m = H_\infty \sqrt{1 - \frac{K_0^3}{\Lambda (4\pi G\rho_0)^2}}, \quad H_\infty = \lim_{a \rightarrow \infty} H = \sqrt{\frac{\Lambda}{3}}$$

# Dependence on Cosmological Parameters



# GR Model

$$ds_{\pm}^2 = -dt_{\pm}^2 + a_{\pm}^2 [d\chi_{\pm}^2 + f_{\pm}^2(\chi_{\pm})d\Omega^2], \quad f_{\pm}(\chi_{\pm}) = \begin{cases} \sin \chi_{\pm} & (k_{\pm} = 1) \\ \chi_{\pm} & (k_{\pm} = 0) \\ \sinh \chi_{\pm} & (k_{\pm} = -1) \end{cases}$$

$$ds_{\Sigma}^2 = -d\tau^2 + R^2(\tau)d\Omega^2$$

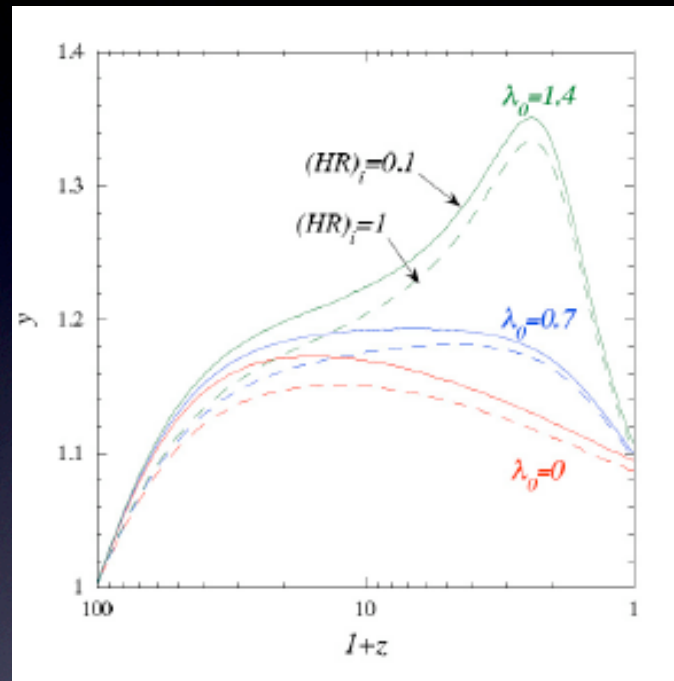
$$H_{\pm}^2 + \frac{k_{\pm}}{a_{\pm}^2} = \frac{8\pi G}{3}\rho_{\pm}, \quad \text{with} \quad H_{\pm} = \frac{\dot{a}_{\pm}}{a_{\pm}}, \quad \text{and} \quad \rho_{\pm} = \rho_{\text{vac}} + \rho_{\pm}^{(m)}$$

$$\rho_{\text{vac}} = \Lambda/(8\pi G)$$

$$R = a_+ f_+ \left( \chi_+^{(\Sigma)} \right) = a_- f_- \left( \chi_-^{(\Sigma)} \right)$$

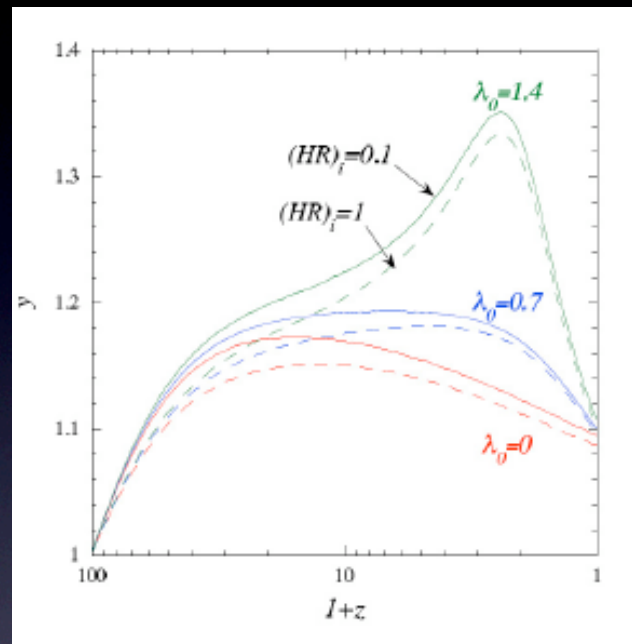
$$d\tau^2 = dt_+^2 - a_+^2(t_+) \left( d\chi_+^{(\Sigma)} \right)^2 = dt_-^2 - a_-^2(t_-) \left( d\chi_-^{(\Sigma)} \right)^2$$

# GR Model



1. The voids always grow faster than Hubble expansion (*i.e.*,  $y > 1$ ) along their evolution. The peculiar velocity starts with a huge burst and decreases asymptotically toward Hubble velocity; the present value is higher than the Hubble flow by about 10%.
2. The smaller the radius gets, the higher the peculiar velocity is. Namely, the void with the largest initial size  $R_i = H_i^{-1}$  grows slightly slower, which provides us with the minimal peculiar velocity.

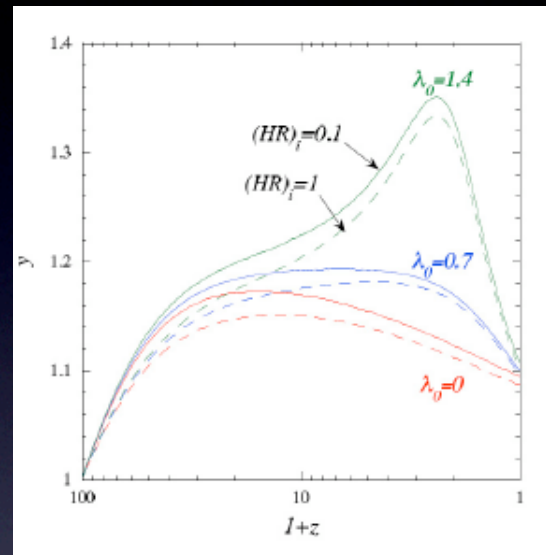
# GR Model



3. While the dependence on  $\lambda_0$  is not significant in the beginning, its effect eventually appears in the evolution, and the correction factor  $y$  increases with  $\lambda_0$ . With  $\lambda_0 = 0.7$ , the peculiar velocity reaches the value higher than the Hubble expansion at  $z \sim 10$  by about 20%.
4. The redshift  $z^*$  when  $y$  reaches its maximum  $y_{\max} = y(z^*)$  decreases with  $\lambda_0$ . It corresponds to  $z^* \sim 40$  for  $\lambda_0 = 0$  and to  $z^* \sim 1.7$  for  $\lambda_0 = 1.4$ . For the intermediate value  $\lambda_0 = 0.7$ , for which the corresponding curve shows a plateau,  $\Lambda$  gives the maximal contribution to  $v$  at  $z^* \sim 1.7$ , which corresponds to 30% of that of the matter density (when it is compared to the case of  $\lambda_0 = 0$ ).

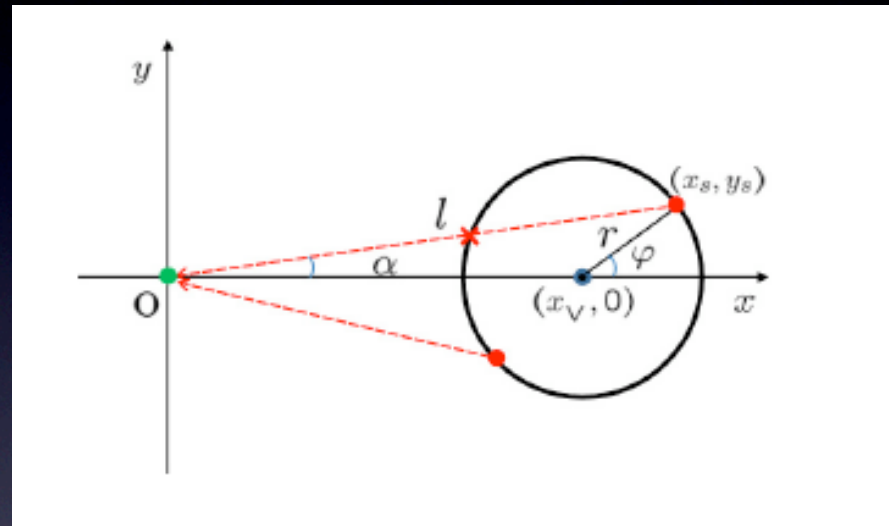


# GR Model



5. Let us pay attention to the presence of a bump on the curve, which becomes visible for  $\lambda_0 > 0.7$  (*i.e.* when  $k > 0$ ). It is caused by the fact that the universe experiences a loitering period of cosmological expansion, while the void then continues its own expansion.
6. The present values  $y_0$  related to  $\Omega_0 = 0.3$  but to different values for  $\lambda_0 \in \{0, 0.7, 1.4\}$  are very close to each other, while the variation of  $y$  with time (or with  $z$ ) depends undeniably on  $\lambda_0$ . In other words, the  $\Lambda$  effect, which accounts for a deviation between these curves, is not substantial nowadays.
7. It is remarkable that the present GR approach for the sub-horizon void confirms the previous result found in the Newtonian dynamics [13]. Even for a relativistic spherical void with a horizon size radius  $R_i = H_i^{-1}$ , the relativistic effect turns out to be weak.

# Void in the redshift-space



The Sachs-Wolfe effect

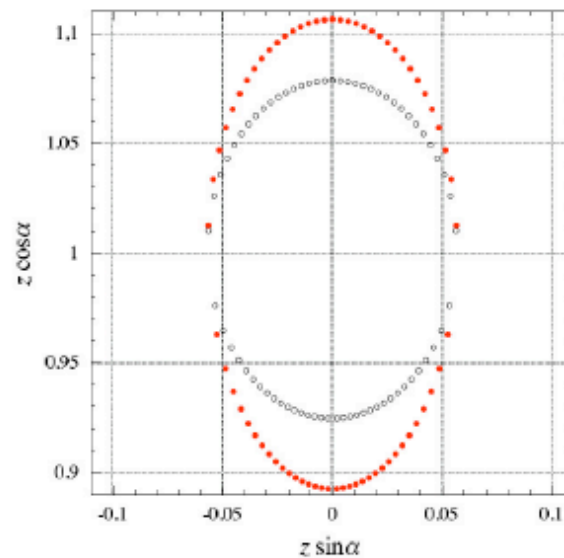
$$\Delta z_s|_{\text{ISW}} \sim (HR)^3$$

and the gravitational lens effect

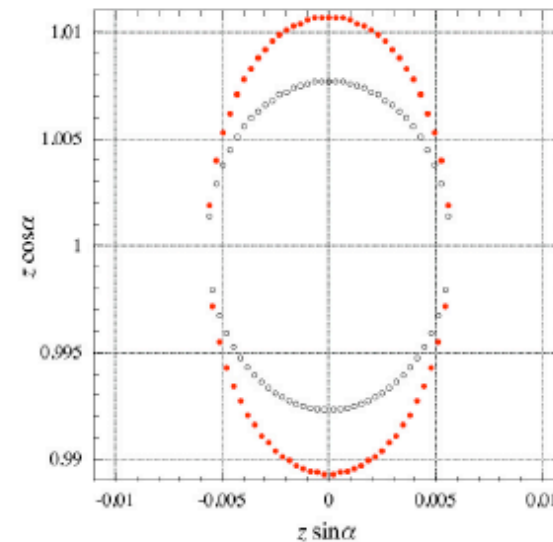
$$\Delta z_s|_{\text{GL}} \approx \frac{R(\Delta\alpha)^2}{ax_V} \approx 0.06 \frac{r}{x_V} (HR)^2 \sin(\varphi - \alpha), \quad \text{for } \Omega_0 = 0.3$$

are weak compared to the “doppler effect”.

# Image of a Void in the redshift-space



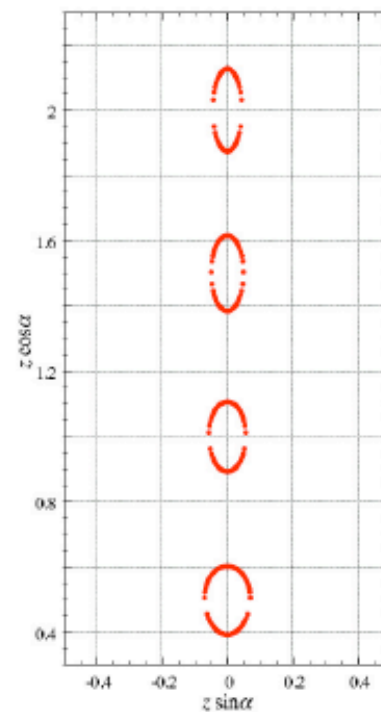
(a)  $R_0 = 0.1H_0^{-1}$ ,  $z_V = 1$



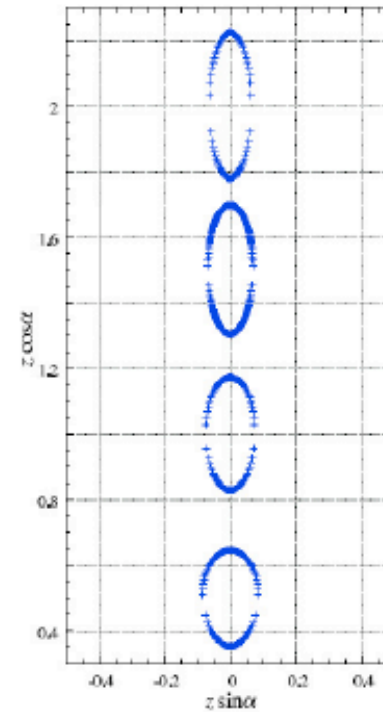
(b)  $R_0 = 0.01H_0^{-1}$ ,  $z_V = 1$

Figure 3. Empty spherical voids in the redshift space. — The images (red dots) of their boundary layers (*i.e.*, the void shell  $S$ ) are depicted for the case of  $\Omega_0 = 0.3$  and  $\lambda_0 = 0.7$ . The present values of their radii are (a)  $R_0 = 0.1H_0^{-1}$  and (b)  $R_0 = 0.01H_0^{-1}$ . The observed radii at  $z_V = 1$  are : (a)  $R_V = 0.0437H_0^{-1}$  and (b)  $R_V = 0.00437H_0^{-1}$ . The images of the standard static spheres (small black dots) are also displayed.

# Dependence on Cosmological Parameters



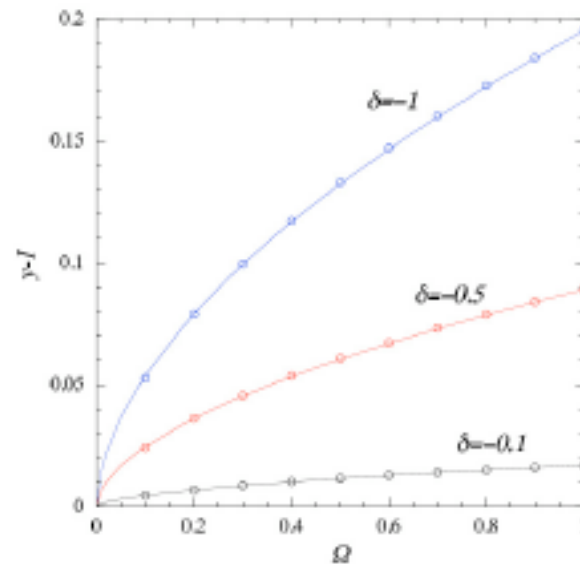
(a)  $\Omega_0 = 0.3$



(b)  $\Omega_0 = 1$

Figure 5. The images in the redshift space of an empty spherical void with present radius  $R_0 = 0.1H_0^{-1}$  at redshift  $z_V \in \{0.5, 1, 1.5, 2\}$  for (a)  $\Omega_0 = 0.3$  and (b)  $\Omega_0 = 1$ .

# Non Empty Voids



Velocity  $y$  versus  $\Omega$  and  $\delta^-$ .

Figure 7. The velocity  $y$  of a non-empty void. — We assume  $\Omega + \lambda = 1$  and  $\delta^- \in \{0.1, -0.5, -0.1\}$ . The circles and the continuous lines correspond respectively to our numerical results and to the fitting formula given in Eq. (3.2).

# *Conclusion*

In addition to provide us with a method for estimating the cosmological parameters, this test enables us to characterize clearly the sign of the spatial curvature of the universe. To improve such an approach for exploiting the observational data may help us to answer the question whether the space is finite or infinite, which is a crucial issue for the primordial cosmology.

Thank you