The FLG model faced to anthropic issues

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Hot Topics in General Relativity and Gravitation

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Outline

- Friedmann-Lemaître-Gamov (FLG) Model
- The Cosmological Constant (Problems)
- Dynamical phases of FLG Model
- The Coincidence Problem

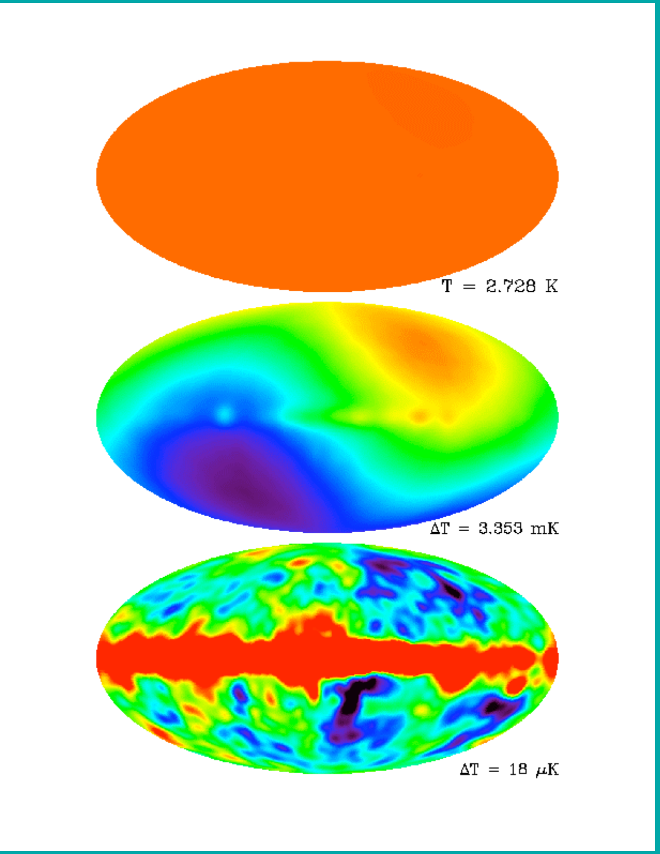
FLG Model

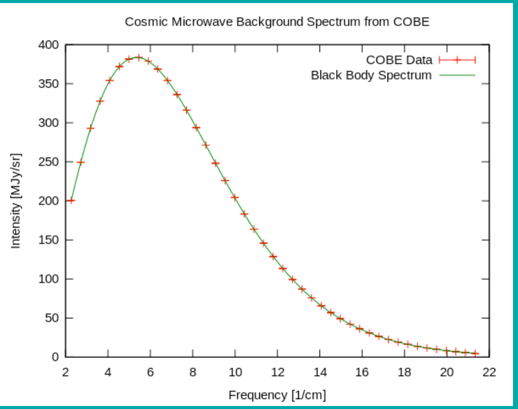
Jean-Marie Souriau (1922-2012)

- **(1944) 1957-65 : (General) Relativity (and Gravitation)**
  - *"Géométrie et relativité", Herman, Paris 1964*
- **1965-78 : GR (principles), Thermodynamics**
  - *"Définition covariante des équilibre thermodynamiques" Nuovo Cimento 1 (1966), no. 4, 203−216.*
  - *"Mécanique Statistique, Groupe de Lie et Cosmologie", Coll. Internationaux du CNRS N°−237 Géométririe symplectique et physique mathématique (1974)*
  - *"Faut-il prendre au sérieux la constante cosmologique?" Journées Relativistes (Bruxelles, April 1976), pp. 215-229. Publ. Univ. Libre de Bruxelles, 1976.*
  - *Fliche & Souriau (79), Fliche,, Souriau, Triay (81),…*

FLG Model

Cosmic Microwave Background (CMB)





The Nobel Prize in Physics

1978 A. Penzias and R. Wilson (1965)
2006 J.C. Mather, G.F. Smoot (1989)
2011: S. Perlmutter, B.P. Schmidt and A.G. Riess
"for the discovery of the accelerating expansion of the Universe through observations of distant SN"

FLG Model

Geometry of the temperature (Planck) vector

$$\nu_1 ds_1 = \nu_2 ds_2$$

$$\frac{\nu_1}{\nu_2} = \frac{ds_2}{ds_1} \qquad \frac{\nu_1}{T_1} = \frac{\nu_2}{T_2}$$

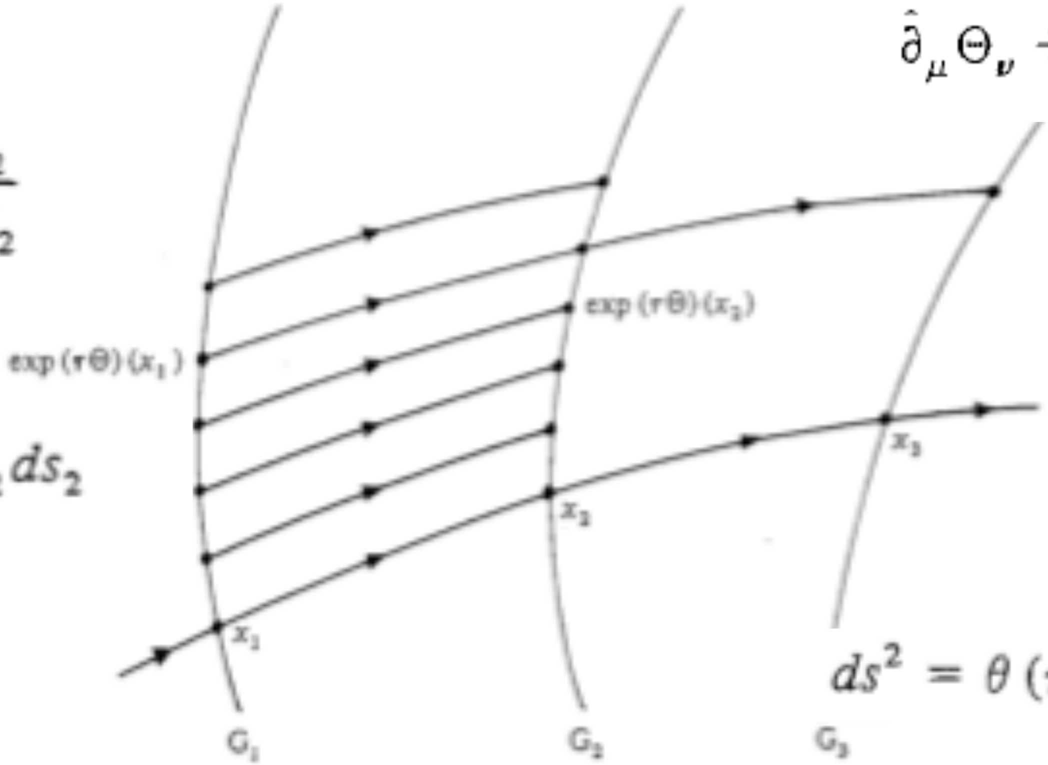
$$\frac{T_1}{T_2} = \frac{ds_2}{ds_1}$$

$$d\tau = kT_1 ds_1 = kT_2 ds_2$$

$$U_1 = \frac{dx_1}{ds_1}$$

$$\frac{dx_1}{d\tau} = \frac{U_1}{kT_1}$$

$$\Theta = \frac{U}{kT}$$



$$\hat{\partial}_\mu \Theta_\nu + \hat{\partial}_\nu \Theta_\mu = \lambda g_{\mu\nu}$$

$$x \mapsto \Theta$$

$$\frac{dx_1}{d\tau} = \Theta(x_1)$$

$$\exp(a\Theta)(x_0)$$

$$ds^2 = \theta(\tau)^2 [d\tau^2 - d\sigma^2]$$

FLG Model

U^μ / kT

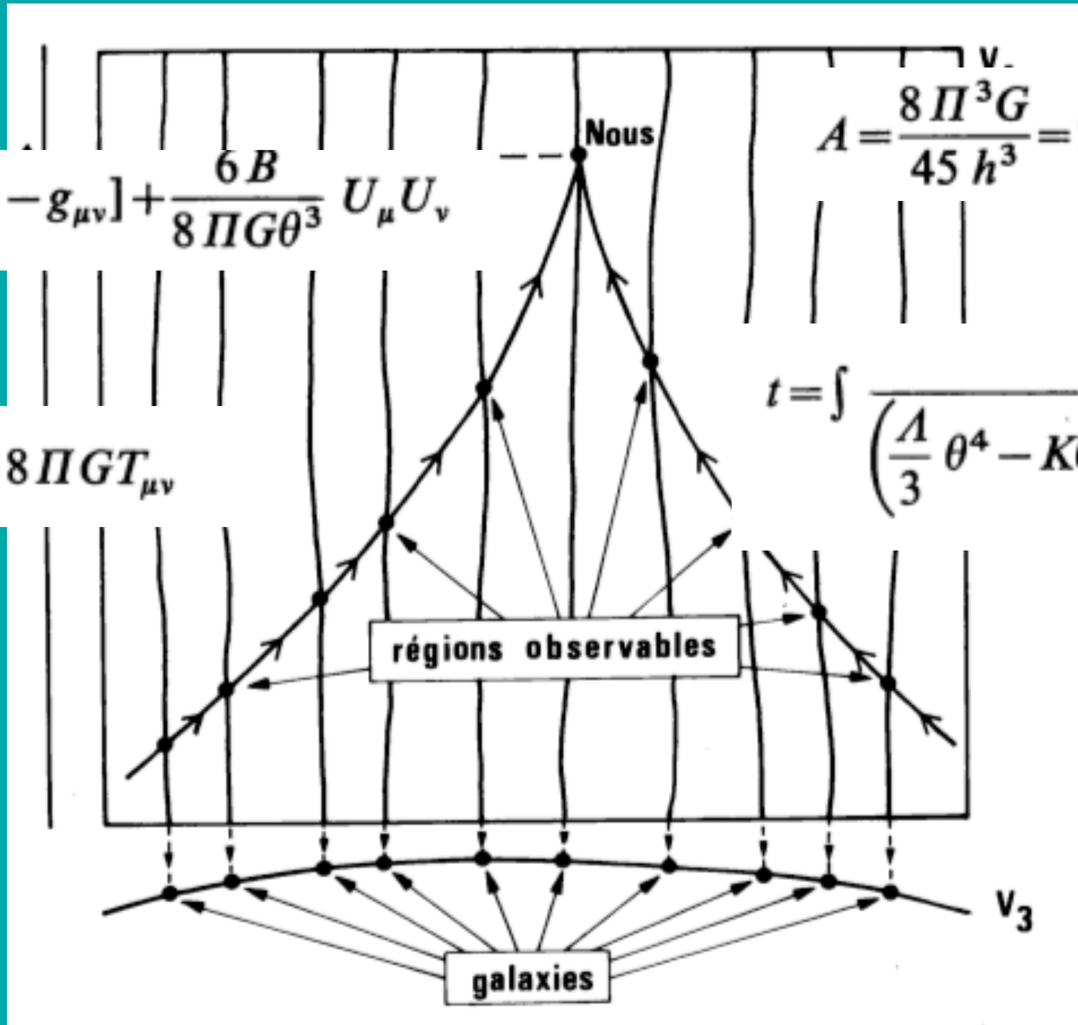
$$T_{\mu\nu} = \frac{A}{8\Pi G\theta^4}[4U_\mu U_\nu - g_{\mu\nu}] + \frac{6B}{8\Pi G\theta^3} U_\mu U_\nu$$

$$S^\mu = \frac{4\pi^2 k^4}{45\hbar^3} T^3 U^\mu$$

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\Pi G T_{\mu\nu}$$

$$ds^2 = dt^2 - \theta^2 d\sigma^2$$

$$1 + z = T_1/T_2$$



$$A = \frac{8\Pi^3 G}{45h^3} = 9.4\ 10^{84}\ \mathrm{g}^{-4}\ \mathrm{cm}^{-2}.$$

$$\rho = \frac{3B}{4\Pi G\theta^3}$$

$$t = \int \frac{\theta d\theta}{\left(\frac{\Lambda}{3}\theta^4 - K\theta^2 + 2B\theta + A\right)^{1/2}}$$

FLG Model

$$a = a(t) = (1+z)^{-1} > 0$$

$$H = \frac{\dot{a}}{a} = H_\circ \sqrt{\phi_\circ(a)}, \quad \phi_\circ(a) = \lambda_\circ - \frac{\kappa_\circ}{a^2} + \frac{\Omega_\circ}{a^3} + \frac{\alpha_\circ}{a^4} \geq 0, \quad a_\circ \equiv a(t_\circ) = 1, \quad \phi_\circ(a_\circ) = 1$$

where $H_\circ$ is the Hubble constant (a scale parameter), $\lambda_\circ$, $\kappa_\circ$, $\Omega_\circ$ and $\alpha_\circ$ are dimensionless quantities that stand for the values of *cosmological parameters*[1] at present epoch $t = t_\circ$. One has : – the reduced cosmological constant $\lambda = \frac{1}{3}\Lambda H^{-2} \geq 0$; – the curvature parameter $\kappa = \iota_\circ|\kappa| = KH^{-2}$, where K is the scalar curvature of the comoving space V_3, its sign $\iota_\circ$ characterizes the dynamics of the cosmological expansion[2]; – the (reduced) density parameter $\Omega = \rho/\rho_c$, where ρ is the specific density of massive particles, $\rho_c = \frac{3}{8}\pi^{-1}\mathrm{G}^{-1}H^2$ being the critical energy density[3]; – the radiation parameter $\alpha = \frac{8}{45}\pi^3\mathrm{G}(\mathrm{k}T)^4\hbar^{-3}H^{-2}$, where T is the CMB temperature, G the Newton constant, k the Boltzmann constant and $\hbar$ the reduced Planck constant. Among these parameters, solely $\alpha_\circ \approx 5\,10^{-5}$ is accurately estimated from CMB data.

FLG Model

It describes the Cosmological expansion
from the decoupling era till now, and in the future (…)

The dimensionless reformulation of Friedmann equation, which reads

$$1 = \lambda - \kappa + \Omega + \alpha$$

can be interpreted as a normalization condition on the cosmological parameters.

$$\lambda = \lambda_\circ \phi_\circ^{-1}(a), \quad \kappa = \kappa_\circ a^{-2} \phi_\circ^{-1}(a), \quad \Omega = \Omega_\circ a^{-3} \phi_\circ^{-1}(a), \quad \alpha = \alpha_\circ a^{-4} \phi_\circ^{-1}(a)$$

If the cosmological parameters lies in the values domain

$$\mathcal{D} \equiv \begin{cases} \Omega_\circ > 0, \quad \lambda_\circ \geq 0, \quad \alpha_\circ \approx 5\,10^{-5} \\ \text{if } \kappa_\circ > 0 \text{ then } \kappa_\circ^2 < \frac{3}{2}\sqrt{3\lambda_\circ}\left(\Omega_\circ\sqrt{\kappa_\circ} + \alpha_\circ\sqrt{3\lambda_\circ}\right) \end{cases}$$

then the dynamics of the expansion relates to a Big-Bang scenario without bouncing/crunch

The Cosmological Constant

the gravitational field, which are characterized by a *vanishing divergence* stress-energy tensor T in the field equations

$$T = \mathscr{A}(g) \tag{1}$$

account for a metric tensor g with signature $(+,-,-,-)$ on the space-time manifold. $\mathscr{A}$ reads as a series of covariant tensors written in term of g and its derivatives

$$\mathscr{A}(g)_{\mu\nu} = -\mathscr{A}_0 F^{(0)}_{\mu\nu} + \mathscr{A}_1 F^{(1)}_{\mu\nu} + \mathscr{A}_2 F^{(2)}_{\mu\nu} + \dots \tag{2}$$

where the $F^{(n)}$ denote $2n$ degree tensors. The $\mathscr{A}_n$ stand for *coupling constants*, their values have to be determined from observations. For $n \leq 1$ these tensors are unique

$$F^{(0)}_{\mu\nu} = g_{\mu\nu}, \quad F^{(1)}_{\mu\nu} = S_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{3}$$

where $R_{\mu\nu}$ is the Ricci tensor and R the scalar curvature. A dimensional analysis of

The Cosmological Constant

The *transition scale* between the first two terms is defined by

$$\tau_{1/0} = \sqrt{|\mathscr{A}_1/\mathscr{A}_0|} \tag{4}$$

It is a useful quantity for disentangling the relative influence of these terms on the gravitational dynamics. The comparison of Schwarzschild solution of Eq. (2) with (modified) Poisson equation provides us with the following identifications

$$\mathscr{A}_1 = \frac{1}{8\pi G} \sim 5\,10^{26}\ \mathrm{g\,cm^{-1}}, \qquad \mathscr{A}_0 = \frac{\Lambda}{8\pi G} \tag{5}$$

and with the Newton acceleration field around a point mass m

$$\vec{g} = \left(-G\frac{m}{r^3} + \frac{\Lambda}{3}\right)\vec{r} \tag{6}$$

This approximation shows that if $\Lambda > 0$ then there is a critical distance

$$r_\circ = \sqrt[3]{3mG/\Lambda} \tag{7}$$

where the gravity vanishes; it is attractive if $r < r_\circ$ and repulsive if $r > r_\circ$.

The determination of consecutive terms $n > 1$ of the expansion in Eq. (2) requires to specify the $F^{(n)}_{\mu\nu}$ by means of additional principles.

The Cosmological Constant

Dimensional analysis

$$c = 1, \qquad 1\,\mathrm{s} = 2.999\,792\,458\,10^{10}\,\mathrm{cm}, \qquad \mathrm{G} = 7.4243 \times 10^{-29}\,\mathrm{cm\,g^{-1}}$$

Only two fundamental units can be chosen, herein denoted respectively by M and L

The correct dimensional analysis of GR sets the covariant metric tensor to have the dimension $[g_{\mu\nu}] = L^2$, and thus $[g^{\mu\nu}] = L^{-2}$, $[R_{\mu\nu}] = 1$ and $[R] = L^{-2}$. Since the specific mass density and the pressure belong to T^{μ}_{ν}, one has $[T_{\mu\nu}] = ML^{-1}$. Hence, the dimensions of A_n are the following

$$[A_0] = ML^{-3}, \quad [A_1] = ML^{-1}, \quad \ldots [A_n] = ML^{2n-3}$$

which shows their relative contributions for describing the gravitational field with respect to scale. Namely, the larger their degree n the smaller their effective scale[1]. Equivalently, the estimation of A_0 demands observational data located at scale larger than the one for A_1, *etc.*... This is the reason why the Λ effect is not discernible at small scale but requires cosmological distances.

The Cosmological Constant

$$T^{\rm vac}_{\mu\nu} = \rho_{\rm vac}\, g_{\mu\nu}\,, \qquad \rho_{\rm vac} = \hbar k_{\rm max}$$

$$\rho^{EW}_{\rm vac} \sim 2\, 10^{-4}\ {\rm g\, cm^{-3}}, \quad \rho^{QCD}_{\rm vac} \sim 1.6\, 10^{15}\ {\rm g\, cm^{-3}}, \quad \rho^{Pl}_{\rm vac} \sim 2\, 10^{89}\ {\rm g\, cm^{-3}}$$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \sim h^2\, 10^{-29}\ {\rm g\, cm^{-3}}$$

A similar problem happens when

$$\Lambda_{\rm vac} = 8\pi G \rho_{\rm vac} \tag{11}$$

is interpreted as a cosmological constant. Indeed, if the quantum field theory which provides us with an estimate of $\rho_{\rm vac}$ is correct then the distance from which the gravity becomes repulsive in the sun neighborhood ranges from $r_\odot^{EW} \sim 2\, 10^{-2} h^{-2/3}$ a.u. down to $r_\odot^{Pl} \sim 3\, 10^{-11} h^{-2/3}$ Å depending on the quantum field theory, see Eq. (7). Obviously, such results are not consistent with the observations.

The Cosmological Constant

$$T_{\mu\nu} = -g_{\mu\nu} + S_{\mu\nu} + A_2 F^{(2)}_{\mu\nu} + \ldots$$

$$l_g = 1/\sqrt{\Lambda} \sim h^{-1}\, 10^{28}\ \mathrm{cm}, \qquad m_g = 1/(8\pi G\sqrt{\Lambda}) \sim 4h^{-1}\, 10^{54}\ \mathrm{g}$$

$h \sim 10^{-120}$ quantum action unit compared to $\hbar = 1$

The Cosmological Constant

$$T_{\mu\nu} + T_{\mu\nu}^{\mathrm{vac}} \approx -A_0 F_{\mu\nu}^{(0)} + A_1 F_{\mu\nu}^{(1)}$$

$$\frac{\Lambda_{\mathrm{eff}}}{8\pi G} = A_0 + \rho_{\mathrm{vac}} \sim h^2\, 10^{-29}\ \mathrm{g\,cm}^{-3}$$

The Cosmological Constant

Modeling the observations

$$A_0 + \rho_{\rm vac} = A_1 = 1$$

$$\hat{l}_g = 1/\sqrt{\Lambda_{\rm eff}} \sim 7h^{-1}\, 10^{27}\ {\rm cm}, \qquad \hat{m}_g = 1/(8\pi G\sqrt{\Lambda_{\rm eff}}) \sim 4h^{-1}\, 10^{54}\ {\rm g}$$

$$T_{\mu\nu} + \rho_{\rm vac}\, g_{\mu\nu} \approx -A_0 g_{\mu\nu} + S_{\mu\nu}$$

$$\rho_{\rm m} \sim 3\, 10^{-1} \qquad \rho_{\rm r} \sim 4\, 10^{-5}$$

whereas the magnitude of quantum vacuum density ranges within

$$2\, h^{-2} 10^{26} \leq \rho_{\rm vac} \leq 2\, h^{-2} 10^{118}$$

The Cosmological Constant

Rescaling to theory

$\rho_{\rm vac} = A_1 = 1$

$$\hat{l}_v = 1/\sqrt{8\pi G \rho_{\rm vac}}, \quad \hat{m}_v = 1/(8\pi G \sqrt{8\pi G \rho_{\rm vac}})$$

$$T_{\mu\nu} + g_{\mu\nu} = -A_0 g_{\mu\nu} + S_{\mu\nu} + \left(A_2 F^{(2)}_{\mu\nu} + \ldots\right)$$

assumed QFT, see eq. (16) : from electroweak scale

$$A_0 \sim -1 + 1\,h^2\,10^{-25}, \quad \hat{l}_v \sim 2\,10^{15}\text{cm}, \quad \hat{m}_v \sim 8\,10^{41}\text{g}$$

up to Planck scale

$$A_0 \sim -1 + 5\,h^2\,10^{-117}, \quad \hat{l}_v \sim 5\,10^{-32}\text{cm}, \quad \hat{m}_v \sim 2.5\,10^{-7}\text{g}$$

Dynamical phases of FLG Model

$$t_2 = H_\circ^{-1} \int_{a_1}^{a_2} \frac{\mathrm{d}a}{a\sqrt{\phi_\circ(a)}} + t_1, \qquad a_i = a(t_i), \quad i = 1, 2$$

$$\mathrm{d}\tau = \frac{\mathrm{d}t}{a} = H_\circ^{-1} \frac{\mathrm{d}a}{a^2\sqrt{\phi_\circ(a)}} = H_\circ^{-1} \frac{\mathrm{d}a}{\sqrt{P_\circ(a)}}, \qquad P_\circ(a) = a^4 \phi_\circ(a)$$

$$1 = \lambda - \kappa + \Omega + \alpha$$

$$a_{\mathrm{rm}} = \frac{\alpha_\circ}{\Omega_\circ}, \quad a_{\mathrm{mc}} = \frac{\Omega_\circ}{|\kappa_\circ|}, \quad a_{\mathrm{cv}} = \sqrt{\frac{|\kappa_\circ|}{\lambda_\circ}}, \quad a_{\mathrm{mv}} = \sqrt[3]{\frac{\Omega_\circ}{\lambda_\circ}}, \ldots$$

$$\eta_{\mathrm{rm}} = \frac{a}{a_{\mathrm{rm}}}, \quad \eta_{\mathrm{mc}} = \frac{a}{a_{\mathrm{mc}}}, \quad \eta_{\mathrm{cv}} = \frac{a}{a_{\mathrm{cv}}}, \quad \eta_{\mathrm{mv}} = \frac{a}{a_{\mathrm{mv}}}, \ldots$$

Dynamical phases of FLG Model

- When $a \approx 0$, one has

$$\lambda \sim \eta_{\rm rv}^4, \quad \kappa \sim \iota_\circ \eta_{\rm rc}^2, \quad \Omega \sim \eta_{\rm rm}, \quad \alpha \sim 1 - \eta_{\rm rm}$$

$$H(a) \sim H_\circ \sqrt{\alpha_\circ} a^{-2} \left(1 + \frac{1}{2}\eta_{\rm rm}\right)$$

where

$$\eta_{\rm rv} = \frac{a}{a_{\rm rv}}, \quad \eta_{\rm rc} = \frac{a}{a_{\rm rc}}, \quad a_{\rm rv} = \sqrt[4]{\frac{\alpha_\circ}{\lambda_\circ}}, \quad a_{\rm rc} = \sqrt{\frac{\alpha_\circ}{|\kappa_\circ|}}$$

The expansion is radiation dominated

Dynamical phases of FLG Model

- When $a \approx 1$ (present epoch), one has

$$\lambda \sim \lambda_\circ \left(1 + 2\left(1 + q_\circ\right)\epsilon\right), \quad \kappa \sim \kappa_\circ \left(1 + 2q_\circ\epsilon\right)$$
$$\Omega \sim \Omega_\circ \left(1 + \left(q_\circ - 1\right)\epsilon\right), \quad \alpha \sim \alpha_\circ \left(1 + 2\left(q_\circ - 1\right)\epsilon\right)$$
$$H(a) \sim H_\circ \left(1 - \left(1 + q_\circ\right)\epsilon\right)$$

where $\epsilon = a - 1$ and

$$q = -1 - \kappa + \frac{3}{2}\Omega + 2\alpha$$

is the deceleration parameter.

Dynamical phases of FLG Model

- When $a \gg 1$:
  - if $\Lambda \neq 0$ then one has a vacuum dominated expansion

$$\lambda \sim \begin{cases} 1 + \iota_\circ \eta_{\text{cv}}^{-2} & \text{if } \kappa_\circ \neq 0 \\ 1 - \eta_{\text{mv}}^{-3} & \text{otherwise} \end{cases}$$

$$k \sim \iota_\circ \eta_{\text{cv}}^{-2}, \quad \Omega \sim \eta_{\text{mv}}^{-3}, \quad \alpha \sim \eta_{\text{rv}}^{-4}$$

$$H(t) \sim H_\circ \sqrt{\lambda_\circ} \times \begin{cases} 1 - \frac{1}{2} \iota_\circ \eta_{\text{cv}}^{-2} & \text{if } \kappa_\circ \neq 0 \\ 1 + \frac{1}{2} \eta_{\text{mv}}^{-3} & \text{otherwise} \end{cases}$$

Dynamical phases of FLG Model

– if $\Lambda = 0$ then $\kappa_\circ \leq 0$ for ensuring an eternal expansion.

∗ If $\kappa_\circ < 0$ then one has a curvature dominated expansion

$$k \sim -1 + \eta_{\rm mc}^{-1}, \quad \Omega \sim \eta_{\rm mc}^{-1}, \quad \alpha \sim \eta_{\rm rc}^{-2}$$

$$H(t) \sim H_\circ \sqrt{-\kappa_\circ} a^{-1} \left(1 - \frac{1}{2} \iota_\circ \eta_{\rm mc}^{-1}\right)$$

∗ if $\kappa_\circ = 0$ (Euclidian space) then one has a matter dominated expansion

$$\Omega \sim 1 - \eta_{\rm rm}^{-1}, \qquad \alpha \sim \eta_{\rm rm}^{-1}$$

$$H(t) \sim H_\circ \sqrt{\Omega_\circ} a^{-3/2} \left(1 + \frac{1}{2} \eta_{\rm rm}^{-1}\right)$$

The Coincidence Problem

$$\lambda_\circ \approx 0.7, \qquad \kappa_\circ \approx 10^{-3}, \qquad \Omega_\circ \approx 0.3, \qquad \alpha_\circ \approx 5\,10^{-5}$$

$$a_{\rm rm} = \frac{\pi^2 (kT_\circ)^4 \hbar^{-3}}{\rho_\circ} \approx 1.7\,10^{-4}, \quad a_{\rm mv} = \sqrt[3]{\frac{\rho_\circ}{\rho_{\rm vac}}} \approx \frac{3}{4}, \qquad \rho_{\rm vac} = \frac{\Lambda}{8\pi G}$$

There is no curvature dominated era because $a_{\rm mv} < a_{\rm mc}$

The reason "why the energy density nearly coincides with the matter density today" [1] has been addressed as being one of the most fundamental problem in cosmology, which is known as *The Coincidence Problem*.

I. Zlatev, L. Wang, P.J. Steinhardt, *Quintessence, Cosmic Coincidence, and the Cosmological Constant*, *Phys. Rev. Lett.* **82**, 896 (1999);
I. Zlatev, P.J. Steinhardt, *A tracker solution to the cold dark matter cosmic coincidence problem*, *Phys. Lett.* **B459**, 570 (1999)

The Coincidence Problem

1. If $a_{\rm mv}$ is an occurrence of a random variable $\tilde{a}_{\rm mv}$ which is uniformly distributed in the interval $]0,1]$ then one has the probability

$$\Pr(a_{\rm mv} < \tilde{a}_{\rm mv} \leq 1) = 1 - a_{\rm mv} = 25\% \tag{3.4}$$

2. Let $t_{\rm mv}$ be the matter-vacuum transition epoch, which is defined by $a_{\rm mv} = a(t_{\rm mv})$, see Eq. (2.9). If it is an occurrence of a random variable $\tilde{t}_{\rm mv}$ which is uniformly distributed in the interval $]0, t_\circ]$, where $t_\circ$ is the Universe's age (present date), then

$$\Pr(t_{\rm mv} < \tilde{t}_{\rm mv} \leq t_\circ) = \frac{t_\circ - t_{\rm mv}}{t_\circ} = 49\% \tag{3.5}$$

3. Similarly as above but with conformal dates. If $\tau_{\rm mv}$, defined by $a_{\rm mv} = a(\tau_{\rm mv})$, see Eq. (2.10), is an occurrence of a random variable $\tilde{\tau}_{\rm mv}$ which is uniformly distributed in the interval $]0, \tau_\circ]$ where $\tau_\circ$ is the Universe's conformal age then

$$\Pr(\tau_{\rm mv} < \tilde{\tau}_{\rm mv} \leq \tau_\circ) = \frac{\tau_\circ - \tau_{\rm mv}}{\tau_\circ} = 9\% \tag{3.6}$$

The Coincidence Problem

4. With the aim to investigate this proximity with respect the entire evolution of the universe, analogous schemas might be envisaged. However, with the present values of cosmological parameters, as given in Eq. (3.1), the universe will expand for ever, the expansion parameter a and the cosmic time t become unbounded quantities. Therefore, the analysis of such a proximity with respect to our future $t > t_\circ$ becomes an ill posed problem when using $\tilde{a}_{\mathrm{mv}}$ and $\tilde{t}_{\mathrm{mv}}$ in their new value domains with a uniform probability law. On the other hand, because $\lambda_\circ > 0$, the conformal time turns out to be bounded[8]

$$\tau_\infty = H_\circ^{-1} \int_0^{+\infty} \frac{\mathrm{d}a}{\sqrt{P_\circ(a)}} < +\infty \tag{3.7}$$

which enables us to use the following schema : If $\tau_\circ$ is an occurrence of a random variable $\tilde{\tau}_\circ$ which is uniformly distributed in the interval $[\tau_{\mathrm{mv}}, \tau_\infty[$, then

$$\Pr(\tau_{\mathrm{mv}} < \tilde{\tau}_\circ \leq \tau_\circ) = \frac{\tau_\circ - \tau_{\mathrm{mv}}}{\tau_\infty - \tau_{\mathrm{mv}}} = 74\% \tag{3.8}$$

The Coincidence Problem

5. By assuming all outcomes are equiprobable in the values domain of cosmological parameter with $\Omega_\circ > 0$ one has

$$\Pr_1(a_{\rm mv} < \tilde{a}_{\rm mv} \leq 1) = \frac{\arctan a_{\rm mv}^{-3} - \arctan a_\circ^{-3}}{\pi/2} = 10\% \tag{3.9}$$

Note that this probability increases if one assumes that the outcome belongs to values domain $\mathcal{D}$, as given in Eq. (2.8), and it is reduced to the half if the constraint $\lambda_\circ > 0$ is ignored.

Conclusion

According to FLG model, the data suggest that today is not an exceptional date in the cosmological evolution of the universe, with respect to any transition epoch chosen.

Thank you