On Negative Mass in de Sitter Spacetime

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- Negative mass has always been an intriguing possibility
- It gives rise to the possibility of anti-gravity, and idea that has captured our imagination from the beginning of time.
- With the understanding of a dynamical theory of gravity afforded by general relativity, we had the tools that are necessary to contemplate negative mass.
- Negative mass particles, if they exists are strange beasts.

- A negative mass particle creates a negative gravitational field, however, because of the equivalence principle, it is attracted to positive mass particles.
- The force is exerted on a negative mass particle $-m$ by a positive mass particle M is:

$$F = \frac{GM(-m)}{r^2}$$

- The acceleration felt by the negative mass particle is:

$$\dot{p} = (-m)a = F = \frac{GM(-m)}{r^2}$$

$$a = \frac{GM}{r^2}$$

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- Obviously the negative mass cancels from both sides of the equation giving:

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- On the other hand, the acceleration felt by the positive mass particle is obtained through:

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- which is clearly negative or repulsive.

- This yields the amusing scenario that a positive and negative mass pair of particles will spontaneously accelerate along the line that joins the two, the positive mass particle being pushed away from the negative mass particle while the negative mass particle being attracted to the positive mass particle.
- This has no inconsistency with momentum conservation, the momentum of the negative mass particle is of course negative, thus the moving pair has net momentum zero.
- If the particles have differing magnitude of masses, the negative mass larger than the positive mass, then they will asymptotically separate and move at constant velocity getting ever further and further apart, while if the negative mass is smaller, it will eventually collide with the positive mass particle.

Dominant Energy Condition

- The dominant energy condition is a very reasonable, local constraint to impose on any physical energy-momentum distribution.
- It enforces that no observer will see energy-momentum is moving faster than the speed of light.
- Consider a spherically symmetric distribution of matter and the ensuing metric in Schwarzschild coordinates

$$d\tau^2 = \left(1 - 2\frac{M(r)}{r}\right) dt^2 - \frac{1}{1 - 2\frac{M(r)}{r}} dr^2 - r^2 d\theta^2 - r^2 \sin^2(\theta) d\phi^2$$

- Here, $M(r)$ is the mass inside a sphere of coordinate radius r.
- Inserting this metric into the Einstein tensor gives the energy-momentum tensor that would create such a metric. It is easily found that:

$$T_0^0 = T_1^1 = \frac{2M'(r)}{r^2} \qquad T_2^2 = T_3^3 = \frac{M''(r)}{r}$$

- The dominant energy condition imposes:

$$T^{0\nu}u_\nu \geq 0 \qquad T^{\mu\nu}u_\nu T_{\mu\alpha}u^\alpha \geq 0$$

- For any time-like or light-like vector u

- In a given Lorentz frame, a general time-like vector is of the form:

$$u^\mu = \frac{1}{(1-a^2-b^2-c^2)^{1/2}}(1, a, b, c)$$

- with $\quad 1 - a^2 - b^2 - c^2 > 0$
- Then the dominant energy condition implies:

$$T^{0\nu}u_\nu = \frac{2M'(r)}{r^2\sqrt{1 - a^2 - b^2 - c^2}} \geq 0$$

- which is equivalent to simply:

$$M'(r) \geq 0$$

- And the second constraint yields:

$$T^{\mu\nu}u_\nu T_{\mu\alpha}u^\alpha =$$
$$\left(\frac{4M'(r)^2}{r^4} - \frac{(b^2+c^2)\left(r^2M''(r)^2 - 4M'(r)^2\right)}{(1-a^2-b^2-c^2)\,r^4}\right) \geq 0$$

- which requires

$$(rM''(r) - 2M'(r))\,(rM''(r) + 2M'(r)) \leq 0$$

- since the denominator in the second term can be arbitrarily small.
- This inequality is equivalent to the two conditions:

$$\frac{d}{dr}\left(\frac{M'(r)}{r^2}\right) \leq 0 \qquad \frac{d}{dr}\left(M'(r)r^2\right) \geq 0$$

Positive Energy Theorem

- The positive energy theorem states that any space-time that is asymptotically flat and contains energy-momentum that everywhere satisfies the dominant energy condition, will necessarily have a positive ADM mass.
- In our specialized spherically symmetric geometry, this requires that:

$$M(r) \to M \text{ and } M \geq 0$$

- Thus we cannot have asymptotic flatness and

$$M(r) \to -M \text{ and } M \geq 0$$

- The Schwarzschild metric is an exact solution of the vacuum Einstein equations which contains one parameter, the mass which can be positive or negative:

$$d\tau^2 = (1 - 2G(-M)/r)\,dt^2 - \frac{dr^2}{(1 - 2G(-M)/r)} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

- The metric contains no energy-momentum, except at the location of its singularity at the origin. For negative mass it has no horizon. The singularity is at a spatial point, and seems to be benign, it is repulsive and can be avoided. Furthermore, it can be smoothed out by replacing:

$$-M \to -M(r) \qquad M(r) \to 0 \text{ as } r \to 0$$

- But it is exactly such a deformation that cannot satisfy the dominant energy condition, since the positive energy theorem tells us that if the dominant energy condition were satisfied, the mass parameter would necessarily have to be positive.
- But we can imagine eschewing the positive energy condition by dropping the constraint of asymptotic flatness. The inflationary phase of the universe or even in principle the present, accelerating universe are both asymptotically de Sitter universes which are not asymptotically flat.

- There is another exact solution of the Einstein equations with cosmological constant given by the metric

$$d\tau^2 = \left(1 - \frac{(\Lambda/3)r^3 - 2GM}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{(\Lambda/3)r^3 - 2GM}{r}\right)} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

$$G_{\mu\nu}[g_{\lambda\rho}] = \Lambda g_{\mu\nu} \qquad T_{\mu\nu} = (\Lambda/8\pi G) g_{\mu\nu}$$

- containing two free parameters Λ and M which can be positive or negative.
- We ask the question: Is there a deformation of the positive Λ but negative M metric that satisfies the dominant energy condition everywhere?

- We find that indeed there is such a deformation. If we take:

$$M(r) = m + \frac{\lambda r^3}{6} - \frac{q}{2r} \qquad r < R_1$$

$$M(r) = M + \frac{\Lambda r^3}{6} \qquad r > R_2 > R_1$$

- And interpolating between we take:

$$M(r) = -\frac{2q \int_{R_1}^{r} z^3 \int_{R_1}^{z} J_{np}(\xi)\, d\xi\, dz}{R_1^5 J_{np}\left(R_1\right)} + \left(m - \frac{q}{2R_1} + \frac{\lambda R_1^3}{6}\right) + \frac{\left(r^3 - R_1^3\right)}{3R_1^2}\left(\frac{q}{2R_1^2} + \frac{\lambda R_1^2}{2}\right)$$

- With

$$J_{np}(r) = \frac{1}{r^5} \int_r^{R_2} (z - R_1)^n (R_2 - z)^p \, dz$$

- Then we find:

$$M = m - \frac{2q}{3} \frac{\int_{R_1}^{R_2} \frac{1}{z} (z - R_1)^n (R_2 - z)^p \, dz}{\int_{R_1}^{R_2} (z - R_1)^n (R_2 - z)^p \, dz}$$

$$\lambda = \Lambda - \frac{q}{R_1^4} \left(1 - \frac{4 \int_{R_1}^{R_2} J_{np}(\xi) \, d\xi}{R_1 J_{np}(R_1)} \right)$$

- A little algebra shows that it is possible to have

$$\Lambda > 0 \quad M < 0 \qquad m > 0 \qquad \lambda > 0$$

- The solution represents a negative mass bubble in a de Sitter background containing a positive mass, charged black hole with cosmological constant that everywhere satisfies the dominant energy condition.

Conclusions

- We have shown that negative mass configurations which everywhere satisfy the dominant energy condition, can exist within non-asymptotically flat space-times.
- This could have important consequences for the early universe, where the inflationary phase corresponds to a de Sitter universe.
- Pair production of positive and negative mass pairs would give rise to a strange gravitational plasma.
- The negative mass particles would chase after the positive mass particles and in principle always exit any Hubble volume. However, for an infinite universe, there would always be such pairs entering the Hubble volume from outside, so the entire system would be stable.
- We speculate that such a plasma would screen gravitational waves rendering the initial singularity always hidden behind an opaque curtain.