Massive gravity and cosmology

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Based on collaboration with
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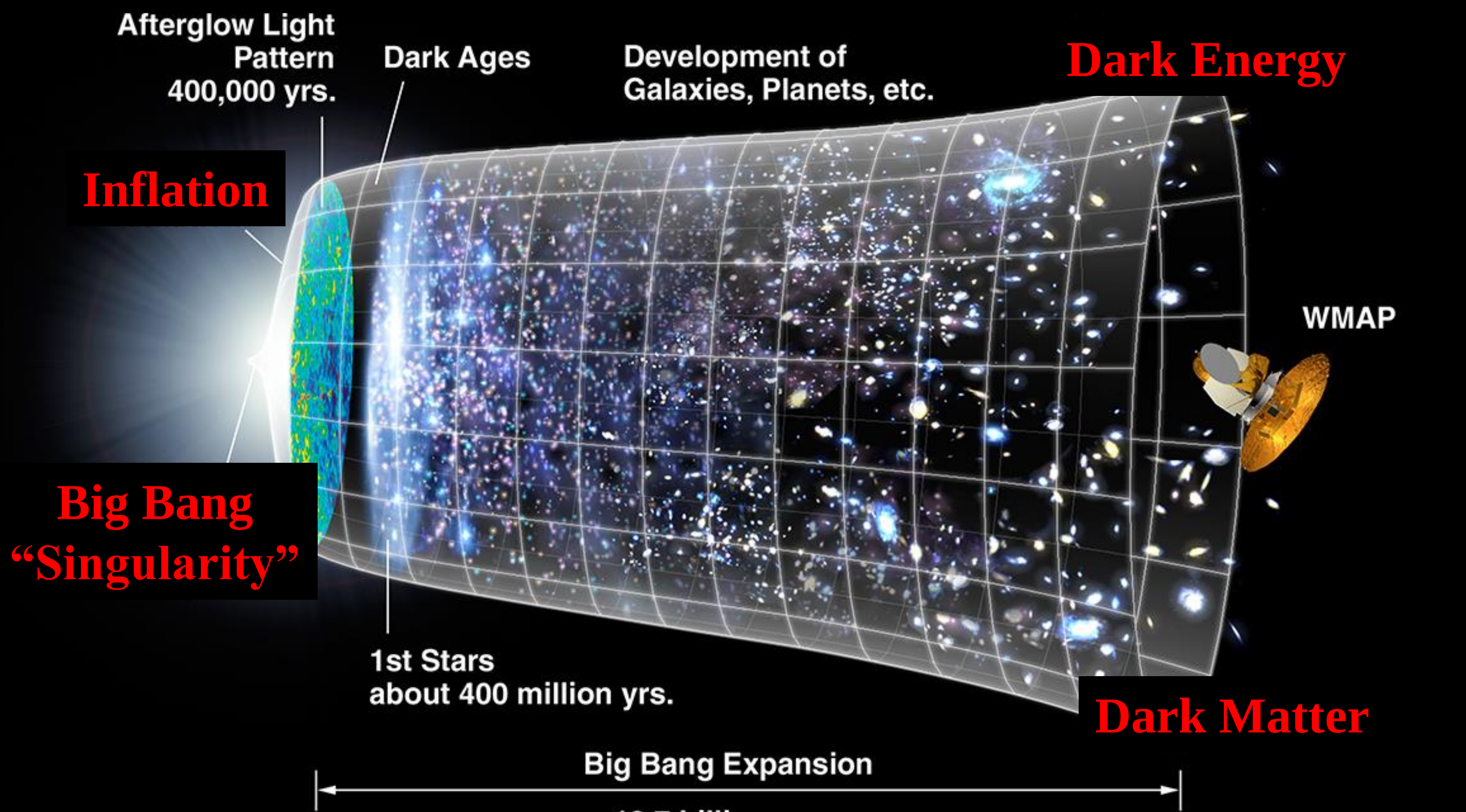
Why alternative gravity theories?



http://map.gsfc.nasa.gov/

Three conditions for good alternative theories of gravity (my personal viewpoint)

1. Theoretically consistent
   e.g. no ghost instability
2. Experimentally viable
   solar system / table top experiments
3. Predictable
   e.g. protected by symmetry

Some examples

I. Ghost condensation
   IR modification of gravity
   motivation: dark energy/matter
II. Nonlinear massive gravity
   IR modification of gravity
   motivation: “Can graviton have mass?”
III. Horava-Lifshitz gravity
   UV modification of gravity
   motivation: quantum gravity
IV. Superstring theory
   UV modification of gravity
   motivation: quantum gravity, unified theory

A motivation for IR modification

- Gravity at long distances

  Flattening galaxy rotation curves

  extra gravity

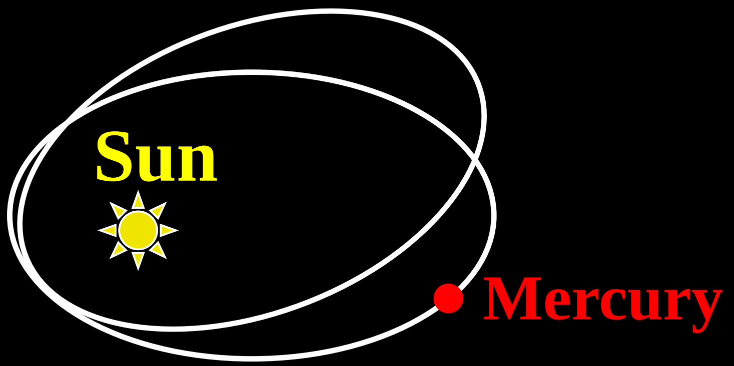
  Dimming supernovae

  accelerating universe

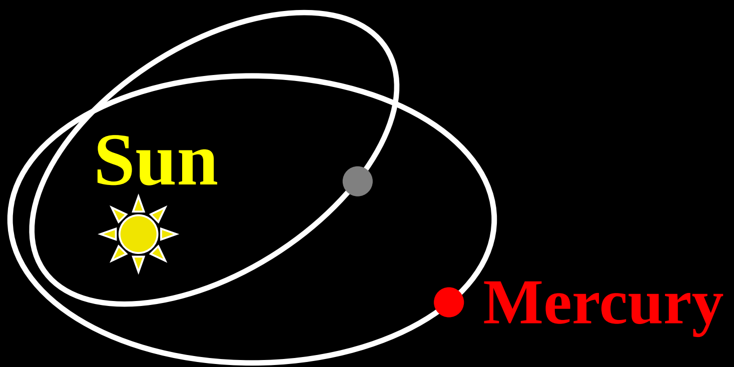
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion observed in 1800's…



which people tried to explain with a "dark planet", Vulcan,



But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Can we change gravity in IR?

- **Change Theory?**
  Massive gravity **Fierz-Pauli 1939**
  DGP model **Dvali-Gabadadze-Porrati 2000**

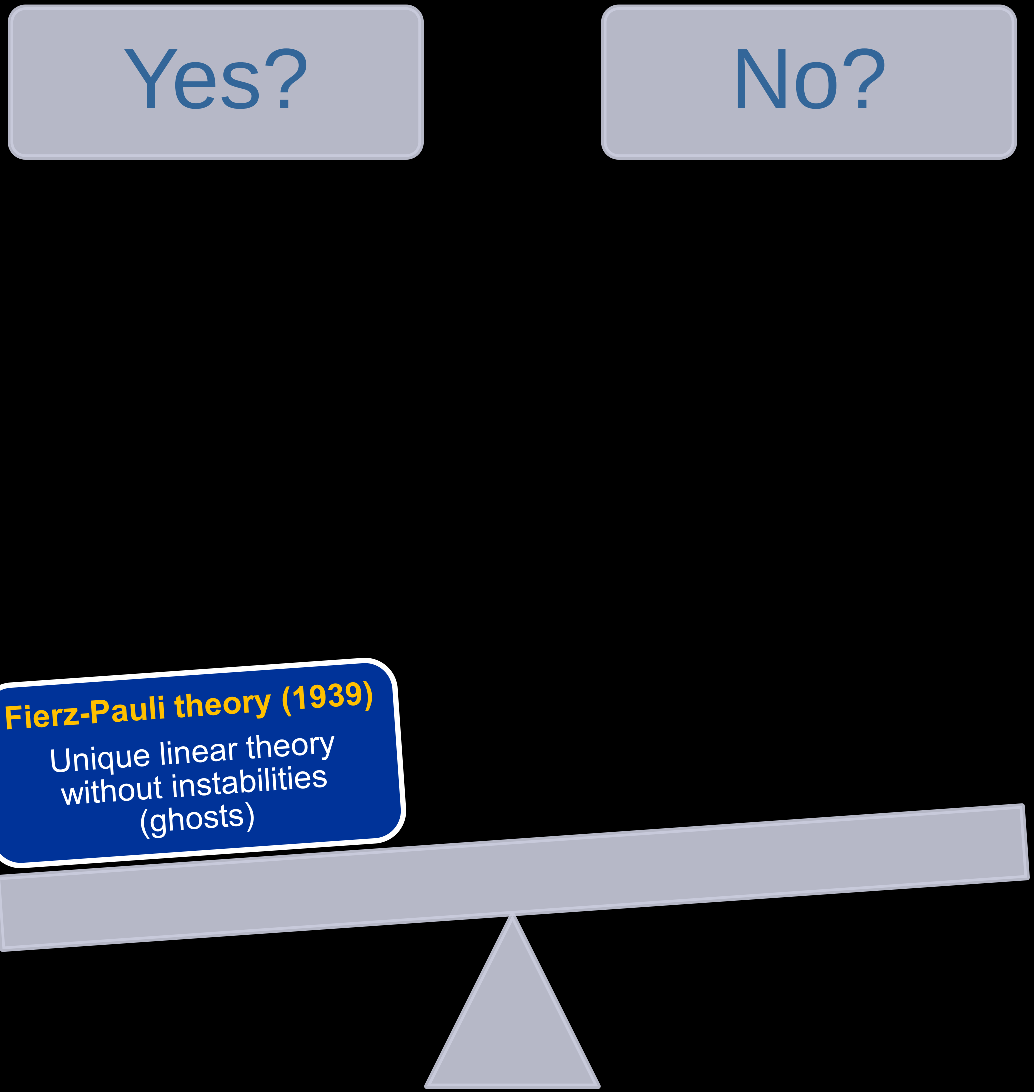
- **Change State?**
  Higgs phase of gravity
  The simplest: Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



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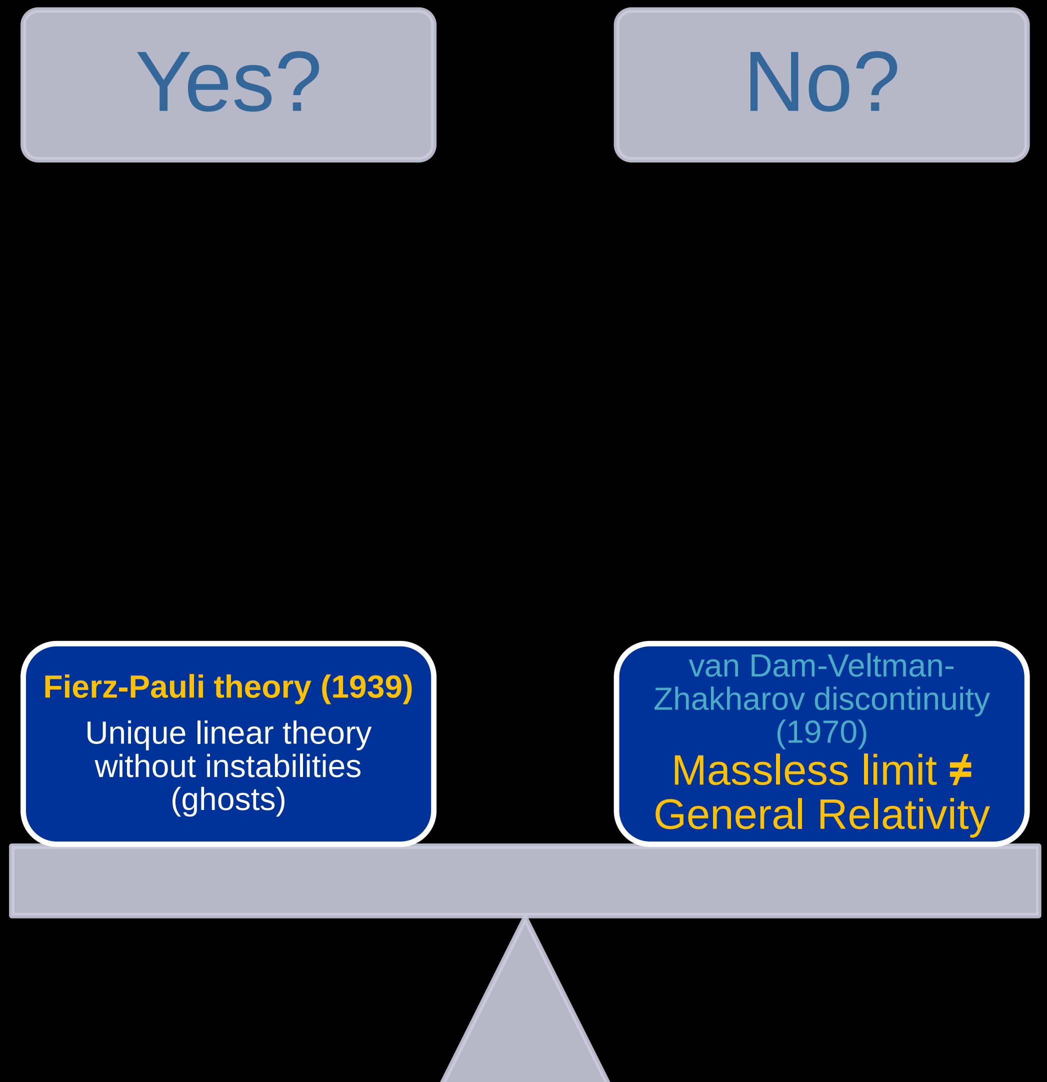

Massive gravity: history

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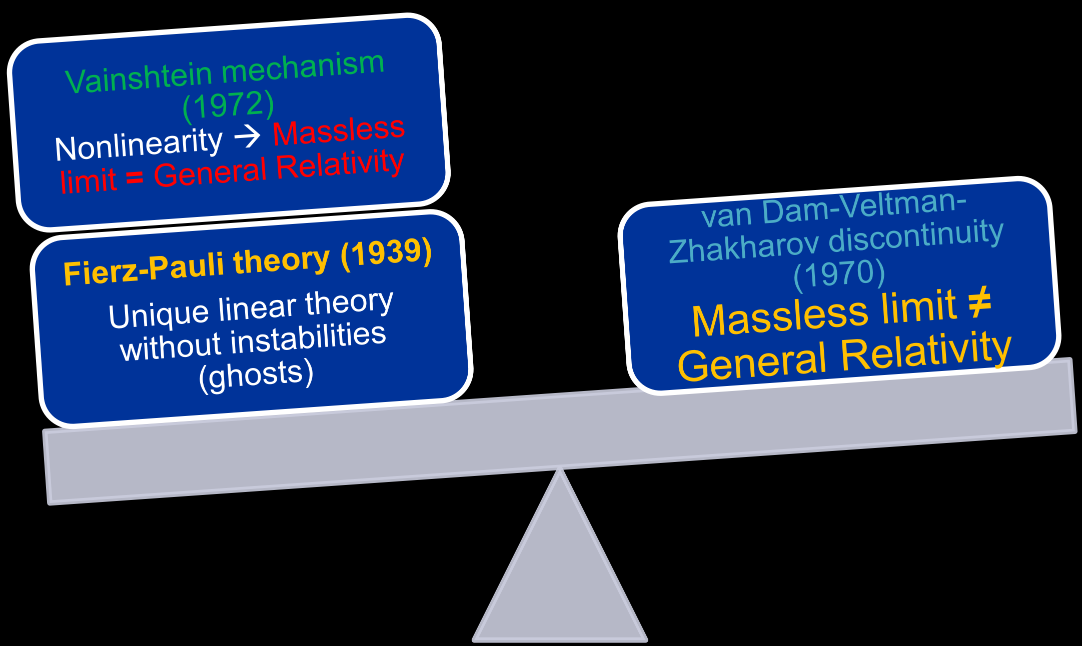
May lead to acceleration without dark energy

Yes? No?



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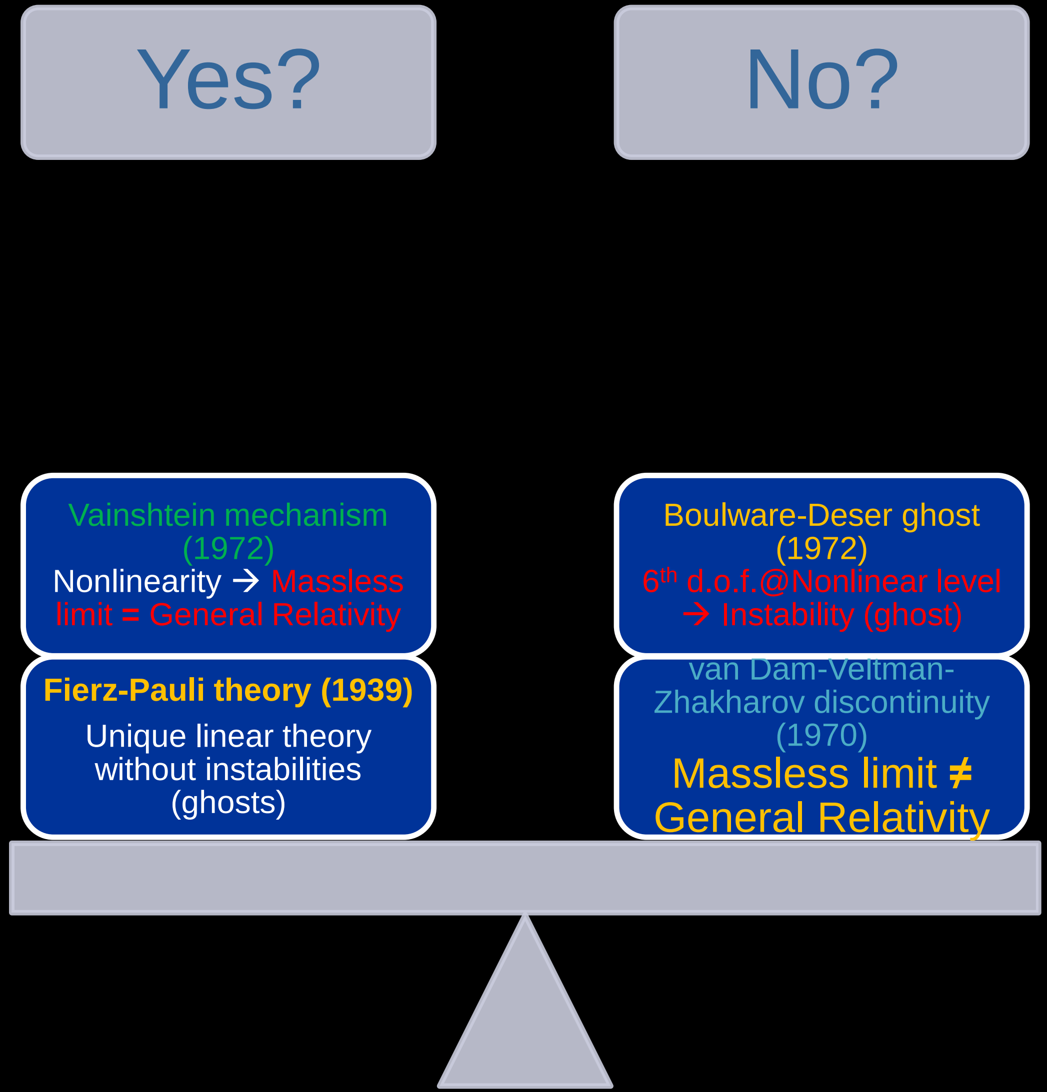

Nonlinear massive gravity

de Rham, Gabadadze 2010
de Rham, Gabadadze & Tolley 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical (but technically natural)
- Properties of 5 d.o.f. depend on background

- **4 scalar fields ϕ^a (a=0,1,2,3)**
- **Poincare symmetry in the field space:**

$$\phi^a \to \phi^a + c^a, \quad \phi^a \to \Lambda^a_b \phi^b$$

→ $$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

Pullback of Minkowski metric in field space to spacetime

Systematic resummation

de Rham, Gabadadze & Tolley 2010

$$I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_{Pl}^2 m_g^2 \int d^4x\sqrt{-g}\,(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4)$$

$$f_{\mu\nu} \equiv \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu$$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$[\mathcal{A}] \equiv Tr\mathcal{A}$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3\,[\mathcal{K}]\,[\mathcal{K}^2] + 2\,[\mathcal{K}^3]\right)$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6\,[\mathcal{K}]^2\,[\mathcal{K}^2] + 3\,[\mathcal{K}^2]^2 + 8\,[\mathcal{K}]\,[\mathcal{K}^3] - 6\,[\mathcal{K}^4]\right)$$

No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

$$\mathcal{K}_{\mu\nu} = \partial_\mu\partial_\nu\pi \;\Rightarrow\; \mathcal{L}_{2,3,4} = (\text{total derivative})$$

No BD ghost away from decoupling limit (Hassan&Rosen)

Massive gravity: history

Simple question: Can graviton have mass?

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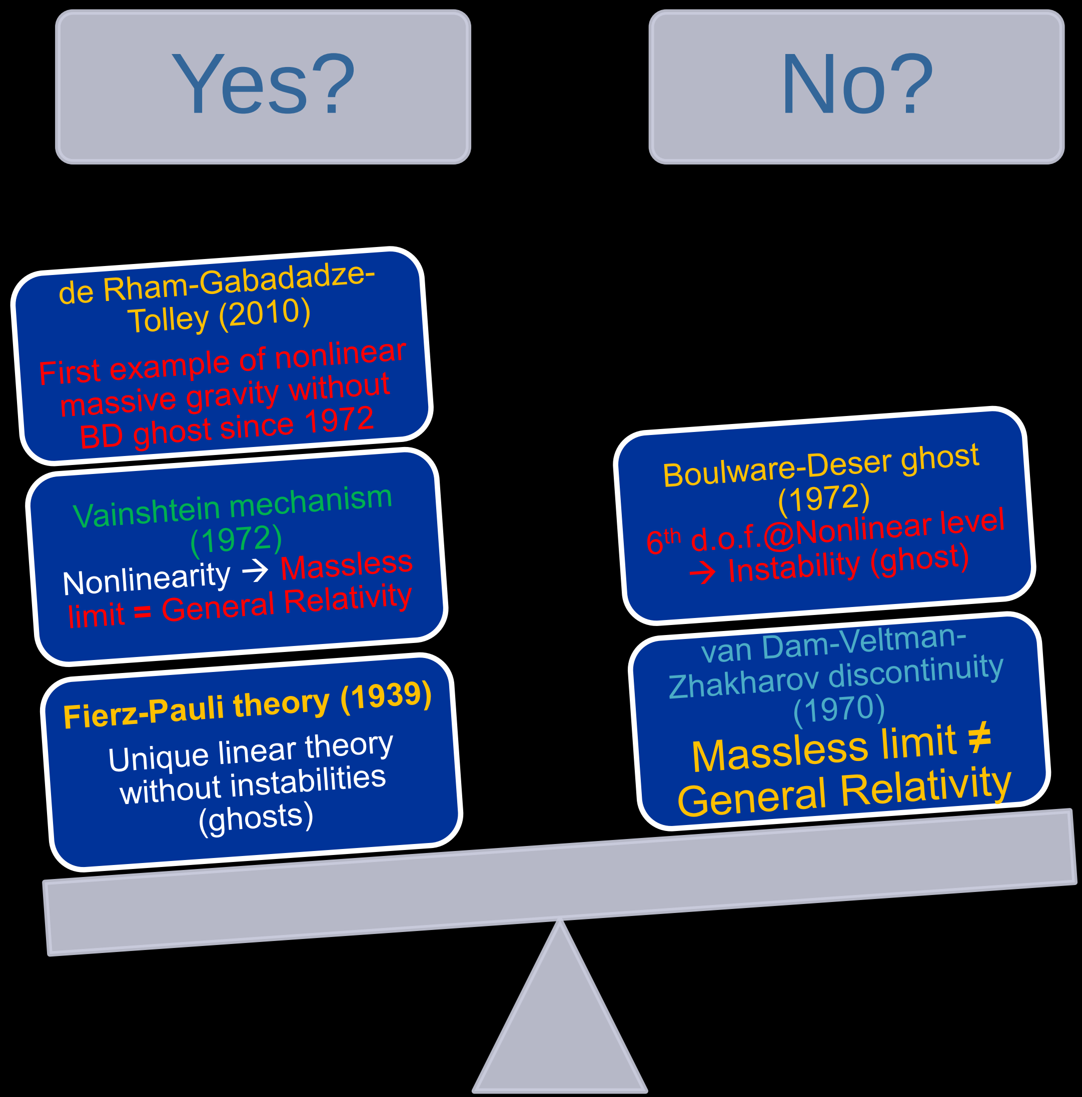


No FLRW universe?

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtshalava, Tolley (2011)

- Flat FLRW ansatz in "Unitary gauge"

  $g_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + a^2(t)(dx^2+dy^2+dz^2)$

  $\phi^a = x^a \quad \Rightarrow \quad f_{\mu\nu} = \eta_{\mu\nu}$

- Bianchi "identity" → $a(t)$ = const.

  c.f. $$\nabla^\mu \left(\frac{2}{\sqrt{-g}} \frac{\delta I}{\delta g^{\mu\nu}} \right) = \frac{1}{\sqrt{-g}} \frac{\delta I_g}{\delta \phi^a} \partial_\nu \phi^a$$

  → no non-trivial flat FLRW cosmology

- "Our conclusions on the absence of the homogeneous and isotropic solutions do not change if we allow for a more general maximally symmetric 3-space"

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?	No?
de Rham, Gabadadze, Tolley (2010) First example of nonlinear massive gravity without BD ghost since 1972	D'Amico et al. (2011) Non-existence of flat FRW (homogeneous isotropic) universe!
Vainshtein mechanism (1972) Nonlinearity → Massless limit = General Relativity	Boulware-Deser ghost (1972) 6th d.o.f.@Nonlinear level → Instability (ghost)
Fierz-Pauli theory (1939) Unique linear theory without instabilities (ghosts)	van Dam-Veltman-Zhakharov discontinuity (1970) Massless limit ≠ General Relativity

Consistent Theory found in 2010 but

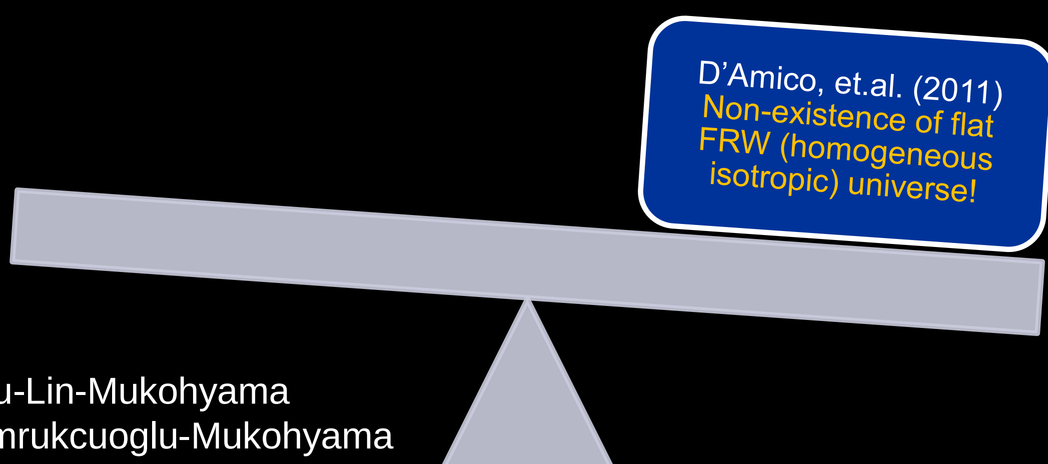
No Viable Cosmology?

Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

Bad?



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- $f_{\mu\upsilon}$ spontaneously breaks diffeo.
- Both $g_{\mu\upsilon}$ and $f_{\mu\upsilon}$ must respect FLRW symmetry
- Need FLRW coordinates of Minkowski $f_{\mu\upsilon}$
- No closed FLRW chart
- Open FLRW ansatz

$$\begin{aligned}
\phi^0 &= f(t)\sqrt{1+|K|(x^2+y^2+z^2)}, \\
\phi^1 &= \sqrt{|K|}f(t)x, \\
\phi^2 &= \sqrt{|K|}f(t)y, \\
\phi^3 &= \sqrt{|K|}f(t)z.
\end{aligned}$$

$$f_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2\, dt^2 + |K|\,(f(t))^2\,\Omega_{ij}(x^k)dx^i dx^j$$

$$\begin{aligned}
g_{\mu\nu}dx^\mu dx^\nu &= -N(t)^2 dt^2 + a(t)^2\Omega_{ij}dx^i dx^j\,, \\
\Omega_{ij}dx^i dx^j &= dx^2+dy^2+dz^2 - \frac{|K|(xdx+ydy+zdz)^2}{1+|K|(x^2+y^2+z^2)}\,,
\end{aligned}$$

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- EOM for ϕ^a (a=0,1,2,3)

$$(\dot{a} - \sqrt{|K|N}) \left[\left(3 - \frac{2\sqrt{|K|}f}{a}\right) + \alpha_3 \left(3 - \frac{\sqrt{|K|}f}{a}\right)\left(1 - \frac{\sqrt{|K|}f}{a}\right) + \alpha_4 \left(1 - \frac{\sqrt{|K|}f}{a}\right)^2 \right] = 0$$

- The first sol $\dot{a} = \sqrt{|K|}N$ implies $g_{\mu\upsilon}$ is Minkowski
  → we consider other solutions

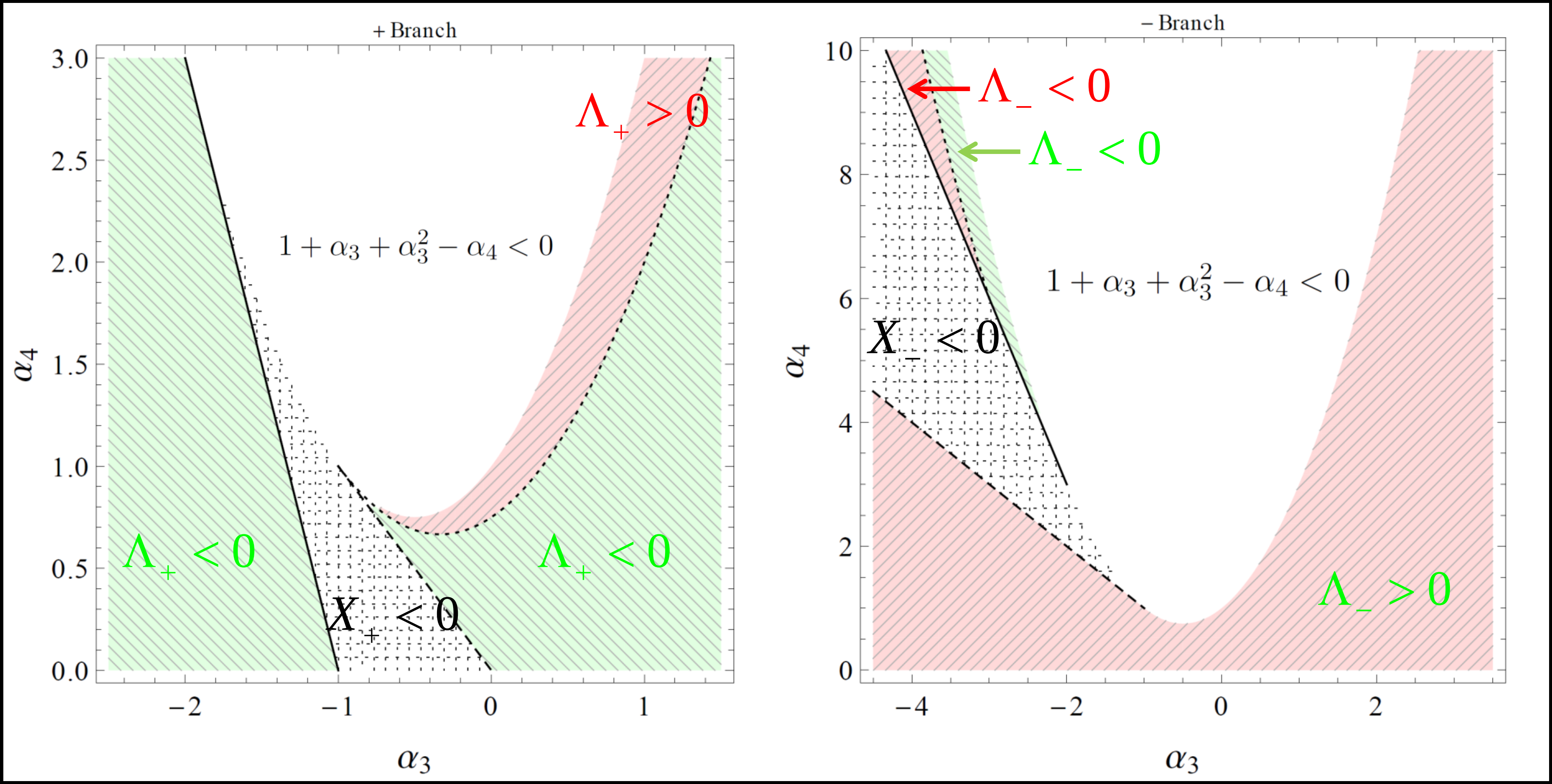
$$f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

- Latter solutions do not exist if K=0
- Metric EOM → self-acceleration

$$3\,H^2 + \frac{3\,K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3 + \alpha_4)^2} \left[(1 + \alpha_3)\left(2 + \alpha_3 + 2\,\alpha_3^2 - 3\,\alpha_4\right) \pm 2\,\left(1 + \alpha_3 + \alpha_3^2 - \alpha_4\right)^{3/2} \right]$$

Self-acceleration



$$f = \frac{a}{\sqrt{|K|}} X_{\pm}, \quad X_{\pm} \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

Our recent contributions

Cosmological solutions of nonlinear massive gravity

Good?

Bad?

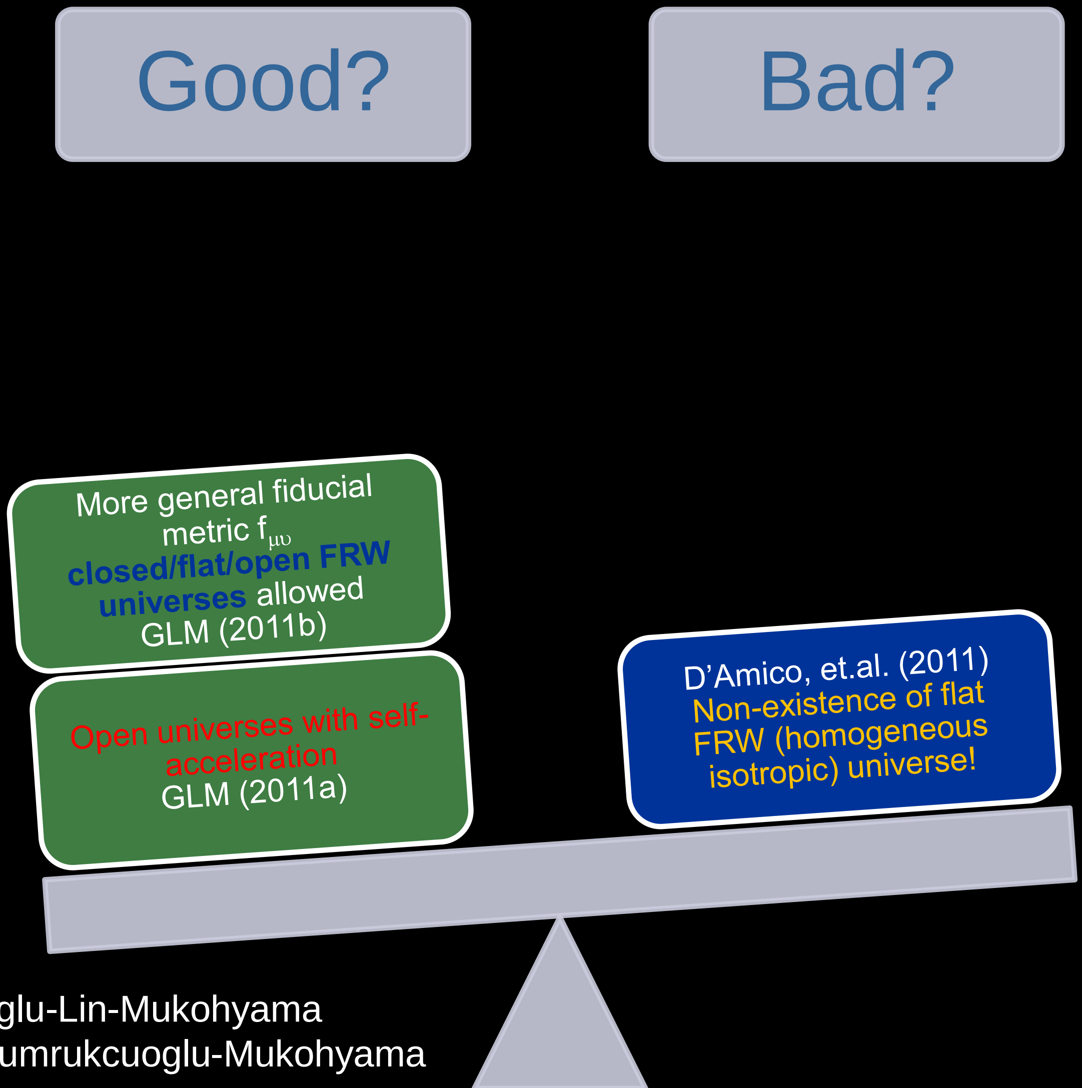
Open universes with self-acceleration
GLM (2011a)

D'Amico, et.al. (2011)
Non-existence of flat FRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Our recent contributions

Cosmological solutions of nonlinear massive gravity



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Summary so far

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass

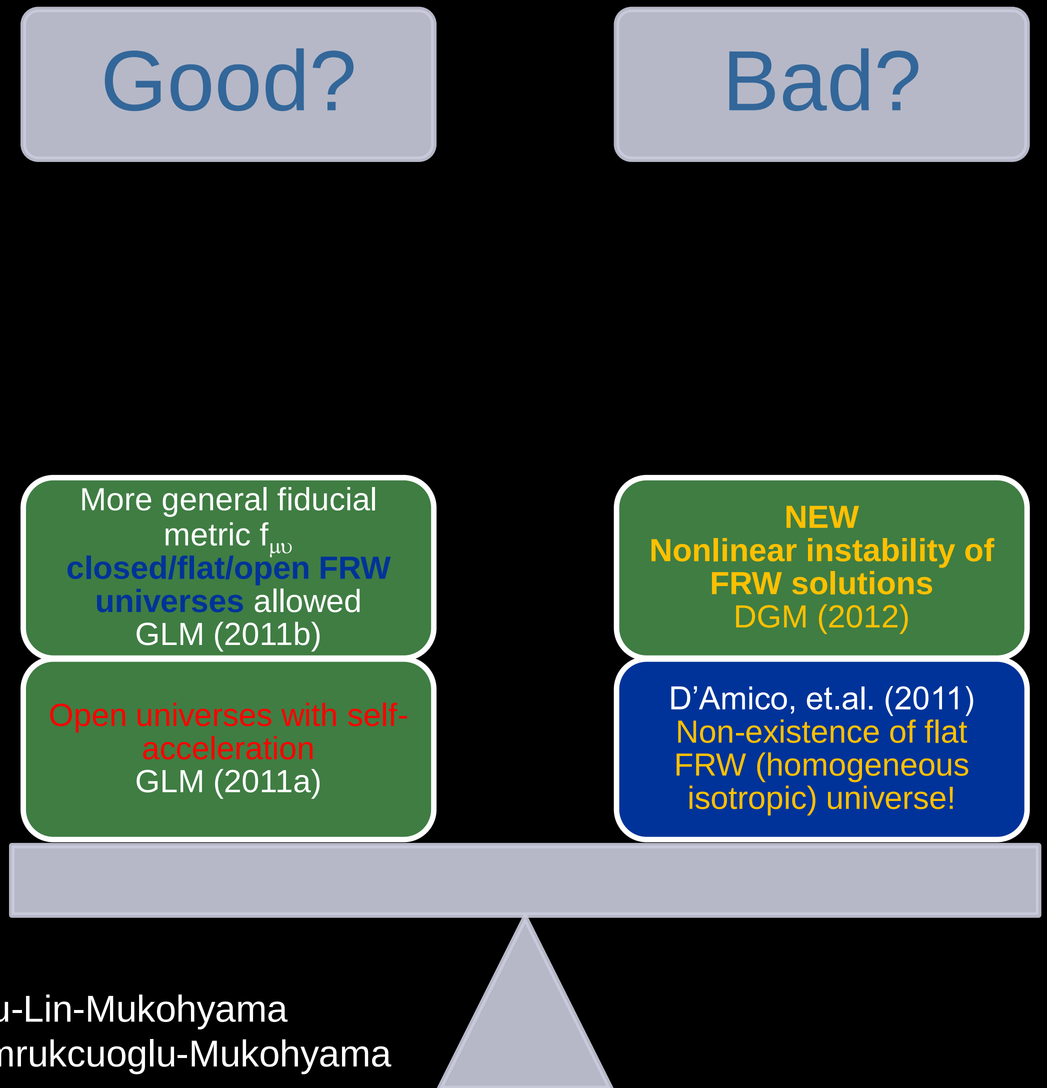
Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc → FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
  → nonlinear perturbation around flat FLRW
- Odd-sector:
  1 healthy mode + 1 healthy or ghosty mode
- Even-sector:
  2 healthy modes + 1 ghosty mode
- This is not BD ghost nor Higuchi ghost.

Our recent contributions

Cosmological solutions of nonlinear massive gravity



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New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]
+ De Felice, arXiv: 1303.4154 [hep-th]

- Healthy regions with (relatively) large anisotropy
- Are there attractors in healthy region?
- Classification of fixed points
- Local stability analysis
- Global stability analysis

At attractors, physical metric is isotropic but fiducial metric is anisotropic.

→ Anisotropic FLRW universe!

statistical anisotropy expected
(suppressed by small m_g^2)

New class of cosmological solution

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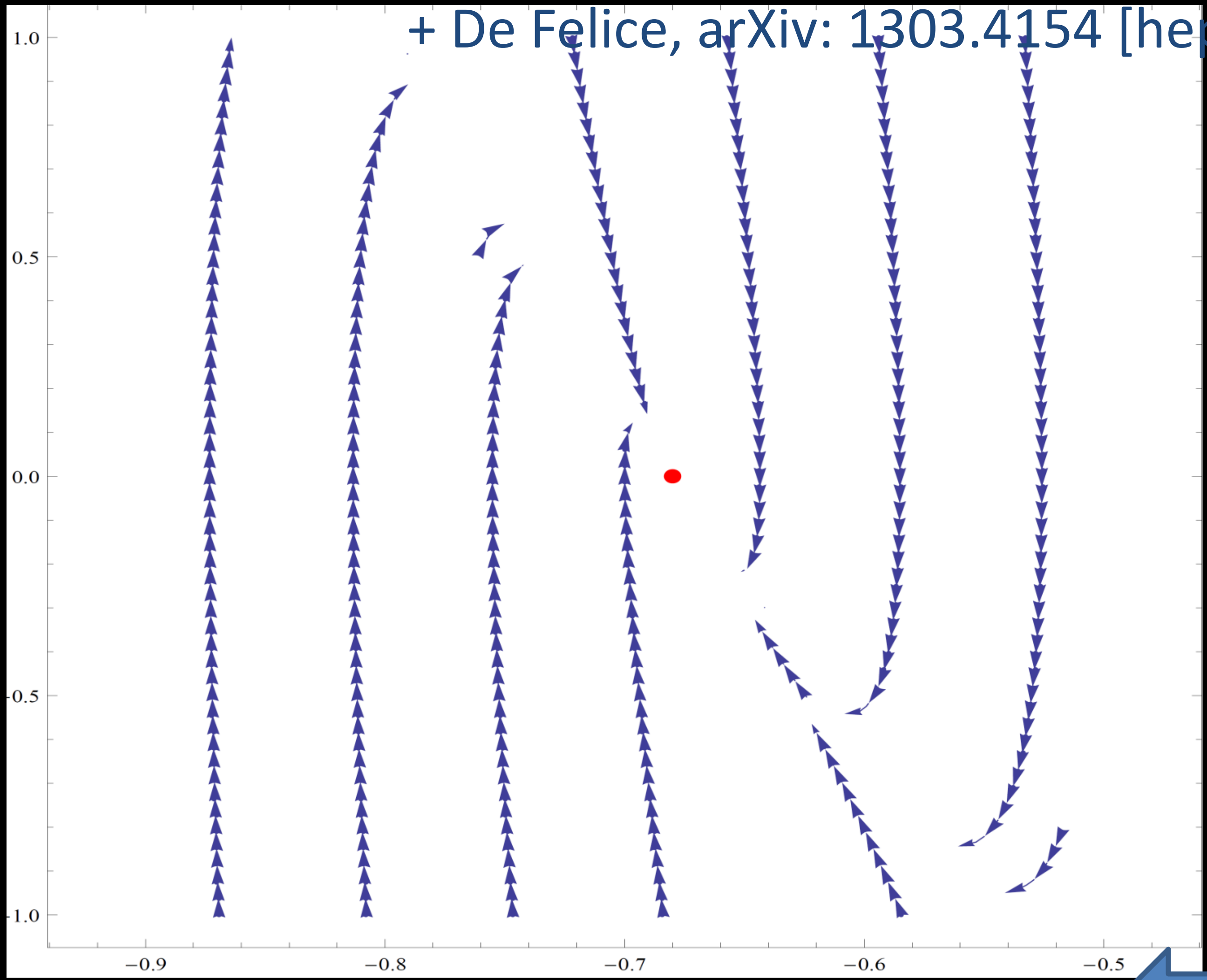
Anisotropy in Expansion



Anisotropy in fiducial metric

Our recent contributions

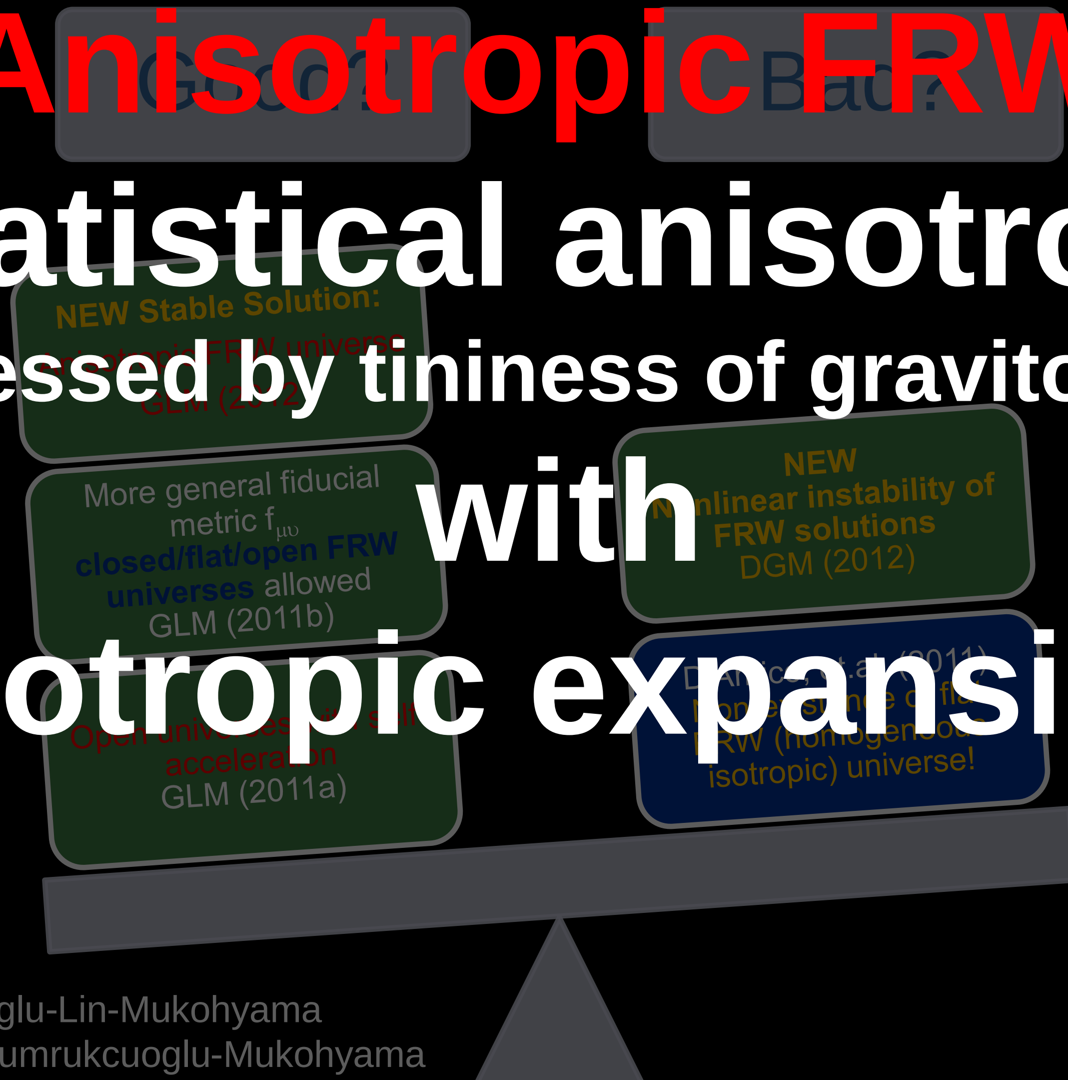
Cosmological solutions of nonlinear massive gravity

Anisotropic FRW:

Statistical anisotropy

(suppressed by tininess of graviton mass)

with
isotropic expansion



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Quasidilaton

D'Amico, Gabadadze, Hui, Pirtskhalava, 2012

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] → (i) new backgrounds, or (ii) extended theories
- Quasidilaton: scalar σ with global symmetry:

$$\sigma \to \sigma + \sigma_0 \qquad \phi^a \to e^{-\sigma_0/M_{\rm Pl}}\,\phi^a$$

- Action

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\Big[R - 2\Lambda - \frac{\omega}{M_{\rm Pl}^2}\partial_\mu\sigma\partial^\mu\sigma + 2m_g^2(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4)\Big]$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - e^{\sigma/M_{\rm Pl}}\left(\sqrt{g^{-1}f}\right)^\mu{}_\nu \qquad f_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

- Scaling solution = self-accelerating de Sitter (H = const > 0 with Λ = 0)

Stable extension of quasidilaton

arXiv: 1306.5502 [hep-th] /w A. De Felice

- Self-accelerating solution in the original quasidilaton theory has ghost instability
  [Gumrukcuoglu, Hinterbichler, Lin, Mukohyama, Trodden 2013; D'Amico, Gabadadze, Hui, Pirtskhalava 2013]
- Simple extension: $f_{\mu\nu} \rightarrow \tilde{f}_{\mu\nu}$

$$\tilde{f}_{\mu\nu} \equiv f_{\mu\nu} - \frac{\alpha_\sigma}{M_{\rm Pl}^2 m_g^2} e^{-2\sigma/M_{\rm Pl}} \partial_\mu\sigma\partial_\nu\sigma$$

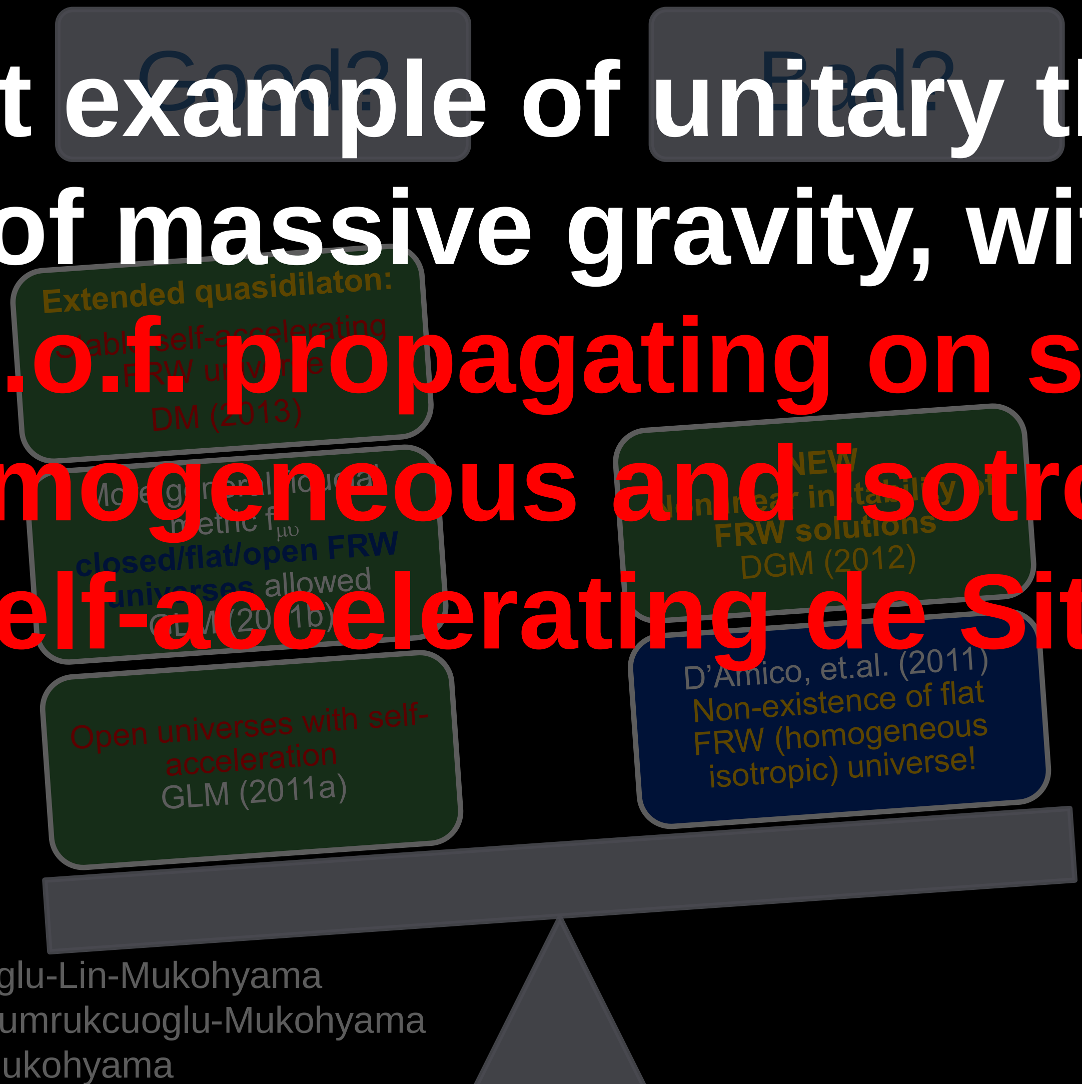
- Self-accelerating solution is stable if

$$0 < \omega < 6 \qquad X^2 < \frac{\alpha_\sigma H^2}{m_g^2} < r^2 X^2 \qquad X \equiv \frac{e^{\bar{\sigma}/M_{\rm Pl}}}{a}$$

$$M_{\rm GW}^2 \equiv \frac{(r-1)X^3 m_g^2}{X-1} + \frac{\omega H^2 (rX + r - 2)}{(X-1)(r-1)} > 0 \qquad r \equiv \frac{n}{N} a$$

Our recent contributions

Cosmological solutions of nonlinear massive gravity



First example of unitary theory of massive gravity, with all d.o.f. propagating on strictly homogeneous and isotropic, self-accelerating de Sitter

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama
DM = DeFelice-Mukohyama

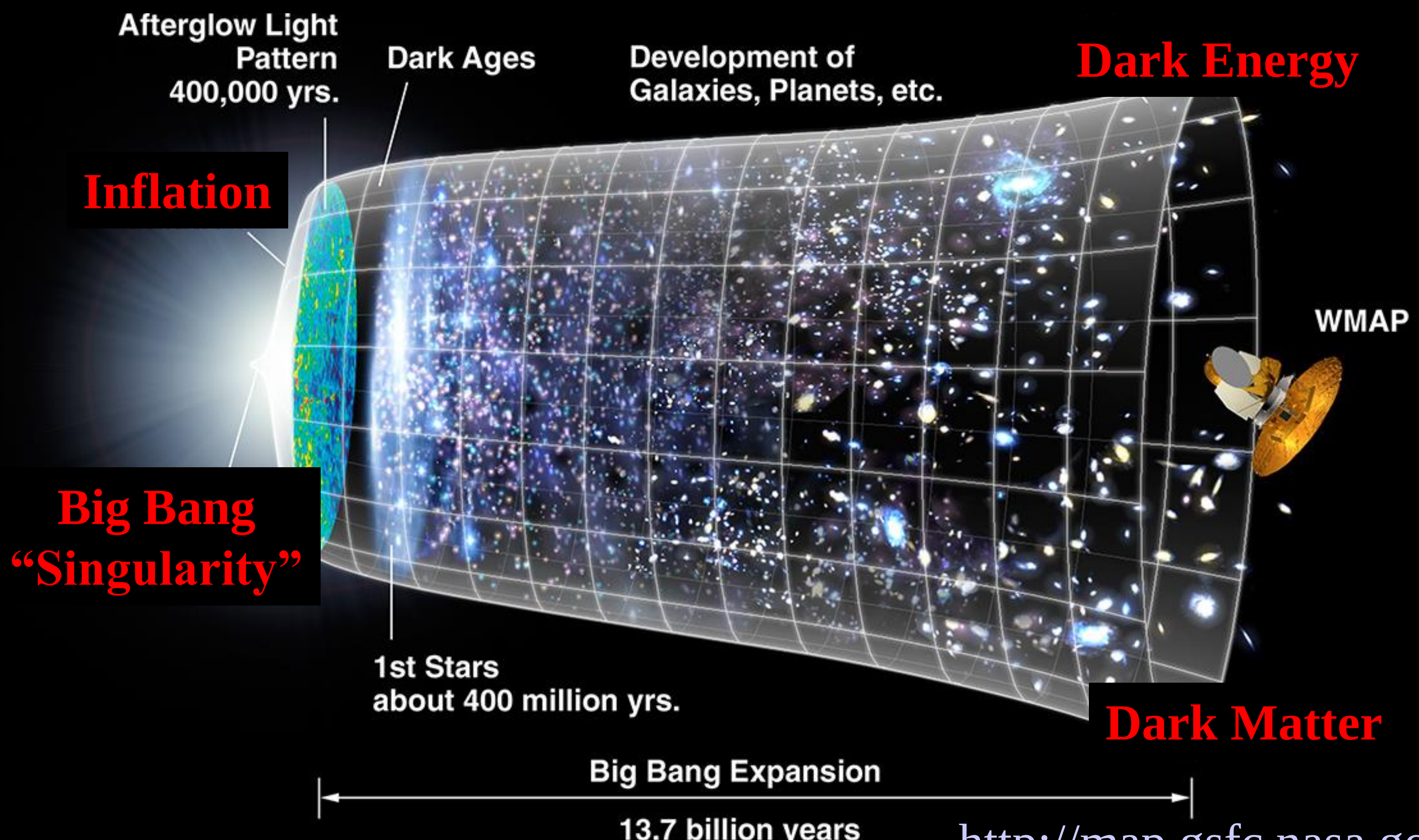
Summary

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass
- All homogeneous and isotropic FLRW solutions in the original dRGT theory have ghost
- New class of cosmological solution:
  anisotropic FLRW → statistical anisotropy (suppressed by small m_g^2)
- Extended quasidilaton: stable self-accelerating FLRW

Comment: Is there acausality in Massive Gravity?

- Deser-Waldron's recent claim (2012) on acausality of massive gravity is due to misuse of characteristics analysis [see Izumi-Ong (2013)]
- Characteristics analysis: det (time kinetic matrix) = 0 AFTER solving constraints → instantaneous mode → acausality
- dRGT eliminates BD ghost → would-be BD ghost has vanishing time kinetic term → det (time kinetic matrix) = 0 BEFORE solving constraints → this does NOT imply acausality
- On the other hand, there are instantaneous modes on self-accelerating branch backgrounds [De Felice- Gumrukcuoglu-Mukohyama (2012)]. So, the issue must be analyzed on each background to establish healthy EFT.
- For example, we can consider an inhomogeneous configuration which connects a self-accelerating branch solution inside and a trivial branch solution outside. If we like, we can consider this as a kind of nonlinear excitation above a trivial branch background. Obviously, this leads to an instantaneous propagation [Deser-Izumi-Ong-Waldron 2013].
- A physical question is really how much energy we need to excite this kind of configuration and whether this energy is above or below the cutoff scale of the EFT on a given background.

Why alternative gravity theories?



http://map.gsfc.nasa.gov/

BACKUP SLIDES

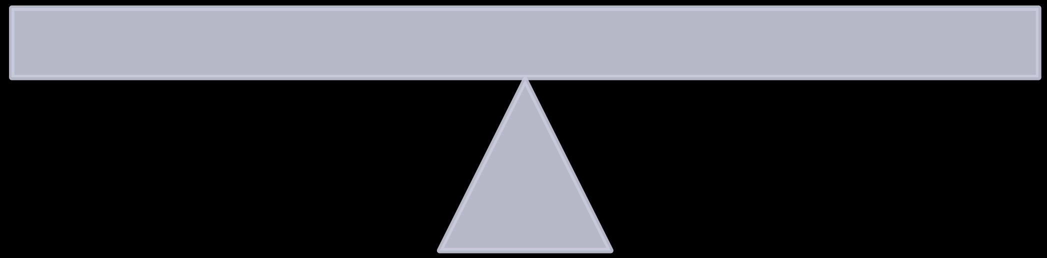
Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



Linear massive gravity (Fierz-Pauli 1939)

- Simple question: Can spin-2 field have mass?
- $L = L_{EH}[h] + \mathbf{m_g^2[\eta^{\mu\rho}\eta^{\nu\sigma}h_{\mu\nu}h_{\rho\sigma}-(\eta^{\mu\nu}h_{\mu\nu})^2]}$
  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Unique linear theory without ghosts
- Broken diffeomorphism
  → no momentum constraint
  → 5 d.o.f. (2 tensor + 2 vector + 1 scalar)

vDVZ vs Vainshtein

- van Dam-Veltman-Zhakharov (1970)
  Massless limit ≠ Massless theory = GR
  5 d.o.f remain → PPN parameter $\gamma = ½ \neq 1$
- Vainshtein (1972)
  Linear theory breaks down in the limit.
  Nonlinear analysis shows continuity and GR is recovered @ $r < r_V = (r_g/m_g^4)^{1/5}$.
  Continuity is not uniform w.r.t. distance.

Naïve nonlinear theory and BD ghost

- FP theory with $\eta^{\mu\nu} \rightarrow g^{\mu\nu}$
  $L = L_{EH}[h] + m_g^2[g^{\mu\rho}g^{\nu\sigma}h_{\mu\nu}h_{\rho\sigma} - (g^{\mu\nu}h_{\mu\nu})^2]$
  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Vainshtein effect (1972)
- Boulware-Deser ghost (1972)
  No Hamiltonian constraint @ nonlinear level
  $\rightarrow$ 6 d.o.f. = 5 d.o.f. of massive spin-2 + 1 ghost

Stuckelberg fields & Decoupling limit

Arkani-Hamed, Georgi & Schwarz (2003)

- Stuckelberg scalar fields ϕ^a (a=0,1,2,3)

  $$g_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b + H_{\mu\nu} \qquad \phi^a = x^a + \pi^a$$

  $H_{\mu\upsilon}$: covariant version of $h_{\mu\upsilon} = g_{\mu\nu} - \eta_{\mu\nu}$
- Decoupling limit

  $m_g \rightarrow 0$, $M_{Pl} \rightarrow \infty$ with $\Lambda_5 = (m_g{}^4 M_{Pl})^{1/5}$ fixed
- Helicity-0 part π: $\eta_{ab}\pi^b = \partial_a\pi$

  sufficient for analysis of would-be BD ghost

Would-be BD ghost vs fine-tuning

Creminelli, Nicolis, Papucci & Trincherini 2005
de Rham, Gabadadze 2010

$$H_{\mu\nu} = -2\partial_\mu\partial_\nu\pi - \partial_\mu\partial^\rho\pi\partial_\rho\partial_\nu\pi \Leftarrow h_{\mu\nu} = 0, \eta_{ab}\pi^b = \partial_a\pi$$

Decoupling limit   Helicity-0 part

- Fierz-Pauli theory
  $H_{\mu\upsilon}^2 - H^2$
  no ghost
- 3rd order
  $c_1 H_{\mu\upsilon}^3 + c_2 H H_{\mu\upsilon}^2 + c_3 H^3$
  no ghost if fine-tuned
- …
- any order
  no ghost if fine-tuned

General fiducial metric

Appendix of Gumrukcuoglu, Lin, Mukohyama, arXiv: 1111.4107 [hep-th]

- **Poincare symmetry in the field space**

  → $f_{\mu\nu} = (Minkowski)_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$
- **de Sitter symmetry in the field space**

  → $f_{\mu\nu} = (deSitter)_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$
- **FRW symmetry in the field space**

  → $f_{\mu\nu} = (FLRW)_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$

Flat/closed/open FLRW cosmology allowed if "fiducial metric" $f_{\mu\nu}$ is de Sitter (or FRW)

→ **Friedmann equation with the same effective cc**

$$3H^2 + \frac{3K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\alpha_3^2-3\alpha_4\right) \pm 2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

Cosmological perturbation with any matter

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1111.4107 [hep-th]

$$I^{(2)} = \tilde{I}^{(2)}[Q_I, \Phi, \Psi, B_i, \gamma_{ij}] + \tilde{I}^{(2)}_{mass}[\psi^\pi,\ E^\pi,\ F^\pi_i, \gamma_{ij}]$$

$$\tilde{I}[g_{\mu\nu}, \sigma_I] \equiv I_{EH,\tilde{\Lambda}}[g_{\mu\nu}] + I_{matter}[g_{\mu\nu}, \sigma_I] \qquad \tilde{\Lambda} \equiv \Lambda + \Lambda_\pm$$

$$M^2_{GW} \equiv \pm(r-1)m_g^2 X_\pm^2 \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4},$$

$$r \equiv \frac{na}{N\alpha} = \frac{1}{X_\pm}\frac{H}{H_f}, \quad H \equiv \frac{\dot{a}}{Na}, \quad H_f \equiv \frac{\dot{\alpha}}{n\alpha}$$

$$\tilde{I}^{(2)}_{mass} = M^2_{Pl}\int d^4x N a^3 \sqrt{\Omega}\, M^2_{GW} \times \left[3(\psi^\pi)^2 - \frac{1}{12}E^\pi \triangle(\triangle + 3K)E^\pi + \frac{1}{16}F^i_\pi(\triangle + 2K)F^\pi_i - \frac{1}{8}\gamma^{ij}\gamma_{ij}\right]$$

- **GR&matter part + graviton mass term**
- Separately gauge-invariant

  Common ingredient is γ_{ij} only
- Integrate out ψ^π, E^π and F^π_i → $I^{(2)}_{s,v} = I^{(2)}_{GR\ s,v}$
- Difference from GR is in the tensor sector only