Dynamics of bigravity and general relativity

- Introduction
- Homothetic solution
- FLRW universe
- Bianchi universe
- Observational consequence ?

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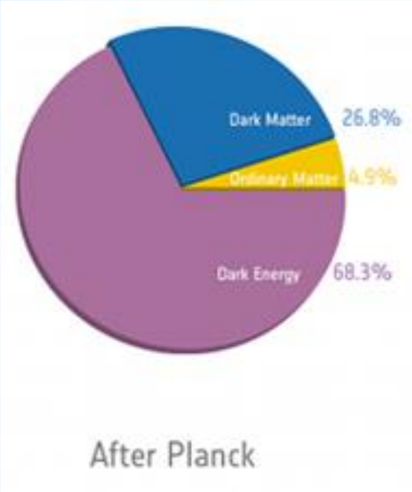
Dark Energy ⟹ cosmological constant Λ



$$\Lambda \leq 10^{-120} m_{PL}^2$$

Modified gravity

General Relativity ⟹ graviton: massless spin 2

massive gravity

Fierz-Pauli ghost-free linear theory

de Rham-Gabadadze-Tolley Non-linear extension

Hassan-Rosen Extension to bigravity

Question:

An accelerating universe is naturally found ?

- Stability of FLRW universe
- de Sitter : attractor ? ⟹ **cosmic no hair conjecture**

In GR with a cosmological constant

- **cosmic no hair theorem** (Wald 83) **for Bianchi spacetime**

  Any anisotropic homogeneous spacetime except for type IX is always isotropized and de Sitter solution is realized.
  (Type IX : ${}^{(3)}R_{\max} < 2\Lambda$ initially → de Sitter)

- Inhomogeneous case → Schwarzschild-de Sitter solution

  large black holes are not formed (Shiromizu et al 93)
  in asymptotically de Sitter spacetime

- GR+ a cosmological constant: attractor ?

content

- Homothetic solution gives (GR+ a cosmological constant)
    - stability against perturbations
- FLRW universe (homogeneous and isotropic spacetime)
    - existence condition of de Sitter solution Aoki, KM (in prep.)
    - matter effect on dynamics
- Bianchi universe (homogeneous but anisotropic spacetime)

  KM, Volkov ('13)
    - homothetic solution is an attractor
    - the shear density drops as a^{-3} (In GR a^{-6})
    - chaotic behaviour in Type IX
- Observational consequence ?
    - Equation of state of DE ?
    - cosmological constant with dark matter ?

Bigravity theory

$g_{\mu\nu}\,,\quad f_{\mu\nu}$ two metrics

$$S[g,f,\mathrm{matter}] = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}\,R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\,\mathcal{R}(f)$$

$$-\frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\,\mathscr{U}[g,f] + S_g^{[\mathrm{m}]}[g,\text{g-matter}] + S_f^{[\mathrm{m}]}[f,\text{f-matter}]$$

$$\kappa^2 = \kappa_g^2 + \kappa_f^2$$

Interaction term

$$\mathscr{U} = \sum_{k=0}^{4} b_k\,\mathscr{U}_k(\gamma), \qquad \gamma^\mu{}_\nu = \epsilon\sqrt{g^{\mu\alpha}f_{\alpha\nu}}\ (\epsilon = \pm 1)$$

$$\mathscr{U}_0(\gamma) = -\frac{1}{4!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 1 \qquad \mathscr{U}_1(\gamma) = -\frac{1}{3!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\nu\rho\sigma}\gamma^\mu{}_\alpha = \sum_A \lambda_A$$

$$\mathscr{U}_2(\gamma) = -\frac{1}{2!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\rho\sigma}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta = \sum_{A<B}\lambda_A\lambda_B$$

λ_A : eigenvalues of $\gamma^\mu{}_\nu$

$$\mathscr{U}_3(\gamma) = -\frac{1}{3!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta\gamma^\rho{}_\gamma = \sum_{A<B<C}\lambda_A\lambda_B\lambda_C$$

$$\mathscr{U}_4(\gamma) = -\frac{1}{4!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta\gamma^\rho{}_\gamma\gamma^\sigma{}_\delta = \lambda_0\lambda_1\lambda_2\lambda_3$$

m : graviton mass

a flat space is a solution

$$\Rightarrow \quad b_0 = 4c_3 + c_4 - 6, \quad b_1 = 3 - 3c_3 - c_4,$$

$$b_2 = 2c_3 + c_4 - 1, \quad b_3 = -(c_3 + c_4), \quad b_4 = c_4.$$

two free coupling constants $c_3, \; c_4$

Basic equations

$$G_{\mu\nu} = \kappa_g^2 \left[T^{[\gamma]}_{\mu\nu} + T^{[\mathrm{m}]}_{\mu\nu} \right] \qquad \mathcal{G}_{\mu\nu} = \kappa_f^2 \left[\mathcal{T}^{[\gamma]}_{\mu\nu} + \mathcal{T}^{[\mathrm{m}]}_{\mu\nu} \right]$$

$$\kappa_g^2 T^{[\gamma]\mu}{}_{\nu} = m_g^2 (\tau^{\mu}{}_{\nu} - \mathscr{U} \, \delta^{\mu}_{\nu}) \qquad \kappa_f^2 \mathcal{T}^{[\gamma]\mu}{}_{\nu} = -m_f^2 \frac{\sqrt{-g}}{\sqrt{-f}} \, \tau^{\mu}{}_{\nu} \,,$$

$$\tau^{\mu}{}_{\nu} = \{ b_1 \, \mathscr{U}_0 + b_2 \, \mathscr{U}_1 + b_3 \, \mathscr{U}_2 + b_4 \, \mathscr{U}_3 \} \gamma^{\mu}{}_{\nu} - \{ b_2 \, \mathscr{U}_0 + b_3 \, \mathscr{U}_1 + b_4 \, \mathscr{U}_2 \} (\gamma^2)^{\mu}{}_{\nu}$$
$$+ \{ b_3 \, \mathscr{U}_0 + b_4 \, \mathscr{U}_1 \} (\gamma^3)^{\mu}{}_{\nu} - b_4 \, \mathscr{U}_0 \, (\gamma^4)^{\mu}{}_{\nu}$$

$$\gamma^{\mu}{}_{\nu} = \epsilon \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

energy-momentum conservation

$$\overset{(g)}{\nabla}_{\mu} \, T^{[\mathrm{m}]\mu}{}_{\nu} = 0, \qquad \overset{(f)}{\nabla}_{\mu} \, \mathcal{T}^{[\mathrm{m}]\mu}{}_{\nu} = 0$$

$$\overset{(g)}{\nabla}_{\mu} T^{[\gamma]\mu}{}_{\nu} = 0. \qquad \overset{(f)}{\nabla}_{\mu} \mathcal{T}^{[\gamma]\mu}{}_{\nu} = 0$$

homothetic metrics: $f_{\mu\nu} = K^2 g_{\mu\nu}$ ⇨ $\gamma^\mu{}_\nu = K\,\delta^\mu{}_\nu$

$$\kappa_g^2 T^{[\gamma]\mu}{}_\nu = -\Lambda_g(K)\delta^\mu{}_\nu\,,\ \ \kappa_f^2 \mathcal{T}^{[\gamma]\mu}{}_\nu = -\Lambda_f(K)\delta^\mu{}_\nu$$

$$\Lambda_g(K) = m_g^2\left(b_0 + 3b_1\,K + 3b_2\,K^2 + b_3\,K^3\right)$$

$$\Lambda_f(K) = m_f^2\left(b_1/K^3 + 3b_2/K^2 + 3b_3/K + b_4\right)$$

$\overset{(g)}{\nabla}_\mu T^{[\gamma]\mu}{}_\nu = 0$ $\overset{(f)}{\nabla}_\mu \mathcal{T}^{[\gamma]\mu}{}_\nu = 0$ ⇨ K: constant

GR with a cosmological constant

$$G_{\mu\nu} + \Lambda_g g_{\mu\nu} = \kappa_g^2 T^{[\mathrm{m}]}_{\mu\nu}$$

$\Lambda_g(K) = K^2\Lambda_f(K)$: quartic equation for K

$$T^{[\mathrm{m}]}_{\mu\nu} = K^2 \mathcal{T}^{[\mathrm{m}]}_{\mu\nu}$$

homothetic solution $\Lambda_g > 0$ ⇨ de Sitter

perturbations around a homothetic solution

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}\,, \quad f_{\mu\nu} := K^2 \tilde{f}_{\mu\nu} = K^2 \left(g^{(0)}_{\mu\nu} + k_{\mu\nu} \right)$$

$$\psi_{\mu\nu} := m_f^2 h_{\mu\nu} + K^2 m_g^2 k_{\mu\nu}$$

$$\varphi_{\mu\nu} := h_{\mu\nu} - k_{\mu\nu}$$

$$\bar{\psi}_{\mu\nu} := \psi_{\mu\nu} - \frac{1}{2}\psi g^{(0)}_{\mu\nu}$$

$$\bar{\varphi}_{\mu\nu} := \varphi_{\mu\nu} - \frac{1}{2}\varphi g^{(0)}_{\mu\nu}$$

de Sitter background

$$\overset{(0)}{\Box}\bar{\psi}^{(\mathrm{TT})}_{\mu\nu} - \frac{2}{3}\Lambda_g \bar{\psi}^{(\mathrm{TT})}_{\mu\nu} = 0$$

massless mode

$$\overset{(0)}{\Box}\bar{\varphi}_{\mu\nu} - \left(\frac{2}{3}\Lambda_g + m_{\mathrm{eff}}^2\right)\bar{\varphi}_{\mu\nu} = 0 \qquad 2\overset{(0)}{\nabla}{}^{\mu}\bar{\varphi}_{\mu\nu} = 0 \qquad \bar{\varphi} = 0$$

massive mode $m_{\mathrm{eff}}^2 = \left(m_g^2 + \dfrac{m_f^2}{K^2} \right) (b_1 K + 2 b_2 K^2 + b_3 K^3)$

⇨ stable

general vacuum homothetic background

$$\overset{(0)}{\Box}\bar{\psi}^{(\mathrm{TT})}_{\mu\nu} + 2\overset{(0)}{C}{}_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\bar{\psi}^{(\mathrm{TT})}_{\alpha\beta} - \frac{2}{3}\Lambda_g \bar{\psi}^{(\mathrm{TT})}_{\mu\nu} = 0$$

$$\overset{(0)}{\Box}\bar{\varphi}_{\mu\nu} + 2\overset{(0)}{C}{}_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\bar{\varphi}_{\alpha\beta} - \left(\frac{2}{3}\Lambda_g + m_{\mathrm{eff}}^2\right)\bar{\varphi}_{\mu\nu} = 0$$

FLRW universe

g-spacetime FLRW spacetime $ds_g^2 = -\alpha^2 dt^2 + a_g^2 \gamma_{ij} dx_i dx_j$

f-spacetime spherically symmetric spacetime

two cases:

- non-bidiagonal $\Longrightarrow$ g- and f- spacetimes are decoupled

  $(\mathrm{GR}(g) + \Lambda_g) \times (\mathrm{GR}(f) + \Lambda_f)$

- bidiagonal

$$ds_f^2 = -\mathcal{A}^2 dt^2 + a_f^2 \gamma_{ij} dx_i dx_j$$

$$\overset{(g)}{\nabla}_\mu T^{[\gamma]\mu}{}_\nu = 0. \Longrightarrow \left(\frac{\mathcal{A}}{\alpha} - \frac{\dot{a}_f}{\dot{a}_g}\right)\left(b_1 + 2b_2 B + b_3 B^2\right) = 0 \qquad B = a_f/a_g$$

$b_1 + 2b_2 B + b_3 B^2 = 0$ special solution

$$\boxed{\frac{\mathcal{A}}{\alpha} - \frac{\dot{a}_f}{\dot{a}_g} = 0}$$

flat FLRW $k=0$

$$\frac{\dot{a_g}^2}{\alpha^2 a_g^2} = \kappa_g^2 \rho_g^{[\gamma]} + \kappa_g^2 \rho_g$$

$$\kappa_g^2 \rho_g^{[\gamma]} = m_g^2 \left(b_0 + 3b_1 B + 3b_2 B^2 + b_3 B^3\right)$$

$$\frac{\dot{a_f}^2}{\mathcal{A}^2 a_f^2} = \kappa_f^2 \rho_f^{[\gamma]} + \kappa_f^2 \rho_f$$

$$\kappa_f^2 \rho_f^{[\gamma]} = m_f^2 \left(b_4 + 3\frac{b_3}{B} + 3\frac{b_2}{B^2} + \frac{b_1}{B^3}\right)$$

$$m_g^2 = m^2 \kappa_g^2 / \kappa^2 \qquad m_f^2 = m^2 \kappa_f^2 / \kappa^2$$

Matter : radiation + dust

$$\rho_g = \frac{c_{gm}}{a_g^3} + \frac{c_{gr}}{a_g^4} \qquad \rho_f = \frac{c_{fm}}{a_f^3} + \frac{c_{fr}}{a_f^4} \qquad B = \frac{a_f}{a_g}$$

$$\Rightarrow \quad \kappa_g^2 \rho_g^{[\gamma]}(B) - \kappa_f^2 B^2 \rho_f^{[\gamma]}(B) + \kappa_g^2 \rho_g(a_g) - \kappa_f^2 B^2 \rho_f(B a_g) = 0$$

$$\Rightarrow \quad a_g = a_g(B)$$

$$\Rightarrow \quad \dot{B}^2 + V_g(B; c_{gm}, c_{gr}, c_{fm}, c_{fr}) = 0$$

[1] vacuum

$$B = B_{\rm vac} \ \ (\text{constant})$$

$$\kappa_g^2 \rho_g^{[\gamma]}(B_{\rm vac}) - \kappa_f^2 B_{\rm vac}^2 \rho_f^{[\gamma]}(B_{\rm vac}) = 0$$

⇨ homothetic solution $f_{\mu\nu} = B_{\rm vac}^2 g_{\mu\nu}$

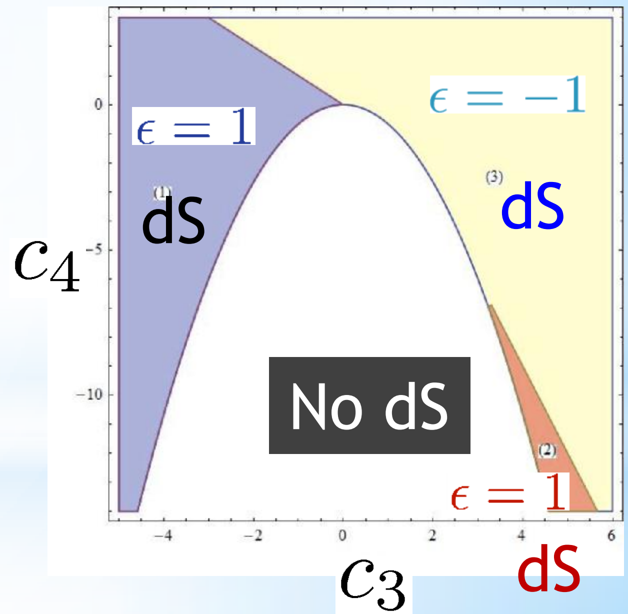
quartic equation

$B_{\rm vac} = 1$ Minkowski (stable)

$\Lambda_g = \kappa_g^2 \rho_g^{[\gamma]}(B_{\rm vac})$ cosmological constant

$\Lambda_g > 0$ 1 dS (stable)

$\Lambda_g < 0$ 2 AdS (unstable)

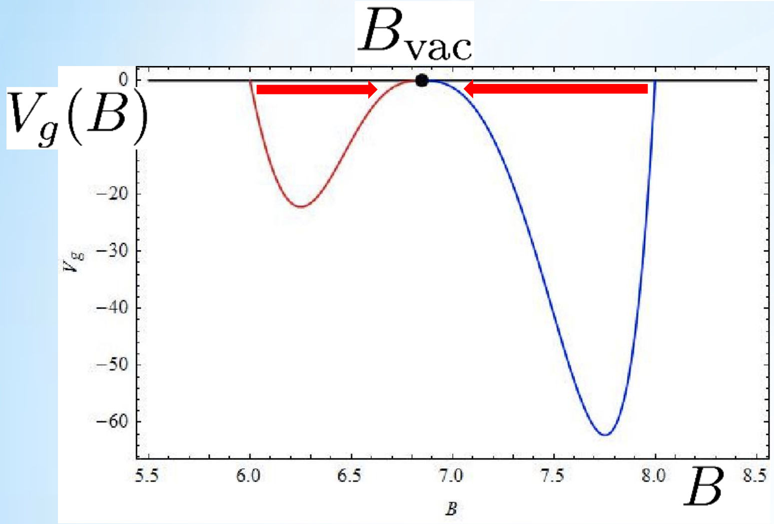


matter : dust

$$\dot{B}^2 + V_g(B, c_{fm}/c_{gm}) = 0$$

$$B = \frac{a_f}{a_g}$$

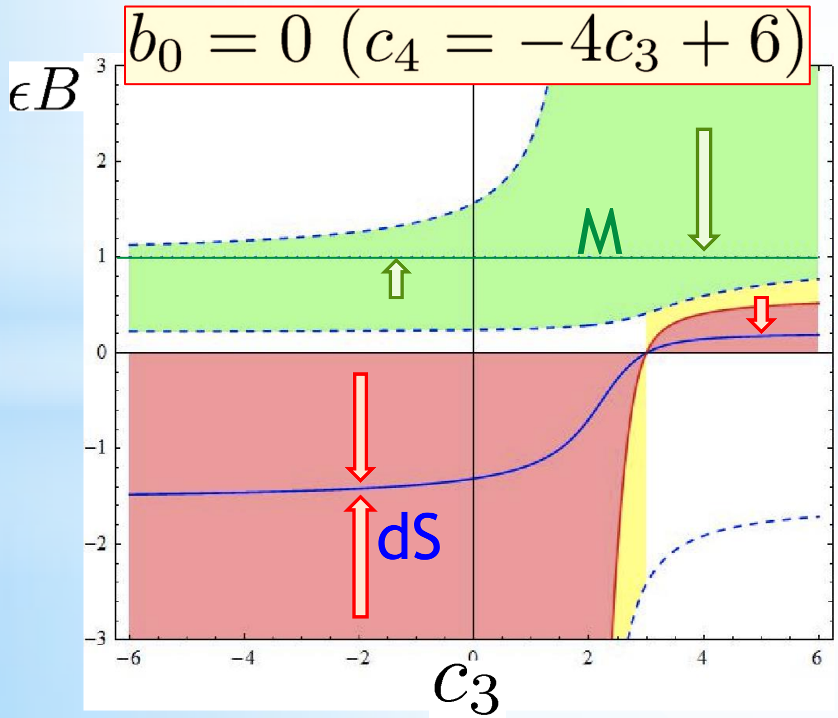
$\frac{c_{fm}}{c_{gm}}$: the ratio of initial matter densities

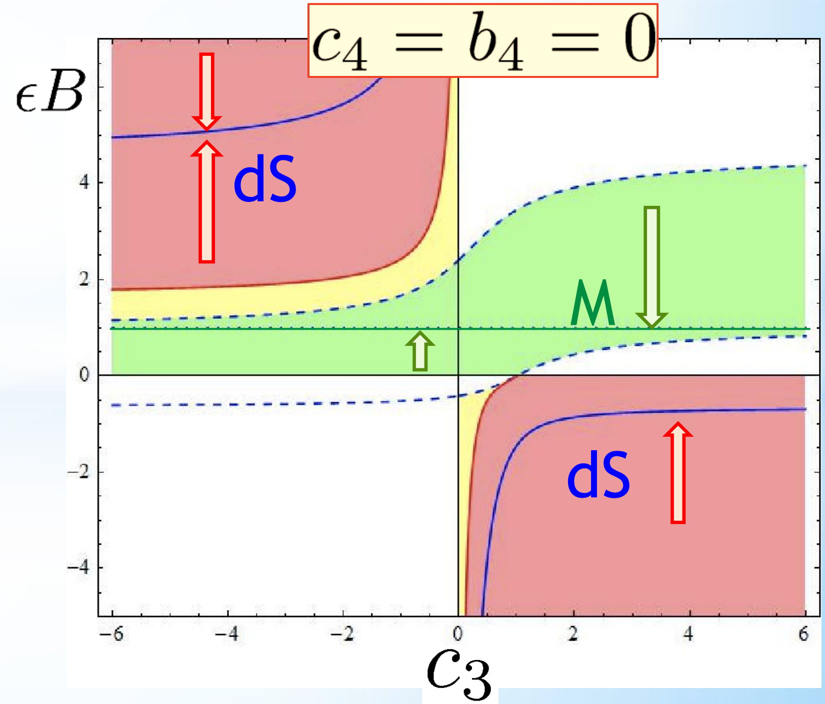


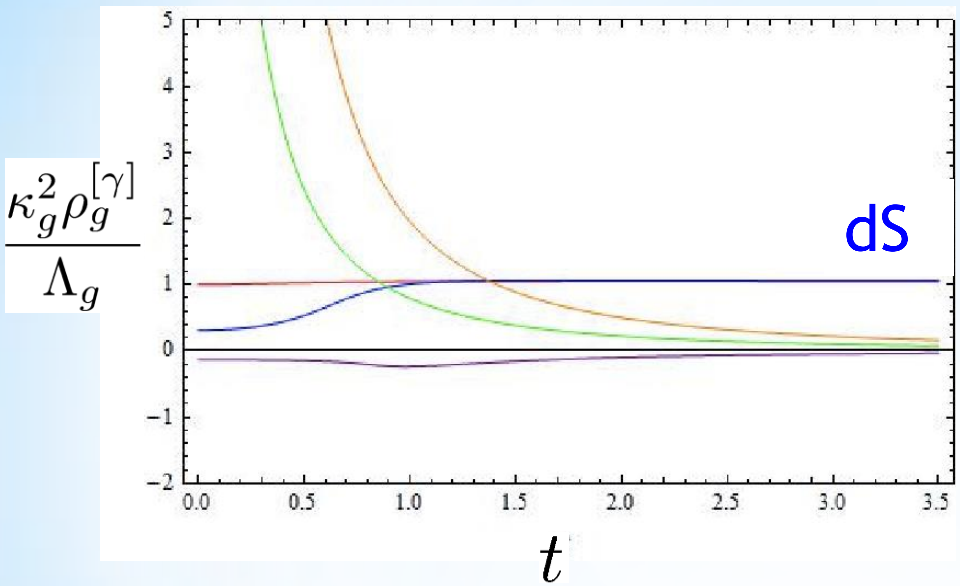
stable spacetimes

Minkowski

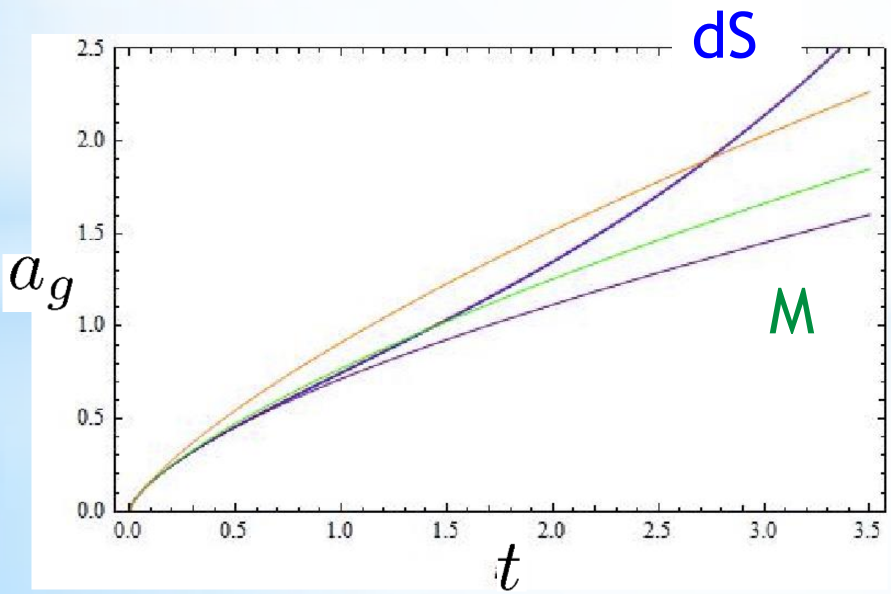
dS







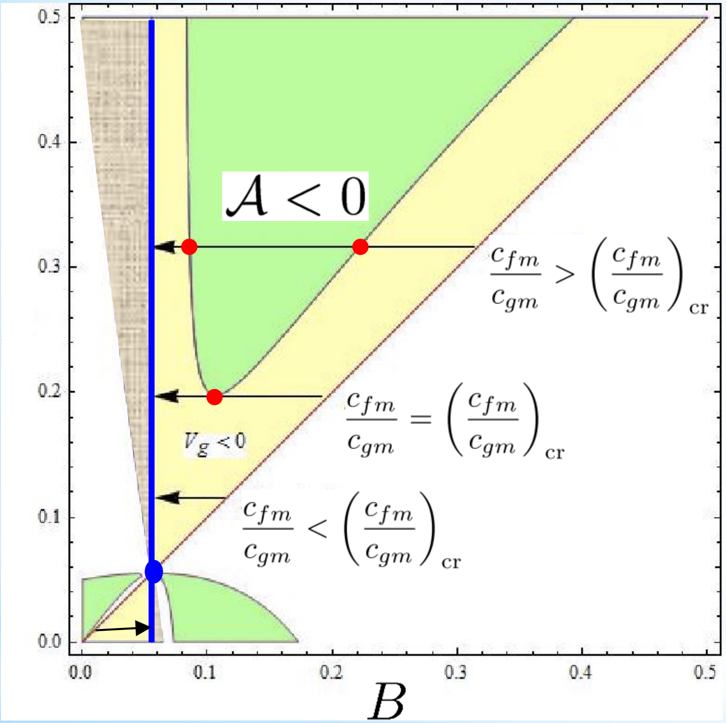
M (zero Λ)

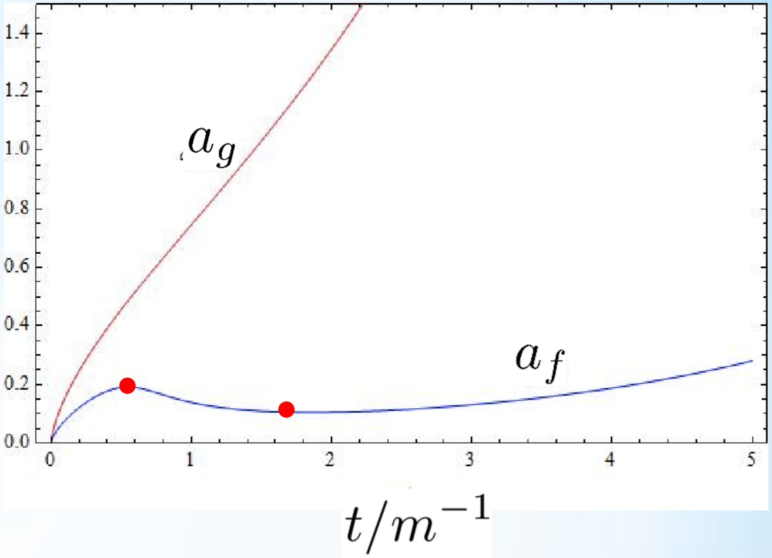


Matter dominant universe

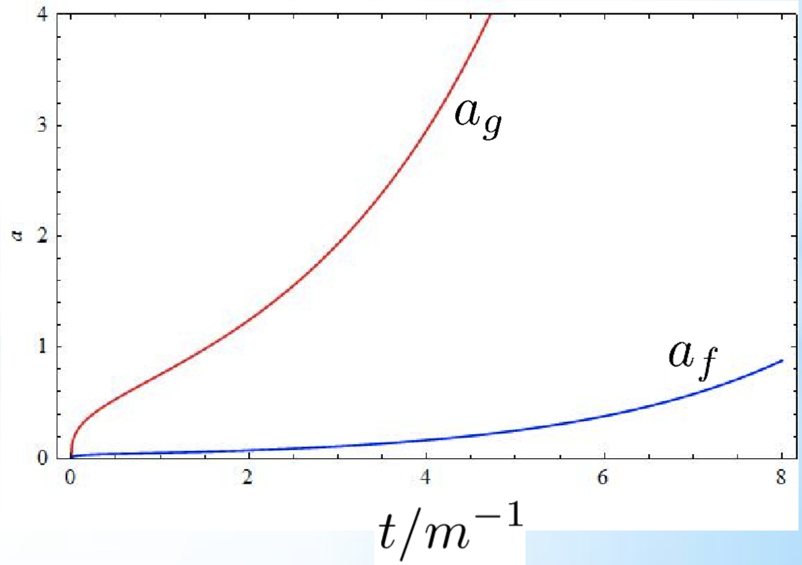
matter effect

dS (homothetic sol)





- singularity



$\frac{c_{fm}}{c_{gm}}$ the ratio of initial matter densities

For the coupling constants with de Sitter homothetic solution,

there always exist a finite range of initial values,
in which de Sitter spacetime is realized asymptotically.

Bianchi Spacetimes

$$[\xi_a, \xi_b] = C^c{}_{ab}\xi_c \qquad \xi_a : \text{Killing vectors}$$

$$C^c{}_{ab} = n^{cd}\epsilon_{dab} + a(\delta^1_a\delta^c_b - \delta^1_b\delta^c_a) \qquad n^{ab} = \mathrm{diag}[n^{(1)}, n^{(2)}, n^{(3)}]$$

$a = 0$ $\Longrightarrow$ Class A

	I	II	VI_0	VII_0	VIII	IX
$n^{(1)}$	0	1	1	1	1	1
$n^{(2)}$	0	0	-1	1	1	1
$n^{(3)}$	0	0	0	0	-1	1

$$ds_g^2 = -\alpha^2 dt^2 + e^{2\Omega}e^{2\beta_{ij}}\omega_i\omega_j \qquad ds_f^2 = -\mathcal{A}^2 dt^2 + e^{2\mathcal{W}}e^{2\mathcal{B}_{ij}}\omega_i\omega_j$$

$$(\beta_{ij}) = \begin{pmatrix} \beta_+ + \sqrt{3}\beta_- & 0 & 0 \\ 0 & \beta_+ - \sqrt{3}\beta_- & 0 \\ 0 & 0 & -2\beta_+ \end{pmatrix}$$

$$(\mathcal{B}_{ij}) = \begin{pmatrix} \mathcal{B}_+ + \sqrt{3}\mathcal{B}_- & 0 & 0 \\ 0 & \mathcal{B}_+ - \sqrt{3}\mathcal{B}_- & 0 \\ 0 & 0 & -2\mathcal{B}_+ \end{pmatrix}.$$

Bianchi I

$$S = \frac{3}{\kappa_g^2} \int d^4x \frac{e^{3\Omega}}{\alpha} \left(-\dot{\Omega}^2 + \dot{\beta}_+^2 + \dot{\beta}_-^2\right)$$
$$+ \frac{3}{\kappa_f^2} \int d^4x \, \frac{e^{3\mathcal{W}}}{\mathcal{A}} \left(-\dot{\mathcal{W}}^2 + \dot{\mathcal{B}}_+^2 + \dot{\mathcal{B}}_-^2\right) - \frac{m^2}{\kappa^2} \int d^4x (\alpha V_g + \mathcal{A}\mathcal{V}_f)$$

$$V_g = -\Big[b_0 e^{3\Omega} + b_3 e^{3\mathcal{W}} + b_1 e^{\mathcal{W}+2\Omega} \left(e^{-2(\mathcal{B}_+ - \beta_+)} + 2e^{\mathcal{B}_+ - \beta_+} \cosh[\sqrt{3}(\mathcal{B}_- - \beta_-)]\right)$$
$$+ b_2 e^{2\mathcal{W}+\Omega} \left(e^{2(\mathcal{B}_+ - \beta_+)} + 2e^{-(\mathcal{B}_+ - \beta_+)} \cosh[\sqrt{3}(\mathcal{B}_- - \beta_-)]\right) \Big]$$

$$\mathcal{V}_f = -\Big[b_1 e^{3\Omega} + b_4 e^{3\mathcal{W}} + b_2 e^{\mathcal{W}+2\Omega} \left(e^{-2(\mathcal{B}_+ - \beta_+)} + 2e^{\mathcal{B}_+ - \beta_+} \cosh[\sqrt{3}(\mathcal{B}_- - \beta_-)]\right)$$
$$+ b_3 e^{2\mathcal{W}+\Omega} \left(e^{2(\mathcal{B}_+ - \beta_+)} + 2e^{-(\mathcal{B}_+ - \beta_+)} \cosh[\sqrt{3}(\mathcal{B}_- - \beta_-)]\right) \Big]$$

perfect fluid $\rho_g^{(\mathrm{m})}, \quad P_g^{(\mathrm{m})} \qquad \rho_f^{(\mathrm{m})}, \quad P_f^{(\mathrm{m})}.$

$$\frac{1}{2}\dot{\Omega}^2 = \frac{1}{2}\left(\dot{\beta}_+^2 + \dot{\beta}_-^2\right) + \frac{m_g^2}{6}\alpha^2 e^{-3\Omega} V_g + \frac{\alpha^2\kappa_g^2}{6}\rho_g^{(\mathrm{m})}$$

$$\ddot{\Omega} - \frac{\dot{\alpha}}{\alpha}\dot{\Omega} + 3\dot{\Omega}^2 = \frac{m_g^2}{6}\alpha e^{-3\Omega}\left[\alpha\left(3V_g + \frac{\partial V_g}{\partial\Omega}\right) + \mathcal{A}\frac{\partial\mathcal{V}_f}{\partial\Omega}\right] + \frac{\alpha^2\kappa_g^2}{2}\left(\rho_g^{(\mathrm{m})} - P_g^{(\mathrm{m})}\right)$$

$$\ddot{\beta}_\pm - \frac{\dot{\alpha}}{\alpha}\dot{\beta}_\pm + 3\dot{\Omega}\dot{\beta}_\pm = -\frac{m_g^2}{6}\alpha e^{-3\Omega}\left(\alpha\frac{\partial V_g}{\partial\beta_\pm} + \mathcal{A}\frac{\partial\mathcal{V}_f}{\partial\beta_\pm}\right)$$

$$\frac{1}{2}\dot{\mathcal{W}}^2 = \frac{1}{2}\left(\dot{\mathcal{B}}_+^2 + \dot{\mathcal{B}}_-^2\right) + \frac{m_f^2}{6}\mathcal{A}^2 e^{-3\mathcal{W}}\mathcal{V}_f + \frac{\mathcal{A}^2\kappa_f^2}{6}\rho_f^{(\mathrm{m})}$$

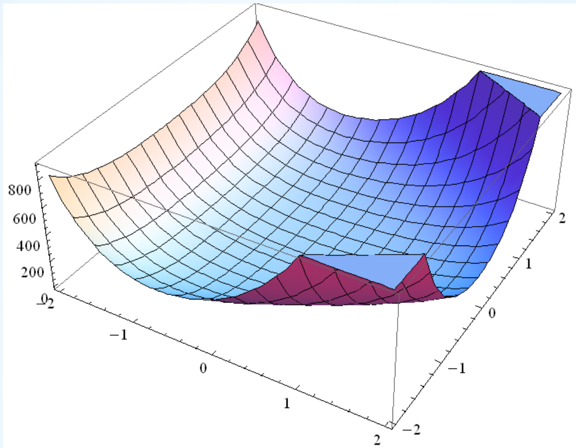
$$\ddot{\mathcal{W}} - \frac{\dot{\mathcal{A}}}{\mathcal{A}}\dot{\mathcal{W}} + 3\dot{\mathcal{W}}^2 = \frac{m_f^2}{6}\mathcal{A}e^{-3\mathcal{W}}\left[\alpha\frac{\partial V_g}{\partial\mathcal{W}} + \mathcal{A}\left(3\mathcal{V}_f + \frac{\partial\mathcal{V}_f}{\partial\mathcal{W}}\right)\right] + \frac{\mathcal{A}^2\kappa_f^2}{2}\left(\rho_f^{(\mathrm{m})} - P_f^{(\mathrm{m})}\right)$$

$$\ddot{\mathcal{B}}_\pm - \frac{\dot{\mathcal{A}}}{\mathcal{A}}\dot{\mathcal{B}}_\pm + 3\dot{\mathcal{W}}\dot{\mathcal{B}}_\pm = -\frac{m_f^2}{6}\mathcal{A}e^{-3\mathcal{W}}\left(\alpha\frac{\partial V_g}{\partial\mathcal{B}_\pm} + \mathcal{A}\frac{\partial\mathcal{V}_f}{\partial\mathcal{B}_\pm}\right)$$

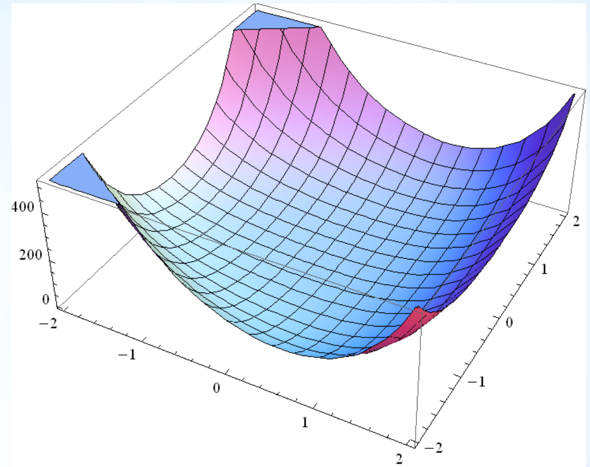
$$m_g^2 = \frac{m^2\kappa_g^2}{\kappa^2}\,, \quad m_f^2 = \frac{m^2\kappa_f^2}{\kappa^2}$$

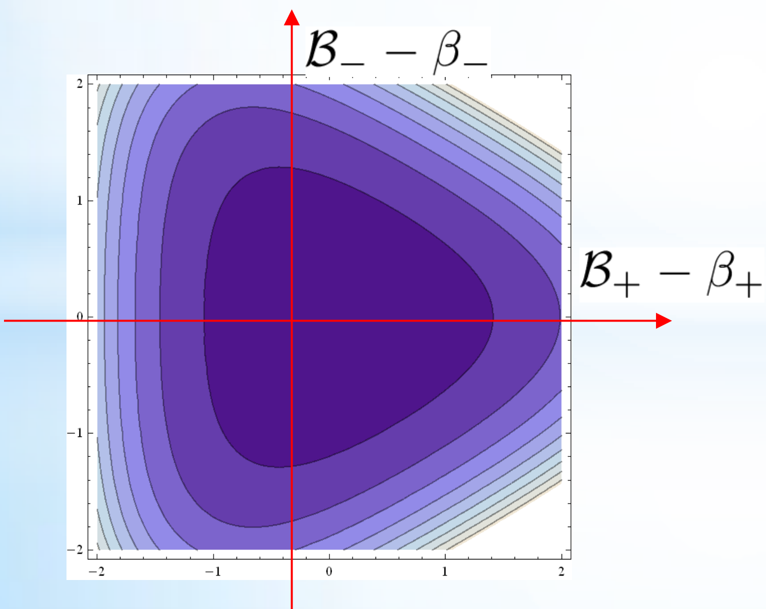
$b_1, b_2, b_3 < 0$

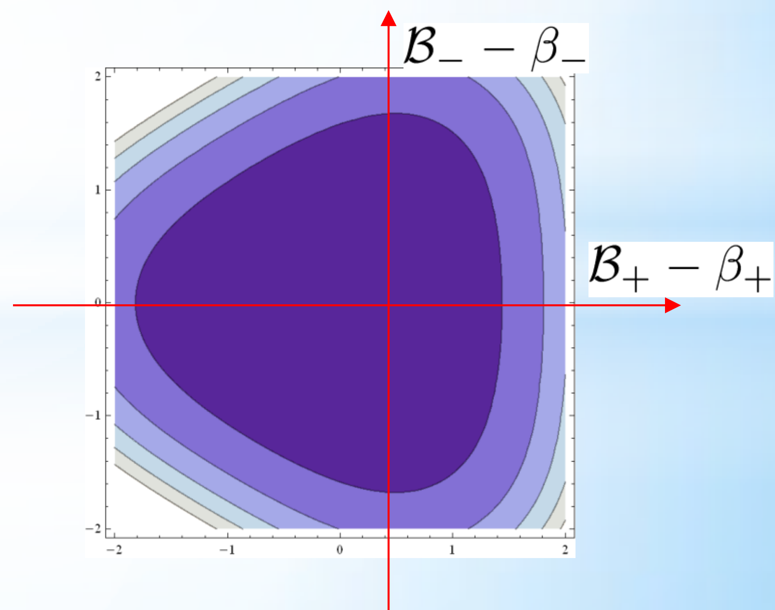
V_g



$\mathcal{V}_f$







$$\mathcal{B}_\pm = \beta_\pm \quad \Rightarrow \quad e^{3\Omega}\,\frac{\dot\beta_\pm}{\alpha} = \sigma_{\pm(0)}\,, \qquad e^{3\mathcal{W}}\,\frac{\dot{\mathcal{B}}_\pm}{\mathcal{A}} = \mathcal{S}_{\pm(0)}$$

$$e^{3(\mathcal{W}-\Omega)}\,\frac{\alpha}{\mathcal{A}} = \frac{\mathcal{S}_{+(0)}}{\sigma_{+(0)}} = \frac{\mathcal{S}_{-(0)}}{\sigma_{-(0)}} \equiv C^2$$

Hamiltonian constraints + EOM ⇨

$$\left[\alpha\left(e^{\mathcal{W}}\right)^{\cdot} - \mathcal{A}\left(e^{\Omega}\right)^{\cdot}\right]\left(b_1 + 2b_2 e^{\mathcal{W}-\Omega} + b_3 e^{2(\mathcal{W}-\Omega)}\right) = 0$$

$$(1)\ \alpha\left(e^{\mathcal{W}}\right)^{\cdot} - \mathcal{A}\left(e^{\Omega}\right)^{\cdot} = 0.$$

$$\Rightarrow \quad f_{\mu\nu} = C^2\, g_{\mu\nu}$$ homothetic $C = K$

$$(2)\ b_1 + 2b_2 e^{\mathcal{W}-\Omega} + b_3 e^{2(\mathcal{W}-\Omega)} = 0 \qquad e^{\mathcal{W}-\Omega} = \xi_0$$

$$\mathcal{A} = \sqrt{\frac{\Lambda_g(\xi_0)}{\Lambda_f(\xi_0)}}\,\alpha$$

$$e^{\mathcal{W}} = \xi_0\, e^{\Omega}$$

$$\rho_f = \frac{\Lambda_f(\xi_0)}{\Lambda_g(\xi_0)}\,\rho_g$$

homothetic solution=vacuum Bianchi I with a cosmological constant Λ in GR

analytic solution

$\Lambda > 0$

$$e^{\Omega} = \frac{1}{2^{1/3}} e^{\pm H_0(t-t_0)} \left(1 - \frac{\sigma_0^2}{H_0^2} e^{\mp 6H_0(t-t_0)}\right)^{1/3}$$

$$H_0 = \sqrt{\Lambda/3}$$

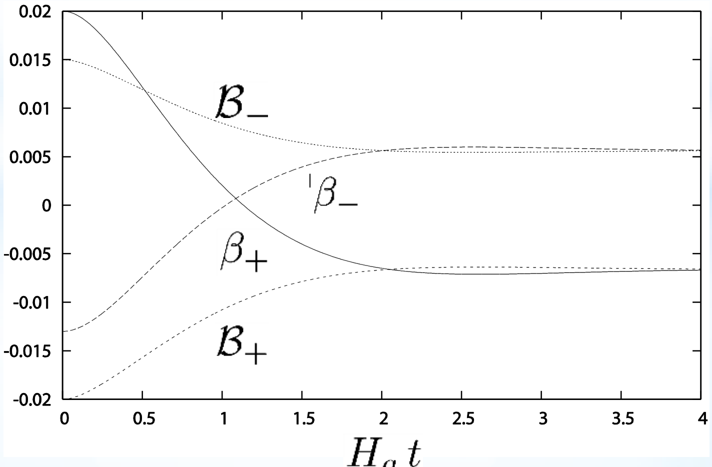
$$e^{\beta_\pm} = e^{\beta_{\pm(0)}} \left(\frac{1 - \frac{\sigma_0}{H_0} e^{\mp 3H_0(t-t_0)}}{1 + \frac{\sigma_0}{H_0} e^{\mp 3H_0(t-t_0)}} \right)^{\pm \frac{\sigma_\pm^{(0)}}{3\sigma_0}}$$

$$\sigma_0^2 = \sigma_{+(0)}^2 + \sigma_{-(0)}^2$$

$$\Sigma^2 = \frac{\sigma^2}{H^2} \propto e^{-6\Omega}$$

Homothetic solution is an attractor in Bianchi I

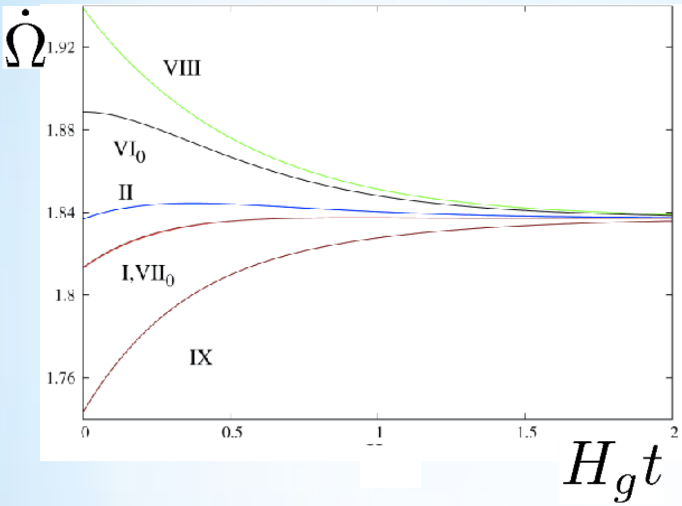
$$\mathcal{B}_\pm - \beta_\pm \to 0$$

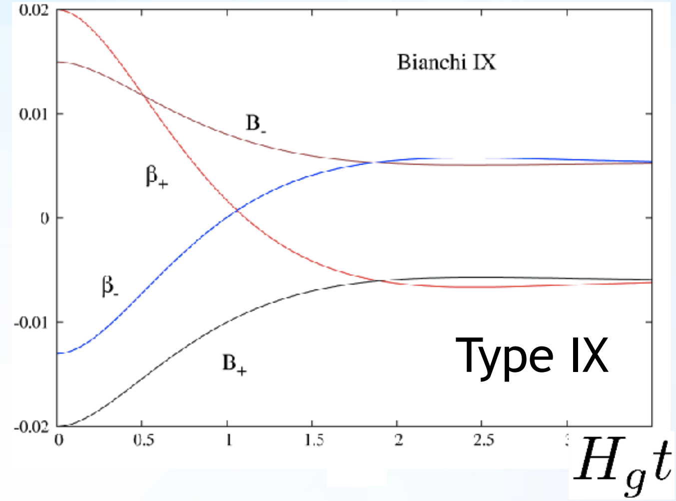


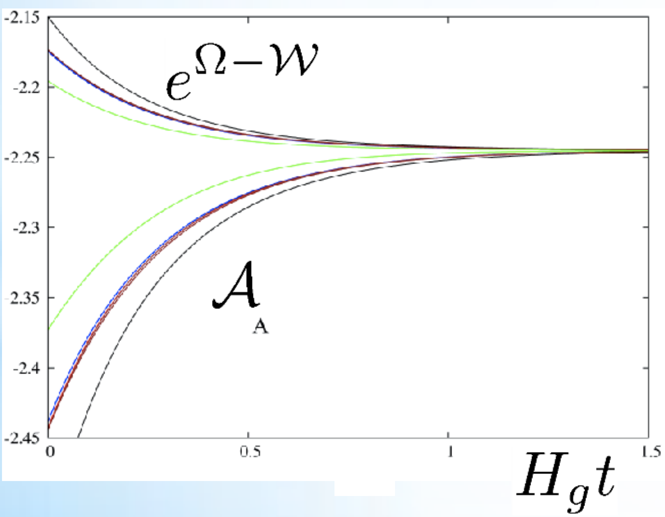
$$H_g = \sqrt{\Lambda_g/3}$$

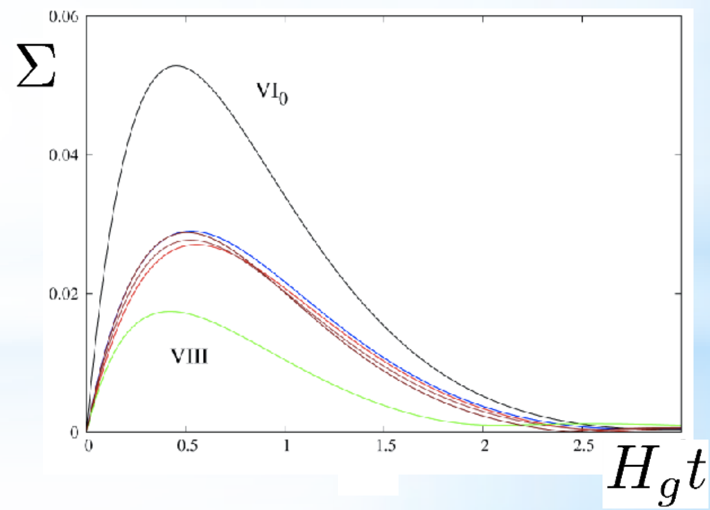
More General Bianchi Types

Numerical Results $c_3 = 1\,, \quad c_4 = 0.3\,, \quad m_g = 0.54m\,, \quad m_f = 0.84m$









Approach to homothetic metrics

homothetic solution $\Rightarrow$ GR with a cosmological constant

Shear drops fast as $\sigma^2 \sim \dot{\beta}_+^2 + \dot{\beta}_-^2 \sim e^{-6\Omega}$

However, it does not drop so fast:

$$\sigma^2 \sim \dot{\beta}_+^2 + \dot{\beta}_-^2 \sim e^{-3\Omega}$$

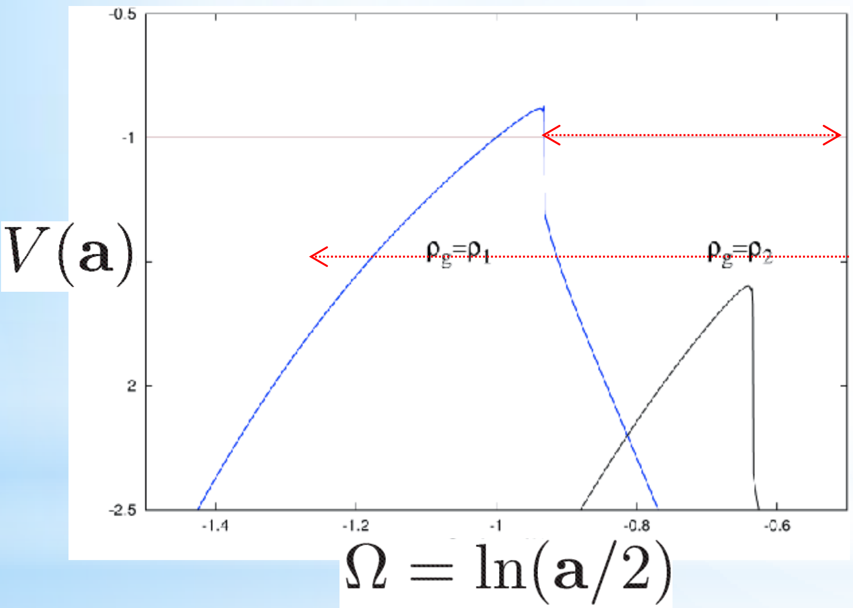
This is the same as matter fluid

Any Observational Effect ?

de Sitter spacetime is not always an attractor

Closed FLRW universe

$$\dot{\mathbf{a}}^2 + V(\mathbf{a}) = -k \quad (k=1)$$



$$\rho_1/m^4 = 0.1 \times e^{-4\Omega}$$

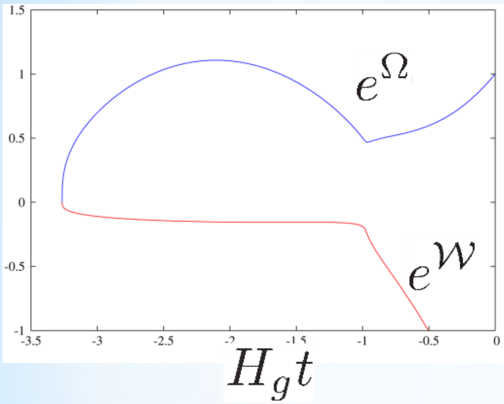
bounce -> de Sitter

$$\rho_2/m^4 = 0.25 \times e^{-4\Omega} + 0.25 \times e^{-3\Omega}$$

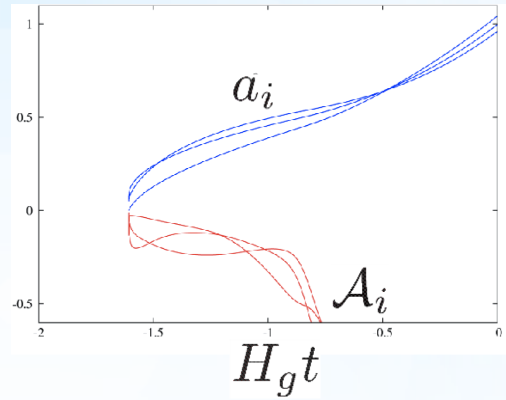
collapse -> singularity

Initial Stage (near singularity)

Bianchi I

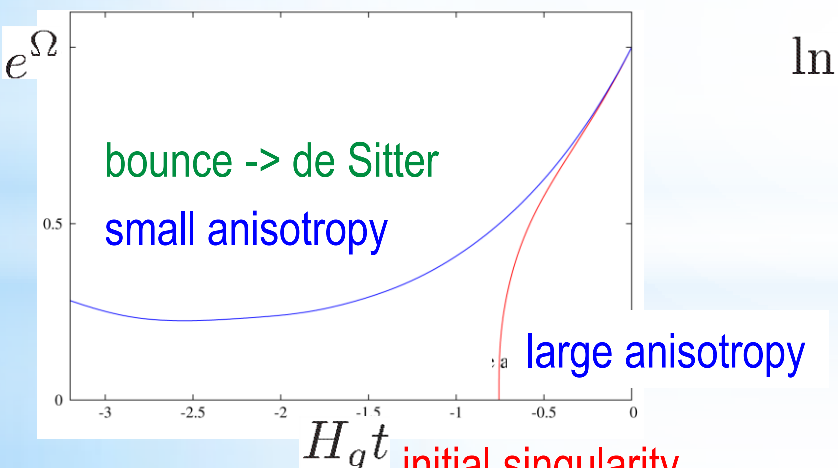


Bianchi IX

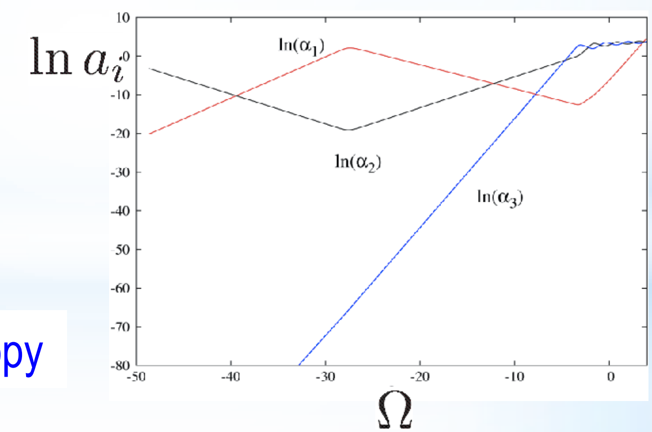


Three spatial components of metrics

vacuum Bianchi IX



initial singularity



chaotic behaviour near singularity

homothetic solution is an attractor

the shear density drops as a^{-3} (In GR a^{-6})

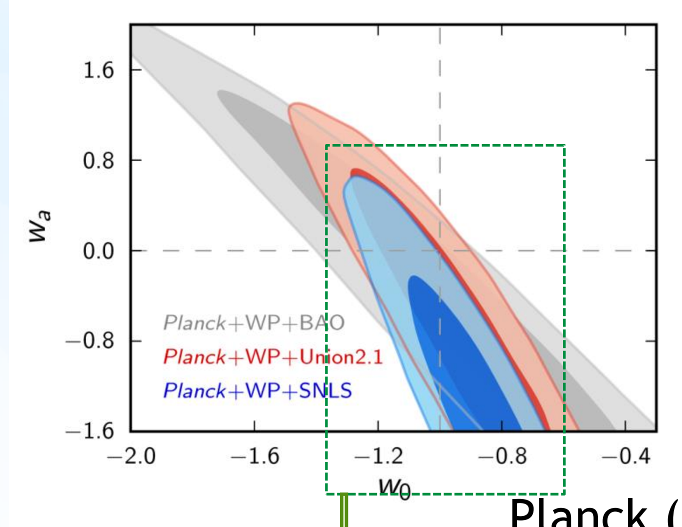
chaotic behaviour near singularity in Type IX

Observational constraints

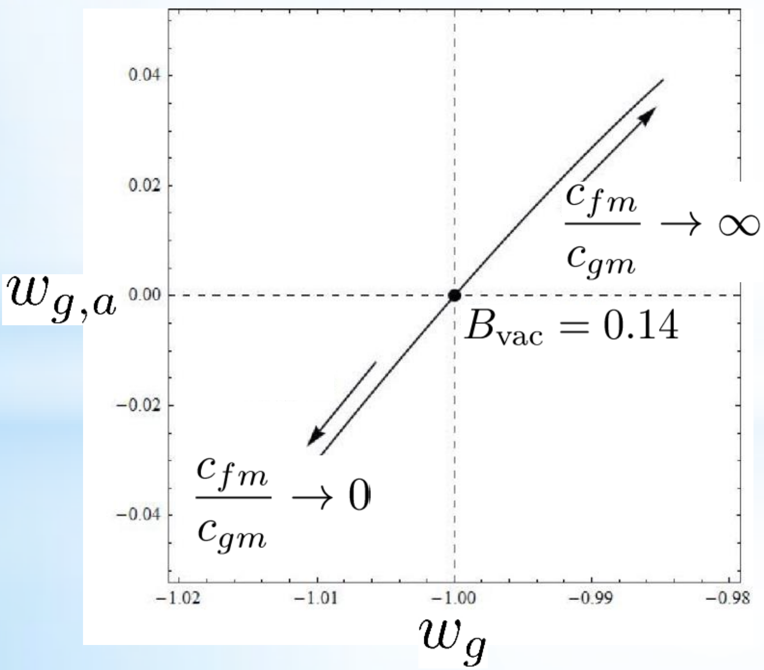
[1] $\rho_g^{[\gamma]}$: dark energy

$$w_g = P_g^{[\gamma]}/\rho_g^{[\gamma]} \to -1$$

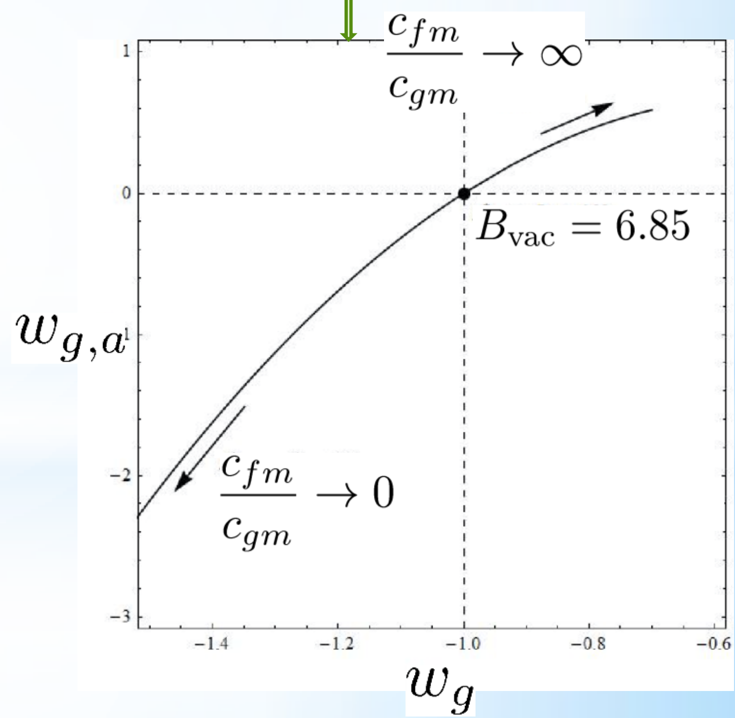
$$w_{g,a} = \frac{dw_g}{da_g}$$



Planck ('13)



$c_3 = 4, c_4 = -12$



$c_3 = -1, c_4 = 0$

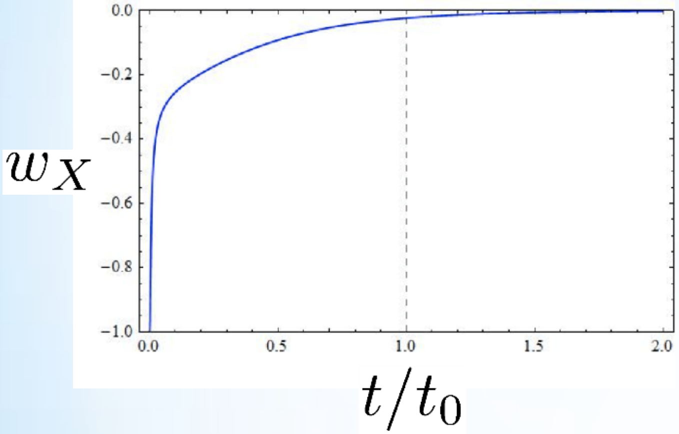
[2] $\kappa_g^2 \rho_g^{[\gamma]} = \Lambda_g + \kappa_g^2 \Delta\rho_g^{[\gamma]}$

exotic matter

$$w_X = \frac{\Delta P_g^{[\gamma]}}{\Delta \rho_g^{[\gamma]}}$$

dark energy = a cosmological constant

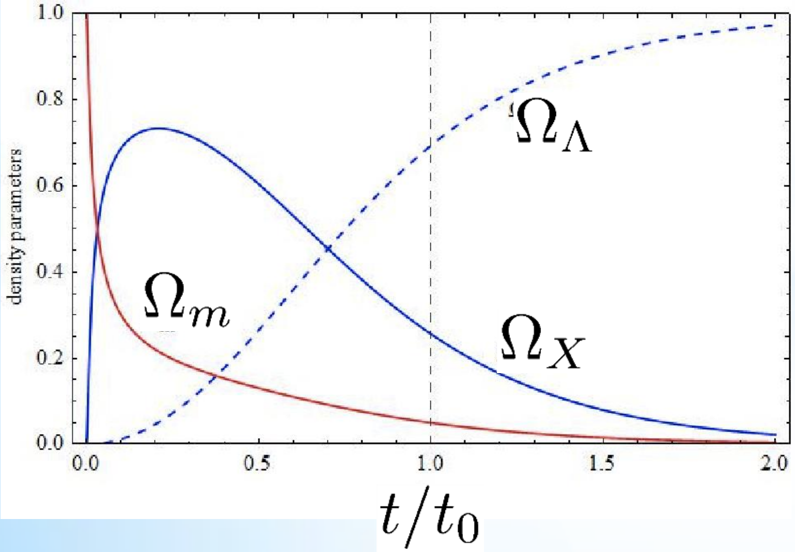
$$\kappa_g^2 P_g^{[\gamma]} = -\Lambda_g + \kappa_g^2 \Delta P_g^{[\gamma]}$$



$w_X \to 0$

"dust fluid"

⇨ Dark Matter ?



$\Omega_\Lambda(t_0) = 0.7$ Dark Energy

$\Omega_X(t_0) = 0.26$ "Dark Matter"

$\Omega_m(t_0) = 0.04$ baryon

Summary

- We discuss dynamics of bigravity

- A homothetic solution corresponds to
  that in GR with a cosmological constant
- The homothetic solution with a positive (or zero) cosmological constant
  is an attractor in FLRW and Bianchi class A spacetimes

$\Longrightarrow$ cosmic no hair

- There may be observational constraints

Further Questions:

- More generic spacetimes

Homothetic solution (GR+Λ) is also an attractor for inhomogeneous spacetimes ?

Cosmic no hair conjecture ?

Black hole uniqueness ?

Thank you for your attention