

Quantum-induced slowdown of highly magnetized neutron stars

B. Lamine, A. Dupays, C. Rizzo and G. F. Bignami



Based on

- A. Dupays, C. Rizzo and G.F. Bignami (EPL, **98** (2012) 49001)
A. Dupays, C. Rizzo, D. Bakalov and G.F. Bignami (EPL, **82** (2008) 69002)

- 1 Electromagnetic properties of vacuum
 - Heisenberg-Euler Lagrangian
 - Vacuum magnetization

- 2 Quantum Vacuum Friction (QVF)
 - QVF in neutron stars
 - Influence on pulsars spindown evolution

Electromagnetic properties of vacuum

Classical EM theory

- Lorentz invariants ($c = 1$)

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{\mathbf{B}^2 - \mathbf{E}^2}{2} \quad , \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu} {}^*F^{\mu\nu} = -\mathbf{E} \cdot \mathbf{B}$$

- Classical (linear) EM (Maxwell) $\rightarrow \mathcal{L}_0 = -\frac{\mathcal{F}}{\mu_0}$

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Quantum corrections in high EM field

- Critical field $B_c = \frac{m^2c^2}{e\hbar} \sim 4 \times 10^9 \text{ T} = 4 \times 10^{13} \text{ G}$.
 - \rightarrow Enough energy to create virtual electron-positron pairs.
 - \rightarrow EM field acquires a charge distribution.
- Quantum correction to the Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$

Heisenberg-Euler Lagrangian (1936)

One-loop correction

- Effective Lagrangian at one-loop

$$\mathcal{L}_1 = 2\alpha \frac{B_c^2}{\mu_0} \int_0^\infty \frac{ds}{s^3} e^{-is} \left\{ s^2 \frac{|\mathcal{G}|}{B_c^2} \cot \left(s \frac{(\sqrt{\mathcal{F}^2 + \mathcal{G}^2} + \mathcal{F})^{1/2}}{B_c} \right) \right. \\ \left. \coth \left(s \frac{(\sqrt{\mathcal{F}^2 + \mathcal{G}^2} - \mathcal{F})^{1/2}}{B_c} \right) + s^2 \frac{2\mathcal{F}}{3B_c^2} - 1 \right\}$$

→ α the fine structure constant.

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Example of weak field approximation

$$\mathcal{L}_1 \simeq \alpha \frac{B_c^2}{\mu_0} \left(\frac{8}{45} \frac{\mathcal{F}^2}{B_c^4} + \frac{14}{45} \frac{\mathcal{G}^2}{B_c^4} \right) \propto (\mathbf{E}^2 - \mathbf{B}^2)^2 + 7(\mathbf{E} \cdot \mathbf{B})^2$$

→ 4-wave mixing phenomenology^a

^a. For a review, see Battesti & Rizzo, Rep. Prog. Phys. 76 (2013) 016401

Vacuum Magnetization

Constitutive equations in QED

- Varying the action $S = \int (\mathcal{L}_0 + \mathcal{L}_1) d^4x$ with respect to A_μ and $\partial_\nu A_\mu$ gives modified Maxwell equations

$$\begin{aligned}\nabla \cdot \left(\epsilon_0 \mathbf{E} + 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{E}} \right) &= 0 \\ \nabla \times \left(\frac{\mathbf{B}}{\mu_0} - 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{B}} \right) &= \frac{\partial}{\partial t} \left(\epsilon_0 \mathbf{E} + 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{E}} \right)\end{aligned}$$

- Effective (non-linear) medium with

$$\mathbf{P}_{qv} \equiv 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{E}} \quad \text{and} \quad \mathbf{M}_{qv} \equiv 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{B}}$$

Magnetization in vacuum

Static effects in a pure magnetic field

- $\mathcal{G} \rightarrow 0$ and $\xi = \frac{B}{B_c}$

$$\mathcal{L}_1 = \frac{2\alpha}{\mu_0} B_c^2 \left\{ \frac{1}{4} + \left(\frac{1}{2} - \xi + \frac{1}{3}\xi^2 \right) \ln 2\xi - \frac{1}{3}\xi^2 + 4\xi^2 \zeta' \left(-1, \frac{1}{2\xi} \right) \right\}$$

→ Still valid in the presence of an electric field if $\mathbf{E} \cdot \mathbf{B} = 0$ (example of a force free magnetosphere)

- Magnetic susceptibility of vacuum

$$\mathbf{M}_{qv} = \chi(\xi) \frac{\mathbf{B}}{\mu_0}, \quad \chi(\xi) = \frac{2\mu_0}{B_c^2} \frac{\mathcal{L}'_1(\xi)}{\xi}$$

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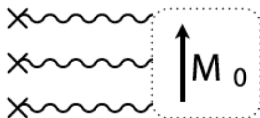
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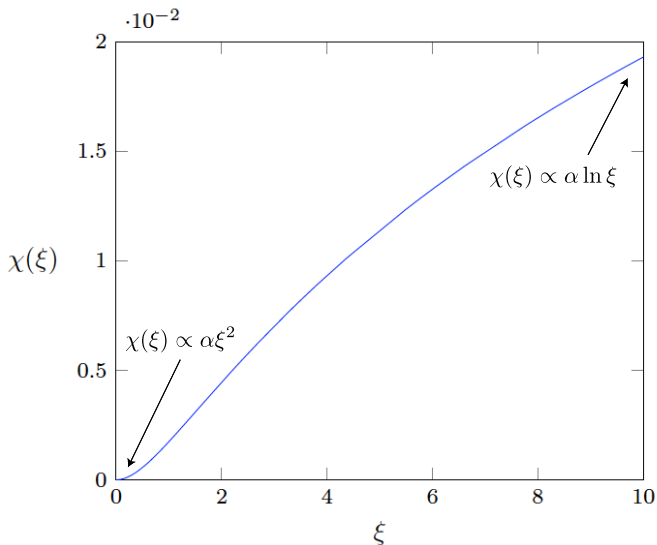
- Weak-field limit

$$\chi(\xi) \underset{\xi \ll 1}{\sim} \frac{16}{45} \alpha \xi^2$$



Magnetization in vacuum

Static effects in a pure magnetic field



Quantum Vacuum Friction

- In our model a neutron star is a compact object with mass $M \sim 1.4M_{\odot}$, a radius $R \sim 10\text{ km}$ and a large magnetic field ($B_0 \sim 10^{10} - 10^{14}\text{ G}$).

NS = **Fast-rotating magnetic dipole moment**

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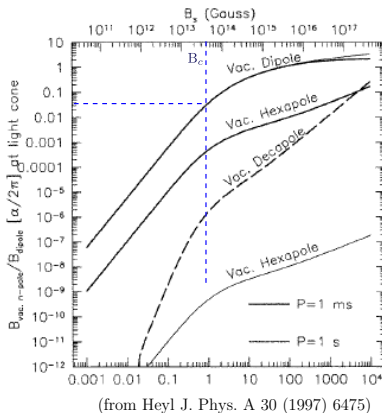
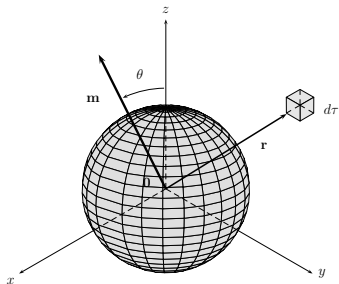
NS = **Fast-rotating magnetic dipole moment**

- The magnetic dipole moment is tilted with respect to the rotational axis.

Non static effects from the vacuum + **backreaction**

QVF in neutron stars

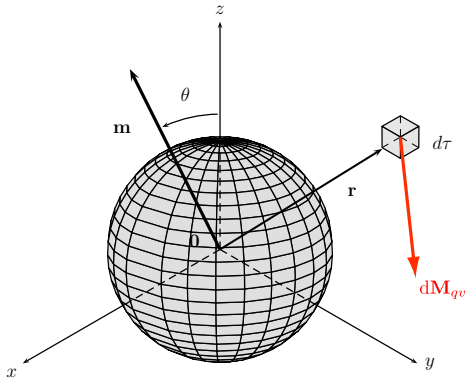
Dipolar approximation



$$\mathbf{B}(\mathbf{r}, t) \simeq \left(\frac{\mu_0}{4\pi} \right) \left[\frac{3(\mathbf{m}(t - r/c) \cdot \mathbf{r})\mathbf{r}}{r^5} - \frac{\mathbf{m}(t - r/c)}{r^3} \right] \quad (kr \ll 1)$$

QVF in neutron stars

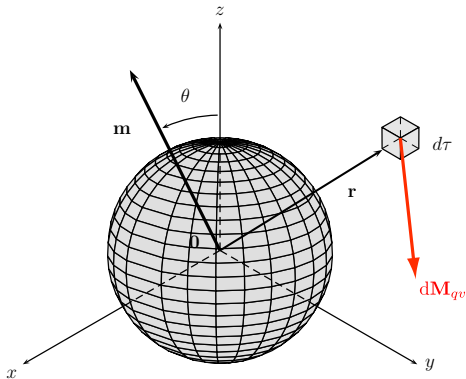
Energy loss rate



- $\mathbf{m}(t - r/c) \rightarrow \mathbf{B}(\mathbf{r}, t) \xrightarrow{HE} d\mathbf{M}_{qv}(\mathbf{r}, t) \rightarrow d\mathbf{B}_{qv}(\mathbf{0}, t + r/c)$

QVF in neutron stars

Energy loss rate



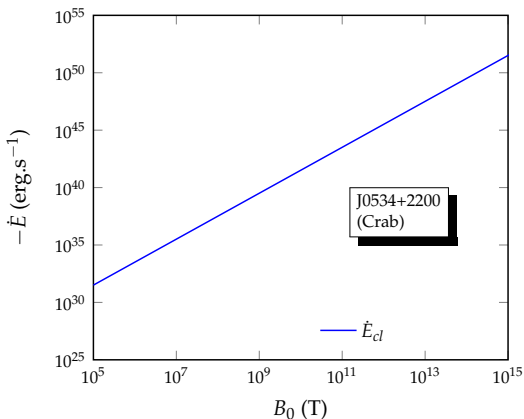
- $\mathbf{m}(t - r/c) \rightarrow \mathbf{B}(\mathbf{r}, t) \xrightarrow{HE} d\mathbf{M}_{qv}(\mathbf{r}, t) \rightarrow d\mathbf{B}_{qv}(\mathbf{0}, t + r/c)$
- Braking torque \Rightarrow loss of energy

$$d\dot{E}_{qv} = - (\mathbf{m}(t + r/c) \times d\mathbf{B}_{qv}(\mathbf{0}, t + r/c)) \cdot \boldsymbol{\Omega} \cdot \mathbf{u}_z \quad \left(\Omega = \frac{2\pi}{P} \right)$$

QVF in neutron stars

Energy loss rate

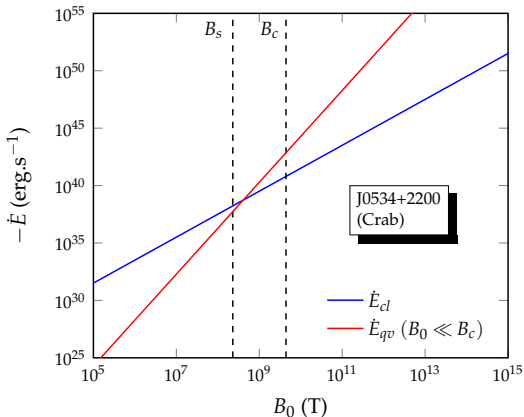
$$\dot{E} = \underbrace{\left(\frac{128\pi^5}{3} \right) \frac{\sin^2 \theta B_0^2 R^6}{\mu_0 c^3 P^4}}_{\dot{E}_{cl}(\text{classical model})}$$



QVF in neutron stars

Energy loss rate

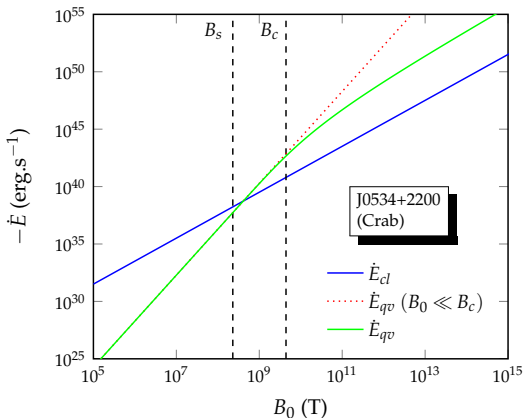
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QVF in neutron stars

Energy loss rate

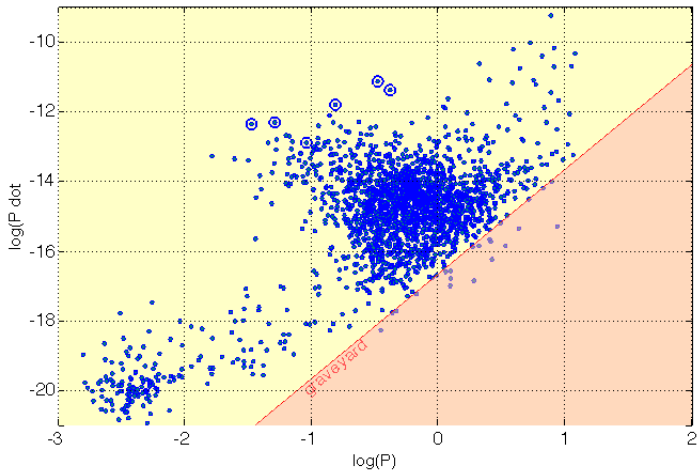
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QVF influence on pulsars spindown evolution

$P - \dot{P}$ Diagram

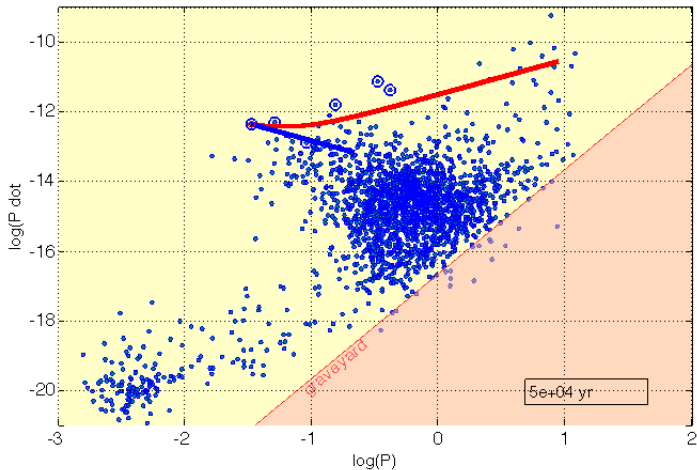
$$E = E_{\text{kin}} = \frac{1}{2} J \Omega^2 \quad \Rightarrow \quad \dot{P} = \frac{\alpha}{P} + \beta P$$



QVF influence on pulsars spindown evolution

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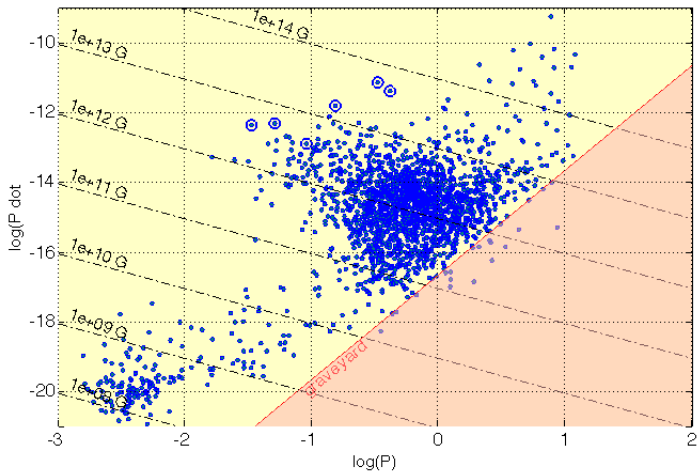
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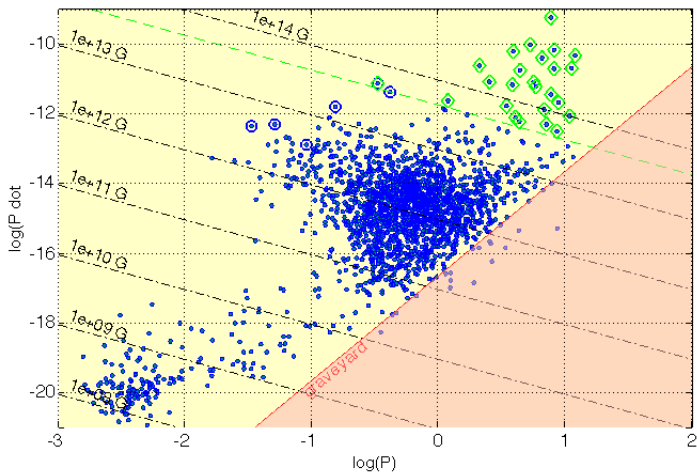
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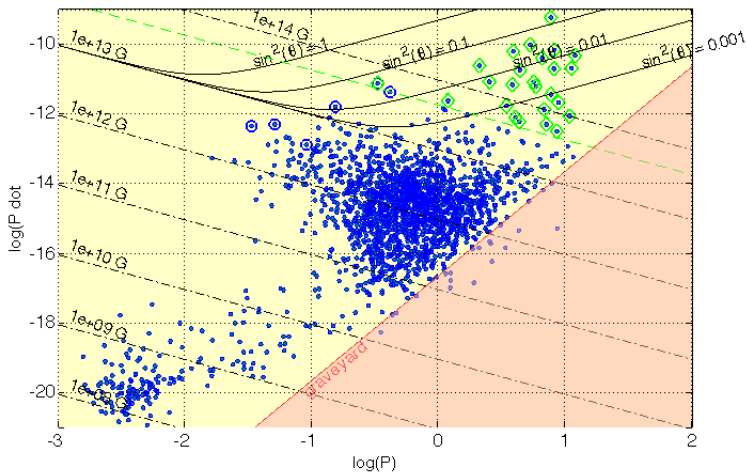
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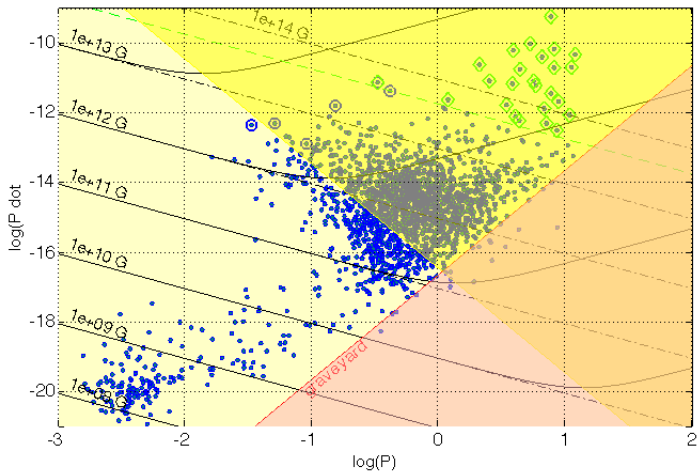
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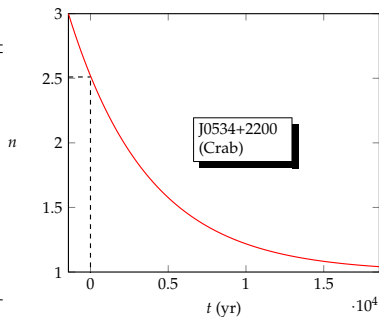


QVF influence on pulsars spindown evolution

Consequence for the braking indice

- Braking index : $\dot{\Omega} = -k\Omega^n \rightarrow n_{\text{obs}} = \Omega\ddot{\Omega}/\dot{\Omega}^2$
- Classical dipolar model (constant B_0) : $n = 3$
- QVF implies that n_{obs} evolves dynamically from 3 to 1

Name	n_0	ν (s $^{-1}$)
J1846-0258	2.65(1)	3.07
J0534+2200 (crab)	2.51(1)	30.2
J1513-5908	2.839(3)	6.63
J1119-6127	2.91(5)	2.45
J0540-6919	2.140(9)	19.8
J0835-4510 (vela)	1.4(2)	11.2





QVF influence on pulsars spindown evolution

Consequence for the braking indice

- Prediction for time evolution of braking indice \dot{n}_0

$$\dot{n}_0 = -(3 - n_0)(n_0 - 1) \frac{P_1}{P_0}$$

Name	\dot{n}_0^{obs} (10^{-12} s^{-1})	\dot{n}_0 (10^{-12} s^{-1})	
J0534+2200	0.26	-9.3	
J1513-5908	-1.2	-3.1	



- Probably other physics involved in the Crab (magnetosphere currents, accretion, pulsar wind etc...)?

QVF influence on pulsars spindown evolution

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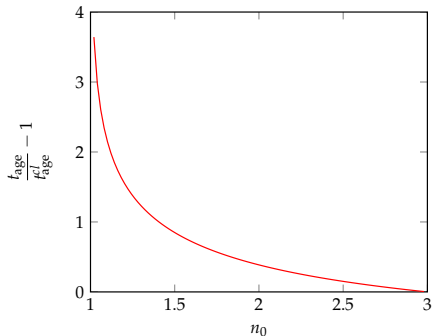
Improving number of confident measure of \dot{n}_0 would be necessary

QVF influence on pulsars spindown evolution

Consequence for the spindown age

$$t_{\text{age}} \underset{P_i \ll P_0}{\sim} \frac{P_0}{(3 - n_0)P_1} \ln \frac{n_0 - 1}{2}$$

Name	t_{age} (kyr)	$t_{\text{age}}^{\text{cl}}$ (kyr)
J1846-0258	.799	.727
J0534+2200 (crab)	1.42	1.24
J1513-5908	1.63	1.56
J1119-6127	1.65	1.61
J0540-6919	2.18	1.67
J0835-4510 (vela)	22.8	11.3

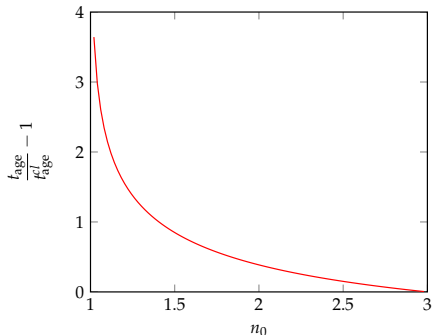


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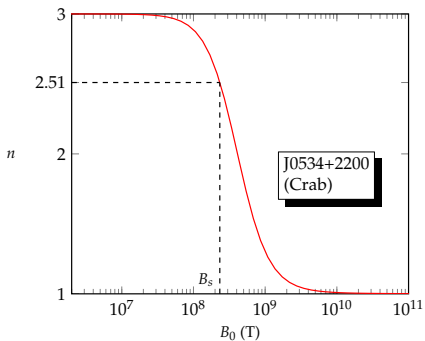


Classical spin-down estimation underestimates the age

QVF influence on pulsars spindown evolution

Magnetic field at the surface of neutron stars

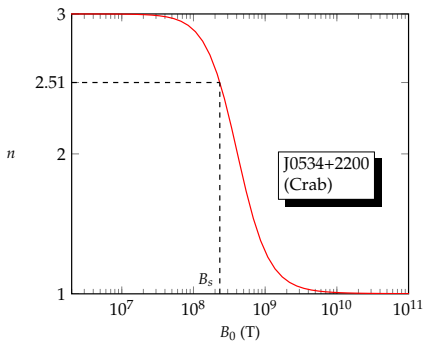
$$n = 3 - \frac{2}{1 + \left(\frac{25\pi^2}{4\alpha}\right) \frac{B_c^2 R^2}{B_0^2 P^2 c^2}}$$



QVF influence on pulsars spindown evolution

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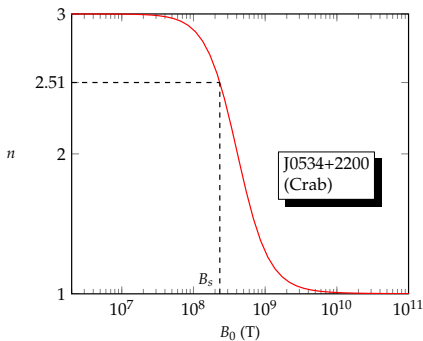


Name	B_0 (10^{12} G)	B_{dipole} (10^{12} G)
J1846-0258	0.19	49
J0534+2200 (crab)	2.3	3.8
J1513-5908	0.26	15
J1119-6127	0.07	42
J0540-6919	2.3	5.1
J0835-4510 (vela)	3.0	3.4

QVF influence on pulsars spindown evolution

Magnetic field at the surface of neutron stars

$$n = 3 - \frac{2}{1 + \left(\frac{25\pi^2}{4\alpha}\right) \frac{B_s^2 R^2}{B_0^2 P^2 c^2}}$$



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Whenever the classical B field estimation is larger than B_c , QVF tends to lower it below B_c .

- QED predicts that Quantum Vacuum can be regarded as a standard medium with its own electromagnetic properties
- Quantum Vacuum Friction is an additional energy loss mechanism that significantly changes the spindown evolution of high magnetized pulsar ($B_0 \gtrsim 10^{11}$ G).
- Confirmation of this effect would be a test of the one-loop correction of QED.
- Independent measurement of magnetic field in magnetars through observations of confirmed cyclotron lines would be a good test.

Backup slides

Magnetar and γ -ray

