Cosmological Perturbations with a Quartet of Scalar Fields (with a Spatially Constant Gradient)

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based on arXiv:1304.7924, SK, S. Kouwn, O. Kwon, P. Oh
Work in Progress: SK, J. Kim, S. Kouwn, P. Oh

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Introduction

- Cosmological principle (homegeneity and isotropy) put strong constraint on the cosmological model building. (degree of anisotropy: $\delta T/T \sim 10^{-5}$)
- Time-dependent background is one easy way to realize the cosmological principle.
- What happened if the background quantities depend on spatial coordinates x^i? (spatial dependent background). Can spatial dependent background maintain the homogeneity and isotropy?
- Maybe yes! In the nonlinear sigma models without the potential, $\sigma^a \sim x^a$ can be obtained.
  Because EoMs contain only 2nd order derivatives in space, the homogeneity and isotropy can be maintained.

Introduction

- $\sigma^m \sim x^m$ $(i = 1, \cdots, N)$ with the extra dimension N were considered to trigger compactificatoin of the extra dimensions. [Omero and Percacci (1980), Gell-Mann and Zwiebach (1984)]
- In this work, we study the cosmological implications of the spatial dependent background, especially focusing on the cosmological perturbations during an accelerating phase in the early times.
- P.S.: We notice that similar idea was realized in Solid inflation, three-form gauge fields ....

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nonlinear sigma model

action for the nonlinear sigma model

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}G_{mn}(\sigma)\partial_\mu\sigma^m\partial_\nu\sigma^n - V(\sigma)\right]$$

where $m, n = 1, 2, 3, 4$ and $G_{mn}(\sigma)$ is the metric on the field space.

In order to realize inflation, we choose the field metric and potential as

$$G_{ab} = f(\varphi)\delta_{ab}, \quad G_{a4} = 0, \quad G_{44} = 1,$$
$$V(\sigma) = V(\varphi)$$

where $\varphi = \sigma^4$ which will play the role of inflaton and $a, b = 1, 2, 3$.

Then the action with a triad σ^a

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}f(\varphi)g^{\mu\nu}\partial_\mu\sigma^a\partial_\nu\sigma^a - V(\varphi)\right]$$

Equations of motion

Energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi + f(\varphi) \partial_\mu \sigma^a \partial_\nu \sigma^a - g_{\mu\nu} \left[\frac{1}{2} f(\varphi) g^{\alpha\beta} \partial_\alpha \sigma^a \partial_\beta \sigma^a + \frac{1}{2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + V(\varphi) \right],$$

field equations

$$\partial_\mu \partial^\mu \varphi - \frac{1}{2} f_\varphi g^{\mu\nu} \partial_\mu \sigma^a \partial_\nu \sigma^a - V_\varphi = 0$$

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} f(\varphi) g^{\mu\nu} \partial_\nu \sigma^a \right) = 0$$

Ansatz for the σ^a

background FRW metric

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$$

Solutions for the σ^a-equation

$$\sigma^a = \alpha x^a$$

where α is an arbitrary constant with $[M]^2$ dimension

- This ansatz satisfies the equation of motion for σ^a

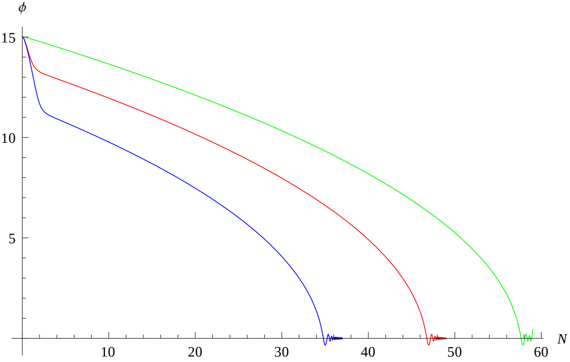
Background Equations of Motion

background EoM

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{3\alpha^2}{2a^2}f + V\right)$$
$$\dot{H} = -\frac{1}{2}\left(\dot{\varphi}^2 + \frac{\alpha^2}{a^2}f\right)$$
$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{3\alpha^2}{2a^2}f_{\varphi} + V_{\varphi} = 0$$

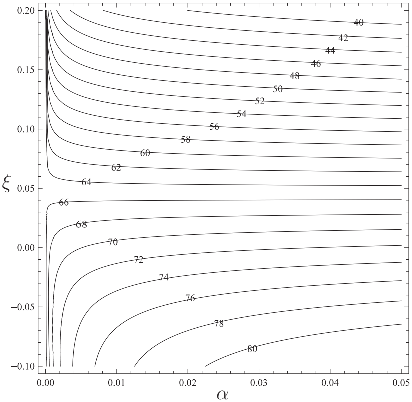
- The homogeneity of the background evolution is not spoiled even though the spatial dependent ansatz for σ^a.
- Because of $1/a^2$, α^2 term effects are important in the early phase of inflation
- $\rho_\sigma = \frac{3\alpha^2}{2a^2}f, \quad p_\sigma = -\frac{\alpha^2}{2a^2}f \Longrightarrow w_\sigma \equiv p_\sigma/\rho_\sigma = -\frac{1}{3}$
- If $f(\varphi) = 1$, α-dependent terms behaves like negative spatial curvature.

Background Evolutions



- α-dominant regime: Since $\epsilon \equiv -\frac{\dot{H}}{H^2} \sim \mathcal{O}(1)$, inflation doesn't occur
- In this work, we assume $\alpha \ll 1$

e-foldings



- $f(\varphi) = e^{2\xi\varphi}$
- e-folding numbers for $V(\varphi) = \frac{1}{2}m^2\varphi^2$

  $\mathcal{N} \simeq \frac{1}{2}\int_{\varphi_e}^{\varphi_i} \varphi\left[1 - \left(\xi - \frac{1}{\varphi}\right)\frac{\alpha^2 f}{a^2 m^2 \varphi} + \mathcal{O}\left(\alpha^4\right)\right] d\varphi$
- For $\xi > \xi_c \equiv 1/\varphi$, as α increases, $\mathcal{N}$ decreases
- For $\xi < \xi_c$, as α increases, $\mathcal{N}$ also increases

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scalar perturbatoins

linearized metric

$$ds^2 = -(1+2A)dt^2 + 2a\partial_i B dt dx^i + a^2[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E]dx^i dx^j$$

linearized scalar fields

$$\varphi(t,x) = \varphi(t) + \delta\varphi(t,x), \quad \sigma^a(t,x) = \sigma^a(x) + \delta\sigma^a(t,x)$$

If we decompose $\delta\sigma^i$ as

$$\delta\sigma^i = \frac{1}{k}\partial_i u + \delta\sigma^i_\perp,$$

then $\delta\sigma^i_\perp$ can be treated as the vector-mode perturbation which doesn't contribute to scalar-mode perturbation.

vanishing of the $\delta\sigma^i_\perp$

perturbed Einstein equation $\delta G^0_i = \delta T^0_i$:

$$2\,\partial_i\left(HA + \dot{\psi}\right) = \left(-\partial_i(\dot{\varphi}\delta\varphi) - \alpha f \delta\dot{\sigma}^i + \frac{\alpha^2 f}{a}\partial_i B\right).$$

If the curl act on both sides, we found that

$$\epsilon_{ijk}\partial^j\delta\sigma^k = 0$$

we can ignore the perpendicular mode $\delta\sigma^i_\perp$

$$\delta\sigma^i = \frac{1}{k}\partial_i u$$

mode equations for scalar mode

For simplicity, we choose $f(\varphi) = 1$ and assuming $\alpha \ll 1$.

Sasaki-Mukhanov eqs

$$\mathcal{V}_k'' + \left(k_1^2 - \frac{\mu_1^2 - \frac{1}{4}}{\tau^2} \right) \mathcal{V}_k = 0$$

$$\mathcal{U}_k'' + \left(k_2^2 - \frac{\mu_2^2 - \frac{1}{4}}{\tau^2} \right) \mathcal{U}_k = 0$$

where $\mathcal{V} = a\left(\delta\varphi - \frac{\dot{\varphi}}{H}\psi\right)$, $\mathcal{U} = a(u - \alpha k E)$.

$$k_1^2 \equiv k^2 - \frac{\alpha^2}{6}, \quad k_2^2 \equiv k^2 + \frac{11\alpha^2}{6},$$

$$\mu_1 \simeq \frac{3}{2} + 3\epsilon - \eta, \quad \mu_2 \simeq \frac{3}{2} + \epsilon$$

mode solutions

initial vacuum state

Choosing the positive frequency mode as an initial vacuum state at $\tau \ll 0$

$$\mathcal{V}_k(\tau) = \frac{1}{\sqrt{2k_1}} e^{-ik_1\tau} e_1(k),$$
$$\mathcal{U}_k(\tau) = \frac{1}{\sqrt{2k_2}} e^{-ik_2\tau} e_2(k)$$

$e_i(k)$ are the independent Gaussian random variables

mode solutions

$$\mathcal{V}_k(\tau) = \frac{\sqrt{\pi}}{2} e^{\frac{i\pi}{2}\left(\mu_1+\frac{1}{2}\right)} \sqrt{-\tau}\, H^{(1)}_{\mu_1}(-k_1\tau)\, e_1(k),$$
$$\mathcal{U}_k(\tau) = \frac{\sqrt{\pi}}{2} e^{\frac{i\pi}{2}\left(\mu_2+\frac{1}{2}\right)} \sqrt{-\tau}\, H^{(1)}_{\mu_2}(-k_2\tau)\, e_2(k)$$

stability of initial vacuum state

stability of initial vacuum

k_1 should be real to have a well-defined quantum state

$$k_1^2 \equiv k^2 - \frac{\alpha^2}{6} \geqslant 0 \Longrightarrow k^2 \geqslant k_{min}^2 \equiv \frac{\alpha^2}{6}$$

: lower bound of the comoving wavenumber

Power spectrum in the large scale limit: $k_{1,2}|\tau| \ll 1$

curvature and isocurvature perturbations

$$\mathcal{R} \equiv \psi - \frac{H}{\rho + p}\delta q, \quad \mathcal{S} \equiv H\left(\frac{\delta p}{\dot{p}} - \frac{\delta \rho}{\dot{\rho}}\right)$$

power spectrum for $\mathcal{R}$ and $\mathcal{S}$

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \frac{H_*^2}{8\pi^2}\frac{1}{\epsilon}\left(1 + (2-2C)\eta + (6C-8)\epsilon - \frac{1}{k^2}\frac{\alpha^2}{\epsilon}\right),$$

$$\mathcal{P}_{\mathcal{S}}(k) \simeq \left(\frac{H_*}{18\pi}\right)^2 \frac{\alpha^2}{\epsilon^2 k^2}\left(1 + 2C - 32\epsilon - 18\eta - \frac{73\alpha^2}{12k^2}\right)$$

where $C = 2 - \ln 2 - \gamma$ and $\gamma \simeq 0.5772$. These results are valid in the following range of α: $\alpha^2 \leqslant \epsilon^2 k^2$

spectral index and running spectral index

spectral index and its running

$$n_{\mathcal{R}} - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k} \simeq 2\eta - 6\epsilon + \frac{2\alpha^2}{\epsilon\, k^2},$$
$$n_{\mathcal{S}} - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k} \simeq -2 + 4\eta - 10\epsilon + \frac{67\alpha^2}{6k^2}$$
$$\frac{dn_{\mathcal{R}}}{d \ln k} = -\frac{4\alpha^2}{\epsilon k^2} + \mathcal{O}(\epsilon^2, \epsilon\eta).$$

- The last terms in $n_{\mathcal{R}} - 1$ and $n_{\mathcal{S}} - 1$ are of the order of ϵ and ϵ^2, respectively.
- While the curvature perturbation is nearly scale invariant, the isocurvature perturbation is proportional to the inverse square of the k.

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summary and discussion

- Motivated from the nonlinear sigma model, we have considered the triad of the scalar fields which depends on linearly spatial coordinates,$\sigma^a \sim x^a$, keeping the homogeneity and isotropy.
- Because of $1/a^2$ term, the effect of σ^a is important in the early phase of inflation.
- We have also considered the scalar mode perturbations and cacluated the power spectrum, spectral index.
- For the stable initial vaccum state, there exists a lower bound of the comoving wavevector (UV cut-off scale).
- We'll leave many things as future works: non-Gaussianity, physical interpretation of α, cosmological perturbations with general $f(\varphi)$, observational implications of momentum cut-off scale $(k_{min}) \cdots$